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H. C. Hege K. Polthier (eds.)

# Visualization and Mathematics

## International Workshop

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# Collected Abstracts

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# Preface

The main goal of this workshop is to bring together mathematicians and experts in scientific visualization to close the gap between both areas.

In recent years significant progress has been achieved both in mathematical disciplines employing computers as experimental device and in computer graphics and scientific visualization. This is not only due to the general progress in hardware and software technology. Computer graphics gains by exploitation of mathematical methods, whereas mathematical disciplines profit by new computer graphical algorithms and visualization methods. Also in mathematics new working methods and research directions arose in connection with visualization, for example experimental mathematics and discrete techniques. This illustrates the mutual benefit to both disciplines.

The focus of the workshop is on new mathematical concepts and on mathematical applications in scientific visualization. The following topics are covered:

- applications in differential geometry and partial differential equations
- algorithmic aspects of adaptive techniques in space and time
- algorithmic representation of objects for storage and exchange
- time control of animated objects and corresponding algorithms
- new techniques for visualizing mathematical objects
- integration of visualization and numerical computation
- natural programming (visual, object-oriented, symbolic approaches).

To our joy, the announcement of the workshop met with a lively response as this collection of abstracts shows. We wish a good course of the workshop with many fruitful discussions over the fences between the different scientific disciplines.

Berlin, May 1995

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# Contents

# A new 3D Graphics Library: Concepts, Implementation and Examples

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**Abstract**

3D graphics libraries play an important role in aiding both mathematicians and engineers to visualize their data and results. One of the most common graphics libraries is given by the GL (resp. OpenGL) implementation [1] by Silicon Graphics, Inc. However, the results from the GL/OpenGL are not acceptable for high-quality images. The reason for this in inadequacy is due to the missing PHONG interpolation of normal vectors [2], the absence of global illumination models and the deficiency of configurable shaders and procedural textures.

We present a new 3D graphics library, which combines both the speed of the OpenGL and the rendering quality of professional commercial products. This improvement was achieved by a flexible and extensible concept which integrates the use of different renderer types, user-definable shading procedures and an optimal adaption to many different hardware platforms. Our graphics library allows to preview a complex scene e.g. on a fast SGI machine and produce a high-quality ray traced image from the same source code by changing one line of code when the previewed image is satisfying.

Several examples built with our graphics library will be presented along with the introduction of our modelling language. The latter is a comfortable and powerful tool for creating hierarchical scenes which can be imported into our graphics library through the concept of display lists. An outlook to future enhancements of our library will conclude the presentation.

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# Towards Interactive Distributed Simulation and Visualisation

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**Abstract**

The contribution focusses on the preliminary version of a distributed environment for the interactive simulation and visualization of unsteady flow problems based on the distributed visualization system *pV3* [1]. Results of the present study are obtained by an implicit block-structured finite-volume procedure for the Navier-Stokes equations. The latter is parallelized using *pvm* [2], hence computations can either be done on a single workstation, a workstation cluster or even a massively-parallel system, e.g. Cray T3D.

A motif-based graphical control tool allows for interactive modifications of the simulation properties, such as boundary conditions, and manages all features of the online visualization. All inter-process communication is done via *pvm*. The visualization runs concurrently with the solver where the graphic *clients* communicate with the graphic *server*. The graphic server runs on a graphics workstation (GWS), the client/server communication is also based on *pvm*. In order to minimize network traffic, the clients send only the so-called *extracts* needed for the visualization instead of whole 3D data sets. Having received the requested data extract, all graphics is consecutively done on the server side which should employ a high performance GWS (24 bit planes, z-buffer). The approach enables the user to 'plug' into the running simulation, check the data, change boundary conditions and switch off the visualization which can be plugged in again anytime later. The flow solver runs almost independently from the graphics part. Since the *pvm* software is public domain, the approach does not require any specific equipment or software except for the GWS (currently only Silicon Graphics machines are supported).

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[2] Al Geist et al: PVM 3 user's guide and reference manual, Oak Ridge National Laboratory.

# Object-Orientation in 3D Graphics: Extensible Modeling and Rendering

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**Abstract**

Existing ISO and industry standards are characterized by a finite set of graphical primitives dedicated to a special kind of rendering. Therefore, the multi-dimensional extensibility (primitives, rendering, interaction) is an important requirement for the up-coming ISO multimedia standard PREMO [1]. The main way to achieve this is by using object-oriented technology providing reusability, configurability and integration of advanced or application-specific semantics in a homogeneous manner. Hence, PREMO will be an extensible set of components primarily offering modeling and rendering facilities. A concrete application will be built on top of this by subclassing and aggregation relations.

Numerous publications stand for the applicability of the object-oriented paradigm in design of graphics systems. [2] describes an approach to integrate modeling and rendering facilities into customized graphical primitives by using multiple inheritance and run-time type information. An example for customized primitives that realize special semantics (visualization of field parameters by their visible representation) on the one hand, but are indistinguishable from built-in primitives on the other hand, is given in [3]. This paper proposes also a class hierarchy for scientific primitives that bases on an integration of abstract geometrical and topological classes by multiple inheritance. A class hierarchy for parametric curves and surfaces is described in [4]. The functionality of this package including blending and derivation operations is far away from the built-in parametric facilities in existing industry or ISO standards like OpenGL and PHIGS PLUS and shows the possibilities of open systems.

The definition of an open and in some sense generic graphics kernel is in progress in the ISO, as written above. Contents of this kernel will be parametric classes of aggregates, topological classes without a concrete representation and abstract classes that define protocols to concrete modeling and rendering components. A prototype realization in this direction is the YART graphics kernel as part of the GOOD project [5] that is in development at the TU Ilmenau. Basing on the generic YART primitive analytical, data-set based, parametric and scientific primitives could implemented very homogeneously.

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# MRT – a Visualization Tool Addressing Problems 'outside' the Classical Rendering Domain

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**Abstract**

We present an object-oriented software architecture for a 3D rendering environment which significantly improves the readability of the underlying algorithms, drastically improves productivity, and, most importantly, consists of building blocks that lend themselves to programmer customization thus making 3D image synthesis more accessible.

