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Optimization of Transportation Systems

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Abstract. The world has experienced two hundred years of unprecedented advances in vehicle technology, transport system development, and traffic network extension. Technical progress continues but seems to have reached some limits. Congestion, pollution, and increasing costs have created, in some parts of the world, a climate of hostility against transportation technology. Mobility, however, is still increasing. What can be done?

There is no panacea. Interdisciplinary cooperation is necessary, and we are going to argue in this paper that *Mathematics* can contribute significantly to the solution of some of the problems. We propose to employ methods developed in the *Theory of Optimization* to make better use of resources and existing technology. One way of optimization is better planning. We will point out that *Discrete Mathematics* provides a suitable framework for planning decisions within transportation systems. The mathematical approach leads to a better understanding of problems. Precise and quantitative models, and advanced mathematical tools allow for provable and reproducible conclusions. Modern computing equipment is suited to put such methods into practice.

At present, mathematical methods contribute, in particular, to the solution of various problems of *operational planning*. We report about encouraging *results* achieved so far.

Keywords. Transportation Systems, Optimization, Discrete Mathematics

Mathematics Subject Classification (MSC 1991). 90B06

1 Around the World in 80 Days (Introduction)

Jules Verne's famous novel *"Around the World in 80 Days"* describes a journey around the globe in the year 1872. It all starts with Phileas Fogg's bet that one can do the trip in at most 80 days. To prove his claim and win the bet, he sets out for his journey. Fogg makes use of the most advanced long range transportation facilities of his time: Hot air ballons, intercontinental railways, and steam driven ocean liners. Despite some unexpected elephant riding and other more traditional ways of locomotion, he manages to arrive back at his starting point in London in time.

Verne's story provides a good impression of the high efficiency that the transportation systems had achieved by 1872. *"The world* has *grown smaller, since a man can now go round it ten times more quickly than a hundred years ago."*, writes Verne. Now, 125 years later, we could do his trip *another forty times* (!) more quickly. What would he say about that?

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The *transportation systems* of today are the result of only 200 years of development, governed by two main forces, namely, progress in *vehicle technology* ("faster") and traffic *network extensions* ("more").

The starting point for the development of modern *vehicle technologies* is the invention of the steam engine by Watt in 1769. Only two decades later, the first steam driven locomotives and ships allowed the transportation of passengers and goods in hitherto unknown quantity and speed. The construction of the automobile by Benz and others in the eighteen-eighties and its mass production by Ford from 1913 on, Siemens & Halske's electric train of 1879, and the first flight with a heavier-than-air plane by the Wright brothers in 1903 mark a few milestones in a dynamic process that continues until today and has widely surpassed even Jules Verne's wildest technological extrapolations.

Each vehicle technology requires an appropriate *network*. Several backbones of such networks date also back to Verne's times. Melodious names for the first intercontinental railways such as the "Orient Express" from Paris to Istanbul, the "Golden Flyer" from London to Paris, or the famous "Union Pacific" and the "Western Pacific" railroads in the US show that these lines were clearly conceived as the beginnings of the global transportation networks of today. A climax of these developments was reached in the nineteen-sixties when mobility, in particular by cars, was equated with freedom.

But gradually *problems* arose: Road traffic started to suffer from congestion, residents complained about noise, pollution, and other environmental problems. Railways and other public transportation facilities were perceived as uncomfortable and tedious to use, while operation costs exploded. To resolve this *crisis*, the first reaction was to apply the successful recipes of the past. Faster cars and trains, emission reduction, and other technical measures, combined with network extensions, helped. However, it became visible that *the benefits from technological progress and from additional investments in traffic networks are limited*. It is, for instance, unacceptable to cover half of the world with transportation facilities.

If mobility and, thus, transportation demand is still rising, what else can be done? Changing "the rules of the market" by legislation to impose artificial restrictions or higher costs on mobility? Perhaps this can help, too. But whatever measure is taken, one action is certain: We have to make better use of our resources, i.e., we must *optimize* our transportation systems. One way to do this is to improve the *design* and the *operation* of transportation systems by better *planning*. And it seems that there are *large unused potentials* for optimization, because planning standards in the transportation sector are often rather low.

> *"As long as the available transportation infrastructure is not used in the most effective way, investments in its better utilization and in the reduction of congestion are more profitable than the construction of new facilities. ... In general, we must improve the cost-benefit ratio, i.e., we must handle necessary transportation with less energy, less time, less costs, and with fewer negative effects on health and on the environment."*
>
> J. Rüttgers, German Minister for Science, Education, Research, Technology, Rüttgers [1997].

Many transportation companies, for instance, still use *manual planning* and heuristic *ad hoc methods*. This is certainly due to the monopolistic market structures of the past which have not provided incentives for better planning. Note the sharp contrast to competitive sectors such as, e. g., industrial engineering, where no company can survive without employing computer simulation, numerical optimization, and computer aided design (CAD)! How many people have ever heard of CAS — computer aided scheduling? The last international conference on this topic, the CASPT'97 in Boston, had fewer than 150 participants, and half of them were from academia!

