Modular Localization, H-Temperatures and the Bethe Ansatz Structure.

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Abstract

The recently proposed construction approach to nonperturbative QFT, based on modular localization, is reviewed and extended. It allows to unify black holes physics and H-temperatures (H standing for Hawking or Horizon) with the bootstrap-formfactor program for nonperturbative construction of low dimensional QFT. In case of on-shell particle number conservation, the equations characterizing the modular localization spaces for wedges are Bethe-Ansatz equation in the form as recently obtained in the treatment of factorizable models.

1 Historical Remarks and Present State

The modular theory of von Neumann algebras is one of the few theories of which the foundations received independent contributions from mathematicians and quantum field theorist; a fact which testifies to the naturalness of the modular concepts. During the 60's in a tour de force the mathematician Tomita obtained the most important statements which were received by his contemporaries with a mixture of surprise and disbelief. Shortly afterwards his Japanese colleague Takesaki [2] corrected and further developed the theory[1], this time already using concepts of the Haag, Hugenholtz, Winnink[1][2] description of the infinite volume (thermodynamic) limit for thermal states on QFT systems which were elaborated at approximately the same time as Tomita's contributions. As usual,

[1]The reader is urged to read the lecture notes of R.Kadison which will be published in the proceedings of the 1997 Summer School in Portugal on "Noncommutative Geometry and Applications".

[2]Whenever references have entered textbooks, we prefer to quote the latter.

the physical context of such a conceptual discovery is somewhat more special. It was the deeper understanding of the so-called "KMS property"[3], which connected the HHW-thermal theory with the modular theory of Tomita and its improvement by Takesaki [2] and finally led to what is nowadays referred to as the Tomita-Takesaki modular theory.

I can only think of one other such *natural* "marriage" between physics and mathematics. This is the closely related subfactor theory in von Neumann algebras of V. Jones [3], in which case the (again much more special) physical counterpart (the Doplicher-Haag-Roberts theory of localized endomorphisms[1]) preceded the mathematical development by almost one decade, although the interconnections were only noticed several years after Vaughn Jones's discovery.

The further development of the T.-T. modular theory (here in the context of algebraic QFT briefly referred to as "modular theory") during the 70's is characterized by the names A.Connes and H.Araki. These authors extended the theory by the concept of "natural cones" and Connes used the theory for his famous classification of type III factors [4]. On the physical side the development during the 70's is characterized by the beginning of understanding of the importance of modular theory for the localization concept in relativistic QFT. Following suggestive ideas of Araki, Eckmann and Osterwalder[5] and later Leyland, Roberts and Testard[6] first recognized the close connection in the context of free fields before Bisognano and Wichmann finally achieved a more general understanding within the setting of QFT as formulated by A.S. Wightman[1]. During the 80's and 90's there have been many mathematical physics contributions extending physical and mathematical aspects of modular theory into different directions. Our own contribution presented in this article is "inverse" to the one of Bisognano and Wichmann[1] i.e. it tries to construct local theories via the concept [9][15] of "modular localization" . As such it has some mathematical aims in common with investigations of M.Wollenberg [7].

In this paper we are interested in the inverse problem: how to obtain localized states and local algebras (and fields) from modular theory. Interestingly enough, our more physically motivated approach leads to the *Main Inverse Problem of QFT*: how to obtain reasonable physical conditions under which a given admissible scattering operator S_s determines uniquely a local QFT. As a kind of side result we obtain new ideas of how the somewhat elusive (in its nonperturbative aspects) crossing symmetry together with its associated "on shell" analyticity property is related to the KMS property of the (Rindler) wedge-localized Hawking-Unruh effect. Our concepts attribute a very fundamental role to the modular reflection. As such our viewpoint has many things in common with recent work of Buchholz and Summers[8] apart from the fact that they use the Tomita $J's$

[3]Originally just a receipe to avoid the calculation of cumbersome traces in favour of analyticity properties combined with boundary conditions which was introduced in the 50's by Kubo, Martin and Schwinger[1]

for the reconstruction of space-time properties and a characterization of a vacuum reference state (or its substitute).

In order to structure our review of some old points in a new light and combine it with the presentation of new viewpoints, we will follow the outline in terms of six sections.

1. **Historical Remarks and Present State**
2. **Liberation from "Field Coordinates"**
3. **H-temperature (Hawking, Unruh, Sewell, Wald, Kay and others)**
4. **Modular Localization and Factorizing Theories..**
5. **General Interactions and Modular Localization.**
6. **Present Conclusions, Outlook and possible Connection with other Ideas.**

The second section describes the path from Wigner's positive energy representations to modular localization subspaces and local nets and avoids the use of "field coordinates" altogether [9]. The localization peculiarities of massless theories with helicities $h \geq 1$ (which constitute the physical origin of the "gauge" phenomenon) as well as those of "continuous" helicity are briefly mentioned and the necessity for noncompact modular localization of anyons is explained.

The third section strengthens the field theoretic interpretation of thermal aspect of the Hawking-Unruh effect as consequences of localization. The big Latin letter H has not been chosen as a result of recent fashions (hiding unfamiliar and insufficiently defined inventions behind familiar sounding letters), but rather represents the intended double meaning of either Hawking or (bifurcated) Horizon depending on the context. In the present context this temperature may also be called "modular localization temperature" and it is associated with its own "temperature Hilbert space" as will be justified in section 3.

The fourth section contains the use of modular localization for the description and explicit construction of interacting theories in d=1+1 dimensions. In these notes the computational successful, but insufficiently understood bootstrap-formfactor approach of Karowski and Weiss[10] , as well as of Smirnovs extensions [11], and in particular the more recent contributions of Babujian, Fring, Karowski and Zapletal [13][14] will be newly interpreted, modified and extended in the light of modular localization [15]. The surprising new result is that the Riemann-Hilbert problem and the Bethe Ansatz method, which appears in the work of the last 4 authors is identical to the properties of the modular localization equations for wedges in relativistic QFT with on shell particle conservation (but arbitrarily complicated off shell nonconservation behavior).

The fourth section contains remarks on higher dimensional systems. In particular for d=1+3 theories, for which on shell conservation of particle number is known to be incompatible with interactions, our remarks are presently very speculative and preliminary indeed.

Finally we use the fifth section to draw some general conclusions about the structure of QFT and compare the modular localization concept to attempts which base quantum physics on more global concepts (string theory, method of noncommutative geometry for gravity and electro-weak interactions).

2 Liberation from Free Field Co-ordinates

As explained elsewhere [9], one may use the Wigner representation theory for positive energy representations in order to construct fields from particle states. For $d = 3 + 1$ space-time dimensions there are two families of representation: (m, s) and $(0, h)$. Here m is the mass and designates massive representation and s and h are the spins resp. the helicites h. These are invariants of the representations ("Casimirs") which refer to the Wigner "little" group; in the first case to SU(2) in which case $s =$ (half) integer, and for $m = 0$ to the little group (fixed point group of a momentum $\neq 0$ on the light cone) $\tilde{E}(2)$ which is the two-fold covering of the euclidean group in the plane. The zero mass representations split into two families. For the "neutrino-photon family" the little group has a nonfaithful representation (the "translative" part is trivially represented) whereas for so-called "continuous h representation" the representation is faithful but allows no identification with known zero mass particles.

In the massive case, the transition to covariant fields is most conveniently done with the help of intertwiners between the Wigner spin s representations $D^{(s)}(R(\Lambda, p))$ which involve the Λ, p dependent Wigner rotation R and the finite dimensional covariant representation of the Lorentz-group $D^{[A,B]}$

$$u(p)D^{(s)}(R(\Lambda, p)) = D^{[A,B]}(\Lambda)u(\Lambda^{-1}p) \tag{1}$$

The only restriction is:

$$| A - B | \leq s \leq A + B \tag{2}$$

which leaves infinitely many A, B (half integer) choices for a given s. Here the $u(p)$ intertwiner is a rectangular matrix consisting of $2s + 1$ column vectors $u(p, s_3), s_3 = -s, ..., +s$ of length $(2A+1)(2B+1)$. Its explicit construction using Clebsch-Gordan methods can be found in Weinberg's book [16].Analogously there exist antiparticle (opposite charge) $v(p)$ intertwiners: $D^{(s)*}(R(\Lambda, p) \longrightarrow D^{[A,B]}(\Lambda)$. The covariant field is then of the form:

$$\psi^{[A,B]}(x) = \frac{1}{(2\pi)^{3/2}} \int (e^{-ipx} \sum_{s_3} u(p_1 s_3)a(p_1 s_3) + e^{ipx} \sum_{s_s} v(p_1 s_3)b^*(p_1 s_3))\frac{d^3p}{2\omega}. \tag{3}$$

where $a(p)$ and $b^*(p)$ are annihilation (creation) operators in a Fockspace for particles (antiparticles). The (only at first sight, as it fortunately turns out) bad news is that in we lost the Wigner unicity: there are now infinitely many $\psi^{[A,B]}$ fields with varying A, B but all belonging to the same (m,s)-Wigner representation and living in the same Fock space. Only one of these fields is "Eulerian" (examples: for $s=\frac{1}{2}$ Dirac, for $s=\frac{3}{2}$ Rarita-Schwinger) i.e. the transformation property of ψ is a consequence of the nature of a linear field equation and which is derivable by an action principle from a Lagrangian. Non-Eulerian fields as e.g. Weinberg's $D^{[j,0]}+D^{[0,j]}$ fields for $j>\frac{3}{2}$, cannot be used in a canonical quantization scheme or in a formalism of functional integration because the corresponding field equations have more solutions than allowed by the physical degrees of freedom (in fact they have tachyonic solutions). The use of formula (3) with the correct u, v intertwiners in the (on shell) Bogoliubov-Shirkov approach based on causality is however legitimate. Naturally from the point of view of the Wigner theory which is totally intrinsic and does not use quantization ideas, there is no preference of Eulerian versus non Eulerian fields.