The rendering platform MRT [1] is *object-based* rather than drawing based and consists of an extensible set of objects that perform a variety of operations. The 3D objects as well as the imaging objects are the building blocks that lend themselves to programmer customization through techniques such as subclassing.

Keeping the functionality local to the individual classes and objects enables software engineers to focus on a specific task rather than spend their time in trying to understand a complex (and sometimes monolithic) package.

Experiences with our (inhomogeneous) user population prove that the system meets its design goal of being highly customizable and extendable. Furthermore, it serves as a compact testbed for various rendering aspects as well as for new algorithms 'outside' of the classical rendering domain.

This is supported by a recently started cooperation with a German mobile comunication network supplier. The development of a prototype package to simulate the 3D distribution of radio waves in urban environments based on MRT could be completed by one of our students within two weeks. The incredibly short development time (considering that we started from scratch) in combination with the fact that the prototype was significantly faster than what was available before made it fairly easy to attract external funding for this project.

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# A 4-dimensional Laterna Magica

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**Abstract**

Mathematicians, geometers especially, are by now quite familiar with fast and versatile computer graphics workstations. Many have used advanced but highly accessible packages, such as Geomview, Grape, AVS, to animate, in real-time and interactively, their favorite mathematical objects and processes. Almost all have seen wonderful video tapes, such as "Not Knot" and "Outside In", made in the traditional way.

Neither technology, nor popular taste, stands still. The integral involvement of my colleagues at the National Center for Supercomputing Applications, the Electronic Visualization Lab, and the Argonne National Laboratory in the development of virtual environments such as the CAVE, the Immersadesk, and the Power Wall, affords a unique opportunity for mathematicians to participate in a substantial way. At SIGGRAPH'94 last July, the four mathematical entries ran for about 16 of the 100 or so hours the 3 CAVEs entertained some 8,000 visitors. This coming November, the Immersadesk, the Power Wall and the I-Way (a transcontinental network linking remote supercomputers and virtual reality theaters) will join the CAVE at Supercomputing'95.

For these technological innovations, the choice of mathematical experiments and the design of graphical software must differ from that which we are familiar with, at least for now. This presentation will address this difference, review our success at SIGGRAPH'95 and, hopefully, reveal the first results from the newest experiments for Supercomputing'95. With the generous collaboration of my mathematical colleagues at many institutions, mathematics will be there again, entertaining, and enlightening scientists and programmers, with some very new eversions of the sphere, and isotopies of knotted surfaces in four-space.

# Surfaces via Mathematica and Geomview

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**Abstract**

In my lecture and demonstration, I will explain and illustrate how classical surfaces can be studied and drawn with Mathematica. Coloring can be done with curvature. I will show how to extend the techniques described in my book "Modern Differential Geometry of Curves and Surfaces".

In addition I will show how to color and animate the surfaces using packages I have written for Geomview and Acrospin.

# Variational Design – a Universal Approach for Modeling with Spline Surfaces

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**Abstract**

When manipulating more complex spline surfaces (e.g. having $50 \times 50$ control points) by hand, it may be difficult to preserve an overall 'fair' shape of the surface. A method, called 'variational design' enables the user to specify only a (small) number of constraints (e.g. boundary conditions, interpolation conditions, tangency conditions etc.). The remaining degrees of freedom are set automatically in such a way that the overall shape of the resulting surface is 'pleasant'. The idea is to minimize an appropriate fairness functional, thus leading to a constraint optimization problem or, a variational problem.

A crucial role in tis approach is the choice of the fairness functional. It has to be chosen such that 1. it produces surfaces of pleasent shape and that 2. the numerical minimization can be performed in a resonable time. The first condition can be obtained, by using functionals that measure the total curvature (square of the mean curvature (in mean square sense), or, sum of the squares of the principle curvatures (in mean square sense)). The variation of the curvature can be used as well. In order to satisfy the second requirement, it is better to use good quadratic approximations to these functionals. This leads to the concept of data dependent fairness functionals.

In the talk we describe the procedure, putting emphasize on the choice of the fairness functional. We also sketch some applications: construction of blend surfaces, fairing of surfaces and interpolation of scattered data.

# Numerical Experiments on Compact Constant Mean Curvature Surfaces


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**Abstract**

Since Wente's discovery of constant mean curvature tori in 1984 the class of tori has been investigated intensivly, both theoretically and numerically. For higher genus the situation is different and we are only at the beginning to understand these surfaces. Kapouleas proved existence of a class of compact constant mean surfaces, and in particular found surfaces for every genus [Kp]. His result does not give an answer to the existence problem for any specific surface though. His surfaces are characterized by their size – they are rather large –, and their geometry – they look like spheres connected with thin handles. It is therefore an open problem to find what the geometry of simpler compact constant mean curvature surfaces is.

In my talk I would like to present the simple and beautiful surfaces found in numerical work joint with K. Polthier [G-BP]. We give examples of genus from 3 to 10 (also 12, 30), that have only few 'bubbles' connected with big handles.

We employ an algorithm of B. Oberknapp and K. Polthier to compute discrete compact constant mean curvature surfaces and make use of the graphic programming environment grape. The algorithm is based on the conjugate surface method introduced by Lawson [L] and later extended by Karcher [1] and the author [G-B].


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# Exploring Plane Hyperbolic Geometry

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**Abstract**

Hyperbolic geometry is a geometry whose Euclidean representations can not be conveniently handled. Therefore straight edge and compass are not the best tools for exploring hyperbolic geometry, the software described in this talk will do a much better job. To find out about a new mathematical structure it is a good way to have a look at the mathematical objects on the one hand and to observe how structure preserving mappings work on these objects on the other. Thus, we have developed software that is capable of drawing geometric primitives like points, lines and polygons, apply geometric transformations like reflections, rotations and translations and measure distances and angles. All of these actions are interactively menu- and mousedriven and the results are simultaneously displayed in the three bestknown models for hyperbolic geometry, the *Poincare disk*, the *Klein model* and the *upper halfplane model*.

