Wigner Representation Theory of the Poincaré Group, Localization , Statistics and the S-Matrix

Bert Schroer
Freie Universität Berlin
Institut für theoretische Physik
Arnimalle 14 14195 Berlin
e-mail schroer@physik.fu-berlin.de

October 1996
Revised Version May 1997
To be published in Nucl. Phys. B

Abstract

It has been known that the Wigner representation theory for positive energy orbits permits a useful localization concept in terms of certain lattices of real subspaces of the complex Hilbert -space. This "modular localization" is not only useful in order to construct interaction-free nets of local algebras without using non-unique "free field coordinates", but also permits the study of properties of localization and braid-group statistics in low-dimensional QFT. It also sheds some light on the string-like localization properties of the 1939 Wigner's "continuous spin" representations. We formulate a constructive nonperturbative program to introduce interactions into such an approach based on the Tomita-Takesaki modular theory. The new aspect is the deep relation of the latter with the scattering operator.

1 Introduction.

The main aim of this paper is the exploration and extension of Wigner's representation-theoretic approach to relativistic quantum theory [1] for the construction of particles and free fields in the context of d=2+1 abelian braid-group statistics (the particles are often referred to as "anyons or "abelian plektons "). In this way one hopes to obtain a more direct understanding of the origin and the physical consequences of plektonic statistics than that by the somewhat vague (and often indirect and complicated) method via imposing Chern-Simons perturbation on standard fermionic matter, using the formalism of functional integrals (in a region where the necessary and sufficient conditions for Feynman-Kac representability of quantum physics are strictly speaking violated).

The main problem in the adaptation of Wigner's theory to this question is "localization". Algebraic QFT taught us that compact localization in d=2+1 dimensions yields permutation group statistics [23]. In order to obtain braid group statistics one must consider semiinfinite spacelike strings i.e. spacelike cone localization in the framework of algebraic QFT [29].

The first attempt to define localization in the framework of relativistic quantum theory is due to R. Newton and E. P. Wigner [2]. We explain in the next section why this localization is not useful if one wants to use Wigner's representation theory as a tool of QFT. The more appropriate concept consists in defining as the subspace of localized state vectors belonging to the localization region $\mathcal{O}$ the dense set of vectors obtaine by applying the smeared fields with support of test functions in $\mathcal{O}$ to the vacuum Ω. This localization concept, if translated back into the Wigner representation theory, leads to real subspaces [3] of the complex Wigner representation space. Bisognano and Wichmann [10] have shown, that in the Wightman framework of QFT for the special wedge localization region, this concept relates to the rather universal and deep mathematical Tomita-Takesaki modular theory for von Neumann algebras [4] which has been known to connect to such diverse looking physical issues as the existence of antiparticles (TCP), the stability of temperature states, the Unruh-Hawking effect [5] and many other structures of "Local Quantum Physics" [7]. The underlying "modular" wedge-localization, unlike the standard tools (as e.g. the Gell-Man-Low perturbation theory in terms of time-ordered products or the functional integral approach), has no counterpart in classical field theory or in nonrelativistic quantum theory. *Among all concepts in QFT it is the most intrinsic one,* and it achieves something which in e.g. in coordinate-free differential geometry was accomplished already a long time ago, namely the separation of intrinsic properties from mainly accidental "coordinatizations". In particular in the context of free theories it helps to recover the unicity of the free QFT for a given mass and spin (or helicity) in the face of myriads of different free field covariant realizations of the Wigner theory.

In section 2 entitled "ancient history" we review those aspects of Wigner's theory which are relevant for our purpose. This theory was the first successful attempt to formulate a framework of relativistic quantum theory *not based on the quantization* parallelism to classical theories. Mainly through algebraic QFT, its spirit of only using intrinsically defined concepts has been kept alive in present day field theory. To most physicists of the younger generation who are familiar with the perturbative aspects of the Lagrangian quantization approach, the Wigner theory remained largely unknown because modern texts often equate QFT with the Lagrangian approach and path integrals.

Besides the (perhaps too brief on this issue) reference [7], the reader can find more extensive material in S.Weinberg's recent book [9]. Although we will often refer to Weinberg, our ideas of how to use Wigner's theory as a quantum field theoretical tool is very different. In Weinberg's book the Wigner approach is used in order to *support* the Lagrangian formulation of QFT and the path-integral approach (although the possibility of other non-Lagragian approaches to interactions is not ruled out, as remarked by the author). Here our aim is quite different, namely to understand areas which are badly described or *not covered at all* (and probably never will be) by the Lagrangian framework.

An immediate gain of our conceptual framework of modular localization is that all free "field coordinates" are equally suited in order to describe the same physical reality; the fact that most of these "coordinates " are not "Euler-Lagrangian" does not make them any less physical. The perturbative causal framework of Bogoliubov and Shirkov can be formulated in any field coordinates and is not restricted to those fields which which fulfil Euler-Lagrange equation of motion.

It is worthwhile to mention that all the pertinent results on chiral conformal QFT as well as a large part of results on massive d=1+1 theories have been obtained by nonperturbative non-Lagrangian methods such as representation theory, S-matrix bootstrap, formfactor program etc. A Lagrangian name, where it appears, usually only served for "baptizing" these low dimensional models [1] in the traditional way, but plays no role in its construction.

In fact the fields which e.g. appeared in the 1974 work on conformal field theory [8] were so remote from Lagrangian-and Euclidean- (and even from Wightman-) fields, that the problem of a systematic model construction was not pursued as a result of Zeitgeist prejudices which at that time pointed into the direction of euclidean theory. Nowadays it is very natural to consider charged fields which have nontrivial source and range projectors onto superselection sectors (and hence violate the so-called Reeh-Schlieder properties of standard QFT), but at that time this appeared as going against the holy grail of QFT [23].

In fact one would be surprised if plektonic d=2+1 fields or those corresponding to Wigner's d=3+1 "continuous spin" representations are not of this new *non-Lagrangian* kind. An educated guess is that all fields in $d \geq 2+1$ which create only states with weaker than compact localization properties are non-Lagrangian. This is in agreement with an old result of Yngvason about the obstructions posed by the d=3+1, m=0 Wigner "continuous spin" representation within a Wightman framework [14].

In section 3 entitled "recent history ", we briefly present those aspects of the Tomita-Takesaki modular theory which are relevant in the present context of "modular localization. In particular we explain, how by introducing *real* subspaces of the Wigner representation space via the Tomita-like involution s, one may implement a localization concept which is more useful for our purpose than the Newton-Wigner localization. These ideas, although known to some experts, unfortunately had never been published in an accessible way [3]. Section 3 also contains a brief sketch of a direct construction of local nets from the so called "wedge localization" [10] which, in the case of free bosonic theories, will be treated in more details and presented within a more general mathematical setting in forthcoming work of Brunetti, Guido and Longo [11].

[1] In the standard approach the use of different field coordinates has sometimes caused an inflatory use of names, viz. the Wess-Zumino-Witten-Novikov model which is just the generalized Thirring model in the multicomponent coupling constant space at the critical point, rewritten in different field group-valued field coordinates. These are evidently more adapted to the differential geometric mathematical exploitation than the Thirring parametrization, whereas the latter formulation is much more useful in condensed matter physics.

The fourth section explains why the adaptation of Wigner's theory for d=2+1 anyonic spin is not compactly localizable, but still falls into the weaker spacelike cone-(or semi-infinite string-) localizable category and presents the corresponding "anyonic" statistics in terms of a "twist " which is necessary in order to balance the opposite quantum localization of the wedge-localized real subspace (i.e. its symplectic complement) with the geometric (causal) opposite in the sense of Lorentz-transformations of fields.

In section 5, we comment on a an interesting topological obstruction against Haag duality for non-simply connected regions which occurs in certain zero mass theories including the free Maxwell theory. These topological obstructions are absent in massive theories, but they are typical of local gauge theories i.e. those Lagrangian theories in which the free field content is associated to the (m=0, h$\geq$ 1) Wigner representations. Although algebraic QFT is not able (for very good reasons in my opinion) to incorporate classical gauge ideas which have their natural formulation in fibre bundle theory, through such duality obstructions for corona regions as discussed in section 5, the algebraic approach at least perceives that there is a deep problem on the level of local quantum physics.

In section 6 we show that Wigner's zero mass "continuous spin" representation falls into this weaker space-like cone localization category. In fact the natural covariant description [14] is in terms of semi-infinite light-like strings, very similar to the covariantization attempt of the d=2+1 "Wigner anyons" mentioned in section 4.

The last section contains some speculative attempts of incorporating an intrinsic notion of interaction via an ansatz for an "interacting" Tomita involution J within the Fock-space setting defined by scattering theory. This Ansatz generalizes the J_0-reflection obtained from the Wigner theory. It does not involve the interaction picture and time-ordering, but uses only nonperturbative concepts of general massive quantum field theory. In this last section I also speculate on several presently insufficiently understood problems which have connections with modular ideas.