It turns out that the above family of fields corresponding to (m,s) constitute the linear part of the associated "Borchers class" [1]. For bosonic fields the latter is defined as:

$$B(\psi)=\left\{\chi(s) \mid [\chi(x),\psi(y)]=0,\ \ (x-y)^2<0\right\} \tag{4}$$

If we only consider cyclic (with respect to the vacuum) relatively local fields, than we obtain transitivity in addition to the auto-locality of the resulting fields. This class depends only on (m,s) and is generated by the Wick-monomials of ψ. A mathematically and conceptually more managable object which is manifestly independent of the chosen (m,s) Fock-space field, is the local von Neumann algebra generated by ψ:

$$\mathcal{O}\rightarrow\mathcal{A}(\mathcal{O},\psi)=\mathcal{A}(\mathcal{O},\chi) \tag{5}$$

Here $\chi\sim\psi$ is any cyclic (locally equivalent) field in the same Borchers class of ψ.

Now we have reached our first aim: the lack of uniqueness of local (m,s) fields is explained in terms of the arbitrariness in the choice of "field coordinates" which generate the same net of (observable) von Neumann algebras. According to the physical interpretation in algebraic QFT this means that the physics does not depend on the concretely chosen (cyclic) field.

Since algebraic QFT shuns inventions and favors discoveries, it is deeply satisfying that there are arguments that every causal net fulfilling certain spectral properties is automatically "coordinatizable". For chiral conformal theories there exists even a rigorous proof [17]. So one can be reasonably sure that the physical content has not been changed as compared to the standard Wightman approach. The use of local field coordinates tends to make geometric localization properties of the algebras manifest. But only if there exist pointlike covariant generators which create charged states (counter example: for Maxwellian charges they do not exist) the localization can be encoded into classical smearing function. The localization concept is "maximally classical" for the free Weyl and CAR algebras which are just function algebras with a noncommutative product structure. For these special cases the differential geometric concepts as fibre bundles may be directly used in local quantum physics. Outside of these special context, the only reliable methods are the von Neumann algebra methods of algebraic QFT. In that case the quantum localization may deviate from the classical geometric concepts and use of field coordinates is less useful.

In the following we describe a way to construct the interaction-free nets directly thus bypassing the use of field coordinates alltogether. We use the d=3+1 Wigner (m,s)-representations as an illustrative example. In case of charged particles (particles≠antiparticles) we double the Wigner representation space:

$$H = H_{Wig}^{p} \oplus H_{Wig}^{\bar{p}} \tag{6}$$

in order to incorporate the charge conjugation operation as an (antilinear in the Wigner theory) operator involving the p-p̄-flip. On this extended Wigner space one can represent the full Poincaré group where those reflections which change the direction of time are antiunitarily represented. For the modular localization in a wedge we only need the standard L-boost $\Lambda(\chi)$ and the standard reflection r which (by definition) are associated with the $t-x$ wedge:

$$\delta^{i\tau} \equiv \pi_{Wig}(\Lambda(\chi = 2\pi\tau)) \tag{7}$$

$$j \equiv \pi_{Wig}(r) \tag{8}$$

These operators have a simple action on the p-space (possibly) doubled Wigner wave functions, in particular:

$$(j\psi)(p) \simeq \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \bar{\psi}(p_0, p_1, -p_2, -p_3) \tag{9}$$

By functional calculus we form $\delta^{\frac{1}{2}}$ and define:

$$s \equiv j\delta^{\frac{1}{2}} \tag{10}$$

This unbounded antilinear densely defined operator s is involutive on its domain: $s^2 = 1$. Its -1 eigenspace is a real closed subspace H_R of H which allows the following characterization of the domain of s :

$$\begin{aligned} dom(s) &= H_R + iH_R \\ s(h_1 + ih_2) &= -h_1 + ih_2 \end{aligned} \tag{11}$$

Defining:

$$H_R(W) \equiv U(g)H_R, \;\; W = gW_{stand} \tag{12}$$

where g is an appropriate Poincaré transformation, we find the following theorem:

Theorem 1 *(Brunetti, Guido and Longo, in preparation): $H_R(W)$ is a net of real Hilbert spaces i.e. $H_R(W_1) \subsetneq H_R(W_2)$ if $W_1 \subsetneq W_2$.*

Although we will not give the proof [18], it turns out to be quite easy, at least if one is familiar with the work of Borchers [19] which already contains the idea on how positive energy translations are related with compressions of algebras.

If we now define:

$$H_R(\mathcal{O}) \equiv \bigcap_{W \supset \mathcal{O}} H_R(W) \tag{13}$$

Then it is easily seen (even without the use of the u,v-intertwiners) that the spaces $H_R(\mathcal{O}) + iH_R(\mathcal{O})$ are still dense in H_{Wig} and that the formula:

$$s(\mathcal{O})(h_1 + ih_2) \equiv -h_1 + ih_2 \tag{14}$$

defines a closed involutive operator with a polar decomposition:

$$s(\mathcal{O}) = j(\mathcal{O})\delta(\mathcal{O})^{\frac{1}{2}} \tag{15}$$

Although now $j(O)$ and $\delta(\mathcal{O})^{i\tau}$ have no obvious geometric interpretation, there is still a bit of geometry left, as the following theorem shows:

Theorem 2 *The $H_R(\mathcal{O})$ form an orthocomplemented net of closed real Hilbert spaces, i.e. the following "duality" holds $H_R(\mathcal{O}') = H_R(\mathcal{O})' = iH_R^{\perp}(\mathcal{O})$*

Here $\mathcal{O}'$ denotes the causal complement, $H_R^{\perp}$ the real orthogonal complement in the sense of the inner product $Re\,(\psi, \varphi)$ and H_R' is the symplectic complement in the sense of $\mathrm{Im}\,(\psi, \varphi)$.

The direct construction of the interaction-free algebraic bosonic net for (m,s=integer) is now achieved by converting the "premodular" theory of real subspaces of the Wigner space into the Tomita-Takesaki modular theory for nets of von Neumann algebras using the Weyl functor:

Theorem 3 *The application of the Weyl functor $\mathcal{F}$ to the net of real spaces:*

$$H_R(\mathcal{O}) \stackrel{\mathcal{F}}{\rightarrow} \mathcal{A}(\mathcal{O}) \equiv alg\,\{W(f)\,|f \in H_R(\mathcal{O})\} \tag{16}$$

leads to a net of von Neumann algebras in $\mathcal{H}_{Fock}$ which are in "standard position" with respect to the vacuum state with a modular theory which, if restricted to the Fock vacuum Ω, is geometric:

$$\begin{aligned} \mathcal{F}(s) &= S, \ S A\Omega = A^*\Omega, \ A \in \mathcal{A}(W) \\ S &= J\Delta^{\frac{1}{2}}, \ J = \mathcal{F}(j), \ \Delta^{i\tau} = \mathcal{F}(\delta^{i\tau}) \end{aligned} \tag{17}$$

The proof of this theorem uses the functorial formalism of [6]

Clearly the W or $\mathcal{O}$ indexing of the spaces corresponds to a localization concept via modular theory. Specifically $H_R(\mathcal{O})+iH_R(\mathcal{O})$ is a certain closure of the one particle component of the Reeh-Schlieder domain belonging to the localization region $\mathcal{O}$. Although for general localization region the modular operators are not geometric, there is one remaining geometric statement which presents itself in the form of an algebraic duality property:

$$\mathcal{A}(\mathcal{O}')=\mathcal{A}(\mathcal{O})', \quad \textit{Haag Duality} \tag{18}$$

Here the prime on the von Neumann algebra has the standard meaning of commutant.

It is very instructive to write the modular wedge localization equations in Fock space more explicitly.. For simplicity we consider the case of d=1+1 free fields because in that case n-particle states can be characterized solely in terms of rapidities θ (there is only one wedge and its opposite). We find:

$$S\psi=\psi\Leftrightarrow\overline{f_n(\theta_1,.....\theta_n)}=f_n(\theta_1+i\pi,....\theta_n+i\pi) \tag{19}$$

where f_n are the n-particle component wave functions in momentum space (rapidities). This is a boundary relation for an analytic function which is analytic inside the multidimensional $i\pi$ strip; the analyticity beeing a consequence of the fact that ψ must be contained in the domain of $\Delta^{\frac{1}{2}}$ because $domS=dom\Delta^{\frac{1}{2}}$. The analytic "master" function f_n has different real boundaries corresponding to the permutations of the standard ordering $\theta_1\leq\theta_2\leq...\leq\theta_n$. As for Wightman functions in x-space, the different orders are obtained by different ways of letting the imaginary parts approach zero.

Whereas in Wigner space the subspace belonging to e.g. a double cone is simply the intersection of the two wedges which define this double cone (at least in case of (half)integer L-spin), these net properties are not true in Fock space. So in the interacting case we cannot expect the existence of a functor from real subspaces of Fock space to local algebras.

In the following we make some schematic additions and completions which highlight the modular localization concept for more general cases.