The *planning problems* that come up in transportation systems range from basic questions (e.g., long term traffic forecast), over strategic issues of system design and extension (e.g., decisions to build new subways, to construct new roads, or to buy a new fleet of busses), to operational problems (such as bus timetabling or vehicle and crew scheduling in public transport). It is evident that such a multitude of complex problems of quite different nature requires the development of a toolbox of specialized planning methods. The danger in such a situation is that only simple ad hoc trial-and-error approaches are developed and no conceptual planning framework that provides a *language* to precisely state the problems and yields *tools* for their solution.

The aim of this paper is to propose *Mathematics*, in particular *Discrete mathematics* and the *Theory of Optimization*, as such a framework. We will point out how the mathematical approach leads to a better understanding of transportation systems, how abstraction allows the unified treatment of different problems with general mathematical methodology and powerful algorithms, and how modern computing technology puts this approach into planning practice. We survey areas where today's optimization techniques can make a contribution and report about encouraging *results* that have been achieved. Our vision is that, in the future, models of discrete mathematics play a similar role in the design and operation of transportation systems as differential equations today in engineering. Phrasing our view in buzzwords: Transportation is a combination of vehicle and network technology, economics, computer science, and mathematical optimization. And we feel that the last two components have not received due attention yet.

Our belief in the significance of mathematics for transportation and mobility may appear ambitious or even provocative. Jules Verne thought the following about that:

> *"But, in order not to exceed it [the deadline], you must jump mathematically from the trains upon the steamers, and from the steamers to the trains again."*
> *"I will jump — mathematically."*
> *"You are joking."*
> *"A true Englishman doesn't joke when he is talking about so serious a thing."*
>
> P. Fogg, British Gentleman, Verne [1873].

2 Transportation and Mathematics

In ancient Greece, the oracle of Delphi was famous for its capability to produce a true answer to any question put forth to it. The oracle was also known for exorbitant prices and for the notorious evasiveness of its answers. It was, thus, extremely important to ask "the right question". Mathematics is not the Delphic oracle of today. It does also give correct answers, but it is not evasive and not expensive. There is, however, an important parallel: The usefulness of mathematics for transportation planning depends on the *identification* of suitable optimization problems.

Mathematics itself does not contribute to this very important task of *problem identification*. This is usually performed by engineers, social scientists, economists, administrators, and experienced practitioners. They have analyzed and structured the area of transportation and have defined the planning problems. The mathematical treatment of such problems is based on this structuring. Mathematics can provide no more and no less than a tool for the solution of well defined questions.

> *"In the modern world, decisions must be made more and more quickly, and decision side effects must be understood in advance. Availability of high performance computing has caused common use of computers as decision aids (spreadsheets); huge progress in computational optimization; and a rich infrastructure of simulation models. ... My thesis is that the basic notion of a spreadsheet can be modified in fundamental ways by using optimization research to produce new tools required by a wide range of the most complex decisions."*
>
> J. Dennis, Plenary Address at the XVI$^{\text{th}}$ Int. Symp. on Math. Programming, Dennis [1997].

We will now provide a closer look at what can what can't be structured in such a way that mathematical methods are applicable.

The Global View. From a very abstract point of view, transportation is caused by *demands* to move something between locations, say, from an origin to a destination. The demands fall into different (not always clearly distinguishable) categories: Transportation of persons or goods, slow or fast, regional or interregional, periodically or spontaneous, etc. Transportation systems are developed to *satisfy* these demands.

It would be perfect to know all such demands (precisely or approximately) for a foreseeable future. Then one could "globally" develop and plan transportation systems that satisfy these requirements at lowest possible total cost. However, the world is not perfect, and thus there is no "world transportation problem" that could be addressed, neither with mathematical nor with other methods.

The Market View. It is much better to look at *markets* where transportation systems have or are being developed in order to satisfy certain *categories* of demand. In this model, each supplier of transportation looks *individually* at "his" market segment (e.g., airlines at long range traffic) and forecasts the relevant demand by estimating so-called origin-destination matrices (O/D matrices) which describe the amount of transportation demand between locations (within certain demand-specific time intervals). Such forecasts are difficult to make on a long-term basis since they have to take local and global trends into account, e.g., the migration into big cities in the third world and out of the big cities in industrialized countries, the increased transportation volume by innovations such as just-in-time production, etc. Nevertheless, O/D matrices are estimated wherever quantitative methods have entered the decision process. Mathematics can help to some extent to improve the forecasting process and correctly interpret the estimated and computed data.