2 Ancient History.

In 1939 Wigner [1] classified the irreducible ray-representations of the Poincaré-group (or what amounts to the same, the irreducible vector-representations of its covering group). His main motivation was to understand in intrinsic physical terms the ever increasing "zoo" of linear relativistic (higher spin) field equations of those days, which were proposed in the aftermath of the Dirac equation. For this purpose he had to extend the Frobenius method of induced representations from finite groups to the non-compact Poincaré-group, a mathematical novelty which gave rise to mathematical developments in group representations, [15]. He first determined all transitive momentum-space orbits under the Lorentz-group, and then classified the fixpoint-group of a conveniently chosen reference vector p_R on the orbit. The "induction" was done with the help of this "little group" and a suitably defined family of "boosts" served to identify the fixpoint-groups at different orbit points.

For the positive energy orbits $p^2 = m^2,\ p_0 > 0$ and $p^2 = 0,\ p_0 > 0$ (the only orbits of relevance for our purpose) in d=3+1, the (coverings of the) little groups are $SU(2)$ resp. $\widetilde{E}^{(2)}(2)$ (the two-fold covering of the two-dimensional Euclidean group).

The massive $[m,s]$-representations are most conveniently described in terms of $2s+1$ component wave-function spaces:

$$H = \{\underline{\psi}(p) | \sum_{s_3} \int |\psi_{s_3}(p)|^2 \frac{d^3p}{2\omega} < \infty\} \tag{1}$$

on which the Lorentz transformation acts as:

$$(U(\Lambda)\underline{\psi})(p) = D^{(s)}(R_W(\Lambda,p))\underline{\psi}(\Lambda^{-1}p) \tag{2}$$

with R_W being the Λ- and p-dependent (formally nonlocal) Wigner rotation.

In the $m^2 = 0$ case one has a greater wealth for the representation theory of the little group. In case the "translations" of $\widetilde{E}(2)$ are mapped to zero, one obtains the family of nonfaithful one-component semi-integers-helicity representations:

$$(U(\Lambda)\psi)(p) = e^{is\Phi_W(\Lambda,p)}\psi(\Lambda^{-1}p) \tag{3}$$

The conversion of the one-component Wigner wave functions ψ into e.g. the standard local helicity description in terms of field strength $F_{\mu\nu}$ is well-known.

Explicit formulas for the Wigner phase Φ_W as well as the previous Wigner rotation R_W are to be found already in the original paper as well as in the books mentioned in the introduction. Also the extensions to the full group, including space and time reflections, may be found in the literature. In the following we will need the formula for the $TCP = \theta$ transformations acting on the (doubled, if particles are not self-conjugate) $[m, s]$-representation as:

$$\theta \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} = D^{(s)}(i\sigma_2) \begin{pmatrix} \psi_-^* \\ \psi_+^* \end{pmatrix}, \quad \pm\text{=(anti)particle doubling} \tag{4}$$

Before we relate this TCP-transformation of the Wigner theory to a new localization concept, some more historical remarks are in order.

Wigner's work, although little noticed at the time (at least by the community of inventers of new relativistic field equations), showed in one stroke that the problem of inventing more general looking field equations was of a somewhat academic nature; what really mattered for the particle content was their irreducible p-space representation structure, and not their covariant appearance in x-space.

Wigner was apparently aware that Poincaré-invariance was not the only physical requirement for relativistic particles, but there were also the important issues of causality and localization. In 1949 he wrote a paper together with R. Newton [2] in which they proposed, what became later known as the Newton-Wigner localization. This localization was not covariant and effectively violated Einstein causality at distances shorter than a Compton wavelength, but it seemed to be the best one could do if one adapts the wave-packet localization of the Schroedinger theory to the relativistic domain.

As a result of these unsatisfactory aspects of this localization, Wigner became increasingly suspicious about the internal consistency of QFT[2]. However a short time later Wightman and collaborators showed that there was no contradiction between the Heisenberg-Pauli canonical quantization approach and the Wigner theory [16]. Nevertheless Wigner's point was well taken, because *localization is the central property in relativistic QFT*[3] for the interpretation of relativistic quantum physics. However the formulation he chose was not the correct one for the transition to QFT. One correct way to come from positive energy representations to pointlike free fields was found some years later [16] [9] [17]. Let us first give a very brief description for the massive positive energy representations.

[2]This is a private remark obtained through R. Haag.

[3] *This is also the case in condensed matter physics.*

With one $[m,s]$-representation one connects a whole family of free fields which all share the same canonical momentum space creation and annihilation operators affiliated (transforming) with the $[m,s]$ Wigner representation:

$$\Psi(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx} \sum_{s_3=-s}^{s} u(p,s_3)a(p,s_3) + e^{ipx} \sum_{s_3=-s}^{s} v(p,s_3)b^*(p,s_3) \right) \frac{d^3p}{2\omega} \tag{5}$$

Here u and v are explicitly known column-vectors in a space of $\geq 2s+1$ dimensions. They represent intertwiners between the Wigner representation and the covariant description of its content:

$$\sum_{s_3'} u(p,s_3')D^{(s)}_{s_3,s_3'}(R_W(\Lambda,p)) = D^{[n,m]}_{covar}(\Lambda)u(\Lambda^{-1}p,s_3) \tag{6}$$

The intertwiner property becomes even more manifest if one combines the 2s+1 u's to a $(n+m)\times(2l+1)$ matrix $U(p)$:

$$U(p)D^{(s)}(R_W(\Lambda,p)) = D^{[n,m]}_{covar}(\Lambda)U(\Lambda^{-1}p) \tag{7}$$

Here the matrix D_{covar} describes a finite-dimensional tensorial (and therefore non-unitary) representations of the Lorentz-group. The indexing of the entries is given by a pair (n,m) of n un-dotted and m dotted symmetrised spinorial indices $(\alpha_1\alpha_2 \ldots.\alpha_n, \beta_1\beta_2\ldots..\beta_m)$. The only restrictions are that $\frac{n+m}{2}$ be semi-integer if s is semi-integer as well as the validity of the inequality $\left|\frac{n}{2}-\frac{m}{2}\right| \leq s \leq \frac{n}{2}+\frac{m}{2}$. The p-dependence of the intertwining relation insures that it takes place inside the Poicaré group and that the nonunitarity of the D_{covar} matrices does not endanger the unitarity of the representation of the Lorentz subgroup inside the Wigner representation. A systematic determination of this infinitely large family of intertwiners for fixed $[m,s]$ is not contained in the original work, but was carried out later by Joos [17] and Weinberg. In Weinberg's recent book [9] the reader finds an exhaustive treatment of this family of local fields which all share the same momentum space creation and annihilation operators.

There one also finds a careful discussion of some peculiarities of the $[0,h]$ photon-neutrino class. In that case the covariantization of these nonfaithful Wigner representation is much more restrictive than for massive theories. Whereas for the latter case one has the above inequality, the helicity for the former obeys the equality $h = \left|\frac{n}{2} - \frac{m}{2}\right|$. For the much more elusive infinite component "continuous spin" faithful zero mass representation, the covariantization was carried out in the 70's [14] where also the lack of the standard localization property was noticed, but the modular concepts for a sharp localization investigation were not yet available.

In all cases whether massive or zero mass, the local covariant fields live in the same Fock space i.e. they share the same momentum space creation and annihilation operators and in addition are local relative to each other in the sense of space-like (anti)commutation relations. Using a very appropriate concept of Borchers [18], one obtains a more concise description of this notion of relative locality. Namely it turns out that these fields are members of an equivalence class of relatively local fields. More specifically, they form a linear subset of the free field $[m,s]$ "Borchers class", an object which has been explicitly computed in the 60's by H.Epstein and the present author [19] and consists precisely of the set of all Wick-polynomials of the free field and its derivatives.

Borchers showed in complete generality that fields, which are local with respect to a given local field, with the latter acting cyclically on a Hilbert space, are automatically local (with respect to themselves). He then proved that this entails the following consequences:

- (i) The cyclically acting members generate the same local von Neumann-algebras, i.e. if $A(x)$ and $B(x)$ are two such fields and $\mathcal{A}(A,\mathcal{O})$ denotes the local von Neumann-algebra generated by the field $A(x)$ smeared with test-functions having support in $\mathcal{O}$ (a natural family of regions $\mathcal{O}$ are the so called double cones on which Poincare-transformations act stably) one has:

$$\mathcal{A}(A,\mathcal{O}) = \mathcal{A}(B;\mathcal{O}) \tag{8}$$

- (ii) The different members of the Borchers class do not only lead to the same local observables, but also entail the same S-matrix i.e. the S-matrix is a class invariant.

This suggests a viewpoint of QFT ("algebraic QFT") which is quite different from the standard one in most of the textbooks, although it is based on the same physical principles. By analogy with differential geometry, the pointlike covariant fields are like co-ordinates and the algebraic net, i.e.the assignment: $\mathcal{O} \to A(\mathcal{O})$ contains all the intrinsic physical information [7]. The terminology "field co-ordinates", which is used freely in the present work, is precisely meant in this sense. The Borchers theory also gave the prominent role as a net invariant to the S-matrix. In the last section we will use the S-matrix as an invariant of the wedge-based modular theory.