- (1) In the case of $m\neq0,s=$ halfinteger, the Wigner theory produces a mismatch between the "quantum" and the "geometric" opposite of $H_R(W)$, which however is easily taken care of by an additional factor i (interchange of symplectic complement with real orthogonal complement). This (via the physical localization property) requires the application of the CAR-functor instead of the CCR-functor as well as the introduction of the well-known Klein transformation K which takes care of the above mismatch in Fockspace:

$$\begin{aligned} J &= K\mathcal{F}_{CAR}(ij)K^{-1} \\ \mathcal{A}(\mathcal{O}') &= K\mathcal{A}(\mathcal{O})'K^{-1}, \ \textit{twisted Haag Duality} \end{aligned} \tag{20}$$

- (2) For $m=0, h=$(half)integer, as a consequence of the nonfaithful representation of the zero mass little group $E(2)$ (the two-dimensional euclidean group or rather its two-fold covering), the set of possible u-v intertwiners is limited by the selection rule: $|A-B| = \pm h$. This means on the one hand that there are no covariant intertwiners which lead to $D^{\left[\frac{1}{2},\frac{1}{2}\right]}$ (vector-potential of classical Maxwell theory), $D^{\left[\frac{1}{2},\frac{1}{2}\right]} \otimes (D^{\left[\frac{1}{2},0\right]} + D^{\left[0,\frac{1}{2}\right]})$ (Rarita-Schwinger potential for massless particles), gravitational potentials etc. On the other hand, all local bilinear expressions in the allowed covariant intertwiners vanish and hence cannot be used in order to rewrite the Wigner inner product (for e.g. $h=1$ in terms of field strength intertwiners $F_{\mu\nu}(p)$). A reasonable compromise consists in relaxing on strict L-covariance and compact (double cone) modular localization but retaining the relation with the Wigner inner product. One then may describe the Wigner space in terms of polarization vector dependent vector-potentials on the light cone which have the following affine Lorentz transformation:

$$\begin{aligned} \left(U(\Lambda)A\right)_{\mu}(p,e) &= \Lambda_{\mu}^{\nu} A_{\nu}(\Lambda^{-1}p, \Lambda^{-1}e) \\ &= \Lambda_{\mu}^{\nu} A_{\nu}(\Lambda^{-1}p, e) + p_{\mu} G(p,\Lambda,e) \end{aligned} \tag{21}$$

where the "gauge" contribution G by which one has to re-gauge in order to refer to the original spacelike polarization vector e is a nonlocal term which follows from the above definitions. This description is the only one if one follows the above logic of implementing the modular localization for the $(0, h=1)$ Wigner representation. After applying the Weyl functor, we obtain a local covariant net theory which is described in terms of slightly nonlocal semiinfinite stringlike field coordinates whose relation to the local $F_{\mu\nu}(x)$ field strengths is given by:

$$A_{\mu}(x,e) = \int_{0}^{\infty} e^{\nu} F_{\mu\nu}(x-es)ds \tag{22}$$

If we now define the modular localization subspaces as before by starting from the wedge region, we find that the (smoothened versions of) the vectorpotentials are members of these subspaces (or their translates) as long as the spacelike directions e point inside the wedges. They are lost if we form the localization spaces belonging to e.g. double cone regions regions. Hence these stringlike localized vector potentials appear in a natural way in our modular localization approach for the wedge regions. Whereas the natural use of such nonpointlike objects in a future interacting theory based on modular localization may be possible, in the present formulation of gauge theories they have not been used. There, one aims for a description in which the affine contribution of the L-boosts is absent. Adapting the Kostant-Sternberg analysis of constraint symplectic geometry[20], one can canonically derive the Kugo-Ojima[21] operator version of the Faddeev-Popov-BRS description of local free photon fields. Preliminary work by G. Scharf, T. Hurt, M Duetsch, F. Krahe, K. Fredenhagen and R. Stora[22] point to the conjecture that perturbative gauge theories can be formulated as deformatios of the free photon situation within the Bogoliubov Shirkov Epstein Glaser[23] framework. Whereas such an operator formulation may be considered as progress as compared to the functional calculus of Faddev-Popov (operators are much closer to local quantum physics than euclidean functional integrals), one should not use the BRS formalism as [24] a soft cushion to rest, because (in my opinion) then one would lose the tremendous enigmatic power which still resides in these half-solved physical gauge problems of finding a conceptually acceptable physical description of interactions involving vector fields with weaker localization properties. It may very well be that this last step[4] is only possible by using modular localization ideas in addition to perturbative deformations.

- (3) $d = 3 + 1, m = 0, h$ "continuous" ; $d = 2 + 1, m \neq 0, s \neq$(half)integer.

The common feature of these cases is that they do not admit compact localization i.e. $H_R(\mathcal{O}) = \{0\}$, but $H_R(W_1 \cap W_2) \neq \{0\}$ for $W_1 \cap W_2 \neq \varnothing$. In particular for d=1+2 this means that the spacelike cones have nontrivial localization spaces. However the attempt to construct a local net of spaces including the spacelike cone regions fails [27] since the intersection of spaces turns out to be genuinely bigger than the localization space belonging to the intersected region. This fits nicely together with observations about scattering theory showing that the multiparticle in-spaces for plektons (including anyons) are not tensor product of Wigner spaces [28].There is another theorem which shows that a field which obeys free field equations cannot fulfill anyonic statistics [26][27], in other words the two-point functions of anyonic operators have off shell creation even if the asymptotic particle number is conserved. Such situations, similar to the d=1+1 factorizing theories, cannot be unraveled by Wigner's theory alone; also the structure of asymptotic multiparticle states of scattering theory is needed. Concerning the d=1+2 continuous helicity situation, the a priori best localization and the "freest" field theory behind it still needs to be investigated. The obligation of a theoretical physicist is not to refer to "nature not making use of these representations"[5] but rather to argue that some localization aspects, which for ordinary particles are required in addition to the irreducibility and positive energy, could be possibly missing. But of course there are also particle-like objects as quarks which are not ordinary particles in the sense of Wigner+compact localization..

[4]The step of avoiding the introduction of ghosts which in a later stage will be eliminated again.

[5]Just imagine how an application of this argument would have influenced the course of supersymmetry in the last twenty years.

3 H-Temperature and Modular Localization

In modular theory the dense set of vectors which are obtained by applying (local) von Neumann algebras in standard position to the standard (vacuum) vector forms a core for the Tomita operator S. The domain of S can then be described in terms of the $+1$ closed real subspace of S. In terms of the "premodular" objects s in Wigner space and the modular Tomita operators S in Fock space we introduce the following nets of wedge-localized dense subspaces:

$$H_R(W) + iH_R(W) = dom(s) \subset H_{Wigner} \tag{23}$$

$$\mathcal{H}_R(W) + i\mathcal{H}_R(W) = dom(S) \subset \mathcal{H}_{Fock} \tag{24}$$

These dense subspaces become Hilbertspaces in their own right if we use the graph norm of the Tomita operators. For the s-operators in Wigner space we have:

$$(f,g)_{Wigner} \to (f,g)_G = (f,g)_{Wigner} + (sf,sg)_{Wigner} \tag{25}$$

$$\curvearrowright (f,g))_{Wigner} = \left(f, \frac{1}{1-e^{2\pi K}}, g\right)_G \tag{26}$$

So the temperature dependence of localized states becomes manifest. The fact that the standard boost K appears instead of the hamiltonian leads to somewhat different energy distribution functions. In particular one is advized to discuss matters of statistics not in Fourier space but rather in the space where they belong, namely spacetime. For those readers who are familiar with Unruh's work we mention that the Unruh hamiltonian is different from 2π by a factor $\frac{1}{a}$ where a is the acceleration.

The very special free field formalism may be generalized into two directions:

- (a) interacting fields
- (b) curved spacetime

For low-dimensional theories (a) will be discussed in the next section. For the generalization (b) to curved space time (e.g. the Schwarzschild solution) it turns out that only the existence of a bifurcated horizon together with a certain behaviour near that horizon matters ("surface gravitation"). In the standard treatment one needs isometries in spacetime. The idea of modular localization suggests to consider also e.g. double cones for which there is no spacetime isometry but only an isometry in H_{Wigner} or $\mathcal{H}_{Fock}$. Of course such enlargements of spaces in order to have a better formulation (or even a solution) of a problem are a commonplace in modern mathematics, particularly in noncommutative geometry[6]. The idea is that one enlarges the isometries by geometrical "fuzzy" ones which only near the horizon may loose their spacetime fuzziness.

[6] Here the unforgettable Gunnar Källen comes to my mind who used to call tricks like this "Methode Erlkönig" which refers to a famous poem of Goethe which contains the line "...und bist du nicht willig so brauch ich Gewalt...".

In this context it would be very important to understand the (nongeometric) modular theory of e.g. the double cone algebra of a massive free field. Fom the folium of states one may want to select that vector, with respect to which the algebra has a least fuzzy (most geometric) behaviour under the action of the modular group. Appealing to the net subtended by spheres at time t=0 one realizes that algebras localized in these spheres are independent of the mass. Since m=0 leads to a geometric modular situation[7] for the pair $(\mathcal{A}_{m=0}(S), |0\rangle_{m=0}$, and since the nonlocality of the massive theory in the subtended double cones is only the result of the fuzzy propagation inside the light cone (the breakdown of Huygens principle or the "reverberation" phenomenon), the fuzziness of the modular group for the pair $(\mathcal{A}_{m\neq 0}(C(S)), |0\rangle_{m\neq 0})$ is a pure propagation phenomenon i.e. can be understood in terms of the deviation from Huygens principle. In view of the recent micro-local spectrom condition we expect such nonlocal cases to have modular groups whose generators are pseudo-differential instead of (local) differential operators [30].

