Coincidences between M(atrix) Theory and Algebraic QFT ?

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Abstract

This is a preliminary account of coincidences between algebraic QFT and some recent observations on "light cone quantization" and applications to "M" theory. The most surprising correspondence is that between the existence of wedge affiliated semiglobal field coordinates without vacuum polarization and those light cone fields (on which "Matrix" theory is based) with analogous properties. There is also an indication that algebraic QFT leads to a generic "Holographic" picture of entropy. As a rule of thumb quasiclassical and geometric pictures abstracted from string theory correspond to nonperturbative quantum properties in the algebraic approach to QFT. The unravelling of the latter requires new physical concepts.

1 Introduction

The aim of this note is the presentation of a surprising parallelism between some recent concepts of nonperturbative (algebraic) QFT [1] and what has been termed "M(atrix) theory" (a somewhat nondescriptive insider name) [2]. This coalescence of various ideas is even more impressive if one reminds oneself that the two areas have developed in complete isolation from each other in all respects; not only are the physical concepts and their mathematical implementations very different, but there is, to the best of my knowledge, no theoretician who has worked on both subjects or even only followed both. The situation is particularly interesting precisely as a result of the lack of a general conceptual agreement despite the parallelism of certain properties observed in this note[1]. Such situations of formal coincidences coupled with conceptual differences have proven to be very fruitful but unfortunately quite rare events in the history of physics; the most famous one was the difference in interpretation between

[1]The history of quantum theory is rich with examples of antinomies, paradoxa and clashes which turned out to be more favourable to progress than harmony.

Lorentz and Poincaré on the one hand and Einstein on the other concerning the Lorentz transformation in Maxwells electrodynamic.

The fastest way to get a glimpse of the present situation is to look at history of post Feynman QFT. The common cradle of all present frameworks of local quantum physics in general, and relativistic particle physics in particular, is Feynman's renormalized perturbation theory and the ensuing understanding of QED. In the aftermath of this work, three schools of thought have emerged. Although their aim was the same, namely to abstract a nonperturbative framework (in order to incorporate strong interactions), the paths taken towards this goal were quite different. The first path which we will refer to as Lagrangian field theory, took the covariant perturbative formalism as a starting point and interpreted it as a realization of an extension of the very successful quantization idea from quantum mechanics to the realm of infinite degrees of freedom. In this parallelism to classical physics, renormalization was understood (following a suggestion of Kramers) in analogy to the selfenergy problems of classical field theory if augmented by models of particles a la Lorentz and Poincaré. The occurrence of certain infinities was natural and, apart from problems of mathematical consistency, their "dumping" into physical parameters was already partially anticipated in the classical theory. The Lagrangian quantization approach led to functional integral representations which, if it would not be for the necessity to perform infinite renormalizations, could serve as nonperturbative definitions of physically relevant objects[2]. In the third section we will argue that there is a certain inner logic which drives this particular approach of QFT towards differential geometry. String theory and speculations on a unifying M-theory would not exist without the sophisticated geometrical framework. The method of (discrete) light cone quantization with the suggestion to make nonperturbative finite matrix approximations ("Matrix" theory) is a recent addition to string theory. Because of its QFT roots a comparison with the algebraic approach is reasonably straightforward. With string theory this is somewhat more difficult. In the third section we will emphasize those aspects which are important for the announced observations.

The second approach starts from the idea that in order to achieve a nonperturbative understanding, it is not sufficient to improve and extend the Feynman approach, but above all, one needs a new conceptual framework which can carry the same physical principles (as those underlying the perturbative approach) into the realm of a nonperturbative and intrinsic understanding and towards a new computational framework. A good test for the quality of such an approach is its capability to incorporate nonperturbative results based on recipes for the non-Lagrangian constructions of the bootstrap-formfactor program and conformal chiral QFT's. We will present the relevant results (for the comparison with the quantization approach) of this algebraic approach in the next section.

[2]In the lattice approach the discrete analogon to euclidean functional integrals is indeed taken as a definition and the renormalization is merged with the herculean task of controlling the scaling limit via second order phase transitions. This task was only accomplished in cases of existence of dynamical variables which are stable under scaling (e.g. the Lieb-Matthis-Schulz Ising model fermions).