Markets work by the interplay between supply and demand and there is, in particular, competition between technical systems. For instance, ocean liners were completely replaced by passenger airplanes within two decades because of clear superiority of air traffic. But the transportation markets are not "free" since the *state* (or supranational units like the European Union) interferes and regulates in many ways: The state may *support* a new transportation system within a certain market, e.g., to promote a new technology and create jobs, as with the controversial installation of the magnetic levitation train between Berlin and Hamburg. The state may regulate by *law*, e.g., to counter negative effects such as pollution by requiring the use of catalytic converters, or the state may regulate by giving only a limited number of licenses as for air traffic operations. The state may create almost *monopolistic markets*, often operated by a state-owned agency, as is frequently done for national railway systems or public transportation within regions. On the other hand, the state may also *create markets* by building, e.g., roads that can be used by all cars, busses, and trucks. This economic market view is a reasonable approach to make the acting forces — supply, demand, the state — visible. But nontrivial mathematics can't contribute much to the solution of the non-quantifyable political, social, and economic decisions that come up at this high level.

Strategic and Operational Planning of Transportation Systems. Planning the *design* and *operating* existing transportation systems, such as airlines, railroads, or public transportation systems, is the area where mathematics enters the picture substantially. Let us consider one example, the planning process for the *public transportation* system of a city. Two phases must be distinguished: A strategic and an operational planning phase.

Based on O/D data on the expected traffic demand between a number of representative points in a city, *strategic decisions* determine the amount of transportation that a city is willing or capable to offer to its residents. The construction of new subway lines, the placement of depots, and the procurement of new vehicles are issues of strategic planning. Various combinations of statistics, stochastic analysis, stochastic optimization, and scenario analysis can significantly improve this important planning phase. However, such quantitative techniques are rarely applied at present.

The subsequent task of *operative planning* step is to provide a maximum of service or a certain level of service at minimum cost with those resources that have been allocated in the strategic phase. Operative planning is commonly organized in a sequential process. The first step is line planning, followed by vehicle scheduling. The vehicle schedule gives rise to a set of tasks to operate the individual vehicles that are next scheduled into duties in the duty (or crew) scheduling step. Finally, the crews are assigned to the duties in a subsequent crew rostering phase. Some coordination among the different units of public transportation is provided by a planning hierarchy that simply first schedules the subways, then the trams and the busses last. Subway timetabling, in turn, often takes railroad timetables as input which provides some coordination with the "neighboring" railway planning process. The problems arising in operational planning almost always come along with massive data and give rise to very challenging optimization tasks: A heaven for mathematical research and a rich source for cost savings.

3 The Rule Approach

The questions of strategic and operative planning in the area of transportation have inspired the development of a large number of problem specific solution methods. Many of these methods can be viewed as approaches of *rule oriented* planning.

Rule planning aims at the definition of a clear *decision process*. The idea is to organize the process as a sequence of elementary steps at some (refinable) level of detail. This results in planning instructions that could read as follows: "First we do a). If the result is b), we do c), otherwise, we do d). Then ..." Such a procedure amounts to a sequence of applications of rules, hence the name. A computer scientist would say that rule oriented planning is nothing else than the specification of a (rather simple) *algorithm* and its subsequent application.

An advantage of the rule approach for the planner is that he can always *justify* his results as a correct outcome of the scheme. Hence, it is difficult to criticize his achievement. This situation is typical for bureaucratic organizations that act process- and not goal-oriented.

If the rules are specified at a sufficient level of detail, the results become also *independent* of the planning person. They are *reproducible* and, as is currently done in many branches of the transportation sector, often also *mechanizable*, i.e., one can encode the rules in a computer program. Verifyability, reproducibility, and, where large amounts of data are concerned, mechanizability are important characteristics of a convincing planning method that have contributed to the long lasting popularity of the rule approach. Many public transportation companies in Germany, for instance, use rule planning to set up their timetables, schedule the vehicles and duties, and to dispatch crews.

The rule approach becomes less convincing when one looks at planning as a way to reach *goals*. Rule planning processes are, of course, designed with an eye on goals, but there is neither a good justification for the individual steps nor for the overall organization of the procedure. From this perspective, a rule oriented decision process is just a *heuristic*, i.e., a method that produces *some* solution but not necessarily (or even provably) one that achieves the overall objective.

4 The Mathematical Approach

The military logistics of the US armed forces had become a stronghold of the rule approach during World War II. Pentagon planners used to organize their decisions in terms of so-called *ground rules* that could be iteratively refined until a particular plan was produced. One of these planners was the young mathematician George Dantzig. In 1946, he had just graduated and become a Mathematical Advisor to the US Air Force Comptroller in the Pentagon, where he tried to "mechanize" the planning of a time-staged deployment, training, and logistical supply program. He describes the situation that he found as follows (see Dantzig [1991]):

"This was the situation before I formulated a model. In place of an explicit goal or objective function, there were a large number of ad hoc *ground rules issued by those in authority to guide the selection. Without such rules, there would have been, in most cases, an astronomical number of feasible solutions to choose from."*

Dantzig soon became unhappy with the ground rules because:

"... objectives were often confused with the ground rules for the solution. Ask a military commander what the goal is and he will say, "The goal is to win the war." Upon being pressed to be more explicit, a Navy man will say, "The way to win the war is to build battleships," or if he is an Air Force general, he will say, "The way to win is to build a great fleet of bombers." Thus the means to attain the objective becomes an objective in itself*, which in turn spawns new ground rules as to how to go about attaining the means such as how to best go about building bombers or space shuttles. These means in turn become confused with goals, etc., down the line."*

