and Z and $\tilde{X}$ are the same as in Theorem 1, cf. (21), (25). Furthermore, $D^kZ(\frac{n}{T}) = O(T^{-k})$ and $D^kZ_1(\frac{n}{T}) = O(T^{-k})$ for all k.

Proof: For $j' = 1$ the announced result is a special case of Corollary 1, namely that where $Y(N) = 1$ for all N. So let us consider the case $j' > 1$. Since $Q_0\Omega_{j'-1}(\frac{n}{T})P_0 = O(T^{-\infty})$,

$$
\begin{aligned}
&\sum_{n=1}^{l} Q_0 U_T^{A^*}(\tfrac{n}{T}) X(\tfrac{n}{T}) U_T^A(\tfrac{n}{T}) \Omega_{j'-1}(\tfrac{n}{T}) P_0 \\
&= \sum_{n=1}^{l} Q_0 U_T^{A^*}(\tfrac{n}{T}) X(\tfrac{n}{T}) U_T^A(\tfrac{n}{T}) P_0 \Omega_{j'-1}(\tfrac{n}{T}) P_0 + O(T^{-\infty}) \\
&= \frac{1}{T}\sum_{n=1}^{l} Q_0 U_T^{A^*}(\tfrac{n}{T}) Z(\tfrac{n}{T}) U_T^A(\tfrac{n}{T}) \Omega_{j'-1}(\tfrac{n}{T}) P_0 \\
&+ \frac{1}{T}\sum_{n=1}^{l} Q_0 U_T^{A^*}(\tfrac{n}{T}) \tilde{X}(\tfrac{n+1}{T}) W_T^A(\tfrac{n}{T}) U_T^A(\tfrac{n}{T}) P_0 T D \Omega_{j'-1}(\tfrac{n}{T}) P_0 + O(T^{-\infty})
\end{aligned}
$$

the last equality following by Theorem 1 with $Y(\frac{n}{T}) = \Omega_{j'-1}(\frac{n}{T})P_0$ provided T is large enough. In fact, as for Corollary 1 one shows that the boundary terms vanish if $T > \frac{1}{\epsilon}$ where $\epsilon > 0$ is chosen such that $\mathrm{supp} X(N) \subset [\epsilon, s-\epsilon]$. Using that, for $j' > 1$, $D\Omega_{j'-1}(\frac{n}{T})P_0 = -U_T^{A^*}(\frac{n}{T})(1 - V_T^*(\frac{n-1}{T}))U_T^A(\frac{n}{T})\Omega_{j'-2}(\frac{n}{T})P_0$ and the intertwining properties (8) one gets

$$
\begin{aligned}
U_T^A(\tfrac{n}{T}) P_0 T D \Omega_{j'-1}(\tfrac{n}{T}) P_0 &= U_T^A(\tfrac{n}{T}) P_0 U_T^{A^*}(\tfrac{n}{T}) T (V_T^*(\tfrac{n-1}{T}) - 1) U_T^A(\tfrac{n}{T}) \Omega_{j'-2}(\tfrac{n}{T}) P_0 \\
&= P_T(\tfrac{n}{T}) T (V_T^*(\tfrac{n-1}{T}) - 1) U_T^A(\tfrac{n}{T}) \Omega_{j'-2}(\tfrac{n}{T}) P_0. \qquad (42)
\end{aligned}
$$

This shows (41) for $j' > 2$. If $j' = 2$ then we may apply Corollary 1 to the term containing $\Omega_{j'-2}(\frac{n}{T})$ obtaining that this term is $O(T^{-\infty})$. It was already shown in the proof of Corollary 1 that $D^kZ(\frac{n}{T}) = O(T^{-k})$. That $D^kZ_1(\frac{n}{T}) = O(T^{-k})$ follows from the discrete Leibniz rule, since all factors satisfy this condition.q.e.d.

Theorem 3 *Suppose that $W_T(N)$ satisfies the conditions 1 and 2 for all k and that $V_T^*(N) - 1$ be supported within $(0, s)$. Then for $N \notin (0, s)$*

$$Q_0\Omega_T(N)P_0 = O(T^{-\infty}). \qquad (43)$$

Proof: Recall that, for all M, $Q_0\Omega_T P_0 = Q_0 R_M P_0 + \sum_{j=1}^{M} Q_0\Omega_j P_0$. We show by induction on j that, for $l \geq sT$ and $j \geq 1$,

$$Q_0\Omega_j(\tfrac{l}{T})P_0 = O(T^{-\infty}). \qquad (44)$$

The theorem then follows from (38). (44) is true for $j = 1$, since, by Corollary 1 applied to $X = 1 - V_T^*$ and $Y = 1$,

$$
\begin{aligned}
Q_0\Omega_1(\tfrac{l}{T})P_0 &= -\frac{1}{T}\sum_{n=1}^{l} Q_0 K_T(\tfrac{n-1}{T}) P_0 \\
&= -\sum_{n=1}^{l} Q_0 U_T^{A^*}(\tfrac{n}{T}) [1 - V_T^*(\tfrac{n}{T})] U_T^A(\tfrac{n}{T}) P_0 = O(T^{-\infty}).
\end{aligned}
$$