We will have little to say about the third approach which developed from the dispersion theory for the S-matrix of the 60^{ies}. In its purest form of the Chew "Bootstrap" it suffered the same fate as Heisenberg's 20 years earlier attempt to formulate an S-matrix theory without localization and fields: the theory aborted due to a total lack of balance from too much ideology and too little computational power. Some good ideas as crossing symmetry together with certain on shell analytic properties found their way via the dual model of Veneziano into string theory and the modern differential geometric based Lagrangian quantization approach. Others, as e.g. an approximate selfconsistency for low energies, were merged with phenomenological ideas (example: PCAC, current algebras) into the low energy description via effective Lagragians as proposed by Weinberg; in this form some of the phenomenological S-matrix ideas survived up to present days.

Whereas it is fairly easy to compare the light cone quantization (and the related DLCQ) with recent findings in algebraic QFT and to obtain an interesting "cross culture" picture of "Holography", a direct comparison with string theory (or its even more elusive incorporation into M-theory) fails. The reason is that nobody has succeeded to give an intrinsic physical definition of the latter (a string field theory could be helpful for this purpose, but unfortunately does not exist). Whereas (algebraic) QFT has a complete intrinsic characterization (independent of its Lagrangian, functional, representation theoretical, bootstrap, etc. way of manufacturing) in terms of correlation function of observable fields or (if one wants to liberate QFT from e.g. the vacuum state in case of e.g. CST) nets of observable operators, no intrinsic characterization of string theory is known (string theory is only defined by its manufacturing process). If in string theory the "spacetime indexing" i.e. a localization structure is applicable, and the claimed successful application of light cone and spacelike quantization ideas [2] supports this, then it would just be a special form of high dimensional algebraic QFT[3]. In this case it should not serve as a model of quantum gravity but at best QFT in CST or a model with spin 2 particles since the speculative ideas about quantum gravity are much more radical (see end of next section) and definitely not compatible with any localization structure as that underlying quantizations based on notions like light cone and spacelike distance. Since we cannot resolve these inherent problematic conceptual points of string theory, we only limit ourselves to compare QFT with what one thinks are consequences of string theory.

The presentation of the recent M-theoretic ideas will be much shorter than the exposition of the algebraic approach in the next section. The reason is threefold: there are several detailed review articles by the inventors of these new ideas, the (M)atrix-theory is much better known (see the many hep-th papers) anyhow, and third, my knowledge, which is based on the reading of two review articles [2], does not go much beyond that of any interested and learned

[3]It would go against all physical intuition to reserve the terminilogy QFT only to its Lagrangian (or functional integral) version because the many nontrivial low-dimensional factorizing or conformal models and all those future constructions within the framework of algebraic QFT follow identical physical principles within different mathematical realizations.

amateur.

2 The Algebraic Approach

In the following I would like to give some background information on algebraic QFT which goes somewhat beyond what is needed for the comparison with the content of the mentioned reviews on M(atrix) theory. An even more extensive account the reader may find in [4]. Among all approaches algebraic QFT is certainly the mathematically most precise one. It is often overlooked that its main aim is the physical conceptual precision. In fact, as a consequence of our present flimsy physical intuition about the nonperturbative regime, the mathematical precision is a well founded temporary safety measure, far from any pedantry, in a still conceptual virgin territory of local quantum physics. It is very illuminating to look at its historical development in relation to standard QFT.

An important step away from perturbation theory was the abstraction of those structures from the renormalized perturbation theory which not only allow a clear interpretation in terms of spectral (positive energy) properties, causality and scattering concepts, but also permitted a rigorous mathematical control. This approach became known under the name Wightman-LSZ theory and (against the pragmatic spirit of most of its authors) was sometimes misleadingly called "axiomatic" QFT. Enriched with the superseletion idea and a natural dichotomy between local observables and charge carrying fields, it developed into the algebraic QFT of Haag, Kastler, Araki, Borchers and others [3]. The guiding principle was to avoid such parallelisms to classical theories as the Lagrangian quantization approach, in order to stay "intrinsic" as much as possible i.e. to avoid nonobservable concepts (as a Lagrangian) which cannot be reconstructed from correlation functions of observables or from the observable nets of algebraic QFT.