When Dantzig *"decided that the* ad hoc *ground rules had to be discarded and replaced by an explicit objective function"* he had done the first step toward a new mathematical method of planning that would soon revolutionize not only Pentagon logistics — *Linear Programming.* And Dantzig was not alone. John von Neumann's *Game Theory*, Wassily Leontieff's 1933 *Input-Output Model of the Economy*, and Leonid Kantorovich's monograph on *Mathematical Methods of Organizing and Planning of Production* of 1939 (the last two scientists received the Nobel Price in economics for their work) were other fundamental contributions in a brand new line of research that aimed at establishing *mathematical methods* as tools for decision support in transportation, production, economics, and in many other areas.

> *"Perhaps it might be desirable to present my own view as to what I obtained in the pre-War years.*
>
> 1. *I indicated a wide class of practical problems of management leading to the same mathematical extremal problem. ...*
> 2. *I found necessary and sufficient conditions for a solution of this mathematical problem to be optimal. ...*
> 3. *On the base of the above mentioned necessary and sufficient conditions I constructed (finite) methods of solution (called* the method of resolving multipliers*) which are on a par, in effectiveness and generality, with subsequently developed methods; for example, the simplex method.*
> 4. *I clarified to some extent the economic meaning of the resolving multipliers ... "*
>
> L. Kantorovich, 1975 Nobel Laureate for Economics, Romanovsky [1989].

What is the power of this approach? We see three sources: Modelling, abstraction, and computation!

- **Modelling.** Stating the planning problem in mathematical terms often provides valuable new insights. It forces the planner to explicitly name all quantities and their relations and to single out those that are important. Only these should enter the model. Mathematical models are unambiguous, they can be communicated easily, and they are quantitative.

- **Abstraction.** Mathematics looks at the general. It aims at unifying problems that look, at first sight, quite different within one and the same "appropriately selected" mathematical theory. This makes it possible to utilize already existing results of this theory or to exploit results developed for one of these problems for the solution of the other ones. The ad hoc approaches to the individual problems can be replaced by a common framework, and a better understanding of the particularities of problems can be achieved.

- **Computation.** Only the quantitative model of a practical problem can provide the basis for the application of solution algorithms. An algorithm is a (in general somewhat sophisticated) system of instructions to perform computations. Successful algorithms are based on mathematical insight.

Let us discuss some of these points. The starting point of the mathematical approach is what one calls a *mathematical model* of a problem. The model is based on the idea of a "space" of *feasible solutions*, i.e., a mathematical representation of all possible results of the planning process. The space typically used is the real vector space $\mathbb{R}^n$ of dimension n depending on the problem characteristics. As the feasible solutions are not known a priori, they are described implicitly by means of so-called *constraints*. A typical constraint is a linear inequality of the form $a_1x_1 + a_2x_2 + \ldots + a_nx_n \leq b$, where b and a_i, $i = 1, \ldots, n$ are given (real) values and the x_i are variables that we seek to determine. The constraints describe conditions that every feasible solution has to meet and, conversely, anything that satisfies all constraints is considered perfectly legal, i.e., a feasible solution. A clear distinction has to be made between the constraints and the *objective* that measures the desirability of a feasible solution. The objective is just a single number that is associated to every solution; costs, time, or combinations of these are, for instance, possible objectives. The key point of this simple concept is that it gives rise to a ranking of all feasible solutions and there is, in particular, one (or several) best feasible solution of the model. Solving the problem means then to find such a best solution.

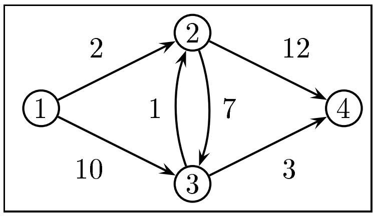

Figure 1: A Directed Graph.

We give a trivial, but fundamental *example* of a simple transportation problem. Consider the directed graph depicted in Figure 1. The nodes of the digraph are labeled 1,2,3,4; we denote the arcs by (i, j). Suppose every arc (i, j) carries the cost value c_{ij} as shown in Figure 1. Suppose that some commodity has to be transported from 1 to 4, i.e., the problem is to find a cheapest directed path from 1 to 4. This basic question is known as the *shortest path problem*.