Progress obtained from this net point of view has been slow, but steady and very solid indeed. The accumulated body of results is quite impressive by now. Its mathematical pillars are the Tomita-Takesaki modular theory [4] and the V.Jones subfactor theory [20], both dealing with structural properties of von Neumann-algebras. It is not an accident that both mathematical theories had their physical (mathematically less general) predecessors: the Haag-Hugenholtz-Winnink [7] description of KMS-states in the first case, and the Doplicher-Haag-Roberts superselection theory [7] [6] in the second, a fact which was of course not known at the time of the mathematical discoveries.

Here we want to show that the seeds for this intrinsic mode of physical thinking are already contained in the Wigner theory. More concretely, the Wigner theory preempts some special aspects of both mathematical theories: localization properties are related to modular properties (explained in detail in the sequel) and duality obstructions related to properties of inclusions (briefly mentioned in section 5). It is interesting to note that this progress occurred precisely about the localization structure, which Wigner, as a result of his work with R. Newton, considered as a questionable aspect of QFT.

Needless to add the remark that the $[m,s]$ fields are in general not Lagrangian fields i.e. the above local free fields are in generally not solutions of an Euler-Lagrange equation. To give an example, for $s=\frac{3}{2}$ the 8-component Rarita-Schwinger field is "Euler-Lagrangian", but the e.g. minimal 4-component field in the same Borchers-class is not. Euler-Lagrange structures are not necessary for formulating *quantum* field theory and the fact that each free field Borchers class contains one Euler-Lagrange representative does not seem to bring about any additional physical insight on the free QFT. With the modular structures we will even move further away from classical properties and quantization prescriptions and arrive at a formulation which avoids field coordinates alltogether.

In Weinbergs book, despite its heavy use of Lagrangians, one finds a formal argument which indicate that invariant (Wick-ordered) polynomial coupling terms lead to perturbations of the S-matrix which are independent of the $[m, s]$ field-coordinates which one uses for the specification of the interaction density. With other words, a given polynomial interaction may be rewritten in terms of any kind of field-coordinates one likes (and it stays polynomial in terms of the new fields). This observation suggests that even in a perturbative approach one should try to avoid these "field-co-ordinates" altogether and aim for a description which *restores Wigner's representation uniqueness* on the level of the associated operator algebras. In this way one could avoid the confusing multitude of field coordinates and hopefully eventually also arriving at a more intrinsic understanding of interactions. The next section takes a step towards this goal.

3 Recent History

For the sake of simplicity let us assume that we are using the Wigner formalism in order to describe a self-conjugate particle situation. Then, apart from a possible sign factor, the previous action of $\theta = TCP$ on the momentum space wave-function simplifies as follows:

$$(\theta\psi)(p) = D^{(s)}(i\sigma_2)\psi^*(p) \tag{9}$$

where $D^{(s)}(i\sigma_2)$ represents the conjugation matrix in the $[m, s]$ Wigner space. Introduce now another conjugation j which differs from θ by a rotation with π around the x-axis :

$$j = R(e_x, \pi) \cdot \theta \tag{10}$$

An elementary calculation shows that this j commutes with the L-boost $\Lambda_{W_{st}}(\chi)$ assciated to the standard wedge (which we have chosen in the 0-1 -plane):

$$U(\Lambda_{W_{st}}(\chi))j = jU(\Lambda_{W_{st}}(\chi)) \tag{11}$$

Whereas the boosts define a unitary subgroup, the continuation to imaginary χ yields an unbounded closable operator:

$$\delta := U(\Lambda(\chi = 2\pi i)) \tag{12}$$

In particular the unbounded operator:

$$s = j\delta^{\frac{1}{2}} \tag{13}$$

turns out to be a closed densely defined antilinear involution. This is as a result of the commutation relation :

$$j\delta^{\frac{1}{2}} = \delta^{-\frac{1}{2}}j \tag{14}$$

which follows from the one before.

In fact the above definition of s agrees with its unique polar decomposition. We now use this involution s in order to define a closed real subspace:

$$H^R_{W_{st}} = \{h \in H \mid sh = h\} \tag{15}$$

The properties of the positive operator δ entail the density of $H_{W_{st}} \equiv H^R_{W_{st}} + iH^R_{W_{st}}$ in the Wigner representation space. This decomposition also allows to introduce a real inner product on $H_{W_{st}}$:

$$(\psi_1 + i\chi_1, \psi_2 + i\chi_2)^R = (j\psi_1, \psi_2) + (j\chi_1, \chi_2) \tag{16}$$

where the application of j transforms a wave-functions from $H^R_{W_{st}}$ into its "symplectic complement" in $H^{R\prime}_{W_{st}} = jH^R_{W_{st}}$ which belongs to the (-1) eigenspace of the adjoint s^*. Both summands on the right hand side are real since:

$$(\psi', \psi) = -(\psi', s\psi) = -\overline{(s^*\psi', \psi)} = -\left(\psi, s^*\psi'\right) = (\psi, \psi') \tag{17}$$

The real structure is the same as the one obtained by using the real part of the complex inner product and then restricting to the subspace $H^R_{W_{st}}$. It is now easy to see that the restriction of the Wigner scalar product to the dense subspace $H_{W_{st}}$ of "modular wedge localization" gives a "temperature scalar product" in terms of the graph Hilbert space of the unbounded operator s:

$$(f, g)_G \equiv (f, g) + \overline{(sf, sg)} \tag{18}$$

In this topology the dense set $H_{W_{st}}$ of modular localized functions becomes a Hilbert space in its own right. Using the above properties of s, δ and j on $H^R_{W_{st}}$ one finds the following "temperature" representation on H_{Wst}:

$$(f, g) = \left(f, \frac{1}{1-\delta^{-1}}g\right)_G, \quad \delta = e^{2\pi K} \tag{19}$$

Here K is the generator of the Lorentz-boost of the standard wedge. This should be compared with the heat bath temperature state of a free field:

$$\begin{aligned} \left\langle \psi(x)\bar{\psi}(y)\right\rangle_{e,\beta,\mu} &= \frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{-ip(x-y)}\sum_{s_3} u(p,s_3)\bar{u}(p,s_3)\frac{1}{1\mp ze^{-\beta p\cdot e}}\frac{d^3p}{2\omega} \\ &+\frac{1}{(2\pi)^{\frac{3}{2}}}\int e^{ip(x-y)}\sum_{s_3} v(p,s_3)\bar{v}(p,s_3)\frac{ze^{-\beta p\cdot e}}{1\mp ze^{-\beta p\cdot e}}\frac{d^3p}{2\omega} \end{aligned} \tag{20}$$

Specialyzing for simplicity to a scalar neutral field A and going into the rest frame of the heat bath, this formula reads after smearing with test functions ($\beta = \frac{1}{kT}$):

$$\langle A^*(f)A(g)\rangle_\beta = \left(f, \frac{1}{e^{\beta H}-1}g\right)_+ + \left(f, \frac{1}{e^{\beta H}-1}g\right)_- \tag{21}$$

where the$\pm$ refers to the forward/backward mass shell (or creation/annihilation part). Here the -inner product refers to a copy (temperature doubling) of the Wigner space with inverted energy spectrum of $\mathbf{H}$. This comparison tells us that the modular localization space which describes the free QFT restricted to the (Rindler) wedge is identical with the Hawking-Unruh free field temperature situation. The boost generator K is nothing but the hamiltonian of an uniformly accelerated observer with acceleration 1 in natural units ($c = 1 = \hbar$) and Hawking temperature $T_H = 2\pi$. The basic difference between a heat bath temperature and a modular localization temperature (both result from loss of information) is that in the heat bath situation the j-reflection is simply complex conjugation without any geometric action. On the level of von Neumann algebras, i.e. after application of the functor (explained below), the heat bath algebra is type I whereas the modolar localization (Rindler) algebra is type III_1.

Conversely one can obtain $H_{W_{st}}$ by introducing a complex structure on $H^R_{W_{st}}$. By applying Poincaré transformations to those spaces, one obtains a whole family of real subspaces which are eigenspaces of densely defined involutions s_W corresponding to the family of wedges obtained from the standard t-x wedge by Poincare-transformations. One then finds the following surprising theorem :

Theorem ([11]): *The family of real wedge subspaces form a covariant net of wedge-localized subspaces.*

This means in particular that one has isotony i.e.$H^R_{W'} \subset H^R_W$ for $W' \subset W$ and it is interesting to note that *this geometric inclusion property is equivalent to the positivity of the energy* [11]. We will not present the proof, but refer to a forthcoming paper.