There were two important ideas which originated outside Feynman's approach, although they were consistent with the latter. One was Wigner's treatment of particles as positive energy ray representations of the Poincaré group, and the other the idea of Wightman Wigner and Wick, that the superposition principle of quantum theory and the von Neumann notion of observables is limited by superselection rules (generalized "charges"). Wigner's work successfully showed on the one hand that a relativistic quantum theory can be successfully formulated in an intrinsic manner, i.e. without reference to an outside classical theory. Related to this is, on the other hand, the even more important message that, contrary to the classical field theory which had to be supplemented by models of particles (pointlike, extended with additional stabilizing forces or soliton like models), the *particle aspect* of relativistic quantum theory is *already contained in the Poincaré covariant local observables,* i.e. there is no necessity and even no place for a model of particles on top of classical fields. In this way the renormalization infinities in Feynman's approach are more a part of a technical "repair job" on a slightly incorrect and too classical starting point inherent in Lagrangians (canonical or path integral quantizations) rather than an intrinsic conceptual problem of relativistic local quantum physics. The message contained in the superselection concept of WWW was even more seminal than that from the Wigner theory. The different superselection sectors invite an interpretation which is analogous to group representation theory: they are to be considered as *inequivalent representation sectors* of an underlying C^*-algebra of observables which are labeled by generalized invariant "charges". Together with the causality[4] and localization structure of QFT (which was not yet considered by WWW), this became in the hands of Haag and collaborators [3] the powerful conceptual framework of "nets of local observables" which provides the most direct link with the causality and stability (spectral, positive energy) principles. For a long time, the main issue of this approach to local quantum physics was the structural insight into nonperturbative QFT (as documented in many deep theorems) as well as the beginnings of a systematic classification of models of local quantum physics which incorporates and extends that of chiral conformal QFT. Only recently, the new concept of "modular localization" [11]also opened the path towards nonperturbative construction of models. We will return to these developments below.

Renormalized perturbation theory for correlation functions belongs to the common ground for which the standard and the algebraic approach yield the same results, a fact which should be kept in mind through the following discussion which emphasizes conceptual differences. In particular the elaboration of the deformation of free fields from the algebraic point of view already anticipate some of the later modular concepts. The deformation point of view of renormalized perturbation theory is also more cautious about existence of theories behind the "Lagrangian baptization" of interactions in terms of the addition of Wick-polynomials to zero order hamiltonians or as the Fock space causal deformation interaction in a Bogoliubov-Shirkov-Epstein-Glaser approach. In fact to avoid any illusion that perturbation is overstatingly interpreted as being a perturbative expansion of an existing theory, we prefer the neutral terminology *"deformation" of free fields*. Different from QM, this way of introducing interactions is beset by all kinds of existence problems which are not less threatening just because many physicist have gotten used to them. There is no reason whatsoever that strictly renormalizable theories exist beyond the deformation theory (even if instanton structures are taken into account!). But even if the Bogoliubov axiomatics has no solution beyond the known superrenormalizable interactions (example: ϕ_2^4), this is not the end of local quantum physics but only a limitation of Lagrangian quantization or the deformation implementation of interaction (by Wick polynomials in the undeformed free fields. There are other ways of introducing interactions; in fact, as explained later "Modular Localization" offers an alternative possibility to give an meaning e.g. to words as "ϕ^4" which is more related to on shell properties and significantly more intrinsic [11]. The apparent Lagrangian link between short distance behavior of fields and ex-

[4] Causality and the ensuing net structure has no counterpart in the group theoretical analogy. Among other things, it allows to relate representations with endomorphisms of the algebra [3].

istence of theories (i.e. the borderline superrenormalizable/renormalizable) is not at all natural from the algebraic viewpoint[5]; e.g. there is no trace of "field theoretic existence threatened by short distance behavior" in the modular approach, and among the QFT's constructed by the d=1+1 bootstrap formfactor method as well as in conformal field theory there are many models with variable interaction strength which have as bad short distance singularities as one wants.

The well known standard deformation approach nowadays usually starts from Feynman-Kac representations involving the (euclidean) action $A = \int \mathcal{L}d^dx$. It is an "off-shell" formalism in which differential identities (as equation of motions) can only be fed back into the Lagrangian with great care; preferably not at all. On the other hand the more algebraic perturbative approach is based on invariant local polynomials $W = Pol_{inv}(\psi)$ where we use ψ as a generic notation representing a collection of free fields and all the free field identities may be used. These $W's$ are invariant polynomials in the Borchers class of free fields [3] which live in a given auxiliary Fock space, i.e. a Fock space which is not yet to be identified with the physical particles. The formal expression for the local (Bogoliubov-Shirkov) unitary transition operator is:

$$S(g) = T \exp iW(g) \tag{1}$$

where $suppg(x)$ is the outer, and $\mathcal{O} = \{x; g(x) \equiv g\}$ the inner localization region[6] and as usual T denotes the time ordering. In view of the expected Poincaré covariance, both localization regions are usually taken from the Poincaré covariant family of compact double cone regions.