We want to formulate this problems in terms of an *integer*, or more precisely, a *0/1 programming model*, one of the models used in discrete optimization. To this purpose, we associate with each arc (i, j) a variable denoted by x_{ij} and stipulate that this variable may attain only two values, namely 0 or 1; $x_{ij} = 1$ means that the path uses arc (i, j), $x_{ij} = 0$ means that the path avoids (i, j). This way we get six so-called 0/1 variables. Now a directed path from 1 to 4 consists of a sequence of arcs with one arc leaving node 1, arc $(1, 2)$ or arc $(1, 3)$, and one arc entering node 4. The directed path need not enter nodes 2 or 3, but if it does, it also has to leave the nodes. One can mathematically formulate these observations by writing down the following linear equations

$$
\begin{array}{ccccccccccccl}
x_{12} & + & x_{13} & & & & & & & & & = 1 & \text{(leave 1)} \\
 & & & & & & x_{24} & & & + & x_{34} & = 1 & \text{(enter 4)} \\
x_{12} & & & - & x_{23} & - & x_{24} & + & x_{32} & & & = 0 & \text{(leave = enter 2)} \\
 & & x_{13} & + & x_{23} & & & - & x_{32} & - & x_{34} & = 0 & \text{(leave = enter 3)}
\end{array}
$$

and by stipulating, in addition, the already mentioned condition

$$x_{ij} \in \{0, 1\} \qquad \text{for all arcs } (i, j).$$

A feasible solution of this *0/1 program* is, for instance, $x_{13} = x_{32} = x_{24} = 1$ and $x_{12} = x_{23} = x_{34} = 0$. It corresponds to the path $1 \to 3 \to 2 \to 4$ and, bringing the objective into play, has cost 23. The cheapest path clearly is $1 \to 2 \to 3 \to 4$ and has cost 12. But there are other solutions, too. In fact, every directed path from 1 to 4 yields a feasible 0/1 solution of the equation system above. Is it also true that all solutions of the above 0/1 program correspond to paths from 1 to 4, i.e., is the 0/1 program a correct integer programming model of the shortest path problem? Not entirely, because not all 0/1 solutions are directed paths. E.g., $x_{12} = x_{23} = x_{24} = x_{32} = 1$ and $x_{13} = x_{34} = 0$ is a feasible solution, but it consists of the directed path $1 \to 2 \to 4$ and the cycle $2 \to 3 \to 2$! Do we have to add inequalities and equations to eliminate such unwanted solutions? It depends! For instance, if all costs c_{ij} are positive then a moment's thought shows that a minimum cost solution will never contain a (superfluous) cycle such as $2 \to 3 \to 2$. Thus, we don't have to care. But if negative costs are possible we have to come up with additional constraints to get rid of these unwanted cycles.

This type of modelling is versatile. Shortest path problems, for instance, often have "additional side constraints", such as restrictions on the maximum path length. In our example we could require that no path contains more than two arcs. It is easy to incorporate this side constraint by adding the inequality

$$x_{12} + x_{13} + x_{23} + x_{24} + x_{32} + x_{34} \leq 2$$

to the model. Now the cheapest of such "short" paths is $1 \to 3 \to 4$ with cost 13. Other conditions can be handled just in the same way and this means that integer programming is a general framework with which many transportation problems with all their variants can be modelled and attacked.

"If I were asked to summarize my early and perhaps my most important contributions to linear programming, I would say they are three:

1. *Recognizing (as a result of my wartime years as a practical program planner) that most practical planning relations could be reformulated as a system of linear inequalities.*
2. *Replacing ground rules for selecting good plans by general objective functions. (Ground rules at best are only a means for carrying out the objective, not the objective itself.)*
3. *Inventing the simplex method which transformed the rather unsophisticated linear-programming approach to economic theory into a basic tool for practical planning of large complex systems."*

G. Dantzig, Inventor of the Simplex Method, Dantzig [1991].

For real world transportation problems, it is in general not easy to find the right level of detail. The modelling process needs the experience and know-how of practitioners and the familiarity of mathematicians with alternative ways of formulating goals, rules, and requirements. Which parameters have to be taken into account? In what way do they interact? What decisions are allowed, what "should not" be done, and what are clearly infeasible actions? What are the goals that we want to reach and what do we do in case of conflicts? How do we decide in case of ambiguities and how do we compare different alternatives actions?

Unfortunately, there is no automatic modelling process that takes care of these considerations. The general concepts of the Theories of Optimization and Discrete Mathematics offer tools to formulate many of the tasks arising in transportation planning. They provide algorithmic frameworks that can, at least in principle, be applied. It is, however, still necessary to come up with a problem specific solution technique for each individual task. Nevertheless, the research done in the recent years has shown that some standard combinatorial optimization problems such as *set partitioning* (see Box 1) or *multicommodity flow* (see Box 2) arise frequently and can be used in many different settings.

Finding a good model is only a first step. Algorithms for their solution are what we are really looking for. It is beyond the scope of this article to give a survey of mathematical optimization methods, but we mention some of the most important concepts. Tools from linear programming such as the primal simplex method, invented by George Dantzig in 1947, the dual simplex method, and various interior point methods form the computational core. These algorithms are combined with cutting plane methods, which are based on theoretical results in polyhedral combinatorics, and branch-and-bound techniques to yield branch-and-cut algorithms. These are currently the most powerful methods for the exact solution of combinatorial optimization problems. If exact solution is out of reach, heuristics of all kinds are employed together with dual bounding procedures such as Lagrangean relaxation and subgradient algorithms. For introductions to mathematical methodologies we suggest the textbook of Chvátal [1980] for linear programming, Schrijver [1986] for integer programming, Ahuja, Magnanti & Orlin [1993] for network optimization, the annotated bibliography of Dell'Amico, Maffioli & Martello [1997] for combinatorial optimization, and Nemhauser, Rinnooy Kan & Todd [1989] for optimization in general.