As already mentioned, it is well known that in case of integer Wigner spin there exists the so called Weyl functor [3] which converts these localized real subspaces into local von Neumann algebras. The generators of these algebras in physicists notation are:

$$W(h) = e^{i(\Psi(h)+\Psi(h)^*)} \ , \quad h \in H^R_W \tag{22}$$

In other words the algebras for the wedge regions can be directly defined in terms of the Wigner theory without reference to "local field coordinates". Algebras of e.g.double cones may be formed through intersections of wedges. Their associated real subspaces turn out to be also the intersection of the real subspaces belonging to the wedges and lead to complexifications which are again dense in the Wigner space. This functorial relation of real subspaces to von Neumann algebras holds for all positive energy Wigner representatios in d=3+1 apart from the "continuous spin" representation. This can be seen e.g. by using the standard pointlike field coordinate formalism based on the properties of the u, v intertwiners of the previous section. A elegant way, more in the spirit of this work consists in avoiding intertwiners and fields alltogether and to use only properties of the Wigner representation. In that case one must understand the analytic characterization of the space $H^R_{W_{st}}$ more explicitely. If the spin is zero, the requirement that the wave function is in the domain of the analytically continued boost $\delta^{\frac{1}{2}}$ (12) is equivalent to Wigner wave function $\psi(p)$ being analytic in the W_{st} related rapidity variable θ in the strip $0 < Im\theta < \pi$:

$$p_0 = (m^2 + p_\perp^2)^{\frac{1}{2}} cosh\theta, \quad p_1 = (m^2 + p_\perp^2)^{\frac{1}{2}} sinh\theta, \quad p_\perp \tag{23}$$

The eigenvalue relation for the unbounded antilinear operator s (15) is the "s-reality condition": the value on the $\theta = \pi$ boundary is the complex conjugate of the value at $\theta = 0$. For s$\neq 0$ the analytic function is obtained by requiring strip analyticity of the vector valued function $D^{(s)}(R_{W_{st}}(\Lambda(\chi), p_r)\psi(\theta+\chi, p_\perp)$ for χ in the imaginary interval $0 < \chi < i\pi$. The s-reality condition is:

$$D^{(s)}(i\sigma_2) \lim_{\chi \to i\pi} D^{(s)}(R_{W_{st}}(\Lambda(\chi))\psi(\theta + \chi, p_\perp) = \overline{\psi(\theta, p_\perp)} \tag{24}$$

Writing the explicitely known matrix contribution (resulting from the analytic continuation in the wedge affiliated rapidity θ of the Wigner rotation) to the right hand side, one obtains a representation of the wave function in the strip in terms of an analytic function. It is easy to see that this analytic characterization of the Hilbert space of modular localized functions is equivalent to the previous graph norm Hilbert space (the Hawking-Unruh inner product) (18). The opposite wedge leads to an opposite analyic strip with a common physical boundary. In $d \geq 2+1$, the existence of a net of wedges gives rise to analytic coherence problems of the "edge of the wedge type". In QFT modular localization turns out to be related to the very elusive momentum space on shell analyticity properties including crossing symmetry.

Physicists familiar with another wedge "miracle" from the quantum physics in curved space-time (for a recent review see [21]) namely the Unruh-Hawking effect for the Rindler wedge i.e. the quantum physics of a uniformly accelerated observer, should take notice that this effect and the above theorem are two sides of the same coin. We facilitated this step by rewriting the restriction of the Wigner theory to the wedge localized sbspace in such a way that the temperature aspect of the this restriction became already manifest before converting the Wigner theory into the full free QFT by applying the CCR or CAR functor (done below).

Behind both miracles lies a very basic and universal theory [5] which, as already mentioned in the introduction, mathematicians refer to as the Tomita-Takesaki modular theory. Physically one expects among other things, that this theory explains basic features of QFT as e.g. crossing symmetry of the S-matrix and formfactors to be derivable from the KMS-temperature properties of the Hawking-Unruh situation. We will here only limit ourselves to some salient features.

This modular theory deals with von Neumann-algebras in "standard position" e. g. weakly closed operator algebras in a Hilbert space possessing a cyclic and separating vector Ω. In local quantum physics the vacuum is a vector which has this property with respect to all those local subalgebras which have a nontrivial causal complement to their localization region [7]. In order to construct the basic objects of this theory, one starts from the *-structure of the von Neumann-algebra A and defines an unbounded but closable involutive operator S :

$$S A\Omega = A^{+}\Omega\,, \quad A \in \mathcal{A} \tag{25}$$

Its polar decomposition:

$$S = J\Delta^{\frac{1}{2}} \tag{26}$$

defines the Tomita conjugation J and the modular operator Δ . The latter gives rise to the modular automorphism σ_t. The *nontrivial part* of the T.-T. theorem is about the behaviour of these operators with respect to the von Neumann-algebra A:

$$J(\mathcal{A}) := J\mathcal{A}J = \mathcal{A}', \quad \mathcal{A}' : commutant\ of\ \mathcal{A}. \tag{27}$$

$$\sigma_t(\mathcal{A}) = \Delta^{it}\mathcal{A}\Delta^{-it} = \mathcal{A}, \quad \sigma_t : \ modular\ automorphism\ of\ \mathcal{A} \tag{28}$$

To a physicist, the K in $\Delta^{it} = e^{itK}$ appears "like a" generalized Hamiltonian and J "like a" generalized TCP-operator of the pair $\mathcal{A},\Omega$. The only physical miracle, as far as the application of this theory to the local algebras of QFT is concerned, is that these modular quantities for the pair A(wedge) , Ω (vacuum) become geometric:

$$\Delta^{it} = U(\Lambda_{W_{st}}(\chi = 2\pi t) \tag{29}$$

$$J = \begin{cases} R(e_x,\pi)\Theta, \ \text{s= integer} \\ \mathcal{K}R(e_x,\pi)\Theta, \ \text{for} \ \ \text{s= semi-integer} \end{cases} \tag{30}$$

Here K is the well-known Klein-twist (not to be confused with the closely related Jordan-Wigner transformation) for fermions: $\mathcal{K} = \frac{1+iV}{1+i}$ with $V = \exp i\pi N_{fermi}$. In the integer spin case the Wigner theory preempts this modular structure through the existence of the previously introduced family of real subspaces H_W^R which are converted into algebras of a bosonic net via the Weyl functor. In the semi-integer spin case the CAR-functor plays an analogous role. In the Wigner theory the Fermi-statistics manifests itself through:

$$jH^R_{wedge} \neq H^R_{opposite\,wedge} \quad \text{for s=semi-integer} \tag{31}$$

The difference between the two sides is only a numerical factor i.

This mismatch is repaired on the level of the algebras by the above Klein-twist K:

$$\mathcal{K}\mathcal{F}(jH^R_{wedge}) = \mathcal{F}(H^R_{opp.wedge}) \tag{32}$$

Here F is the CAR functor. K restricted to the Wigner-space of fermions is just a numerical factor i, which is precisely the obstruction factor between j and the π-rotation. So the Klein factor just permits to express the Tomita J in terms of geometrical objects. The rest consists in applying the CCR resp. CAR functor which maps the net of Hilbert spaces into the net of von Neumann algebras. Both functors furnish a noncommutative structure on classical function spaces (in our case the Wigner spaces) and according to the above remark, the selection of the functor is already preempted by geometric obstruction (31) in the Wigner theory. For a detailed functorial presentation of these algebras we refere to [30] [31]

Note that all recent contributions of modular theory to the understanding and construction of Borchers classes (including the present one) could have been given two decades ago, ever after the prominent role of wedge algebras was discovered by Bisognano and Wichmann [10] and the relevance to the Hawking problem was recognized by Sewell [5]. But as it often happens, conceptual gains need a longer time for mental digestion than gains in formalism.

4 Fractional Wigner-Spin and Statistics of Anyons.

In d=2+1, the little group of a point on the forward mass shell is the abelian U(1) and therefore the Wigner theory allows (at least a priori) for *any* value of spin, i.e.one expects "anyons" (the more restrictive non-abelian plektons will only be mentioned at the end of this section). Using the methods of the previous section, and checking the prerequisites for the existence of a TCP operation on the direct sum of particle-antiparticle Wigner spaces, one again establishes the properties of a family of real subspaces which can be associated with localization and statistics properties of field theoretic two point functions. However the difference between the modular complement $jH^R_W = (H^R_W)'$ and the geometric complement $H^R_{W'} = Rot(\pi)H^R_W$ is bigger than in the previous fermionic theory i.e. the Klein transformation, which accounts for this difference, is more complicated.

In order to keep the Klein-twist simple, let us imagine that we are dealing with a Z_N-spin i.e. we assume that $s=\frac{1}{N}$. Then the Klein factor which corrects the mismatch between localization via commutativity and the geometric localization in analogy with conformal QFT [22] is expected to be a suitable "square root" of the action of the $2\pi-$rotation in space of scattering states

$$K=\sum_n e^{-i\pi sn^2}P_n \tag{33}$$

This is in agreement with the nonlinear composition of anyonic spin. These kinematical facts suggest that the scattering space cannot have the tensor product structure as for Bosons and Fermions. A physically relevant question is: what is the a priory best possible localization of the anyonic algebras? What can we learn from the modular localization in the Wigner representation space for $s\neq$semiinteger? It turns out that a compact localization as in the previous section is not possible, i.e. in H_W^R there are no nontrivial compactly localized wave functions. If such a wave function would exist, one could perform a 2π-rotation such that the support remains inside one wedge for all angles, however the nontrivial phase created by such a rotation contradicts its affiliation to the real subspace H_W^R.