In contrast to the above "display-models" a fourth model, the *Minkowski model*, is introduced as a "storage-and-calculation-model". This model is used to describe the geometric objects and their constraints in a uniform way. Furthermore the calculations for distance- and angle-measurement and for the geometric transformations have to be done only once and in a rather convenient way (by applying $4 \times 4$-matrices).

In contrast to previous software [2, 3, 5] that focuses on the structure and visualization of hyperbolic three-space, our software focuses on the geometric transformations of the hyperbolic plane. In particular, the user can apply various hyperbolic transformations (reflections, rotations, translations, parallel displacements, and glides) intertactively and watch the results simultaneously in three different display models. We believe that this kind of software greatly helps to develop an intuitive understanding of hyperbolic geometry that can not be obtained from simply looking at pictures or movies.

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# Resolution Independent Vector Field Visualization

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**Abstract**

Line integral convolution, introduced in 1993 by Cabral and Leedom [1] building on work of v. Wijk [2], is a powerful technique for generating striking images and animations from 2D vector data. Based upon locally filtering an input texture along a curved stream line segment in a vector field, it is able to depict directional information of the vector field up to pixel resolution. The method has found many application areas, ranging from scientific visualization [3] to computer arts.

We present a new method for efficiently computing line integral convolution images. The algorithm minimizes the total number of stream lines to be computed and thereby reduces computational costs by an order of magnitude compared to the original algorithm. It guarantees numerical and visual accuracy by utilizing fast, error-controlled numerical ODE integrators [4]. Decoupling the characteristic lengths in vector field grid, input texture, and output image, filtered images can be computed at arbitrary resolution. Zooming continously into regions of interest is of great significance for exploration of vector fields. This can also be utilized in computer animation where takes with varying camera focus require textures with different spatial resolutions.

Changing the shape of the filter kernel over time allows to animate the resulting image texture [5]. The apparent motion is well suited to envision vector field orientation in addition to pure tangential information. We present methods for improved texture animation, employing constant filter kernels only. To obtain an optimal motion effect, spatial decay of correlation between intensities of distant pixels in the output image has to be controlled. This is achieved by blending different phase shifted box filter animations and by adaptively rescaling the contrast of the output frames.

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# Visual Tools for Integrable (Nonlinear) Systems

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**Abstract**

During the last few decades more and more problems in geometry and physics have been found to be related to integrable nonlinear (partial differential) equations. Twenty years ago one would have been quite satisfied to describe the solutions of such systems by reducing them to finite dimensional ODEs and then expressing them, in a sense explicitly, in terms of special functions. Nowadays the speed of modern computers and their graphic capabilities allow a much more detailed exploration of solutions of these integrable equations. Up to now, however, the necessary numerical and graphical tools had not been developed to accomplish such a task.

This work fills this gap and provides the necessary numerical theories and tools to investigate interactivly integrable systems which allow explicit solutions in terms of theta functions.

# Volume Rendering on Irregular Grids

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**Abstract**

*Direct volume rendering* is an attractive method to visualize three-dimensional data fields by depicting them as density clouds of variable transparency [1]. The resulting pictures provide an informative overview of the whole data distribution.

In computer simulation data fields are often computed on an underlying irregular grid. This is the case in adaptive multi-level finite element methods [2] where tetrahedral grids are employed. While very efficient algorithms for volume rendering exist for processing regular data grids, a fast *and* exact image computation of irregular grids still turns out to be problematic.

We take an object-oriented approach to build a framework in which various algorithms for volume rendering on irregular grids can be used to weigh up between speed and image precision. As we mainly deal with data on tetrahedral finite element meshes the algorithms implemented so far concentrate on this kind of unstructured grids. Of the different types of algorithms for direct volume rendering we consider the *projection approach* most suitable for irregularly distributed data. Several projection algorithms being applicable to irregular grids have been proposed in the last five years [3, 4, 5].

Our system for volume rendering on irregular grids implements several variations of these algorithms ranging from a *splatting* method for fast previewing to high quality rendering employing more exact numerical solutions to the *equation of transfer*. The system is easily extendable and also serves as a framework to experiment with different color mapping and shading strategies.

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# Periodic Tilings

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**Abstract**

The theory of Delaney symbols, introduced by Andreas Dress (see Dre84), supplies a useful combinatorial language for describing, and a convinient data structure for computing 2-, 3- or higher dimensional periodic tilings, see e.g. DHM93 or Hus94. Based on this, a program has been produced that can be used to systematically generate and visualize all possible periodic tilings of the plane (see DH95). Making use of the Delaunay triangulation program Detri (by E. Mücke, see EM94), and working with N. Dolbilin (Moscow), the speaker has developed a program that does the following: It takes as input a periodic 3-dimensional point set $\Gamma \cdot X$, where $X$ is a finite set and $\Gamma$ is one of the 230 space groups, and produces as output the corresponding periodic Delaunay tiling $Del(\Gamma \cdot X)$, and it's Delaney symbol $(\mathcal{D}, m)$. This program may have applications in chemistry or crystallography.

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# Energy based adaptative time step and inertia-matrix based adaptative discretisation for fast converging dynamic simulation

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**Abstract**

Dynamic systems are capable to simulate objects behaviours (motion, deformation, collision) and to study their contact interaction with the environment. They have initially developed in the computer graphics field[1] in order to do the behavioural animation. In robotics field, we do not find the notion of a compleat dynamic model. One uses a geometrical model in addition to a partial physical model[3] to detect contact [6], to represent collision [5], or to avoid obstacle [4]. Compleat dynamic models become important in the robotics field to study a complex contact interaction between the robot and its environment[2].

Whatever the used dynamic model: objects have to be discretised in sub-objects, else they will not be able to deform or to interact, the position and the speed of each sub-object is obtained in solving a differential equation system which the general form is the following:

$$m_i \ddot{\vec{P}}{}^i_t = \vec{FrcEx}{}^i_t + f(\vec{P}^j_t, \dot{\vec{P}}{}^j_t, \vec{P}^i_t, \dot{\vec{P}}{}^i_t)$$

Where $P^i_t$ is the position at the instant $t$ of the sub-object $i$, $m_i$ is its mass, $FrcEx^i_t$ is the external forces applied on $i$ and $\dot{\vec{P}}$ is the derivation of the function $\vec{P}$.