All the developments sketched above would not help without sufficient computing power. Fortunately, the mathematical solution approach benefits considerably form the enormous progress in computing technology in the recent years.

5 The Present and Future of Transport Optimization

In this section, we sketch a few *areas* where mathematical optimization techniques have successfully been applied to problems in transportation. For further and more detailed information on applications in transportation, we refer the reader to the CASPT proceedings Daduna & Wren [1988], Desrochers & Rousseau [1992], and Daduna, Branco & Paixão [1995], while the series Burkard, Ibaraki & Queyranne [1995, 1997], and Burkard, Ibaraki & Pulleyblank [1998] as well as the collection Hoffmann, Jäger, Lohmann & Schunck [1997] (in German) discuss, more generally, topics on the use of mathematics in industrial systems. Actual *installations* of mathematical optimization systems in the transportation sector are not so frequent. This is somewhat surprising. But there are reasons.

In the past, many areas of transportation have been protected by *monopolistic structures*. (Typical examples are national railways or local public transportation systems.) Incentives for optimization were little. Consequently there was only low interest in the development of mathematical tools to aid in the decision process. But this situation has changed. In Europe, e.g., the deregulation of the transportation sector by the Maastricht II treaty has or will put the transportation companies under serious competition. Similar developments take place elsewhere.

A second point is of mathematical nature. Most transportation problems arising in practice are really *large*. Hundreds of thousands of constraints and millions of variables are not uncommon. Problems of such dimensions were simply out of reach until very recently. Mathematical advances and advances in computing machinery, however, have changed the picture. Now it is possible to attack transportation problems of sizes that were beyond imagination just a decade ago, see, e.g., Box 2. Thus, *we are at the point in time to put the approach described here into practice*. Decision support based on mathematical models can make a difference!

It is impossible to provide here a complete survey of the successful applications of mathematics to transportation. We mention a few (representative) *examples* where the operational planning was greatly improved by the use of mathematical optimization techniques.

- **Airline Industry.** This is probably the most competitive sector in transportation — due to early deregulation. Operations Research and Optimization have a long history in this area; mathematical decision support techniques have been continuously employed and extended over the last 30 years. Leading companies use optimization techniques for daily, weekly, and monthly planning such as fleet assignment, crew scheduling, and crew rostering. Many airlines have created divisions or subsidiary companies to provide the necessary mathematical knowledge, consulting capacity, software tools, and computing machinery, see the box below.

  Two articles describing the mathematical treatment of the whole chain of strategic and operational planning are Bachem, Monien, Prömel, Schrader & Voigt [1996] and Bachem et al. [1997]; a recent collection of articles on the use of operations research in the airline industry is Yu [1998]; particular issues are discussed in Hoffman & Padberg [1993] and in Barnhart et al. [1994].

> *"The idea that Delta [Airlines] would even attempt to develop a Coldstart fleet assignment model — that is, start with a raw schedule without any fleetings wedded to it, run it through an L[inear]P[rogram] and fleet it — was greeted with skepticism by the rest of the airline industry. The general consensus was that it couldn't be done. Delta proved them wrong.*
>
> *Coldstart was successfully implemented and remains fully operational. Scheff [Coldstart project leader at Delta Decision Technologies] estimates the new system saved the airline between $50.000 and $100.000 a day during the initial start-up period in September and October of 1992. While most of the identifiable savings occurred up front, the system continues to pay significant dividends as the schedule is fine-tuned over time."*
>
> P. Horner, Editor of OR/MS Today, Horner [1995].

- **Rail Transport.** The use of mathematical methodology in this subsector is, at present, not as widespread as in air traffic; many railroad companies have only recently observed the potential of mathematical decision support. (E.g., in 1994 the Ferrovie dello Stato SpA of Italy was the first railway company in Europe to run a competition — the FASTER contest — for the best solution of particular "set covering" crew scheduling problems to find a partner for future collaboration.) From the mathematical side, the planning process has been analyzed; models for the various stages exist; codes for several subproblems ranging from line planning, timetabling, rolling stock and crew scheduling are under development, see, e.g., Bussieck, Winter & Zimmermann [1997] and Caprara, Fischetti, Toth, Vigo & Guida [1997] for surveys. These recent articles contain pointers to the relevant literature.

Vehicle Scheduling in Handicapped People's Transport belongs to a class of transportation problems that involve complicated planning rules. The handicapped customers of a dial-a-ride system have to be transported with a fleet of mini-busses that are rented on demand. Labour regulations for bus drivers, vehicle renting contracts, and the special service needs of the handicapped give rise to a set of complex restrictions on the feasibility of possible vehicle tours. The objective is to service all requests at minimal costs.