The Hilbert space setting of modular localization offers also a deeper physical understanding of the universal domain $\mathcal{D}$ which plays a rather technical role in the Wightman framework In the modular localization approach the necessity for such a domain appears if one wants to come from the net of localization spaces which receive their natural topology from the (graphs) net of Tomita operators $\bar{S}(\mathcal{O})$ to a net of (unbounded) polynomial algebras $\mathcal{P}(\mathcal{O})$ such that:

$$dom\ \bar{S}(\mathcal{O}) \cap \mathcal{D} = \mathcal{P}(\mathcal{O})\Omega = dom\ \mathcal{P}(\mathcal{O}) \tag{27}$$

this domain is of course also expected to be equal to $\mathcal{A}(\mathcal{O})\Omega$. Here we used a more precise notation which distinguishes between the operator S defined on the core $\mathcal{A}(0)\Omega$ and its closure $\bar{S}$ which is defined on $\mathcal{H}_R(\mathcal{O}) + i\mathcal{H}_R(\mathcal{O})$.

4 Modular Localization and Factorizing Theories

"In diesem Fall und ueberhaupt, kommt es ganz anders als man glaubt" (W.Busch).

[In this special case, as almost always, things happen completely different to expectations].

In this section we will show that the ranges of spaces obtained by applying all $\mathcal{O}$-localized (Wightman) fields (or the operator algebra $\mathcal{A}(\mathcal{O})$) to the vacuum, can be used for the nonperturbative construction of QFT's. This somewhat unexpected state of affairs comes about through modular localization. Although the modular localization concept is a general structural property of QFT[8], its constructive use is presently limited to factorizable (integrable) QFT models.

[7] The modular group is a one-parametric subgroup of the conformal group.

[8] Its remoteness from perturbative structures may be the reason why it was only discovered rather late.

We remind the reader that "factorizable" in the intrinsic physical interpretation of algebraic QFT means "long-distance representative" (in the sense of the S-matrix) in a given superselection class, in other words each general d=1+1 theory has an asymptotic companion which has the same supersection sectors ($\simeq$ same particle structure or incoming Fock space) but vastly simplified dynamics associated to a factorizing S-matrix [15]. In this paper our goal will be limited to the modular interpretation and basic field theoretic understanding of the bootstrap-formfactor program which presently is largely a collection of plausible cooking recipes [11]. Its computational power in enlarging the class of soluble models will be shown in seperate future work.

All applications of modular localization to interacting theories are based on the observation that in asymptotically complete theories with a mass gap, the full interaction resides in the Tomita operator $J(W)$, whereas the modular group $\Delta^{i\tau}(W)$ for wedges (being equal to Lorentz boosts) is blind against interactions (the representation of the Poincarè group is already defined on the free incoming states). In other words the interaction resides in those disconnected parts of the Poincaré group which involve antiunitary time reflections. The Haag Ruelle scattering theory together with the asymptotic completeness easily yield (for each wedge):

$$J = S_s J_0, \quad \Delta^{i\tau} = \Delta_0^{i\tau} \tag{28}$$

where the subscript 0 refers to the free incoming situation and we have omitted the reference to the particular wedge. The most convenient form for this equation is:

$$S = S_s S_0 \tag{29}$$

where S and S_0 are the antiunitary Tomita operators and S_s is the scattering operator. Therefore the scattering operator S_s in relativistic QFT has two interpretations: it is a global operator in the sense of large time limits and a modular localization interpretation of measuring the deviation of J or S from their free field values. This modular aspect is characteristic of local quantum physics and has no counterpart in nonrelativistic theory or quantum mechanics. The modular subspace of $\mathcal{H}_{Fock} = \mathcal{H}_{in}$ for the standard wedge:

$$\begin{aligned} S_s S_0 \mathcal{H}_R &= \mathcal{H}_R & (30) \\ S_s S_0 \psi &= -\psi, \ \ \psi \in \mathcal{H}_R & (31) \end{aligned}$$

for general S-matrix is a rather unmanageable object. However for scattering matrices S_s which commute with the incoming particle number and have the Yang-Baxter structure it will be shown that these equations take on the form of Bethe-Ansatz equations which can be solved by the (nested) Bethe-Ansatz method. Before we explain this we will look at the simplest of such d=1+1 models which is the Federbush model. The model is so simple that it can be solved by any field theoretic method including the Lagrangian method. The model consists in coupling two species of Dirac fermions via a (parity violating) current-pseudocurrent coupling [31][32]:

$$\mathcal{L}_{int} = g : j_\mu^I j_\nu^{II} : \varepsilon^{\mu\nu}, \quad j_\mu =: \bar{\psi}\gamma_\mu\psi : \tag{32}$$

One easily verifies that:

$$\begin{aligned} \psi_I(x) &= \psi_I^{(0)}(x) \vdots e^{ig\Phi_{II}^{(l)}(x)} \vdots \\ \psi_{II}(x) &= \psi_{II}^{(0)}(x) \vdots e^{ig\Phi_I^{(r)}(x)} \vdots \end{aligned} \tag{33}$$

where $\Phi^{(l,r)} = \int_{x' \lessgtr x} j_0 dx'$ is a potential of $j_{\mu 5}$ i.e. $\partial_\mu \Phi \sim \varepsilon_{\mu\nu} j^\nu = j_{\mu 5}$ and the superscript l,r refers to whether we choose the integration region for the line integral on the spacelike left or right of x. The triple ordering is needed in order to keep the closest connection with classical geometry and localization and in particular to maintain the validity of the field equation in the quantum theory; for its meaning we refer to the above papers. This conceptually simpler triple ordering can be recast into the form of the analytically (computational) simpler standard Fermion Wick-ordering. Although in this latter description the classical locality is lost, the quantum exponential do still define local Fermi-fields; in the case of relative commutation of ψ_I with ψ_{II} the contributions from the exponential (disorder fields) compensate. Despite the involved looking local fields (33), the wedge algebras are of utmostl simplicity:

$$\begin{aligned} \mathcal{A}(W) &= alg\left\{\psi_I^{(0)}(f)U_{II}(g), \psi_{II}^{(0)}(h); suppf, h \in W\right\} \\ \mathcal{A}(W') &= \mathcal{A}(W)'_{Klein} = alg\left\{\psi_I^{(0)}(f), \psi_{II}^{(0)}(h)U_I(g); suppf, g \in W'\right\} \end{aligned} \tag{34}$$

i.e. the two wedge-localized algebras (W denotes the right wedge) are generated by free fields "twisted" by global $U(1)$ symmetry transformation of angle g (coupling constant)[9]. This follows from the observation that if the x is restricted to W one may replace the exponential in ψ_I (which represents a left half space rotation) by the full rotation since the exponential of the right halfspace charge is already contained in the right free fermion algebra etc. The computation [32] of the scattering matrix S_s from (33) is most conveniently done by Haag-Ruelle scattering theory [1]:

$$\begin{aligned} S_s \left|\theta_1^I, \theta_2^{II}\right\rangle &= S_s^{(2)} \left|\theta_1^I, \theta_2^{II}\right\rangle = e^{i\pi g} \left|\theta_1^I, \theta_2^{II}\right\rangle \\ S_s^{(n)} &= \prod_{pairings} S_s^{(2)} \end{aligned} \tag{35}$$

These formulae (including antiparticles) can be collected into an operator expression [32] :

$$S_s = \exp i\pi g \int \rho_I(\theta_1)\rho_{II}(\theta_2)\varepsilon(\theta_1 - \theta_2) d\theta_1 d\theta_2 \tag{36}$$

[9]The equality of the $\mathcal{A}(W)$ net (34) to the net obtained by the subsequent modular method adapted to the Federbush model is not a very easy matter.

Where $\rho_{I,II}$ are the momentum space charge densities in the rapidity parametrization.

The surprising simplicity of the wedge algebra as compare to say double cone algebras consists in the fact that one can choose on-shell generators. This is not just a consequence of the on-shell particle number conservation, but requires the energy independence of the elastic scattering. Another example is the Ising field theory.

For the more interesting factorizing models with energy dependent S-matrices we find the distinction between diagonal and nondiagonal $S_s$very helpful. Examples for the former are the Z_N-models whereas the various Gross-Neveu models have nondiagonal two-particle S-matrices.

Considering first the diagonal situation $S^{(2)} = e^{i\delta(\theta)}$ (with δ denoting the scattering phase shift), we define (assuming for simplicity charge neutrality) operators b in terms of the incoming $a^{\#}$:

$$\begin{aligned} b(\theta) \quad &: \quad = a(\theta) \exp -i\pi \int_{-\infty}^{\theta} \delta(\theta-\theta')a^*(\theta')a(\theta')d\theta' : \\ b^J(\theta) \quad &: \quad = a^*(\theta) \exp -i\pi \int_{\theta}^{+\infty} \delta(\theta-\theta')a^*(\theta')a(\theta')d\theta' \end{aligned} \tag{37}$$

It is easy to check that they fulfill the relation of the Zamolodchikov algebra [33]:

$$b(\theta)b(\theta') = S^{(2)}(\theta-\theta')b(\theta')b(\theta) \quad etc. \tag{38}$$

Combining the b-operator with its "j-adjoint" b^J , we obtain the following nonlocal but TCP-invariant (and therefore weakly local) operator:

$$B(x) = \int (b(\theta)e^{-ipx} + b^J(\theta)e^{ipx})d\theta \tag{39}$$

where b^J has been defined in 37. Its use facilitates greatly the construction of modular wedge localized states because it turns out that the closure of the real space:

$$\int f_n(x_1,....x_n) : B(x_1)....B(x_n) : \Omega, \quad suppf_n \in W^{\otimes n}, \ f_n \ real$$

is the desired $\mathcal{H}_R^{(n)}(W)$ i.e. solves the -1 eigenvalue equation for the "interacting" Tomita operator $S = S_s S_0$. The formfactors of local fields:

$$^{out}\left\langle p_1', ...p_m' \left| A(0) \right| p_1, ...p_n \right\rangle^{in} \tag{40}$$

can be expressed in terms of a vacuum expectation value which involves n+m fields $B^{\#}$ and one (at this instance still unknown) local field A :

$$\left\langle B(x_1)^{\#}...B(x_m)^{\#}A(x)B(x_{m+1})^{\#}...B(x_{n+m})^{\#}\right\rangle \tag{41}$$