The main problem of dynamic models are the large computational time needed to achieve a simulation. This comes from two principals reasons for which we propose solutions:

**The time step:** The external forces, in the general case, are not continuous and they are not known in advance. So one can not use an implicit method[4] to solves the above-mentioned differential equation system. Only explicit methods[5] are possible to be used. These methods need a very small time step to ensure correctness and stability. Also , in the general case, there is no way to determine this time step. Our solution is to use an adaptive approach based on the notion of the energy conservation. In this approach the stability can be characterised by $\Delta E < 0$ and correctness can be characterised by $|\Delta E| < \epsilon$. Experiments show that the incurred error in position is proportional to $\epsilon$. At each iteration one choice the largest time step which verify the needed constraint. The gain of this method is very important and it can reach a very big values such as 10000 for a very stiff collision.

$$Gain = \frac{\text{The greatest time step}}{\text{The average time step}} > 1$$

[3]A partial physical model is a model which represent the motion, the deformation, or the interaction but not the three aspects at the same time

[4]Implicit methods to solve a differential equation, can be alway stable whatever the used time step

[5]using this method the current state of the system $State_t$ depend only to its past.

**The discretisation:** The correctness of the behaviour of the modeled object depends on the fiability of its representation. A fin discretisation is a solution but it is very expensive in calculation time. Our solution is to decompose an object into sub-objects which do not subjected to a big deformations during the motion. Then we represent each sub-object by nine point masses which are sufficient to identify their inertia-matrixes and their inertia-centers with the real ones. So we need a fin discretisation for the very deformable parts of the object but not for the rigid ones. Using this approach the object becomes hollow, so a dynamic surface/surface contact interaction is proposed to solve this problem.

These two adaptative approaches and surface/surface contact interaction method are integrated with the dynamic modeling system $Robot_{\Phi}$[3]. They are the subject of this paper.

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# Efficient Calculation of Subdivision Surfaces for Visualisation


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**Abstract**

Subdivision of surfaces is one possible way to describe free form patches used in geometric modelling. Starting with an initial mesh of facettes (polygons), a sequence of refinements is generated, which converges towards a smooth surface. Examples for subdivision surfaces are those of Catmull & Clark [CC78] and Doo & Sabin [DS78].

The main shortcoming of these algorithms is, that the number of facettes of a mesh grows exponentially with the number of refinement steps. This leads to exponential time and space requirement, if the refinements are successively evaluated, as proposed in the literature.

We show that the problem of exponential space requirement may be reduced by processing the refinements in a depth-first manner. A facette of the finest refinement is immediately reported, e.g. drawn on the screen for visualisation, and is removed from memory. The main goal of this contribution is to work out the details of this depth-first approach, which results in asymptotically the same time as the well-known breadth-first algorithms but in considerably less space requirement.

The subdivision is split into a calculation rule and a rule of topological assignment. The latter describes the relation between facettes (polygons) of the mesh and facettes of its refinement. This assignment is represented by a calculation graph. This point of view permits a modular implementation. Thus an easy application to different subdivision algorithms is supported, for example to those refered to above.

Independent of the calculation rule and the topological assignment several strategies for depth-first processing of the facettes of the refinements do exist. For instance, algorithms for polygon filling can be used in a slightly modified way. This implementation allows to apply several strategies.


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# Numerical Methods, Simulations and Visualization

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**Abstract**

Our main interest concerns the theory and the numerical treatment of partial differential equations and systems as a mathematical model for realistically physical problems in particular in 3-D. In order to analyse and to interpret the results of complex numerical simulations it is absolutely necessary to use the tools of a powerful visualization environment. In the first step the visualisation software should be able to show the same 'pictures' as one gets from the physical experiments. But beyond it also the inherent structures like local vorticity vectors or complex shock interactions , which cannot be studied in the physical experiment without extensive and difficult measurements, should be visualized. It is obvious that a good visualization is also necessary for the development and implementation of a complex pde solver and for the presentation of the results. These goals are attainable only if the vizualization software is adapted to the structures of the numerical methods. In particular the data structures produced by modern techniques like local adaptive concepts on structured and unstructured grids, multigrid methods and parallel computing are a new challenge for the visualization. For the problems concerning adaptive grids we have already developed some effective tools and we are working on the other ones.

In this lecture we shall report on some projects we are working on in our group. In the first examples we consider local refinement zones which are moving with time dependent shock patterns on structured and unstructured grids. This process is controlled by different grid indicators . The underlying numerical method is an upwind finite volume scheme, also on dual cells. The time evolution of these structures will be shown in a video.

In collaboration with a car company we have studied the inviscid flow of a compressible fluid through a cylinder in 3-D as in a two stroke engine. Since the situation of a moving piston is included the computational domain is timedependent. The underlying grid consists of tetrahedrons and local grid refinement near the in- and outflow channels is necessary. The cells and those parts of the cells which have been cut off by the moving piston are neglected and the computations have been done on the remaining part of the grid. For stabilization small fragments of the cells which also remain have to be put together with larger cells. The implementation of this part was nearly impossible without the visualization of the local structures. In a movie we shall present the flow field during one period .

In an other project we treat the deformation of an elastic shell in 3-D. The forces which are responsible for this deformation are acting in singular points were the grid has to be refined. We shall show some first results of this work.

Finally we shall give an overview on the future work. For further details and the basic concepts concerning the visualization we refer to the lecture of M. Rumpf.

# Time Control of Objects in Scientific Visualization

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**Abstract**

The emergence of visual representations has been one of the most significant recent trends in computing in the last years. Computer animation, graphical user interfaces and visual programming systems are commonly used for data visualization. An essential feature of multidimensional data visualization problems is that the data typically have no immediate geometric interpretation.