This formal expression allows a characterization in terms of the already mentioned Bogoliubov axiomatics (which is based on an apparent strengthened form of causality) and the latter permits a deformation theoretic solution in terms of a power series in g. An important technical aspect of this deformation method is the Epstein Glaser formalism for the extension of time ordered off diagonal distributions to the diagonal of coalescent points. This (Hahn-Banach) extension process replaces the cutoff or regularization method of the standard approach. The latter (cutoff, regularization) is often misunderstood as the physicists tool for verifying the existence of a deformed operator formalism in the ambient Fock space. Whereas regularizations may lend a certain amount of plausibility to such an idea, if it comes to proof, there is no regularization dependent substitute or short cut for the Epstein Glaser approach. In spite of popular opinion, physically interpretable theory with a cutoff or regulator simply do not exist, this is at best a formal mathematical device[7]. A *deformation solution* of the

[5]In the algebraic approach the individual field coordinates and their short distance behaviour loose their threatening power about the existence of theories and the scaling and short distance behaviour in the sense of algebraic QFT [8] becomes relevant.

[6]Here the constant g represents a set of coupling constants (deformation parameters); one for each invariant monomial contribution.

[7]No interpretable QFT which violates Einstein causality has ever been constructed. Only the increasing lack of conceptual precision in the last quarter of this century (in particular the vageness by which many physicist the very subtle and limited relation with euclidean theories) has created this false impression. One attempt to manipulate a relativistic cutoff into any of

causal unitary Bogoliubov axiomatics as provided by Epstein and Glaser does of course say nothing about the convergence of the so obtained power series.

This formalism may be extended to the deformation construction of fields and their composites. In fact it may be shown that one obtains a *deformation theoretic* definition of a causal operator net (a formal *-algebra in the sense of power series) inside the inner localization region of g, and by inductive limit also gets the full net [5]. With other words, the deformed net (considered as the deformation of the original free field net) is already formally correct (in the sense of deformation theory) inside the inner localization region without having to face the difficult issue of the "adiabatic limit" i.e. the true particle content and (by reconstruction in this limit) the physical Hilbert space as opposed to the auxiliary Hilbert space for the net. The existence of a deformation adiabatic limit requires the absence of infrared divergencies and leads to the possibility of choosing the Hilbert space of the coupled free fields identical to the physical reconstructed Hilbert space (the incoming Fock space of scattering theory).

The advantage of a dichotomic approach, in which the algebraic (net) structure is separated from the issue of constructing the relevant states (ground state, temperature states) on this net is the better positioning of the physically delicate points and a greater flexibility of choosing between ground or thermal states within one fixed algebraic formalism. Thanks to this deformation theoretic finite approach it is not too difficult to see the validity of interacting equations of motions in the form of differential identities between the "elementary" fields and their composites. Instead of a deformed Wick product formalism (called "normal products"), the latter may also be defined in terms of point split spacetime limits (natural intrinsic regularization). In fact these point split field equations may be used as the starting point of a completely finite perturbation theory for fields as advocated by Schwinger and illustrated on the Schwinger model of massless QED_2. The resulting operator expressions correspond formally to the iteration+renormalization of the Yang Feldman equations.

As a result of the peculiar nature of the noncompact zero mass Wigner little group, the deformation theory for zero mass fields with helicity ≥ 1 poses a localization problem [1]. The only presently known technical trick for incorporating such cases into the deformation formalism is to enforce formal pointlike locality by leaving the realm of quantum theory through indefinite metric[8] extensions which lead to Lorentz covariant (formally) local pointlike vector fields. These fields serve the deformation mathematics but hide aspects of physical localization. Such constructions would be physically worthless if Becchi Rouet and Stora would not have provided the BRS formalism, a regularization independent cohomological method to *return to local quantum physics*. The associated aux-

the existing nontrivial d=1+1 factorizing theories would bring anybody back to reality: no physical impunity for violating causality, or the principles of the beginning of this century win against certain pictures of wrong real time interpretations of the renormalization group formalism of the 70^{ies}.