From the general structure of algebraic QFT we expect that the spectral gap leads to a (arbitrarily thin) space-like cone localization. In d=2+1 only genuine braid group statistics is able to exhaust this possibility, whereas permutation group statistics resulting from semi-integer spin leads back to the compact localization. Since the core line of a semi-infinite spacelike cone is characterized by an initial point x and a spacelike unit direction e, we expect a string-like localized wave function depending on x and e. The following pedestrian approach to the anyonic localization problem within Wigner's representation theoretical setting highlights the difficulties of anyons with respect to the standard u,v covariantization procedure.

Starting from the Wigner wave function which transforms according to (ψ is one-component) :

$$(U(g)\psi)(p)=e^{is\Phi_W(g,p)}\psi(\Lambda^{-1}(g)p), \qquad g\in\widetilde{SO(2,1)} \tag{34}$$

with $\Phi_W(g,p)$ being the (nonlocal) Wigner phase. One looks for a factorization into covariant factors in analogy with the u-v intertwiners of the previous section. Following ideas of [12] this is achieved by [13] defining:

$$\psi_{covar}(p,g) = F(L^{-1}(p)g)\psi(p) \tag{35}$$

where any function F on $\widetilde{SO(2,1)}$ is acceptable as long as it fulfills the equivariance law:

$$F(\omega \cdot g) = e^{is\omega}F(g) \qquad \omega \in R \subset \widetilde{SO(2,1)} \quad g \in \widetilde{SO(2,1)} \tag{36}$$

The equivariant function F is a formal analogon to the u, v intertwiners but, as we will see, with much weaker localization properties. As a consequence of (36) we find the covariance law:

$$(U(g')\psi_{cov})(p,g) = \psi_{cov}(\Lambda^{-1}(g')p, gg') \tag{37}$$

With this covariant wave function we now affiliate an (improper) plane wave state vector which, as usual, is created from the vacuum:

$$|p,g\rangle = a^*(p,g)\Omega \tag{38}$$

It obeys the contragradient transformation law:

$$U(g)a^*(p,g')U(g)^{-1} = a^*(\Lambda(g), g'g^{-1}) \tag{39}$$

Transforming to x-space fields (the subscript cov will be omitted in the sequel):

$$\psi(x,g) = \int \frac{d^2p}{2\omega}(e^{ipx}a(p,g) + e^{-ipx}a^*(p,g)) \tag{40}$$

one obtains the desired transformation law:

$$U(g')\psi(x,g)U(g')^{-1} = \psi(\Lambda(g')x, g'g) \tag{41}$$

The two-point function is a quadratic expression in the function F:

$$(\Omega, \psi(x_1,g_1)\psi(x_2,g_2)\Omega) = \int \frac{d^2p}{2\omega}e^{ip(x_1-x_2)}\overline{F(L^{-1}(p)g_1)} \cdot F(L^{-1}(p)g_2) \tag{42}$$

Choosing x_2 and g_2"opposite" to $x_1$and g_2 i.e. such that:

$$x_2 = Rot(\pi)x_1 \qquad g_2 = Rot(\pi)g_1 \tag{43}$$

the covariant transformation law gives:

$$(\Omega, \psi(x_1, g_1)\psi(x_2, g_2)\Omega) = e^{2\pi i s}(\Omega, \psi(x_2, g_2)\psi(x_1, g_1)\Omega) \tag{44}$$

for the would-be field-theoretic correlation function which is in agreement with the expected anyonic statistics of the non-compact localization.

In order to obtain fields which are localized on semi-infinite strings, one has to chose a model for F. The choice :

$$F(g) = e^{isg(0)} \tag{45}$$

with $g = (\gamma, \omega)$ acting fractionally on the line $u \in R = \widetilde{S^1} \subset \widetilde{SO(2,1)}$ as:

$$(\gamma, \omega)(u) = \omega + \arg \frac{e^{iu} + \gamma}{1 + e^{iu}\overline{\gamma}} \tag{46}$$

The two-point function specializes to:

$$\langle \psi(x_1, u_1)\psi(x_2, u_2)\rangle = \int \frac{d^2p}{2\omega} e^{ip(x_1-x_2)} e^{-is(L(p)^{-1}(u_1) - L^{-1}(p)(u_2))} \tag{47}$$

and the difference between the left and right hand side in (44) replaces the bosonic commutator function for our anyonic case:

$$\Delta(\xi, u_1, u_2) = \tag{48}$$

$$= \int \frac{d^2p}{2\omega} \left\{ e^{ip\xi} e^{-is(L(p)^{-1}(u_1) - L(p)^{-1}(u_2))} - e^{-ip\xi} e^{-is(L(p)^{-1}(u_2) - L(p)^{-1}(u_1))} e^{4\pi i s(\left[\frac{u_1-u_2}{2\pi}\right] + \frac{1}{2})} \right\} \tag{49}$$

where ξ is the difference of the x's and the square bracket indicates the nearest larger integer.

Δ has the property of L-covariance:

$$\Delta(g\xi, gu_1, gu_2) = \Delta(\xi, u_1, u_2) \tag{50}$$

The vectors u_1 and u_2 on the unit circle correspond to a wedge W and its spacelike complement W'. The simultaneous stability group of u$_i$ leaves this wedges invariant. For each $\xi \in W \cup W'$ there exists a transformation

K$\in \widetilde{SO(2,1)}$ which is in the conjugacy class of the $\pi-$rotation which reflects ξ and flips the $u_i's$:

$$K\xi = -\xi \qquad Ku_1 = u_2 \qquad K^2 = 2\pi - rotation \tag{51}$$

Under the action of this "square root" of the 2π-rotation the Δ behaves as:

$$\Delta(K\xi, Ku_1, Ku_2) = \Delta(-\xi, u_2, u_1 + 2\pi) = e^{-2\pi is}\Delta(\xi, u_1, u_2) \tag{52}$$

Consistency between the two transformation formulas yields:

$$\Delta(\xi, u_1, u_2) = 0, \qquad for \;\; \xi \in W \cup W' \tag{53}$$

i.e. as long as $u_1$is different from u_2 any separation ξ of the string starting points x and y, such that a string crossing is avoided, will lead to a vanishing Δ function. A representation in terms of known functions is presumably easier than for the analog problem of the d=3+1 continuous spin representation (presented below).

This situation corresponds to light-like strings . This essentially means that our attempt to use a substitute for u, v intertwiners led to localization informations which do not go beyond the wedge localization (which we knew all along as a result of modular theory).The description does not reveal in a manifest way, if these anyons permit the sharper spacelike cone localization. Only the formation of *wave packets* in the *light-like string direction* e or better a covariantization based directly on space-like strings (using e.g. a de Sitter space representation of the $\widetilde{SO(2,1)}$) could reveal sharper localizations inside wedges. We presented these rather explicit calculations because the covariantization of the d=3+1, m=0 continuous spin representations in section 6 is completely analogous, albeit analytically more complicated [14]. After our pedestrian method revealed its weakness, let us return to the modular localization methods.

Our argument in favour of spacelike cone localization for anyons is based on the observation of non-triviality of intersections of wedge spaces H_W^R. Let $W_1$and W_2 be two wedges and form the dense set of states obtained by averaging with smooth functions of compactly localized Fourier transform:

$$\int dsdtf(t,s)\delta_{W_1}^{it}\delta_{W_2}^{is}\Phi\,, \qquad \Phi \in \mathcal{H} \tag{54}$$

This set of vectors is certainly in the domain of $\delta_{W_1}^{\frac{1}{2}}$. In order to see that also $\delta_{W_2}^{\frac{1}{2}}$can be applied, we need a commutation relation of $\delta_{W_2}^{\frac{1}{2}}$ with $\delta_{W_1}^{it}$. For orthogonal wedges such a commutation relation is well-known from SO(2,1) group theory:

$$\delta_{W_2}^{\frac{1}{2}}\delta_{W_1}^{it}\delta_{W_2}^{-\frac{1}{2}} = \delta_{W_1}^{-it} \tag{55}$$

and the case of W's in a more general position may be reduced to this orthogonal situation and in this way one proves the existence of a simultaneous dense domain for the $\delta's$ associated to different L-boosts[4] and this yields the density of $H_{sc}^R + iH_{sc}^R$, where H_{sc}^R is the real Hilbert space associated with spacelike cone sc modular localization.

At this point one may be tempted to think that our one-particle analysis, which relates to the field theoretic two-point function, may be generalized to the standard commutation relation between creation and annihilation operators as:

$$a(p,u)a^*(q,v) = e^{4\pi is([\frac{u-v}{2\theta}]+\frac{1}{2})}a^*(q,v)a(p,u)+2\omega\delta(p-q)e^{-is(L^{-1}(p)(u)-L^{-1}(p)(v))} \tag{56}$$

The free field equation in x-space turns out to be inconsistent with such an algebraic structure if the spin $s\neq$semiinteger [24]. The analytic techniques of the proof are identical to those of theorem 4-15 in [16].