Introducing the natural parametrization for the 2-dim wedge W in terms of the radius r and the x-space rapidity χ, the KMS property reads:

$$\begin{aligned} & \left\langle B(r_1,\chi_1)^{\#}...B(r_m,\chi_m)^{\#}A(x)B(r_{m+1},\chi_{m+1})^{\#}...B(r_{n+m},\chi_{n+m})^{\#}\right\rangle \\ = & \left\langle B(r_2,\chi_2)^{\#}...B(r_m,\chi_m)^{\#}A(x)B(r_{m+1},\chi_{m+1})^{\#}...B(r_1,\chi_1-2\pi i)^{\#}\right\rangle \end{aligned} \tag{42}$$

for $A=1$ and $n+m=4$ we obtain:

$$\begin{aligned} & \left\langle B(r_1,\chi_1+i\pi)B(r_2,\chi_2)B(r_3,\chi_3)^{\#}B(r_4,\chi_4)^{\#}\right\rangle \\ = & \left\langle B(r_2,\chi_2)B(r_3,\chi_3)^{\#}B(r_4,\chi_4)^{\#}B(r_1,\chi_1-i\pi)\right\rangle \end{aligned} \tag{43}$$

Expressing this in terms of the momentum space rapidity variables we get a special case of the well-known crossing symmetry for the S-matrix:

$$S_s(\theta)=S_s(i\pi-\theta) \tag{44}$$

The more general case of particles$\neq$antiparticles relates S_s^{pp} with $S_s^{p\overline{p}}$ and can be similarly discussed. A generalization to higher dimensions for which the analogon of the auxiliary fields B is unknown (and for which the presence of infinitely many L-transformed wedges gives rise to a consistency problem) is presently unknown, although the validity of the KMS property with its strip analyticity and the (not so well established) on shell crossing symmetry are certainly not independent analytic "symmetries" of QFT. They are both fundamentally linked with the issue of antiparticles. I expect that progress on this subject will relate modular localization to the ill-understood on shell analytic properties of QFT.

The most interesting case is the nondiagonal case where the "b-trick" does not seem to be available. In that case our strategy will be to abstract a complete set of analytical properties for the wedge-localized n-particle wave functions:

$$\begin{aligned} f^A(\theta_1,...\theta_n) &= \langle\Omega\left|A\right|p_1,...p_n\rangle^{in} = \langle\Omega\left|A\right|a^*(p_1),...a^*(p_n)\rangle \quad for\ \theta_1>...>\theta_n \\ &= \langle\Omega\left|A\right|b^*(p_1),...,b^*(p_n)\Omega\rangle \end{aligned} \tag{45}$$

where the ordering has been chosen because such incoming particles do not cross in the future, i.e. S_s acts trivially and the purpose of the second line is only to suggest the right boundary conditions for the meromorphic function in arbitrary θ-order:

$$\begin{aligned} f^A(\theta_{P(1)},...,\theta_{P(n)}) &= \lim_{\operatorname{Im} z_{P(1)}>...>\operatorname{Im} z_{P(n)}\rightarrow 0} f^A(z_1,...,z_n) \\ f^A(...,\theta_i,\theta_j,...) &= f^A(...,\theta_j,\theta_i,...)S_s^{(2)}(\theta_i-\theta_j) \end{aligned} \tag{46}$$

The second important suggestive role of the b-representation stems from the (right wedge) KMS-property (42):

$$\begin{aligned} & \langle\Omega\left|AB^*(r,\chi_1)...B^*(r_{n-1},\chi_{n-1})B^*(r_n,\chi_n-2\pi i)\right|\Omega\rangle \\ = & \langle\Omega\left|B^*(r_n,\chi_n)AB^*(r,\chi_1)...B^*(r_{n-1},\chi_{n-1})\right|\Omega\rangle, \quad A\in\mathcal{A}(W) \end{aligned} \tag{47}$$

or equivalently:

$$\begin{aligned} & \langle\Omega\,|AB^*(r,\chi_1)...B^*(r_{n-1},\chi_{n-1})B^*(r_n,\chi_n-\pi i)|\,\Omega\rangle \\ = & \langle\Omega\,|B^*(r_n,\chi_n+i\pi)AB^*(r,\chi_1)...B^*(r_{n-1},\chi_{n-1})|\,\Omega\rangle \end{aligned} \tag{48}$$

The *on-shell nature of the B-fields* permits an easy transformation of this KMS condition to momentum rapidity space ($\mathrm{p}_i \to \theta_i$):

$$\begin{aligned} & \langle\Omega\,|Ab^*(\theta_1)....b^*(\theta_{n-1})b^*(\theta_n+i\pi)|\,\Omega\rangle \\ = & \left\langle\Omega\,\middle|b^J(\theta_n-i\pi)^*Ab^*(\theta_1)....b^*(\theta_{n-1})b^*(\theta_n+i\pi)\middle|\,\Omega\right\rangle \end{aligned}$$

The quantum field theoretical interpretation of the fouriertransform of the on shell KMS condition is: the analytic continuation to the upper rim of the strip of the wedge-localized wave function of A is the crossed matrix element:

$$\begin{aligned} & \left\langle\Omega\,\middle|b^J(\theta_n-i\pi)^*Ab^*(\theta_1)....b^*(\theta_{n-1})\middle|\,\Omega\right\rangle \\ = & \langle\Omega\,|b(\theta_n)Ab^*(\theta_1)....b^*(\theta_{n-1})b^*(\theta_n+i\pi)|\,\Omega\rangle \end{aligned} \tag{49}$$

By iteration we obtain:

$$\begin{aligned} & \langle\Omega\,|Ab^*(\theta_1)....b^*(\theta_{k-1})b^*(\theta_k+i\pi)...b^*(\theta_n+i\pi)|\,\Omega\rangle \\ = & \langle\Omega\,|b(\theta_k)...b(\theta_n)Ab^*(\theta_1)....b^*(\theta_{k-1})|\,\Omega\rangle \end{aligned} \tag{50}$$

For nonselfconjugate particles the crossed operators on the left must be replaced by antiparticle annihilation operators. If the S-matrix has matrix indices (particle multiplets with nondiagonal 2-particle S-matrix), the f^A carry n matrix indices. Denoting by θ now the rapidity together with the multiplet index and using $\bar{\theta}$ for the rapidity of the antiparticle we have:

$$\begin{aligned} f^A(\bar{\theta}_n,\theta_1,\theta_2,...,\theta_{n-1}) &= \left\langle\Omega\,\middle|b^J(\theta_n-i\pi)^*Ab^*(\theta_1)....b^*(\theta_{n-1})\middle|\,\Omega\right\rangle \\ &= f^A(\theta_1+i\pi,\theta_2,...,\theta_n)=f^A(\theta_n-i\pi,\theta_2,...\theta_1) \end{aligned} \tag{51}$$

In terms of the original functions $f^A(\theta_1,...\theta_n)$ it is very instructive to group the boundary relations as follows:

$$\overline{f^A(\theta_1+i\pi,...,\theta_n+i\pi)}=f^A(\bar{\theta}_1,...,\bar{\theta}_n) \tag{ML}$$

$$f^A(\theta_n-2\pi i,\theta_1,...\theta_{n-1})=f^A(\theta_1,\theta_2,...\theta_n) \tag{KMS}$$

$$f^A(\bar{\theta}_n,\theta_1,...,\theta_{n-1}):=f^A(\theta_1,\theta_2,...\theta_n+i\pi) \tag{Def}$$

The last equation is the (iterative) definition of the crossed channel of the (right most) particle and the second relation is the analytic property of crossing, alias KMS. The first equation is the most general of all: the spatial modular localization relation. Although states of the form $A\Omega$ with $A\in\mathcal{A}_{s.a.}(W)$ fulfill this relation, it also holds for the rapidity wave functions of any vector ψ which lies in the Tomita graph closure (i.e. in the natural wedge localized space) of $\mathcal{A}(W)\Omega$. In that case the relation reads:

$$\overline{\langle\psi \mid \theta_1 + i\pi, ...\theta_n + i\pi\rangle^{out}} = \langle\psi \mid \theta_1, ...\theta_n\rangle^{in} \tag{52}$$

Unlike the special case of $\psi = A\Omega$ there are no furthergoing analytic properties beyond the π-strip analyticity (i.e. no globally meromorphic functions.

The other relation involving KMS property are only meaningful on modular localized vectors of the form $A\Omega$. The additional relation for factorizable systems which only hold in the absence of real particle relation are those which express the n! different boundary values distinguished by the order of $\theta' s$ i.e. their deviation from the reference order $\theta_1 > ... > \theta_n$ via a commutation relation (46). The above relations ML, KMS together with the definition Def and the commutation relations (46) appear precisely in this form as postulates in the recent work of Babujian Fring and Karowski [14][15]. Since the values on the upper strip boundaries are related by known analytic functions which depend on the ordering of the $\theta' s$ to the corresponding lower boundaries one may say that modular theory explains why many of the analytic techniques for factorizing models in momentum space are very similar to x-space analytic properties in Wightman's formulation of QFT in particular of conformal QFT.