[8]For purposes of mathematical controll it is better to use the equivalent method to stay with Hilbert spaces but sacrifice the unitary- in terms of pseudounitary-representations of the Poincaré group.

iliary factor space $H_{phys.} \subset H_{pos.semidef.}$, which generalizes that of the Gupta Bleuler method (a method without cohomological aspects which only applies to the abelian case), has a rather complicated (interaction dependent) position relative to the embedding space, at least for nonabelian gauge theories. In fact the restriction of the inner product to this factor space is positive only in the weak sense of positivity for formal power series [6]. It is the advantage of the algebraic versus the Lagrangian concepts that the relation between this weak positivity and the BRS cohomology becomes clearer. Only if the physical theory has a particle interpretation and a scattering theory one can choose the auxiliary Fock space equal to the physical one, and for this choice the range and kernel subspaces of the BRS charge have a fixed position and weak positivity becomes standard positivity. There are good reasons to view these magic of the BRS cohomology as an indication of nonunderstood aspects of a physical nonlocality (which shows up in a more detailed discussion of modular localization and Haag duality for multiple connected corona regions) of gauge theories which is put into a unphysical but formally local straightjacket as demanded by a stable deformation. What seems to be missing presently is an understanding of an alternative route to renormalized observables which avoids the pointlike interaction polynomials in the pointlike free fields; this is what the BRS ghost "catalyzer" i.e. the result of first introducing ghosts and then getting rid of them at the end seems to tell us.

A somewhat similar situation one meets in the case of interactions W involving massive spin ≥ 1 theories. In that case there is no gauge aspect originating in the Wigner theory, however in $d = 1+3$ there are no local "field coordinates" (even for vectormesons the transversality condition enhances the operator dimension by one i.e. $dimV_{\mu} = 2$ instead of ones as for zero mass) which allow a W with $dimW \leq 4$, which is the limit set by renormalizability. In that case the same method of extension by ghost fields succeeds to introduce an extended auxiliary formal interaction W_{aux} with $dimW_{aux} = 4$. In order to maintain the mass nonzero and at the same time keep the nilpotency of the BRS charge, it is sufficient (probably also necessary) to add another scalar non-ghost field, the perturbative Higgs field. All the statements refer to the deformation method and should not be interpreted as a e.g. proof in favor of a Higgs particle as an intrinsic attribute of a theory of "nonabelian" massive vectormesons (self-interacting vectormesons contributing to their own sources). In this massive case the formal ghost approach hides a nonunderstood synthesis of locality with renormalizability which goes beyond the naive use of pointlike composites in the free field Borchers class[9]. The algebraic approach is expected to have a better chance to unravel why the standard deformation approach requires the introduction of ghost fields, only in order to get rid of them after they fulfilled their mysterious purpose (and whether a nonperturbative understanding asks for similar concepts). The present understanding of this ghost issue goes against the Bohr Heisenberg spirit of intrinsic physical comprehension; the reference to

[9]The deformation of Borchers class of pointlike local fields may not exhaust the possibilities of realizations of locality in the interacting theory.

the beauty of classical fibre bundles is of no help in relation to this quantum physical phenomenon. As long as one keeps this in mind, there is no danger to future physical progress in appreciating the mathematical beauty and the present physical utility of the BRS formalism. These brief remarks about the algebraic interpretation of perturbative gauge theory are in no way an replacement for a recent systematic account [6]