In agreement with our previous observations, the correct multiparticle space from scattering theory has a different structure from a tensor product Fock space [32]. This was to be expected fom the nonlinear spin fusion as mentioned before. The anyonic momentum space creation and annihilation operators associated with incoming scattering states have source and range projections which have to match the superselection charges of the state vectors. Lacking a functor from the Wigner space to von Neumann algebras, one is forced to study the problem of modular localization of free anyons in the space of incoming scattering states with conserved number of incoming particles. In such a situation one expects a "kinematical" S-matrix which consists of piecewise constant phase factors (abelian R-matrices) and hence trivial cross sections [25]. In analogy to factorizing theories in d=1+1 one

[4] A more detailed treatment of common domain problems of modular operators will presumably be contained in [11]

expects a situation real particle (on shell) conservation and virtual particle (off shell) creation. In particular the two point function for localizable free anyonic fields are expected to have continuous contribution beyond the one particle Wigner contribution. We hope to come back to this interesting problem in the near future.

As a consequence of the appearance of the directional degrees of freedom for elementary strings (i.e. strings which cannot be represented in terms of line integrals of other fields as in Mandelstam type exponential line integrals) the Wigner anyonic states are more analogous to infinite component L-covariant wave functions. Their relation to "Chern-Simons anyons" is presently not clear. Note that the geometric Chern-Simons pictures about anyons suffers from a lack of concreteness: there are no operator formulas and it is even not clear how to arrive at them. Algebraic QFT with its intrinsic concepts is expected to be a more suitable place for their understanding than either via Chern-Simons or through quantum mechanics [26].

Collecting results, we saw that the modular localization of the Wigner theory for d=2+1 anyons is in agreement with the structural results about semiinfinite string-like in algebraic QFT. The mentioned inconsistency with the standard free field structure means that anyonic fields have not only semiinfinite string localization properties, but also show the phenomenon of "virtual particle creation" i.e. the two-point function has spectrum above the mass shell. This is a well known feature of d=1+1 factorizable models which show real particle (on shell) conservation and virtual particle (off shell) creation. The principles which determine the contributions beyond the mass shell and their use in a constructive approach are presently under intense investigations. In connection with the Wigner theory the crucial problem of anyons is this: do the modular localization spaces of intersections of d=2+1 wedge spaces really the geometric net property $H_{W_1} \cap H_{W_2} = H_{W_1 \cap W_2}$ for spacelike cones or is there a "defect": $H_{W_1 \cap W_2} \subset H_{W_1} \cap H_{W_2}$? If no such defect exists, then there seems to be no obstruction against the existence of a an anyonic functor from nets of space-like cone localized real subspaces H_{sc}^{R} to a net of von Neumann $\mathcal{A}(O_{sc})$. But this would contradict the expected presence of "virtual particle creation" in the two-point functions of (even "free") anyons. My hope is that the understanding of this inclusion may be helpful for a construction of a theory of free anyons. The systematic way appears to be quite cumbersome. It consists in characterizing the real subspaces $\mathcal{H}_W^R$ in:

$$\mathcal{H}_{scattering} = H^{(0)} \oplus H^{(1)} \oplus \sum_{i=2}^{\infty} H^{(i)} \qquad (57)$$

which obey $S_W \left|\psi\right\rangle = \left|\psi\right\rangle$. Here the multiparticle spaces are not simply tensor products of one particle spaces, but the formula for the inner products in terms of one particle states is known [32]. This approach uses modular theory directly in the known scattering space of anyons. It is similar to the modular approach for the factorizing models in d=1+1 [33] and we hope to take up this constructive idea for anyons in the near future.

Before closing, another brief comment about the d=1+1 situation is in order. In this case the localization properties of free fields depend on the "Lorentz-spin" i.e. on the value of s in the one-dimensional L-representation factor $\exp s\chi$ with χ being the rapidity. All $s > 0$ representations may be obtained in the $s{=}\frac{1}{2}$ Fock space of fermions by using appropriate intertwiners u and v. But only for (half)integer s does one obtain pointlike localized covariant fields. At generic values one does not get beyond wedge localization. The bad localization property does not improve in the zero mass limit. The pointlike localizable fields of chiral conformal field theory have a different origin which is further removed from the Wigner representation theory. They owe their existence to the peculiar structure of current operators which lead to Weyl algebras with a nontrivial center. It seems that also the structurally rich plektonic theories (nonabelian braid group statistics) can be traced back to this property [26]. Their two-point functions have a continuous mass distribution in their Kallen-Lehmann spectral function.

5 Haag Duality and E.M.Duality

In free field theories Haag duality for the vacuum net of wedge localized von Neumann algebras:

$$\mathcal{A}(W') = \mathcal{A}(W)'$$

is a direct consequence of the easily checked geometric nature of the symplectic complement:

$$H_{W'}^R = \left(H_W^R\right)'$$

It is easily generalized to all regions which can be obtained by intersecting $W's$ (star-shaped), in particular double cones.

Massive free fields obey Haag duality not only for double cones, but also for topologically more complicated (multiple connected) localizations e.g. toroidal regions. Algebras associated to massless fields for helicity $s \geq 1$ however cause a topological obstruction resulting in a breakdown of toroidal Haag duality [3]. Let T be the causal completion of a spatial torus we refer to this Minkowski space region as a "corona" [26] . The size of the corona is chosen in such a way that the causal complement $\mathcal{T}$ of consists of a double cone $\mathcal{T}'$ of diameter r and a "double cone at infinity": $|\vec{x}| \geq R + |t|$ causally separated from the former by the T with width: $R - r \geq 0$ region in between. Then one obtains the following proper corona- inclusion:

$$
\begin{aligned}
H^R(\mathcal{T}) \subset & H^R(\mathcal{T}')' \\
\curvearrowright \mathcal{A}(\mathcal{T}) \subset & \mathcal{A}(\mathcal{T}')'
\end{aligned} \tag{58}
$$

where $H^R(\cdot)'$ denotes the previously defined symplectic complement and the A's denote the corresponding von Neumann-algebras as obtained from the $H^R(\cdot)$ by the Weyl construction.

This "classical" obstruction, formally related to the appearance of δ' in the E-H canonical commutation relation, can be physically understood in terms of a (suitably regularized) magnetic flux through a surface which stretches from a circle inside the torus into the space-like separated region inside. Such a flux does not change if one passes through another surface subtended from the same circle. Hence such a flux, also not being localizable within the 4-dim toroidal region nevertheless belongs to the symplectic complement of the spacelike complement of the corona consisting of two spacelike seperated pieces. This entails the above violation of Haag duality for the corresponding algebras. A more systematic approach in the spirit of Wigner consists in rewriting the inner product in terms of tensorial object . This time, unlike the massive case, *there are no covariant intertwiners which lead to a local nondegenerate inner product.* In particular there is no way to rewrite the Wigner inner product in terms of a local bilinear expression in the field strengths.The best one can do is to introduce a partially covariant inner product in terms of a (non boost covariant) vector potential:

$$
\int |\psi(k,\pm)|^2 \frac{d^3k}{2\omega} = \int \bar{A}_\mu(k,n,\pm) A^\mu(k,n,\pm) \frac{d^3k}{2\omega}, \quad \omega = \left|\vec{k}\right| \tag{59}
$$

$$A_\mu(k,n,\pm) \;=\; \frac{n^\nu F_{\nu\mu}(k,\pm)}{n\cdot k - i\varepsilon}, \quad F_{\nu\mu}(k,\pm) = Ak_\nu\varepsilon_\mu(k,\pm)\psi(k,\pm)$$

where A denotes the antisymmetrization in μ,ν and $\varepsilon_\mu(k,\pm)$ the polarization vectors. The singularity in k-space corresponds to the semiinfinite line integral along n in x-space.

$$\begin{aligned} A_\mu(x,n) &= \int_0^\infty n^\nu F_{\upsilon\mu}(x+ns)ds \\ &= \int (e^{-ikx}\sum_{i=\pm} A_\mu(k,n,i) + h.c.)\frac{d^3k}{2\omega} \end{aligned} \tag{60}$$

This vector potential has the following obvious properties:

$$\begin{aligned} \partial_\mu A_\nu - \partial_\nu A_\mu &= F_{\mu\nu} \\ (U(\Lambda)A)_\mu(x,n) &= \Lambda_\mu^\nu A_\nu(\Lambda^{-1}x,n') \\ &= \Lambda_\mu^\nu A_\nu(\Lambda^{-1}x,n) + \partial_\mu G(x) \\ G(x) &= \int e^{ikx}\frac{1}{(kn-i\varepsilon)(kn'-i\varepsilon)} n\cdot F(\Lambda^{-1}k)\cdot n'\frac{d^3k}{2\omega} \end{aligned} \tag{61}$$

i.e. the Lorentz transformation which acts on the Wigner wave function in terms of a nonlocal Wigner phase and covariantly on the $F_{\mu\nu}$ tensor, transforms the potential covariantly *except an additive "gauge term"*. The nonlocality of the vectorpotential is made manifest by this additive contribution , although in the scalar product formula above it drops out. This peculiar "gauge" behaviour is a consequence of the nonfaithful helicity representation of the noncompact "little group" $E(2)$. In particular *the quantum origin of the gauge concept has little to do with the notion of classical fibre bundles* as most of the books allege. This is one of the more interesting clashes between quantization and an intrinsic quantum based approach. The quantization method (from the viewpoint of the Wigner method) would trade the physical nonlocality of A_μ with (physically artificial) formal elegance by the introduction of an indefinite metric (Gupta-Bleuler). In this way the additive term would loose its significance related to Lorentz transformations and the zero mass noncompact little group $E(2)$ and becomes the gauge concept of the mathematicians and of classical Maxwell theory. This is the method in which covariant renormalized perturbation theory is carried out. One profits from the formal geometric elegance at the prize of a conceptionally questionable link to quantum physics [5].