We try to show how a flexible environment for the visual exploration of time-based multidimensional digital data can be build around a data flow visual programming system. We start by exposing the problems specific to the visualization of multidimensional digital data as produced by sensors or numerical simulations. We then describe an object-oriented framework for constructing visual applications where the animated scene on the screen is generated by various processes and input devices. This framework is based on a general treatment of time and event modeling for visualization in scientific computing. It consists of the following elements:

- A data model derived from the theory of fiber bundles.
- A dataflow system that supports for each time slice, the mapping of the data model onto graphical primitives with the help of a glyph editor and binder.
- A time model that supports local time-warps and time-interpolation of the data sources.
- A dialogue model for guided exploration or video production. This model uses an auguemented transition network with possibly disconnected active components representing non-deterministic scenarios whose transitions are triggered by events generated by processes or input devices.
- A dataflow system driven by the dialogue model and supporting multimedia composition for immersive exploration and multimedia document production.

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# Nonlinear Electrodynamics

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**Abstract**

We study Martin's nonlinear model for electrodynamics [3], [2]. Martin's model is formally similar to the Born-Infeld type models [1] and has its origins in the formulation of classical electrodynamics described by Souriau [5]. Recently it has been shown that this model predicts quark-like structures [4].

In this model the cotangent bundle of space-time is replaced with an eight-dimensional $M$ endowed with a neutral metric $g$. The metric $g$ induces a bundle isomorphism $h : TM \to T^*M$. The dynamical structure determines nondegenerate 2-vector fields $\Lambda$ on $M$ that satisfy $\mathcal{C}(\Lambda, h(\Lambda)) = -1$ where $\mathcal{C}$ denotes the contraction of the last index of $\Lambda$ with the last index of $h(\Lambda)$. These equations linearize to the classical Maxwell equations along Lagrangian submanifolds.

Once an Artinian ambient metric is chosen the generalized Maxwell PDE determines a pair of totally null distributions that can be parameterized by four three-component vector fields which can be identified as electric and magnetic fields.

Using equivariant geometry (and computers) we find solutions to these PDE's and study their behavior. Integrals of these solutions would in theory correspond to particles.

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# Solving Problems in Singularity Theory using Computer Graphics

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**Abstract**

In this paper we will illustrate the power of computer visualisation to help solve mathematical problems in singularity theory and differential geometry. We will concentrate on two problems concerning the configuration of certain curves in space [1], [2]. In both cases the results obtained from computational examples appeared to contradict mathematical conjectures about the configurations, bringing the computer results into question. By adopting a strict methodology it is possible to construct powerful arguments about configurations, by purely graphical means. In the two cases we study here the graphical arguments were powerful enough to stimulate further study of the mathematics where flaws where eventually found.

In the first part of this paper we will describe a flexible easy to use interface (The Liverpool Surface Modeling Package) in which mathematical problems can be easily investigated and experimental investigations are encouraged.

The production of convincing examples can often involve more than simply typing an equation. In particular if we are interested in a whole family of surfaces, rather than just a single example then we will run up against problems with genericity - is the example we have chosen degenerate in some way? Even if we have a generic example the image on the screen may be misleading, for instance certain important details may be too small to be visible at a given scale. The second part of the paper deals with such problems, and outlines techniques which by which they can be overcome.

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# Integrating 3D Visualization with the World Wide Web

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**Abstract**

The World Wide Web is now expanding beyone the 2D world of hypertext to incorporate 3D graphics. The VRML [3] standard provides a mechanism for hyperlinks in 3D: that is, associating a 3D object with a URL. Integration of 3D graphics with the Web offers new challenges for both designers and users of visualization software. Although most systems make use of a local interactive 3D viewing program, the distribution of data throughout the Net introduces interactivity constraints.

We will present the WebOOGL [4] system, which integrates the Geomview [2] interactive 3D viewer with the Web. Geomview was designed for mathematical visualization [1], although its flexibility and power have also led to its adoption by researchers and educators in other fields. We will discuss aspects of Geomview's design that facilitate both mathematical visualization and Web integration. Our work in progress on visualizing the structure of the Web itself in hyperbolic space illustrates the need for tools which encompass both domains.

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# Computing Discrete Constant Mean Curvature Surfaces

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**Abstract**

The computation of constant mean curvature (CMC) surfaces is a difficult problem. Only few methods that work under restrictive hypotheses are known. Anderson used the direct numerical approach — to minimize the area under a volume constraint — for graphs in tetrahedra, alternatively other integrals can be minimized, e.g. using Brakke's surface evolver.

The algorithm for computing discrete CMC surfaces developed by the author and K. Polthier [1], [2] extends the ideas of the algorithm by Pinkall and Polthier [3] from minimal surfaces and their conjugates to spherical minimal surfaces and their euclidean CMC surfaces. The algorithm simulates Lawson's conjugate surface construction [4]: the free boundary value problem for a CMC surface patch is reduced to a Plateau problem for a geodesic polygon in $S^3$, conjugation of the solution yields a CMC patch bounded by planar symmetry arcs. See [5] for details of the construction.

The algorithm uses the technique of discrete surfaces and the discrete description of their properties: instead of approximating the smooth case the problem is reformulated on a discrete level. The algorithm consists of two main parts. To compute the Plateau solution as a discrete minimal surface in $S^3$ the algorithm of Pinkall and Polthier is used. The second part is the conjugation procedure which computes the discrete CMC surface from the spherical minimal surface. The conjugation is exact on the discrete level, the CMC surface is computed directly from the discrete data obtained by the minimization.

The algorithms were implemented using GRAPE, the object-oriented GRAphics Programming Environment developed at the SFB 256 in Bonn. Starting with geometric data for the CMC surface the user can interactively construct a starting triangulation, apply local refinement in regions of high Gaussian curvature, and visually control the minimization and conjugation.

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# Visualization of 3-D Nonstationary Navier-Stokes Solutions by Particle Tracing

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**Abstract**

With the availability of increasing computer power in computational fluid dynamics it is possible to consider models and flow geometries of practical interest where the flow and transport processes are fully three dimensional. There is a need to develop fast algorithms and software tools providing flexible visualization of the solutions of these problems. We use a finite element discretization of the three-dimensional incompressible Navier-Stokes equations in primitive variables and solve the resulting finite dimensional problem by a multigrid method ([4]). In order to achieve high flexibility we use locally refined grids.