The nonperturbative picture on $d = 1 + 3$ gauge theories coming from the algebraic approach is interesting, albeit not yet rigorous and very preliminary indeed. Finite energy requirement for particle states lead, at least in the presence of a mass gap, to a semiinfinite string like localization as the a priori best possible localization [5]. The standard point like localization is a special case. A finite energy sector which saturates the allowed semiinfinite (Mandelstam-like) localization is called a "topological charge" sector [3] (in the sense of algebraic QFT). In the case of a mass gap the asymptotic direction of the semiinfinite string is undefined (i.e. it fluctuates strongly). This picture coalesces with what one thinks about the so called unbroken nonabelian gauge theories in the standard approach. Unconfined abelian (Maxwellian) charges behave differently. Structural investigations show that their semiinfinite localization is "frozen", which is in accordance with the picture one has about the noncompact localization of infrared photon clouds surrounding a physical electron state with an almost sharp velocity [7]. In the case of the algebraic counterpart of spontaneously broken gauge invariance which leads to a localization improving mass gap generation, one is invited to think about (gauge invariant) semiinfinite charge creators which loose their charge and condense into the vacuum. This picture is much closer to Schwinger's original proposal [9] about the mechanism of charge screening in Maxwell like theories than to the vacuum condensation of Higgs fields (with all the conceptual problems surrounding the notion of gauge invariant condensates) [4]. In fact in order to get this point across, Schwinger invented his soluble QED_2-model. A recent investigation [8] of this model (with all the conceptual care) revealed, that the above interpretation of the Schwinger-Higgs formalism in terms of condensed strings is indeed correct. Whereas the condensates are extrinsic i.e. depend on conventions and cannot be read off from the gauge invariant content of the model, the charge screening property together with the picture of reemerging charges in the scaling limit is intrinsic. The model also showed that the two-step picture of a "fattened Goldstone particle" is limited to the perturbative folklore; although in two dimensions there is no Goldstone mechanism for continuous symmetry breaking, there exists a perfectly healthy Schwinger-Higgs mechanism.

In the realm of perturbation theory the results of different approaches to gauge theories of course agree and the main reason for presenting this issue at all, was the difference in interpretation. It should be clear from the mentioning of the Lorentz+Poincaré versus Einstein interpretation of the Lorentz transformations, that problems of concepts and interpretations are very important and that at the end of this century they should be expected as important as they were at the beginning.

After having prepared some ground, we are now ready to sketch those nonperturbative ideas and results which we would like to confront in the last section with LCQ and (M)atrix theory. For this purpose, we use yet another theory which starts with the magic letter M, namely the (Tomita-Takesaki) Modular Localization theory. Although being linked inexorably with the principles of relativistic local quantum physics [3], it is (contrary to differential geometry) not so visible from the standpoint of perturbation theory and, as we have argued at the beginning of this section, it remains even somewhat hidden in free field theory. We just list the results together with some comments. For a QFT with a mass gap with a complete scattering interpretation, one can derive the following facts [1][11].

- The modular theory of wedge algebras is geometric with the modular group Δ^{it} being the wedge associated Lorentz boost Λ_w of the incoming particles and the modular reflection J being related to that of the incoming free field theory J_{in} through the scattering operator (S-matrix) S_{scat}:

$$\Delta^{it} = U(\Lambda_w(2\pi t)), \;\; J = S_{scat} J_{in} \tag{2}$$

  The dense set of wedge localized state vectors can be represented in the form:

$$\begin{aligned} H_{loc}(W) &= H_R(W) + iH_R(W) \\ H_R(W) &\equiv \overline{\{\psi; S\psi = \psi\}}, \; S \equiv J\Delta^{\frac{1}{2}} \end{aligned} \tag{3}$$

  where $H_R(W)$ is the real closed subspace generated by the $+1$ eigenvectors of the antilinear unbounded Tomita operator which is involutive on its domain $S^2 = 1$. This brings the thermal Hawking Unruh aspects, which one usually relates with black holes, into ordinary QFT [11][12].

- As a standard reference wedge W_{stan} we may take the z-t wedge in which case we call z,t the longitudinal and x,y the transversal coordinates. This situation suggests to decompose the Poincaré group generators into longitudinal, transversal and mixed generators

$$P_\pm = \frac{1}{\sqrt{2}}(P_0 \pm P_z), \; M_{0z}; \; M_{12}, \; P_i; \; G_i^{(\pm)} = \frac{1}{\sqrt{2}}(M_{i0} \pm M_{iz}), \, i = 1,2 \tag{4}$$

  The generators $G_i^{(\pm)}$ are precisely the "tranlational" pieces of the euclidean stability groups $E^{(\pm)}(2)$ of the two light vectors $e^{(\pm)} = (1,0,0,\pm 1)$ which appeared for the first time in Wigner's representation theory for zero mass particles. More recently these "translations" inside the homogenous Lorentz group appeared in the structural analysis of "Modular Intersections" of two wedges [14]. Its role is analogous to that of the true translations $P_\pm$ with respect to halfsided "Modular Inclusions" [14]