The obstruction against equality in 58 contains a very interesting conceptional message. Whereas violations of Haag duality for simply connected regions are the hallmark of spontaneous symmetry breaking (in fact they may be used for a model independent definition of that concept in the setting of algebraic QFT.), the violation for not (simply) connected regions has two different physical origins. The most common one is the mechanism of "charge split" into causally disjoint regions. In this case the commutant is bigger than the geometric complement suggests because the charge split mechanism on a neutral observable algebra is not incorporable into a geometric picture. Of course one is always invited to enlarge the observable algebra to the field algebra for which there is harmony with the geometrical picture. The second mechanism is the one at hand: a "quantum" caused mismatch between the geometrical complement and the quantum theoretical opposite in the sense of local commutativity resp. of symplectic structure. The defect dimension of the two real Hilbert spaces in 58 is:

$$dim\left[H^R(\mathcal{T}')' : H^R(\mathcal{T})\right] = 1 \tag{62}$$

The obstruction is caused by the presence of just one object:

$$\oint_{C\subset\mathcal{T}} A_\mu^{reg}(x,n)dx^\mu \tag{63}$$

$$A_\mu^{reg}(x,n) = \int \rho(\vec{x}-\vec{y})A_\mu(\vec{y},x_0,n)d^3y$$

The integration is over a closed path C inside $\mathcal{T}$ and we regularized the vector potential with a smooth function of small support $supp\rho \in B_\varepsilon$ so that one maintains normalizability and remains inside $\mathcal{T}$. The line integral represents the class of expressions of this kind, any two such elements differ only by field strength localized in $\mathcal{T}$. The line integral is a L-invariant and may be expressed in terms of a magnetic flux through any surface S with the C boundary. It is precisely this floating surface stretching beyond C, which in the quantum setting of commutativity (or symplectic orthogonality) prevents the affiliation with $H^R(T)$ and makes it a member of the nongeometric

[5] In the terminology used in one may say that one gains the geometric "Diracian elegance" at the expense of the conceptual "Bohr-Heisenberg" elegance..

$H^R(T')'$. This is of course an intrinsic property of the theory which cannot be removed by the indefinite metric formalism.

In order to find a relation of this corona obstruction to superselection rules, let us describe the Wigner photon space in terms of a degenerate bilinear form in the vector potential. As a model we take all transversal vector-valued functions on the light cone with the L-covariant transformation property:

$$\begin{aligned} (A,A') &\equiv -\int A_\mu^*(k)A'^\mu(k)\frac{d^3k}{2\,|k|} \geq 0 \qquad (64) \\ k^\mu A_\mu(k) &= 0, \quad \left(U(\Lambda)A\right)_\mu(k) = \Lambda_\mu^\nu A_\nu(\Lambda^{-1}k) \end{aligned}$$

It is well known that this degenerate description passes to the Wigner description by forming equivalence classes with nullvectors:

$$H_{Wigner} = H_{deg}/H_{null} \qquad (65)$$

We may apply our modular localization construction which starts with the net of wedge localized spaces and tries to come to compactly localized nets by intersections and causal complements. We know from the necessity of the Gupta-Bleuler approach that in a positive semidefinite space it is not possible to descend to pointlike local covariant fields. So there is an interesting question why these modular nets have no local covariant field coordinates of the A_μ. This would amount to a local understanding of the obstruction caused by the noncompact zero mass little group of the Wigner theory.

In the presence of a nullspace, the CCR algebra develops a nontrivial center in nonvacuum representations. The L-group does not leave the zero modes individually invariant, hence these higher sectors are not invariant under boosts. This is very interesting in view of the fact that the charged sectors of QED are non L-covariant (invariant only under the subgroup of 4-translations and 3-rotations). The intuitive picture is that a physical electron is inexorably stuck to an infinite photon cloud which breaks L-covariance. In the standard language, this cloud is a necessary attribute of the local gauge invariance together with the global $U(1)$-transformation law. In the algebraic QFT understanding of QED the Gauss law has several implications of equal importance: weaker localization properties (than in massive theories), infraparticle properties and lack of Lorentz covariance. Of course there can be no charges in the free theory described by (64), but there are infrared nontrivial representations of massless fields. If the presence of such non L-covariant representations is the explanation for the corona obstruction is not clear but an appealing possibility. In that case all violations of vacuum Haag duality for disconnected or multiple connected regions would be understood as signalling the presence of other sectors.

It is a much more difficult question as to what becomes of this topological obstruction in the presence of interactions. The reader can find some remarks in the last section. It is tempting to interpret this obstruction as indicating the necessity of an interaction [7] i.e.of the presence of non-vanishing electric or magnetic (or both) currents.

$$\partial^\mu F_{\mu\nu}(x) = j_\nu(x), \;\; \partial^\mu \tilde{F}_{\mu\nu}(x) = \tilde{j}_\nu(x) \tag{66}$$

The (not tested) idea[6] is that interactions are necessary to restore perfect Haag duality which is violated in the free theory for the corona region. Such a point of view would attribute a very distinguished role to electromagnetic duality i.e. those superselection rules which originate from the quantum version of the Maxwell structure and may well be the physical concept behind the semi-classical "gauge principle". This issue of problematizing the notion of "magnetic field" on the same level of depth as the notion of "charge" in the DHR superselection theory is presently ill-understood in QFT.

In low dimensional QFT the analogous issue of order-disorder duality and the connection with Haag duality is much better understood. There, even in free theories, it is not possible to have *no* charge sectors for both order and the dual disorder charge, the appearance of both charges being related in d=1+1 to the zero mass limit. So the previous idea of maintaining corona duality would bring the interacting Maxwell-like theories closer to the 2-dimensional situation.This analogy is another reason to believe that the free Maxwell situation is peculiar. The remaining three non-peculiar cases, namely the appearance of objects with e.-, m.- or e.m.-charges have an infrared structure, whose implications for localization properties are outside the present scope of understanding. A better insight into the connection between these properties and the modular theory (in the vein of the remarks about interactions in the last section) seems to be essential for future progress.

As in the massive case the algebras of the corona inclusion may be constructed solely in terms of the Wigner theory supplemented by the modular

[6]Even if one does not subscribe (as the present author) to such a idea, it is interesting to note the conceptual richness of algebraic QFT.

theory for wedges, avoiding covariant pointlike like field co-ordinates $F_{\mu\nu}$ alltogether. Helicities h≥ 1 present similar corona structures.

6 The Localization Properties of the Positive Energy Continuous Spin Representations.

Already in the late 60's the question of how to covariantize and incorporate Wigner's zero mass continuous spin representation into existing frameworks of QFT arose some interest notably with physicists who were familiar with the spirit of algebraic QFT [14]. Whereas in standard QFT based on quantization and Lagrangians these representations were usually dismissed as uninteresting because "nature appearently does not make use of them" [7], the spirit in which algebraic field theorists approached this problem was more "Wignerian". They asked whether these representations fulfill the *localization* properties which are *inexorable attributes* (in addition to their *indecomposability* expressed in the irreducibility requirement) *of particles.* It was found that they do not permit a compact localization as the standard Wightman fields do. But with the impossibility of compact localization (singly localized clicks in a counter), it is not immediatly clear how such objects physically manifest their presence.

Looking again at the old computations with the hindsight of the non-compact space-like cone localization of section 4, one easily realizes that their natural covariantization leads to the same wedge-localized light-like strings with an infinite component unitary representation on light-like directions and complex variables ξ_1, ξ_2 taking over the role of the u in the case of the anyonic representation of section 4. The identification of complex 2-vectors with light-like directions is done with the help of the Pauli matrices:

$$l_\mu = \xi^+ \sigma_\mu \xi \tag{67}$$

the intertwiners for this light-like covariantization are:

$$u(p,\xi) = f_\lambda^{\rho\chi}(\xi B_p), \quad f_\lambda^{\rho\chi}(\xi) = \left|\xi_2\right|^{2c-2} e^{-i\lambda\Phi_1} J_{l_0-\lambda}(\frac{\rho}{\kappa}\left|z\right|) e^{il_0\Phi_2} \tag{68}$$

[7]Strangely enough supersymmetry is never dismissed in this way, even though its relation to physical principles is more questionable than Wigner's representation theory.

$$z = \frac{\xi_1}{\xi_2}, \quad \Phi_{1,2} = \pm\pi + \arg\xi_1 \mp \arg\xi_2$$

The $f_\lambda^{\rho\chi}(\xi)$ corresponds precisely to the equivariant function F in section 4.