By tracing particles their behavior in a fluid can be visualized to give us a better understanding of the fluid motion. A starting set of particles moves in the velocity field according to the magnitude and direction of the vectors. This technique can display the global features of the flow. The code was developed on a SPARC workstation using the graphics program AVS, which provides interactive 3D graphics and ray tracing. It includes a number of visualization techniques which the user can invoke simply by selecting them from a menu. Visualization applications can be build up by combining software components of the toolbox, called 'modules', into executable 'flow networks'. Networks are applications based on a connected group of modules. These connections represent the flow of data among the modules. We wrote a number of modules to integrate our own finite element structures (FEAT3D [2] and DEAL [1]), which allows for the use of both multigrid procedure and locally refined grids.

Our main task was to develop a particle tracer making extensive use of the hierarchical structure of the grids and the neighbor relationships resulting from our finite element toolboxes. Therefore, we are able to implement fast search and tracing routines and it is possible to handle complex geometries for our particle tracer.

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# On 3D Representation of Pavings

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**Abstract**

The modelling of objects in three dimensions and assembly of a finite number of solids have many applications, in particular in CAO and in molecular modelling. The solids are generally represented by their boundaries. The idea of map introduced by Edmonds [1] enables us to represent an oriented surface, and in particular the surface of a solid. By assembling these maps, we get the notion of paving [2, 3] which resembles the models given by Lienhardt [4] and by Arques and Koch [5]. A paving is a data structure permitting the representation of space subdivisions and is constituted by a set B of darts (which are the basic elements), an involution and two permutations. Every partition of the three-dimensional space into a finite number of cells can be modelled with a paving.

We give a method for the representation of pavings. Non-Boolean operations enable us to assemble elementary solids according to a construction tree related to CSG (Constructive Solid Geometry) [6]. The operations studied are internal operations in pavings: the merging which suppresses a face between two solids and puts them together into a single solid, the sticking along faces and the sticking along edges which enable us to assemble solids. By composition, other operations are defined. In particular, we thus realize the inverse operation of the merging. We study more precisely what becomes of cells, faces, edges and vertices with such transformations.

In conclusion, we give results on the characteristic of pavings, defined by the number of cells minus the number of faces plus the number of edges minus the number of vertices.

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# The Oorange Project

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**Abstract**

Oorange is a new environment for programming and running scientific computer experiments that need sophisticated visualization. The major achievements are:

- A dependency network consisting of objects being linked together via dependency graphs. If one object has changed, it notifies its dependend objects so they can update themselves. The network can be graphically edited.
- The precise way an object reacts to update messages is not hard-wired but controlled by a script written in interpreted Objective-C. This script can be edited at runtime.
- User definable inspector and control panels for each object give complete access to each object in the network.
- Completely object-oriented design. Extensions by the user (new classes or new methods for old classes) are dynamically loaded during runtime.
- Oorange comes with state-of-the art tools for handling 3D-geometry as well as images.
- A builtin WWW-Browser makes data on the net (HTML, Images, MPEG, 3D-scenes in VRML format) or Oorange network-scripts on the net directly available for further processing. This will greatly faciliate the cooperation between research groups.

Oorange is completely based on freely available software. A first release for Irix or Linux will be available in the fall.

# Particle Tracing Algorithms for Flow Visualization

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**Abstract**

In this talk we will cover two topics related to particle tracing for flow visualization: a comparative analysis of particle tracing algorithms in curvilinear grids, and a technique for visualization of turbulent flow using particle tracing.

Standard particle tracing is based on stepwise integration of the motion equation of a fluid particle in a velocity field. We have investigated two types of algorithms: those directly operating in the curvilinear grid in physical space (P-space), and those using a transformation to a Cartesian grid in computational space (C-space), as commonly used in computational fluid dynamics for solving the flow equations. The local transformation to map P-space to C-space and vice versa uses the Jacobian matrix describing the deformation of a cell in P-space into a Cartesian unit cell in C-space. This transformation is the critical factor for both accuracy and speed. For a vector field defined in P-space, the P-space algorithms are superior in both respects.

Particle tracing can also be extended for visualization of turbulent fluid motions. Turbulent flows can be simulated by direct numerical simulation, large-eddy simulation, or by statistical turbulence models. We have developed a method for visualization of data from a statistical model, based on Reynolds decomposition of the flow field into convective and turbulent motion. The simulation generates a mean velocity field and a turbulence intensity field. To generate turbulent particle motions, stochastic perturbations are added to local particle positions resulting in random walk motions of particles, directly reflecting local turbulent motion. The method allows an integrated visualization of mean velocity and turbulence intensity data.

# Graphical Visualization of the Stokes Phenomenom

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**Abstract**

In this talk, we present a complete package for the study of Linear Ordinary Differential Equations, which computes a basis of **formal** solutions in the neighborhood of a point [1], performs **numerical calculus** to find *actual* solutions [2,5] and enables the graphical representation of these solutions in the complex domain.
The numerical part is a toolkit of methods, which can be achieved according to the nature of the solutions (convergent, divergent, $k$-summable, multi-summable series).
In the graphical part, two representations are available :

- **a surface representation** : to have a global idea of the behavior of the complex function $f$, we can represent it by the relief of its modulus, colored according to the argument (i.e. by a surface of points $(\rho\cos(\theta), \rho\sin(\theta), |f(\rho e^{i\theta})|)$, displayed with a color corresponding to $arg(f(\rho e^{i\theta}))$ [3];
- **a curve representation** : to study more precisely the behavior in the neighborhood of a singularity (translated at the origin), we represent the image by the function $f$ of a circle or a circular arc around the origin. Without representing the domain, the co-domain is then plotted in the complex plane, in two dimensions, each point $f(z)$ displayed with a color corresponding to the argument of $z$ [4].

In both cases, the domain, where the function is calculated, and the numerical method used to compute it, are interactivelly chosen. So we can compare the results obtained by different methods and visualize their respective domain of validity.
We will use these two types of representation to illustrate graphically the **Stokes Phenomenom**, which is well known by mathematicians and physicists, who deal with differential equations and asymptotic expansions near irregular singular points, and we will **show** that, even to characterize an entire function, it is, in some parts of the complex plane, very efficient to use divergent series and multivalued functions.

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# What Should a Surface in 4-Space Look Like ?

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**Abstract**

The question "What should a surface in 4-space look like ?" can be posed as a mathematical question and also as a visualization question. We address both of these.