1. As one reads off from the C.R., $P_i, G_i^{(+)}, P_\pm$ have the interpretation of a central extension of a transversal "Galilei group"[10] with the two "translations" $G_i^{(+)}$ representing the Galilei generators, P_+ the central "mass"

[10] This G's are only Galileian in the transverse sense; they tilt the wedge so that one of the light like directions is maintained but the longitudinal plane changes.

and P_- the "nonrelativistic hamiltonian". The longitudinal boost M_{0z} scales the Galilei generators $G_i^{(+)}$ and the "mass" P_+. Geometrically the $G_i^{(+)}$ change the standard wedge (it tilts the logitudinal plane) and the corresponding finite transformations generate a family of wedges whose envelope is the halfspace $x_- \geq 0$. The Galilei group together with the boost M_{0z} generate an 8-parametric subgroup $G^{(+)}(8)$ of the 10-parametric Poincaré group:

$$G^{(+)}(8): \; P_\pm, \; M_{0z}; \; M_{12}, \; P_i; \; G_i^{(+)} \tag{5}$$

The modular reflection J transforms this group into an isomorphic $G^{(-)}(8)$. All observation have interesting generalizations to the conformal group in massless theories in which case the associated natural spacetime region is the double cone.

- The position of the subspace $H_R(W)$ within the incoming Fock space allows to define a modular Møller operator $U(W)$ which intertwines the wedge affiliated Tomita involution S with that of the corresponding incoming involution:

$$SU(W) = U(W)S_{in} \tag{6}$$

and leaves the vacuum unchanged. The interacting wedge algebra $\mathcal{A}(W)$, which together with the vacuum vector has Δ^{it} and J as its modular data, is defined in terms of $\mathcal{A}_{in}(W)$ as:

$$\mathcal{A}(W) \equiv U(W)\mathcal{A}_{in}(W)U^*(W) \tag{7}$$

The Haag's theorem prevents the existence of analogous intertwining unitaries for the type I equal time canonical algebras:

$$A_{can} \equiv \cap_\varepsilon A(\varepsilon) \tag{8}$$

which in the algebraic approach are represented as the intersection of time slice algebras of thickness ε. This leads to the nonexistence of the interaction picture in local quantum physics and the necessity of the artificial infinite volume limiting procedure involving a quantization box (which is unfortunately not related to modular localization). The above intertwining relation between the unique hyperfinite type III_1 algebras (all localized subalgebras in local quantum physics which have a nontrivial causal complement are of this kind) is protected against such No Go theorems. The existence of the *modular* Møller operator [15]$U(W)$ (in QFT it does not seem to be possible to define a "scattering" Møller operator) leads to the existence of generators of $\mathcal{A}(W)$ which are localized in $\mathcal{A}(W)$ but allow no smaller localization inside W i.e. they are nonlocal inside W. They are "on shell" i.e. contain a negative frequency part which annihilates the vacuum. Formally they are given as $A_W(x) = U(x)U(W)A_0(0)U^*(W)U^*(x)$. It is essentially the absence of vacuum polarization pairs i.e. the mass shell support of their Fourier transform which makes these generators[11] of $\mathcal{A}(W)$ extremely helpful. In factorizable d=1+1 theories, the positive and negative frequency components of these semiglobal operators fulfill the Zamolodchikov Faddeev algebra [11]. Be aware that the x in the U transformed fields has nothing to do with localization around x inside W. Rather localization has to be constructed via smaller algebras defined by intersecting wedge algebras:

$$\begin{aligned} \mathcal{A}(\mathcal{O}) &= \bigcap_{W \supset \mathcal{O}} \mathcal{A}(W) \\ \mathcal{A}(W) &= U(\Lambda)\mathcal{A}(W_{stand})U^*(\Lambda) \\ W &= \Lambda \cdot W_{stand} \end{aligned} \tag{9}$$

where W_{stand} is the standard x-t wedge and the net of W's is generated from W_{stand} by Poincaré transformations. It is very important for the interacting case to realize that $U(W)$ depends on W i.e. $U(W_{stand})$ does not commute with the spacetime transformations $U(L)$ except with the W-associated Lorentz boost. It should be clear that ideas about how to construct such $U's$ should not be viewed in the setting of the perturbative split $H = H_0 + H_{int}$ (the free incoming situation does not correspond to H_0). Whereas the representations of the Poincaré group of the interacting situation agrees with that of the incoming fields, this is not so for the unperturbed theory belonging to H_0. In fact the latter theory does not even live in the same Hilbert space (only its local folium of states agrees with that of the interacting theory).