There is one more equation in the BFK scheme of axioms which from or point of view is a kind of one particle structure property relating the wave functions or formfactors for different n [14][15]:

$$f^A(\bar{\theta}_n, \theta_1 ..., \theta_{n-1}) \underset{\theta_1 \to \theta_2 + i\pi}{\simeq} \frac{2i}{\theta_n - \theta_{n-1} - i\pi} f^A(\theta_1, ..., \theta_{n-2})(1 - S_{n-2,n-1} ... S_{1k,n-1}) \tag{53}$$

These "kinematical poles" outside the physical strip (but approaching its boundary have their physical origin in the so called one particle structure in QFT as first noticed by Stueckelnberg. In factorizing theories they take their above specific form. For nonselfconjugate particle situations the residuum on the right hand side is only different from zero if permitted by the charge superselection rules (i.e. one of the right hand particles must be an particle which can be contracted with the antiparticle symbolized by $\bar{\theta}_n$). Inside the physical region the S-matrix may have bound state poles. In that case one has to enlarge the scattering space by the Fock space of these new incoming particles. Their presence is also felt in the formfactors of the original particles; they have bound state poles and their residua are determined by the S-matrix poles and crossing (KMS). We do not need the concrete formulae from the one particle structures since we will not enter concrete model constructions in this paper. The method of finding the wedge localized Hilbert spaces is now the following. In case of particle multiplets (say SU(n)) we first decompose the vector $A\Omega$ into irreducible representations (highest weight vectors). Assuming that we already have a special solution f_w^A for the highest weight w, the most general solution which defines the localized wave function spaces is:

$$f_w^A : \mathcal{H}_0^{(n)} \to \mathcal{H}_w^{(n)} \tag{54}$$

$$\begin{aligned} g_w(\theta_1, ..., \theta_n) &= f(\theta_1, ...\theta_n) f_w^A(\theta_1, ...\theta_n) \\ f &\in \mathcal{H}_0^{(n)}, \ g_w \in \mathcal{H}_w^{(n)} \end{aligned} \tag{55}$$

Here $f \in \mathcal{H}_0^{(n)}$ ranges through the space of solutions of the above equations with trivial scattering S-matrix. It is easy to see that this space consists of periodic functions with no poles but an arbitrary number of zeros. The space is identical to that of formfactors of local operators in free theories. There are now two crucial questions to be answered:

1. Are the wedge algebras $\mathcal{A}_1(W)$ and $\mathcal{A}_2(W)$ of two theories the same if the spaces (i.e. their θ-space wave functions) are identical $\mathcal{A}_1(W)\Omega = \mathcal{A}_2(W)\Omega$? Since both wedge algebras are hyperfinite type III_1 factors, they are not only algebraically isomorphic but also unitary equivalent: $\mathcal{A}_2(W) = U\mathcal{A}_1(W)U^*$. So the question is $U \in \mathcal{A}'(W)$?

2. How can one obtain a special solution of the full equations for factorizing models?

The first question has a surprisingly simple answer. If the algebras on the vacuum are identical, then (thanks to the crossing relations!) the action on arbitrary incoming particle vectors is the same implying identity of the algebras. This answers the question of uniqueness of the field theory for a given admissable S-matrix i.e. the *uniqueness of the inverse problem.* In particular it shows that $S = 1$ with Bose- or Fermi-statistics has only the free field solution. This was a long standing problem; only for zero mass a solution was known [12].

The second question i.e. the existence for factorizable QFT is more difficult. In the diagonal case it would be very easy to write down special solutions, if it would not be for the pole structure. This is a problem for which an efficient method still has to be found. It turns out that the more difficult case of non-diagonal S-matrices for particle multiplets can be reduced to the previous case by the *nested Bethe-Ansatz method.* Depending on the weight, the solution can be shown to admit the algebraic Bethe[10] representation [13]:

$$f_{1\ldots n}(\underline{\theta}) = \sum_{\underline{u}} B_{1\ldots n,\beta_m}(\underline{\theta}, u_m)...B_{1\ldots n,\beta_1}(\underline{\theta}, u_1)\Omega_{1\ldots n} g^{\beta_1\ldots\beta_n}(\underline{\theta}, \underline{u}) \tag{56}$$

$$\Omega_{1\ldots n} = e_1 \otimes ... \otimes e_1 \in V \otimes ... \otimes V \tag{57}$$

[10]We use the terminology *Bethe Ansatz* (or representation) whenever state vectors of an interacting system are written as a superposition of products of known (matrix) operators. In the case at hand these operators are the collection (labeled by β_i) of $\underline{\theta}$-dependent B-matrices which depend on the additional parameter u. Whereas the perturbative Feynman "machine" exists in a fully developed form, we presently only have a rudimetary knowledge of the Bethe machine.

Let us restrict to SU(2) multiplets. In that case V is 2-dimensional. The $B's$ are known matrix-valued functions acting on $\Omega_{1\ldots n}$ which have an extra index β. They are expressed in terms of a suitably defined n-fold tensor product of the two-particle S-matrix [13] i.e. in terms of the so called algebraic Bethe Ansatz. The number m of B-factors is related by a simple formula to the highest weight of the representation. If we chose the coefficients g as:

$$g^{\beta_1\ldots\beta_m}(\underline{\theta},\underline{u}) = \prod_{i=1}^{n}\prod_{j=1}^{m}\psi(\theta_i - u_j)\prod_{1\leq i<j\leq m}\tau(u_i-u_j)f^{(1)(\beta_1,\ldots\beta_m)}$$

with ψ and τ being ratios of Γ-functions and $f^{(1)(\beta_1\cdots\beta_m)}(\underline{u})$ =constant [13], then $f_{1\ldots n}(\underline{\theta})$ solves the Riemann-Hilbert problem i.e. the set of boundary conditions without the formulae which relate the residuum of one-particle poles to lower particle $f's$[11]. The problem is not to find a special solution with a given pole structure, but rather the tuning of to the residua as demanded by the formfactor interpretation. This problem is absent in the case of trivial S-matrix $S=1$. We expect that this tuning is not necessary if we are only interested in the position of the wedge localizes Hilbert space $\mathcal{H}(W)$ inside the Fock space.

Conjecture 4 *The dense set of wedge localized states is the direct sum of n-particle components (55) where f^A now denotes a special solution of Riemann-Hilbert problem without the residuum condition on the pole structure.*

Our physical picture underlying this conjecture is that the S-matrix S_s and hence the wedge Tomita operator $S(W)$ commutes with the (on shell) particle number. But for interacting theories this cannot hold for smaller regions i.e. $[S(O),\mathbf{N}]\neq 0$ for double cones. This can be verified in the Federbush model and corresponds to the physical picture that localization beyond the wedge region requires "virtual particle creation". This is also the reason why factorizable models have such a rich virtual particle structure[12] even though there is no on shell creation. In fact the above conjecture suggests to sharpen the picture of the local Wightman domain (or algebraic range $\mathcal{A}(W)\Omega$) as:

$$\begin{aligned}\mathcal{A}(W)\Omega &= \mathcal{D}(W)\subset\mathcal{H}(W) \\ \mathcal{D}(W) &= \bigcup_{W\supset O_1} domS(O_1)\end{aligned} \tag{58}$$

In this conjectured formula $\mathcal{D}(W)$ denotes the (open) union of all domains of Tomita operators for double cones ranging over all unit double cones inside W.

[11] For n-particle multiplets obeying the SU(n) symmetry, the $f^{(1)(\beta_1,\cdots\beta_m)}$ for fixed $\beta' s$ are matrices acting on a tensor space $V^{(1)}\otimes\ldots\otimes V^{(1)}$ where $dimV^{(1)}=n-1$. In that case one needs a nested Bethe representation, i.e. one has to repeat the Bethe Ansatz n-1 times.

[12] For example their two-point function always have momentum space contributions above the Wigner one particle mass shell.

$\mathcal{H}(W)$ is the (closed in the temperature topology of section 3) Hilbert space of modular wedge localization. Accepting the correctness of this conjecture, the relation between the n-particle components originates from the noncommutativity of the Tomita operators $S(O)$ with the particle number and $\mathcal{D}(W)$ inherits this property. This is the origin of the rich virtual particle structure of local fields and of the interrelation of the n-particle components for different n of vectors obtained by applying these fields to the vacuum.

In case of existence of the Zamolodchikov operators B the component structure of $\mathcal{H}(W)$ is obvious since the commutation structure of the creation and annihilation parts of B with the particle number operator is the same as that of a free field and $\mathcal{H}^{(n)}(W)$ is obtained by n-fold application of the smeared $B's$ where the support of the smearing function is in W[13].