Figure 3: A Telebus Picks up a Customer.

Problems of this type can be attacked with a set partitioning approach. The idea is to enumerate in a first step all possibilities to schedule tours for individual vehicles and to determine the optimal combination of these tours in a second step.

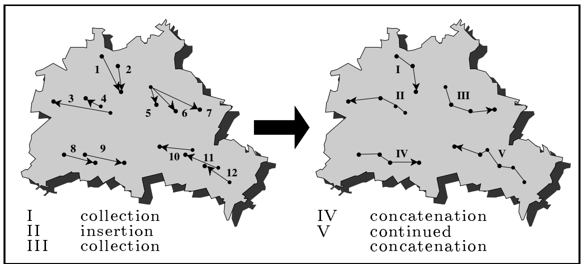

Figure 4: Constructing Vehicle Tours.

The construction of tours for individual vehicles leads to the consideration of shortest paths in space-time graphs that satisfy additional constraints. The problem to construct a best vehicle schedule from these tours can be formulated as an integer program:

$$\min \sum_{j=1}^{n} c_j x_j \qquad (1)$$

subject to

$$\sum_{j=1}^{n} a_{ij} x_j = 1, \quad \text{for all } i = 1, \ldots, m \qquad (2)$$

$$x \geqslant 0 \text{ and integral.} \qquad (3)$$

Each column j of this program corresponds to one out of n possible vehicle tours, each row to one out of m requests. a_{ij} is 1 if tour j services request i and 0 else. Equations (7) state that each request is serviced by exactly one tour, and the objective (6) accounts for the sum of the tour costs c_j.

Dial-a-ride problems arising at Berlin's Telebus service for handicapped people involve 2,000 and more requests per day and hundreds of thousands of vehicle tours. For problems of this size and complexity, the use of mathematical scheduling techniques can result in impressive benefits: Berlin's Telebus can, e.g., transport today about 30% more requests at the same cost as in 1992, while simultaneously the quality of the service (punctuality, transportation time, etc.) could be improved drastically.

These results do not only carry over to other demand responsive transportation systems, set partitioning methods apply also to general vehicle routing problems, and to all kinds of crew scheduling and rostering problems, such as, e.g., duty scheduling for bus drivers.

Box 1: Vehicle Scheduling in Handicapped People's Transport, Borndörfer [1998].

- **Public Transport.** Interest in the optimization of the operational planning has existed, world wide, for more than twenty years. This is, e.g., documented by a series of international conferences on this topic. The last three proceedings of these CASPT meetings Daduna & Wren [1988], Desrochers & Rousseau [1992], and Daduna, Branco

Vehicle Scheduling in Public Transit is the task to service a set of given timetabled trips (defined by lines and their service frequencies) to carry passengers. The primary objective is to find a minimal fleet solution, which aims at minimizing fixed costs for the vehicle fleet. Subordinate, one is also interested in minimizing operational costs among all minimal fleet solutions.

We use a multicommodity flow formulation as the underlying mathematical model. The figure below displays a small multicommodity flow network defined by a two depot instance ('r' and 'g') with five timetabled trips ('a' to 'd'). While the nodes in this network are defined by the depots and timetabled trips, the arcs are given by the possible links, called unloaded trips, without passengers.

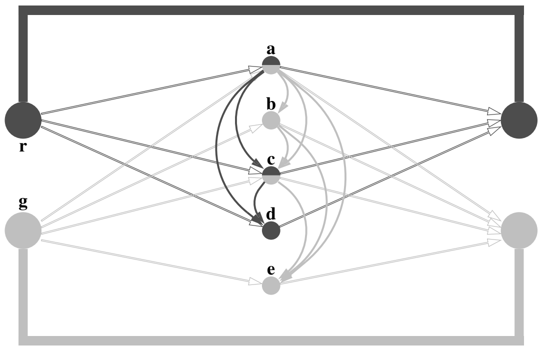

Figure 2: Multicommodity Flow Model.

Let $\mathcal{D}$ denote the set of depots, $\mathcal{T}$ denote the set of timetabled trips, A_d denote the set of arcs for each depot d, and κ_d denote the capacity of depot d. We introduce a 0/1 variable x_{ij}^d for each arc (i,j) and each depot d. x_{ij}^d indicates whether the unloaded trip (i,j) is run by a vehicle of depot d (in this case $x_{ij}^d = 1$) or not ($x_{ij}^d = 0$).