The step from wedge-localization to space-like cone localization is also analogous to section 4. Instead of two orthogonal wedges one now considers three. The smoothening with testfunctions of compact Fourier-transform:

$$\int dtdsduf(t,s,u)\Delta_1^{it}\Delta_2^{is}\Delta_3^{iu}\Phi, \ \Phi \in \mathcal{H} \tag{69}$$

together with commutation relations between the three boosts analogous to the ones used in section 4 will give a simultaneous dense domain for $\Delta_i^{\frac{1}{2}}$ i=1,2,3 . However it is not clear if the case of three wedges in general position can be reduced to the orthogonal case. Even if one succeeds to find common localization domains, it is (like in the previous case of anyons) not clear that the localizations have all the desired net properties. As in the case of anyons, I would expect the associated "free" field theories to have a non trivial virtual particle structure.

It is interesting to compare this space-like cone localization with that established by Buchholz and Fredenhagen on the basis of the spectral gap assumption [7]. In their massive case it is also not possible to realize this in a theory with a on mass-shell supported two-point function. But in their case one would expect, as in all d=3+1 theories which have a well-defined S-matrix, even a real (on shell) particle creation.

7 Intrinsic Understanding of Interactions? Programmatic Remarks.

Since the wedge regions have a preferential status with respect to the construction of interaction-free algebras, it is tempting to think that this may be helpful in obtaining some intrinsic insight into interactions. Let us take a helping hand from scattering theory. So we exclude zero mass phenomena by assuming isolated mass shells which assures the existence of an S-matrix. From scattering theory follows that out- and ingoing- fields share the same Poincaré transformations with the interacting fields i.e. they both possess the same modular transformations Δ^{it} for the wedge region. Only the TCP conjugation is sensitive with respect to interactions. In order to see this we will derive the following representation for S which is valid in an asymptotically complete theory :

$$S = \theta \cdot \theta_0 = J \cdot J_0 \tag{70}$$

J= modular conjugation for interacting wedge algebra

J_0 = modular conjugation for the interaction- free incoming wedge algebras.

The formula follows from the $\Theta = TCP$ transformation of Heisenberg fields. Taking the LSZ limit on this transformation formula and noticing that both sides approach different (in and out) limits (and remembering that the spatial rotation factors between Θ and J are independent of interactions), one obtains the above representation.

The interacting and incoming wedge algebras are of the same type, in fact they are expected to be type III_1factors .Since both are living in the same Hilbertspace, their isomorphism amounts to a unitary equivalence. Unfortunately this kind of argument does not lead to a "natural" unitary operator which we expect to be some kind of natural "square root" of S i.e. some kind of "algebraic" Møller operator. On a very formal level the method of Bogoliubov and Shirkov leads to such unitaries, but this would bring us back to the interaction picture and the formal time ordered operator expressions $S(g)$ which according to all our experience do not exist (i.e. the Bogoliubov-Shirkov axiomatics for d=3+1 has probably no solution).

Let us therefore be more modest and just ask for a modular net of real Hilbert-subspaces of the incoming Fock-space which belongs to a given S. If we pose this problem in two space-time dimensions, we could take a J operator which is different from J_0 by one of those rather simple rapidity dependent factorizing S-matrices of the "bootstrap construction" which are the long-distant limits of the class of theories with the same superselection rules [26]. Here our modular proposal is expected to give a more field theoretic understanding of the so-called formfactor bootstrap program and the Bethe-ansatz approach . Both the formfactor program and the present "modular program" point into the same direction: the construction of local fields respectively of local nets from a given S-matrix. Some results will be presented in a forthcoming paper [33].

Whereas the formfactor program has only been formulated for factorizable S-matrices, the modular idea in principle does not suffer from such a restriction. Presently it is not known if the latter leads to a unique net of *wedge algebras*; the above argument only yields unique net of real local *wedge subspaces* of the Fock space of scattering states. The uniqueness modulo normalizations (corresponding to the different composite fields) of the formfactor program suggests strongly the uniqueness of the general inverse scattering problem in algebraic QFT.

Note that in higher dimensions such a starting point with a model S-matrix is not available since the above long distance limit gives S=1 and hence free fields[8], in agreement with the d=3+1 Coleman-Mandula theorem or the theorem that interaction in the sense of $S \neq 1$always implies the presence of inelastic real processes.

In this case one could contemplate to start with a unitary Poincaré-invariant operator S_{aux} which only fulfills the TCP-invariance and the cluster decomposition property. This alone already leads to the existence of a net of wedge subspaces of the Fockspace: $\mathrm{H}_{W,F}^{R} \subset \mathcal{H}_F$. The requirement that these wedge spaces contain real subspaces describing states localized in noncompact space-like cones or compact double cones i.e. that certain intersections of wedge spaces have nonempty and isotonic net properties is expected to yield analytic on shell restrictions on the S-matrix which S_{aux} will not obey. Therefore S_{aux} will not be the the correct S-matrix. This may be rephrased by saying that only the correct S-matrix of a local QFT will have the property that the modular definition 70 agrees with the scattering theory (LSZ or Haag-Ruelle) definition. One would hope, that such restrictions resulting from localizations which are sharper than the original wedge localization, may give rise to an inductive procedure (a kind of algebraic perturbation theory in which the unitarity relations are fulfilled in every order). Assuming that the net generated by S_{aux} has a scattering theory (this could happen even if the net is not local) one could compute $S_{scat}^{(1)}$ and use this unitary matrix as the input for a new J operator associated with $S_{scat}^{(1)}$ with a new net of wedge spaces and so on. Even if such a program succeeds to yield a limiting net of local subspaces of $\mathcal{H}_F$, it is not clear that a map from these real subspaces of the Fock space to non Neumann algebras exists (thus generalizing the free CCR and CAR functors). Nevertheless, since one is combining the deep modular properties of algebraic QFT with the startling nonperturbative successes of low dimensional QFT, this way of thinking offers an irresistable

[8]It may however lead to a solid proof of the underlying "folklore" statement that S=1 leads necessarily to the free field Borchers class.

temptation[9]. The starting auxiliary S_{aux} could be a unitary operator as in Heisenberg's ill-fated S-matrix theory [28] but unlike in Heisenberg's attempt or in Chew's bootstrap proposal the $S-$matrix would be totally subservient to QFT.

The use of S-matrix properties for the purpose of constructing localized fields and algebraic nets is however not available if infrared properties wreck the mass shell i.e. if the LSZ-asymptotes vanish as in the case of particles carrying Maxwellian charges. Whereas these problems are no serious obstacle for theories which can be considered as scaling limits of massive theories with spectral gaps (e.g. 2-dim. conformal theories as limits of integrable massive models), Maxwellian theories with Maxwellian electric or magnetic charges pose a more serious problem since the lack of a particle reference space and a standard S-matrix (the cause of the electro- magnetic infrared problems) prevent the straightforward use of modular localization. In fact the structural insight into such theories is so poor, that even nonperturbative rigorous arguments in favour of the relation of *spin and statistics* and *TCP* have not been worked out.

A particularily simple distinguished class of S-matrices for this program are the piecewise constant matrices S which were mentioned in section 4. In d=1+1 this is related to the construction of (dis-)order fields [33] associated to the free field Borchers class. In d=2+1 however "free anyons" cannot be represented in the Fock-space of Fermions and Bosons by such simple modifications of the free Borchers class and one expects their formfactor construction to require concepts similar to the factorizing models even if their S-matrix (believed to be piecewise constant in energy) is simpler [25].

The modular approach does not use most of the mathematical formalism of standard QFT. Whereas hamiltonians, actions, functioal integrals, interaction picture, time ordering etc. appear indispensable in the latter, the modular approach rests on the idea that all the content of interaction resides in the L-invariant Tomita reflection J together with the localization concept. In a forthcoming publication we will show that the working hypothesis of the d=1+1 bootstrap-formfactor program follow from the modular properties. Whereas none of the mentioned properties of the standard approach are

[9]Up to now each property in low dimensional QFT had a generalization or specialization (e.g. permutation group statistics instead of braid group statistics) to higher dimensions. "Low dimensional pathologies" belong to the QFT folklore.

characteristic (they also hold in nonrelativistic quantum theories), the concepts of the modular approach are limited to local Quantum Physics only.

Finally we want to emphasize that our modular proposal based on the S-matrix does not cover zero mass theories. Whereas for those massless theories which can be viewed as scaling limits of massive theories (e.g. chiral conformal QFT) this poses no serious problem (since the conceptual complication is compensated for by an analytic simplification), the physically interesting cases in which the electric and/or magnetic charges are derived from Maxwellian field strengths remain presently outside the modular approach. In this case neither the conceptual nor the analytic aspects are simple.

It is surprizing that a theory which existed for more than 60 years as QFT, is capable of producing a fundamentally new concept. Perhaps the reason for the rather late appearance of modular localization and the related constructive ideas is its distance to quantization and Lagrangians as well as euclidean Feynman-Kac representations.

Modular properties require a certain amount of non-commutativity. However its use in order to solve deep problems of local Quantum Physics and the hope that it may lead to explicite constructions has presently nothing to do with the fashionable attempt to find physical realizations of the formalism of noncommutative geometry.

A large part of this work has been done at the UFES in Vitoria, Brazil. I am deeply indebted to some of my colleagues at the UFES (in particular to proof. Julio Cesar Fabris) for their kind hospitality. I am also indebted to Detlev Buchholz for several valuable informations.

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