Interesting surfaces in 4-space exhibit knotting behavior.

Mathematically our approach is to represent knotted surfaces by embeddings which have (locally) minimal knot energy.

We discuss and show in video various techniques for using computer graphics to aid in the visualization of knotted surfaces. These include projection, slicing, coloring and texturing.

# A Procedural Interface for Numerical Data Visualization beyond Data Formats

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**Abstract**

Recent numerical methods to solve partial differential equations in scientific computing are based on a variety of advanced kinds of domain discretizations and appropriate finite dimensional function spaces for the solutions. The scope of grids under consideration includes structured and unstructured, adaptive and hierarchical, conforming and nonconforming meshes. The function spaces might be of Lagrangian or Hermitian type with higher polynomial degree and possibly discontinuous over element boundaries. Unfortunately, the rendering tools in scientific visualization are mostly restricted to special data structures which differ substantially from the data formats used in the numerical application. This forces users to map and interpolate their data, which is time consuming, storage extensive, and accompanied with interpolation errors. We present an interface between numerical methods on various types of grids and general visualization routines which overcomes most of these disadvantages. It is based on a procedural approach managing a collection of arbitrary elements and a set of functions describing each element type. The numerical data is not mapped onto new data structures. Providing a small set of procedures and functions the user will furtheron work on his own data structures. These routines temporarily and locally manage the access to data of interest, like information about a single element. Compared with display routines on a specialized data structure, this general interface does not produce much cpu overhead.

# Animation of Algebraic Surfaces

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**Abstract**

Algebraic surfaces and their deformations have been studied for more than one hundred years. Visualization of these surfaces has always been regarded important and traditionally plaster models were used for this purpose. Recently, of course, computer graphical studies have been carried out. In [2] Hanrahan investigates the raycasting approach to the rendering of algebraic surfaces. The central computational task is twofold. First, one has to efficiently convert the equations of the surface and a given ray into a single equation, i.e., a polynomial in one variable. Secondly, a numerical procedure must be employed to compute the smallest positive root of the polynomial. Hanrahan chose a method developed by Collins and Loos, which is based on Descartes rule of signs. In [3] the approach was tested with other root finding methods and in this case study we extend the method for animation.

An important example of a deformation of algebraic surfaces was proposed by Kummer in the last century. It is given by a parametrized family of fourth-order polynomials in affine coordinates $x$, $y$ and $z$. In this deformation a double sphere is transformed into Steiner's roman surface and then into a tetrahedron. Although individual surfaces from this family had been visualized before, only just recently a sequence of (raycast) images illustrating the entire deformation was given by Barth and Endraß in [1]. In this work we present corresponding computer animations. It is the purpose of this work to demonstrate a feasible approach to rendering animations of mathematical objects suitable not only for the computer graphics specialists but also for students and researchers in mathematics without such background. Thus, we show how to combine public domain software with common off-the-shelf hardware to arrive at the desired product.

The animation is based on the original method of Hanrahan and supported by the pd raytracer *rayshade* of Kolb. This approach requires supplying a corresponding program module for the ray surface intersection calculations. Individual frames are JPEG compressed at a conservative compression ratio. The compressed data can be played back in real time on an SGI Indigo2 workstation equipped with the CosmoCompress option. In this way we can preview the entire animation at full resolution and video frame rate, thus, eliminating the time intensive and error prone frame-by-frame recording. There are several more advantages of this all-digital approach to computer animation as compared to conventional video taping, video frame store and video laser disk: individual frames can easily be retouched, playback of arbitrarily rearranged frames is possible, scene duration is limited only by the available disk space, and animations can be transmitted over the net.

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# Wavelet Algorithms for Illumination Computations

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**Abstract**

The computation of illumination in synthetic environments is one of the central problems of computer graphics. Direct effects are easily computed but the accurate computation of indirect illumination, responsible for many subtle effects, is so expensive as to prevent its widespread use. In this talk I will describe some recent advances in the solution of the illumination problem which yield asymptotically faster algorithms.

The underlying problem can be stated as an integral equation, relating the outgoing radiance (Power per unit area per unit solid angle) at a surface to the outgoing radiances at all other surfaces. Both point sampling techniques such as Monte Carlo and finite element approaches such as the Galerkin method can be used to solve this integral equation. Motivated by recent advances in the numerical analysis community I have pursued Galerkin methods with wavelets as basis functions. These techniques have been applied to the case of diffuse interreflection (radiosity) and glossy interreflection (radiance). They generalize earlier hierachical approaches, based on n-body algorithms, to higher orders. The resulting algorithms have been implemented on workstation class machines. In the talk I will give an introduction to the underlying technology and report on the results achieved in our implementation.

# Visualization of Spline Curves and Surfaces

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**Abstract**

Within the last two decades, splines have established themselves as the leading tool for the description of curved objects in computer graphics and CAD/CAM. In this talk we will survey some recent new developments in curve and surface design using splines. In particular, we will discuss hierarchical splines, splines over arbitrary triangulations, and surface fairing.

We will then describe an object-oriented framework that allows to visualize these different curve and surface types within a single software package. Our design starts with an abstract class of differentiable curves and surfaces and in turn refines this design to curves and surfaces that are explicitly given in paramentric form. The curve and surface types mentioned above are then derived from these abstract classes.

# Simulation and Visualization of Structure Formation in three-dimensional Excitable Media

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**Abstract**

Excitable Media, like oscillating chemical reactions, show a variety of interesting spatio-temporal structures. An example is the well-known Belousov-Zhabotinski reaction that can produce spiral waves in two spatial dimensions.

To simulate such systems on a computer, Cellular Automata (CA) are often used. Here a CA is defined by an entity of discrete variables located on the cells of a square lattice, together with local and deterministic updating rules. So a CA is a dynamic system that is discrete in value, space and time. Compared to differential equations, CA can be faster to compute and may have fewer parameters to adjust when simulating chemical or physical phenomema.

There exists a CA model for a two-dimensional oscillating chemical reaction (see ref. [1]). The so-called Hodge-Podge Machine uses about 200 states per cell and has four numeric parameters.