The integer linear programming formulation of this vehicle scheduling problem is

$$\min \sum_{d \in \mathcal{D}} \sum_{(i,j) \in A_d} c_{ij}^d \, x_{ij}^d \tag{1}$$

subject to

$$\sum_{d \in \mathcal{D}} \sum_{(t,j)} x_{tj}^d = 1, \quad \text{for all } t \in \mathcal{T}, \tag{2}$$

$$\sum_{d \in \mathcal{D}} \sum_{(t,j)} x_{tj}^d - \sum_{d \in \mathcal{D}} \sum_{(i,t)} x_{it}^d = 0, \quad \text{for all } t \in \mathcal{T}, \ \text{for all } d \in \mathcal{D}, \tag{3}$$

$$\sum_{(d,j)} x_{dj}^d \leqslant \kappa_d \quad \text{for all } d \in \mathcal{D}, \tag{4}$$

$$x \geqslant 0 \text{ and integral.} \tag{5}$$

The "two stage objective" is formulated by (1), the condition that each timetabled trip must be serviced exactly once is given by (2), the condition that each vehicle must return to its depot is implied by (3), and depot capacities are controlled by (4).

Large-scale instances from practice define problems with more than 125,000 side constraints of type (2) and (3) and up to 70 million 0/1 variables. Nonetheless, these problems can be solved to proven optimality (or close to optimality) with mathematical optimization.

The possible savings using such optimization tools are immense. Compared with a manual planning process, tests with real world data indicate savings of about 20 % of the vehicle fleet and about 14 % of the operational costs. For instance, the Berliner Verkehrsbetriebe (BVG) expects to save about DM 100 million per year employing mathematical optimization software, see Schmidt [1997].

Box 2: Multiple Depot Vehicle Scheduling in Public Transit, Löbel [1998].

& Paixão [1995] provide a good overview on the state-of-the-art. Several software companies offer products to support stages of or even the overall planning process. Parts of this software are still "rule based", heuristic, or just a graphical aid for manual planning. Due to significant algorithmic advances, see, e.g., Boxes 1 and 2, optimization strongly enters the area and uncovers further potentials for savings and service improvements.

- **Vehicle Routing.** This is a classical area for optimization that includes, e.g., the famous travelling salesman problem, see Lawler, Lenstra, Rinnooy Kan & Shmoys [1985] and Jünger, Reinelt & Rinaldi [1997] for surveys. But vehicle routing problems rarely come up in this "pure" version in practice. Many legal and technical side constraints lead to a great variety of similar looking, but, in fact, quite different routing problems that have to be attacked with tailor-made solution methods. For instance, there is a special literature on street sweeping, garbage collection, or postal delivery problems. Surveys on vehicle routing give Laporte [1997] and Bachem, Hamacher, Moll & Raspel [1997]. We should mention here also some closely related problems such as the sizing of vehicle fleets, see Desrosiers, Sauvé & Soumis [1988], and the location of vehicle service facilities or distribution centers, see Labbé & Louveaux [1997]. A further problem type that is currently coming more into focus are online (rolling horizon) optimization problems where, for instance, the planning can't start from a "standard" state (e.g., all trucks are parked at a central location at night) but where dispatching decisions for vehicles and crews have to be made as soon as new tasks arrive. New theory (stochastic optimization) is being developed to mathematically cope with such situations of high uncertainty, see, e.g., Cheung & Powell [1996] and Powell, Jaillet & Odoni [1995].

- **Traffic Control.** An important problem in road traffic is to manage the flow of vehicles such that congestion is small and vehicles are not forced to make big detours. Recently several ways to model the flow on highways and within cities have been proposed. These are based on cellular automata or analogies to fluid dynamics. Simulation tools exist with which the validity of these models and their assumptions is tested, see Rickert, Nagel, Schreckenberg & Latour [1998] and Krauß [1998]. These models are able to predict congestions based on measured traffic data and try to reroute parts of the traffic to improve the overall flow. These investigations are still in an early stage and transfer into practice is only experimental yet.

The interested reader may have noticed that, except for our own applications described in Boxes 1 and 2, we did not mention *magnitudes of potential savings*. The articles cited above (and almost all others) usually contain the mathematical models used, descriptions of the algorithms designed, details about implementations, and reports about computational performance — but it is difficult to find quantitative statements about the cost savings achieved. The reason for this is simply that the industrial project partners do not want such data to become public. (Our partners, see Boxes 1 and 2, hesitated as well!) Occasionally, savings are reported in the *press* when reporters or (shareholders) insist on precise information. This, in fact, was the case in Schmidt [1997]. On the other had, the article Thurston [1994] on furloughing further 101 pilots due to better scheduling (after having grounded 448 pilots in the preceding fifteen months) by Delta Airlines was obviously used by the company to exert pressure on the Air Line Pilots Association (ALPA) that was negotiating for pay raises.

> *"In laymen's terms, [the new system] allows us to schedule more efficiently. It reduces sit-time, or nonproductive pilot time. The consequence is a surplus of pilots."*
>
> H. Alger, Delta Airlines Executive Vice President-Operations, Thurston [1994].

We did also not address "more traditional" engineering optimization tasks such as the design of airplane wings, the body of a ship, vehicles with low air resistance, energy efficient motors, etc. The use of mathematical models, optimization, and numerical simulation in these areas is well established.

The near future will see further progress in our capabilities to solve operative and other aspects of transportation. In the long run, research aims at integrated approaches, combining, e.g., vehicle scheduling with duty scheduling, etc. Competition will drive these methods and algorithms into industrial practice.

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