The most interesting part is passing from the localized states to *local* fields and algebras. Let me only explain this for the (diagonal) case where the $B's$ have been explicitly constructed. These fulfill all the modular covariance properties with respect to $J, \Delta^{i\tau}$ and S except that the vacuum is not separating for the B-algebra (related to their nonlocality). Consider now operators of the following form:

$$\begin{aligned} A &= \sum \int g^{(\#)}(y_1, ...y_n) : B^{\#}(y_1)...B^{\#}(y_n) : \qquad (59) \\ W &\ni \quad supp\, g^{(\#)}(x; y_1, ...y_n) \end{aligned}$$

Here we use # to denote the operator or its Hermitian adjoint and the superscript on g indicates that the coefficient functions depend on which $B's$ are replaced by Hermitian conjugate. Note that, contrary to free fields, the $B's$ are not commutative under Wick-ordering. As a result of the on shell nature of the $B's$, the above formula gives upon Fourier transformation:

$$A = \sum \int \tilde{g}_n^{(\#)}(p_1, ...p_n) : \tilde{B}^{\#}(p_1)....\tilde{B}^{\#}(p_n) : d\theta_1...d\theta_n \qquad (60)$$

Where now # stands for the momentum space annihilation or creation part. The J (TCP) -covariance and the Δ^{it} covariance gives a simple restriction on the coefficient functions. The only property of A which is missing in order to qualify as a generator for a wedge algebra in standard position is separability. This can however be enforced if the pure annihilation part of the nth contribution $\tilde{g}$ is related to the creation part so that a vanishing of the creation parts without a vanishing of the annihilation contribution is impossible, hence an A which annihilates Ω vanishes as an operator. Precisely this is guarantied if the different frequency parts of the nth contribution are identified with the various would be n-particle formfactors of A, as required by the definition of the $B's$. As already stated, separability of the wedge $A(W)$-algebra together with cyclicity and the

[13]The $B's$ can however not be used in order to generate Hilbert spaces for smaller than wedge localizations. Their auxilary role is strictly confined to modular localized wedge spaces.

above covariance property will enforce the "standardness" of its representation relative to the vacuum. Since S is then the Tomita operator of this separable subalgebra of the algebra generated by the $B's$ restricted to the wedge, the relative locality of the algebra for the opposite wedge is a consequence of modular theory. This separability argument is a significant analytic simplification and conceptual clarification as compared to the existing recipe for checking locality [11]. The previously presented Federbush model offers a nice explicit illustration for the usefulness of the $B^{'}s$ in the construction of the local wedge algebras and the local fields $\psi^{I,II}$.

Here the question remains if modular wedge localized states in factorizable models can always be represented in terms of nonlocal on shell operators B with positive and negative frequency parts which fulfill a Zamolodchikov algebra with a nondiagonal $S^{(2)}$-structure constants. I conjecture that this is the case. The fact that the Ψ_n for different n are related by (53) is favorable for such a conjecture, but even if one can show existence there is still the problem of finding the explicit representation in terms of incoming fields. It would be highly desirable to have a reconstruction theorem for the B-operators from the existence of wedge localized n-particle vectors and their formfactors. Closely related to the existence of the $B's$ is the existence of a "modular Møller operator" U defined by:

$$UJ_0U^{-1} = S_sJ_0 = J \tag{61}$$

The existence of this "square root" of the S-matrix S_s is easily shown to be equivalent to the existence of an "interacting" wedge algebra:

$$\mathcal{A}(W) = U\mathcal{A}_{in}(W)U^{-1} \tag{62}$$

which together with the vacuum vector belongs to the modular object $J, \Delta^{i\tau}$. In applying this modular Møller operator to the free field coordinates of incoming fields it is very important to realize that the point x of the resulting U-transformed field does nor denote localization around x. Although this field transforms in the standard manner under Poincaré transformations, its localization is spread (the "fuzziness" in section 3) all over the wedge. Compact localization can only be achieved by intersecting wedge algebras, and not by further physically interpretable covariant transformations. Furthermore the U (against naive expectations) cannot commute with the Poincaré transformations since otherwise the $\mathcal{A}(W)$ net is the same (i.e. the local algebras are unitarily equivalent with the same unitary) to the $\mathcal{A}_0(W)$ net [14]. However the requirement that the commutant be geometric i.e. $JA(W)J = PA(W)P^{-1}$, where P is any geometric transformation linking the right with the left wedge (in d=1+1 it is a parity symmetry in higher d there are also π-rotations). We will investigate the uniqueness (modulo the above freedom) and the physical significance of this

[14] In fact, as was shown by Wollenberg [7], the existence of such a U is equivalent to $(\mathcal{A}(W), U^{*}\Omega)$ having the modular object $(J, U^{*}\Delta^{\frac{1}{2}}U)$.

modular Møller operator U, as well as its computability in factorizable models and its possible relation to the Zamolodchikov operators in a separate paper.

5 General Interactions and Modular Localization.

We mentioned already that e.g. "free" d=1+2 anyons do not have a Fock space structure. So the functorial method of section 2 relating the Wigner space directly to operator algebras will not work, even though there is no genuine interaction. Since the inner products of multiparticle momentum states and the action of the Poincaré group is however explicitely known, one can start a program of computing n-particle wedge localization spaces and think about spaces of spacelike cone localization spaces and the associated operator algebras. This will not be done here, the main reason being that presently there are no substantial results on this program.

Instead we will address the important question whether the concept of modular localization can be expected to lead to a nonperturbative approach for d=1+3 interacting theories of Fermions and Bosons. Let us try to think about a scenario consisting of three steps.

1. Start with an auxiliary $S^{(0)}$-matrix which is $\mathcal{P}$-invariant (including TCP and J reflections) and fulfills some weak cluster properties. An example would be:
$$S^{(0)} = e^{i\eta}, \quad \eta = g \int : A^{(0)}(x)^4 : d^4x \tag{63}$$
where $A^{(0)}$ is a scalar free field. We use this initial S-matrix for baptizing the theory "A^4-theory"

2. Try to construct a modular localization space $\mathcal{H}_R(W)$:
$$S^{(0)} J \Delta^{\frac{1}{2}} \psi = \psi, \quad \psi \in H^{in}_{Fock} \tag{64}$$
without on shell conservation laws (which before lead to the Bethe Ansatz structure), this seems to be a tough mathematical problem.

3. Try to obtain more refined localization spaces. If multilocal spaces spaces could be constructed then the ideas of scattering theory could lead to an $S^{(1)}$-operator which would agree with the starting S-matrix $S^{(0)}$ if the the theory would be local (which it is not). Use $S^{(1)}$ as a new modular input. By playing the modular S-operator iteratively against a scattering operator one hopes that the iteration $S^{(0)}, S^{(1)}$.....could lead to a limiting local situation for which the modular and the scattering S agree.

The difference of such a hypothetical iteration to a deformation approach as standard perturbation theory would be that in this approach the operator properties (except locality) would be valid in every step. We expect the modular Møller operator U (61) to play an important role, since its existence is equivalent to the existence of an interacting wedge algebra.

As expected from the infrared aspects of standard perturbation theory, the zero mass theories also show a special behavior in the modular localization approach. In the latter case one has to face the physically more serious problem of a missing free reference theory resulting from the vanishing LSZ asymptotic limits of "infraparticles"[15]. This problem already arises in chiral conformal QFT with noncanonical scaling dimensions. The dialectic tension between analytic simplicity and conceptual complexity of massless QFT is one of the fascinating phenomena of contemporary research in QFT.

6 Resume and Outlook

Low-dimensional models of QFT, as e.g. chiral conformal theories or massive $d = 1 + 1$ models, have, apart from possibly condensed matter applications, no direct use in physics. However they are excellent laboratories for theoretical ideas about elementary particle physics. The standard perturbative approach has not only led to the well-known successes, but also created some folklore about nonperturbative aspects which are sometimes not entirely correct and even prejudicial. In the following we compile few of **these incorrect statements** which low-dimensional models solved by nonperturbative methods are able to correct:

- "The existence of QFT is endangered by bad short-distance behavior".

$D = 1 + 1$ soluble models show that this is not so. This is already clear for the simplest of them, as the Federbush model used in these notes, for which one does not even have to invoke the modular localization and the related bootstrap-formfactor-program. By changing a deformation parameter (the coupling constant), the inverse short distance powers may be chosen as large as desired. In the algebraic approach, based on nets of local algebras, the short distance properties are hidden and only appear via associated scaling algebras. It is presently not entirely clear which short-distance aspects are intrinsic net features, and which are attributes of particular generating "field coordinates".

- "QFT suffers from short distance divergencies".

This is closely related to the previous folklore. Is not even correct in perturbation theory. The divergencies in Feynman integrals are a result of the

[15]"Infraparticles" are "below" Wigner particles in the sense that their contribution to the two-point function is less singular than the Wigner (on shell) contribution.

Lagrangian (or canonical) quantization methods which starts from a slightly illegitimate picture about the nature of quantum fields and which needs repair by infinite renormalization. As Poincaré and Lorentz showed at the beginning of the century, these infinities are genuine in the classical theory because the particle picture has to be imposed on the classical field theory. However in local quantum physics, the particle properties are part of the Poincaré transformation properties of fields and as such follows from those. Indeed, if one changes the formulation slightly and aims first at a formally unitary source dependent perturbatively accessible operator $S(g)$ in an auxiliary Fockspace which fulfills the Bogoliubov axiomatic, then the problem of infinities is traded with the problem of Hahn-Banach extension of time ordered functions which are originally defined for noncoinciding arguments. An even more convincing finite methods is the split-point treatment of the nonlinear terms in operator field equation. Some of these manifestly finite methods are not very practical and therefore I recommend to stay with the original Feynman methods but to avoid to draw wrong philosophical conclusions from it. The nonperturbative modular localization method is obviously free of divergencies and even in the higher dimensional cases without an initial physical S-matrix where one expects at best an iterative procedure to succeed, one does not see how infinities could possibly enter. Certainly there is no place for short-distance infinities in a net approach. Neither is there a place for time-ordered products or a Dirac type "interaction picture". Instead one meets a new structure: auxiliary "on shell" fields and related algebras which are between the local interacting and the free incoming fields and which in special cases define a physical realization of the Zamolodchikov algebra and in more general cases seem to be related to a Bethe-Ansatz structure.

- "Lagrangians and actions are indispensable tools of constructive QFT".

With the exception of the Federbuch model and some similar almost trivial models, none of the models constructed by the modular bootstrap-formfactor method has been constructed by using such properties. In fact, even if one can affiliate a Lagrangian with such a model, as in the case of the Sine-Gordon or Thirring Lagrangian, it is mainly used for "baptizing" the model in a conventional manner and plays no role in its nonperturbative construction. In most cases, especially those without deformable coupling constants, a Lagrangian is not even known.

