Abstract

The thesis treats pointwise conjugate representations of finite groups and their rings of polynomial invariants, in particular as unstable modules over the Steenrod algebra.

Given an n-dimensional linear representation of a finite group G over a field of positive characteristic, assumed for the most part not to divide the order of the group, G has an action on the polynomial ring in n variables. We are concerned with the ring of invariant polynomials, and conections between it's structure and that of the group. In particular, it is shown that the invariant rings of two groups are isomorphic as unstable modules over the Steenrod algebra $\mathcal{P}^*$ if and only if the group representations are *pointwise conjugate*. This means that there is a set bijection between the matrices representing the two groups such that corresponding matrices are conjugate in the general linear group. The invariant rings are isomorphic as unstable *algebras* over $\mathcal{P}^*$ if and only if the representations are conjugate – which in particular implies that the groups are isomorphic.

In the case of pointwise conjugacy, this is not true. It is shown that there is a purely group-theoretic condition equivalent to the existence of pointwise congugate representations, namely that of *conformality*. This is the condition that both groups have the same number of elements of each order. Some examples are given, and, in the Appendix, there is a table showing the relative abundance of such pairs of groups. A number of conditions on the groups rings are also shown to be equivalent to pointwise conjugacy.

The Steenrod algebra oringinates in algebraic topology, and an application of the above results is given concerning cohomology of classifying spaces. A large class of examples of such spaces is constructed which have cohomology rings isomorphic as modules over the topological Steenrod algebra, but which are not homotopy equivalent.

Finally the modular case, where the characteristic of the ground field divides the orders of the groups, is examined. It is shown that, as is often the case, the nice results of the non-modular situation do not hold in general.