For a simulation of sytems with three spatial dimensions the speaker further reduced the computing time and the number of parameters. The examined CA uses only three states per cell and has only one parameter: *A Cell in state 1 goes into state 2 if and only if at least p of the eight nearest neigbours are in state 2,* where p = 7 (p = 3 in two dimensions). The rules for states 2 and 3 are obtained by cyclic permutation $1 \to 2 \to 3 \to 1$. This CA produces a great variety of patterns, among others there are planar and spherical waves as well as different kinds of spiral rolls.

Real-time visualization is achieved by techniques of volume rendering, displaying those cells which are in one certain state. To keep the geometry simple, the surfaces of the rectangular cells are only optically smoothed by Gouraud shading, which allows a frame rate of 4-5 f/s for computing and displaying a sytem of $31^3$ cells on an SGI Indigo2 Extreme (R4400, 200MHz).

## References

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# Front Tracking by Embedded Surfaces: The Level Set Method for Propagating Interfaces

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**Abstract**

The level set method is a numerical technique for tracking the motion of complex interfaces as the move under speeds that depend on such factors as local curvature, normal direction, and underlying physics. It handles complex changes in topology, breaking and merging, sharp cusp formation, and naturally handles the entropy conditions that must be enforced in propagating interfaces. The technique works by embedding the front as the zero level set of a higher dimensional function; the associated equation of motion is then solved by exploiting numerical schemes borrowed from hyperbolic conservation laws.

Since the introduction of this technique in 1988, it has has been used in a wide variety of settings. We will review such work, and show video tapes and graphics of applications of the method to the construction of minimal surfaces, shape recovery and recognition in medical imaging, image processing, two and three dimensional grid generation, and etching, deposition and lithography in microchip manufacturing.

# Supporting Numerical Simulations in Medical Treatment Planning

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**Abstract**

We present a new software system designed to integrate complex numerical simulation methods with state-of-the-art visualization and interaction techniques. The system, called HyperPlan, is currently being used as an experimental environment for planning hyperthermia treatments in cancer therapy [1]. In this application adaptive multi-level finite element methods [2] are used to compute electromagnetic fields radiated by microwave antennas as well as power and temperature distributions inside the patient's body. These computations are performed on a three-dimensional patient-specific tetrahedral grid. Special data structures, visualization methods and interaction techniques are provided to support this kind of numerical simulation. However, the flexible design of the system allows it to be used in many other application areas as well.

In contrast to conventional data-flow oriented systems like AVS [4] or Iris Explorer in HyperPlan data objects are emphasized rather than computational modules and algorithmic dependencies. Special-purpose editors allow to operate on data objects interactively. In this way for instance contours may be drawn into tomographic images or a tetrahedral grid may be modified manually. To visualize a data object any number of matching display modules may be linked to it. So-called computational modules allow to implement numerical algorithms and other procedures which do not require user interaction.

The system is built in an object-oriented fashion. It utilizes several powerful class libraries. In particular the Open Inventor toolkit [3] is used for graphical output. This package makes it easy to implement sophisticated 3D-interaction concepts as well as advanced hardware-accelerated rendering techniques. Other features of the HyperPlan system include dynamic loading and scripting interfaces for all modules.

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# Using the Evolver for Geometric Optimization Problems

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**Abstract**

It is easy to modify Brakke's Surface Evolver [1] to minimize energies other than surface area. We have used it to study Willmore surfaces [3] and knot energies [4].

The Willmore energy is the integral of mean curvature squared, an elastic bending energy. This is now available in the Evolver, and has been used to study lipid vesicles, as well as for mathematical research. We have computed previously unknown Willmore surfaces of higher genus.

There are several energies for knots and for embedded surfaces which arise from the replusive potential of charges spread over the knot. We have used the Evolver to compute minimizing configurations of all small knots and links under one of these energies, which was shown by Freedman, He, Wang [2] to be Möbius invariant.

Since the Willmore energy is also invariant under this conformal group, in both cases we have to deal with this large and visually nonintuitive symmetry. It is especially helpful to use the Geomview graphics facilities, since we have added facilities to Geomview for applying Möbius transformations to objects being rendered. Geomview can display objects in the three-sphere either as they would be seen by an internal observer, or under stereographic projection to Euclidean space. Geomview in this mode gives us interactive motions in the full 10-dimensional Möbius group.

When we find a minimum-energy configuration of a knot or Willmore surface, we want to see all conformal representatives to find the nicest picture or to detect when two noncongruent configurations are Möbius equivalent; the conformal viewing option of Geomview permits this.

The Evolver also has features to enforce symmetry. This allows us to compute unstable Willmore surface, as in the minimax sphere eversion, and allows computation of triply periodic structures like the new equal-volume foams related to TCP crystal structures.

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# See what I Mean? Using a Dataflow Toolkit to Visualise Numerical Data.

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**Abstract**

We describe some of our experiences with IRIS Explorer, a commercial visualisation package which is built around the so-called dataflow model and which uses a visual programming paradigm in its user interface. When working with this package, users construct visualisation applications by connecting software modules together in a way which defines the flow of data from one module to the next. Recent work in developing new modules for version 3.0 of this package is discussed in some detail, with particular emphasis on development which makes use of other numerical and graphics libraries in the construction of the module. We also discuss some future directions for the program, including integration with AXIOM, a mathematical symbolic manipulation package. The work is illustrated with a number of examples, drawn from user applications and research performed at NAG.

# Deformation of Doubly and Singly Periodic Minimal Surfaces of Genus Two

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**Abstract**

In [2] the author constructed the first one-parameter family of examples of genus two, doubly periodic embedded minimal surfaces by adding handles to a one-parameter family of genus one examples found by Karcher [1]. Using the same method, one can also add handles to a second, less symmetric, family of Karcher's surfaces and thus get a new family of genus two doubly-periodic minimal surfaces. The construction of this extended family of genus two examples involves two dimensional period problem in the Weierstrass representation. We will show how the deformation is used to solve the period problems and how the doubly-periodic surfaces deform to a one-parameter family of genus two singly periodic embedded minimal surfaces that look like the result of adding handles to the classical Riemann's singly-periodic examples.

## References

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