- "Supersymmetry is a rich symmetry with many physical consequences"

Whereas in low dimensions the issue of internal/external symmetries become inexorably intermingled, in the d=1+3 world there seemed to be no nontrivial way to marry internal and space-time symmetries. In some way supersymmetry seemed to lead to such mildly nontrivial looking marriage. Supersymmetry turned out to be a mathematically apparently powerful symmetry whose physical value is however highly questionable, to say the least. Using our best classification methods for symmetry [16], namely the Tomita-Takesaki modular, there is no trace of supersymmetry. In order to see that we are dealing here with an accidental symmetry, i.e. a symmetry of the field algebra which is not related to any additional physical insight obtained beyond the mentioned method of classifying charges we do not even have to invoke the modular bootstrap-formfactor program. A glance at the simpler situation in chiral conformal field theory already shows that e.g. in the case of the tricritical Ising model and similar supersymmetric models, the supersymmetric formalism does not play any role neither in their definition nor in their solution as members of a discrete or continuous family of non-supersymmetric models. They are also not in any way distinguished by short distance or other properties from their nonsupersymmetric neighbors.. Of accidental symmetries one expects of course instabilities under perturbation. Indeed, it has been shown recently that super-symmetry suffers a "collapse" in temperature KMS states, a situation totally different from any internal or genuine external (example Lorentz-symmetry) symmetry which suffers at most a spontaneous symmetry breaking related to the formation of phases in the theory of phase transitions. Supersymmetry has been used to allege the perturbative existence of a $d = 1 + 4$ conformally invariant gauge theory. Even within the context of perturbation theory such nontrivial gauge invariant correlation functions in $d = 1+3$ would be very interesting if not to say sensational. But within its more than 15 years of folkloric existence, nobody has calculated (or been able to calculate) such correlation functions, even in lowest order i.e. those calculations which have been done for standard theories and (nonsupersymmetric) gauge theories. Here one has some reason to suspect something underneath the carpet.

Our criticism concerning the alleged rich physical content of supersymmetry in no way applies to its use in mathematics.

We now turn to some *new results and problems*

- **"Cooking recipes" of the bootstrap-formfactor program permit now a more profound understanding.**

In a tour de force Smirnov has compiles a list of formal requirements which are designed to allow the extension of the Karowski-Weisz work on two-particle form factors to arbitrary many particles. He demonstrated the viability of these axioms by computing high formfactors of several models. These "axioms" were recently brought into a physically somewhat more transparent form within the so-called LSZ framework of QFT. By realizing that the postulated "crossing symmetry" property is reducible to the KMS property of modular wedge localized rector states, one found a way in which this important construction

[16] The space-time symmetry can be directly constructed from the modular groups of the family of wedge algebras, whereas the endomorphisms associated with charged representations leading to internal symmetries, fulfill the Takesaki "devissage" property with respect to the Tomita-Takesaki modular structure.

approach becomes incorporated into algebraic QFT. The traditional method of aiming at formfactors of fields has become somewhat cumbersome. From our experience with the Federbush model we hope that a direct construction (i.e. avoiding "field coordinates") of the wedge algebra may be simpler.

- **Understanding of a Bethe Ansatz-like nonperturbative approach to QFT.**

Bethe presented a technique by which certain low-dimensional problems in lattice and continuum QFT could be solved. Although it always looked like a shiny part of potentially impressive nonperturbation "machine", the Bethe Ansatz method never reached the same maturity and perfection as the perturbative Feynman "machine". Recently several authors have shown the usefulness of (appropriately adapted) Bethe Ansatz techniques in the bootstrap-formfactor program [13][14].

It appears that the constructive use of the modular localization method may now change the hole picture since it leads to Bethe-Ansatz structures in the most direct conceptual manner. I am using the terminology (generalized) Bethe-Ansatz machine also for the yet unknown analytic formalism behind the more general scenario of the previous section which I expect to emerge as a new constructive formalism in (non factorizable) local QFT.

- **Emergence of Geometry from domains of operators and ranges of algebras.**

One of the most surprising aspects of this new way of looking at local quantum physics is the encoding of geometric data in the net of domains of the modular S operators and the ranges of $\mathcal{A}(O)\Omega$. The initially nongeometric principles of local quantum physics are in this subtle way transformable into geometric properties. It is only through the detailed understanding of this connection that one can utilize differential geometry in physically correct way. Arguments of consistency of (global) geometrical structures with one or the other form of QFT formalism is not enough (example: the global vacuum structure for vacua which do not arise from spontaneous symmetry breaking as in Seiberg-Witten duality discussion).

Finally we comment on possible **relations of modular localization to other approaches** as Alain Connes noncommutative geometry scenario of gravity and electro-weak interactions and string theory.

To say simply that modular theory also occurs in Connes theory is not very revealing since its euclidean use just amounts to the euclidean charge conjugation and not to localization. In fact Connes recent attempts at gravity are probably more related to replace localization by another principle but it is presently completely unclear what this could mean in physical terms. For people who believe in the power of analogies one should perhaps point out that the DFR model, in which the spacetime labeling of nets is replaced by noncommutative spacetime, leads to the same double-sheetedness as in Connes theory. The spacetime uncertainty relations in the DFR work are of course not characteristic, since e.g. string theory also yields such relation. The strength and beauty of the DFR[34] approach lies in the fact that the authors are fully conscientious that they are trying to tinker with the number one principle of QFT namely localization, whose unsubtle removal would also destroy the physical interpretability. Therefore their proposal has the very desirable feature of being refutable by further future work. The physical alternative to causality as needed in Quantum Gravity is not *no causality* or *cutoffs in certain integrals,* but rather a new principle which does not wreck interpretation.

This brings us to a very important issue: a physical theory should carry its own interpretation with it. Local Quantum Physics fulfills this requirement; it has a localization concept from which one can derive scattering theory and a wealth of formulae relating to observations. A theory which only exists in euclidean (imaginary time) form cannot yet be called physical because one cannot derive this wealth of formulas but only plug in some extrapolated euclidean expressions. String theory was born as a proposal for a crossing symmetric S-matrix. Its later use in interpreting it as a kind of extended field theory resulted from purely formal games with the formalism and not by a conceptual analysis which would include the important issue of localization (or its unknown potential substitute). This leaves the question: why does string theory have this unreasonably seeming mathematical power if it comes just from physical formalism "running amok" (being the formalism of the old dual model augmented by differential geometry and "chased up" to 10^{19} GEV[17].) together with geometrical interpretation instead of physical concepts and principles? My tentative answer to this perplexing question is that behind that mathematical success there is an yet invisible form of the modular theory and the closely related subfactor theory. Since I am not an expert on string theory, I can only base my arguments on facts which have been seen in the string theoretic mode of thinking but which can be checked in low dimensional QFT models. Among the many illustrations which come to my mind, I will only mention one: the modular relation (in the Gepner-Witten and Capelli-Itzykson-Zuber[35] sense of the word "modular") for chiral conformal correlation function which relate the would be euclidean correlations associated to the torus and which for the zero-point characters was proposed on geometric reasoning by Verlinde. Note that we are avoiding to say "on the torus", since this would be dangerously close to "localized on" which is the wrong physical picture. These modular identities for chiral correlation

[17]As a student of Harry Lehmann I remember the following words of my adviser: "the chance that a particular formalism implementing a physical concept will still be valid after moving up in energy by two orders of magnitude will be smaller than 50%". It seems to me that a new sociological effect was left out: the popularity and reputation of a mathematically attractive theory grow slowly with its decreasing experimental verificability and this curve after sufficiently many orders of magnitude crosses the previous decreasing curve.

functions are very profound properties because they generalize the well known "Nelson relations" of temperature correlations of box-quantized fields which is the rigorous version of the formal symmetry of the Feynman-Kac representation under interchange of space with euclidean time: the period caused by the temperature may be interchanged with the size of the interval without changing the correlations. Although my past attempts to derive the geometrical modular identities from T.T.-modular theory failed [36], R.Longo was at least able to prove a weaker theorem from T.T.-modular theory. This illustrates that by geometrical consistency arguments one is able to discover a relation whose proper quantum interpretation (and proper physical analogies in higher dimensions) requires the modular construction of a (euclidean) dual euclidean theory. The physical weakness of such a geometric consistency arguments is explained by the fact that it has not been understood as a result of the principles of Local Quantum Physics.

Returning to Alain Connes method of noncommutative geometry, one realizes that there is a very surprising, almost philosophical connection with some of its recent mathematical output [29]: the impressive distinguished role of type III hyperfinite von Neumann factors in Connes extended Galois theory and penetrating study of the Riemann ζ-function. In order to see this from the physical side, let us remind ourselves of the "philosophical" underpinnings of algebraic QFT as Rudolf Haag expressed them at one occasion. The standard most widely accepted picture of physical reality is that of Newton and Einstein: a space-time manifold with a material content. On the other hand Leibnitz picture was different: "Monades" which have no individuality by themselves but which create reality by their interrelations. This fits precisely the nets of observables of algebraic QFT where a monad corresponds to the hyperfinite III_1 factor ("if you have seen one, you know them all"). Whether the fundamental appearance of type III factors in Connes extended Galois theory harmonizes with the same philosophy about physical reality is certainly food for future thoughts. In any case, the concepts and formalism developing around modular localization constitute a change of paradigm in QFT, and should be critically confronted in any attempt to go beyond "Laboratory QFT" into the direction of Quantum Gravity.

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