- The Møller operator $U(W)$ can be explicitly computed for d=1+1 factorizible models and it is intimately related to the Riemann-Hilbert properties of the modular localized real subspace $H_R(W)$. The mass shell components of the nonlocal generators $A_W(x)$ turn out to satisfy [11] the Zamolodchikov-Faddeev algebra and the modular localization property defines a thermal KMS state on this algebra.

- The rich physical structure emerging from inclusions and intersections of local algebras in the net. Algebraic QFT generates the external (spacetime) and internal global symmetries from how one algebra is positioned with respect to another one.

    1. **"Shallow" inclusions**. These are inclusions $\mathcal{N} \subset \mathcal{M}$ which posses a (noncommutative) conditional expectation i.e. $\mathcal{N} = E(\mathcal{M})$ of the kind studied by Vaughn Jones. The local endomorphism of algebraic QFT of the DHR theory and its extension to low dimensional QFT

[11] Strictly speaking these polarization free operators are only well-defined objects in the thermal Hilbert space associated with the modular wedge localization. However they are very valuable as a basis for the pointlike local fields which allow for a nonsingular extension outside the wedge [11].

are typical examples. The relation between the algebraic QFT endomorphism and the Jones setting is well understood. Shallow inclusions are related to inner symmetries[12]. For d=1+3 dimensional theories this leads to Fermi-Bose Statistics and compact internal group symmetry whereas for low dimensional theories this yields the class of (physically) admissable unitary braid group representations through Markov traces on the braid group B_∞.

2. **"Deep" inclusions**. By this one means inclusions which have no conditional expectation but obey a modular restriction which lead to spacetime symmetries. There are two "modular inclusions" whose geometric consequences have been studied: halfsided modular inclusions and modular intersections. The first case is illustrated by two touching wedges, a situation resulting from a light like shift of a wedge into itself. A halfsided modular inclusion leads to noncompact group isomorphic to the longitudinal (d=1+1) Poincaré group. The second case of modular intersections is illustrated by two wedges which have one light ray in common. In that case the intersection data lead among other things to the above Galilei generators. The full Poincaré group is obtained by the relative "modular position" of a finite number of algebras (the minimal number depends on the spacetime dimensionality). In this way one may generate the whole net from a finite "modular skeleton net".

Some comments are in order.

Let us add to these rigorous results two conjectures which are important for the later comparison with (M)atrix theory. These conjectures are related to the Hawking-Unruh issue of horizon physics of quantum matter in black hole solutions. Even though our comments are only conjectures, we will try to stick to the conceptual rigor of the rest of this article. For this reason we will not use the word "quantum gravity" in this discussion and emphasize the fact that the physical origin of the Hawking temperature is the *modular localization* in Minkowski- or curved- spacetime and not primarily a black hole horizon. The latter mainly plays the role of enforcing a natural localization (by creating a bifurcated horizon via the e.g. black hole metric) and constitutes a special case of the former. The notion of a bifurcated horizon through modular localization is more abstract, since it is not related to metric Killing vectors but rather to isometries in the space of wave functions or the underlying Hilbert space of QFT [11]. It nevertheless leads to the same physical consequences of thermal behavior and Hawking radiation. For this reason the main concepts which are usually attributed to gravitation theory can be perfectly understood in terms of *thermality through localization* (instead of the standard heat bath thermal

[12] Note however that in low-dimensional QFT's there si no sharp distinction between inner and spacetime symmetries [10][4]..

behavior). The main difference of the two thermalization concepts can be traced back to that of the two sided spectrum of the modular localization operators (e.g. the Lorentz boost) versus the one sided spectrum of the heat bath hamiltonian which leads to the boundedness of $e^{-\beta H}$.

This raises the question whether modular localization also leads to a fundamental algebraic notion of entropy. Here it is helpful to mention the "degrees of freedom" counting in local quantum physics which deviates in an interesting and significant fashion from that in e.g. Schrödinger quantum mechanics [3]. In the latter case one learns, that the phase space cells (standard notion of localization and momentum restriction) leads to a finite number of degrees of freedom per $2\pi\hbar$ size phase space cell. The first attempt in algebraic QFT by Haag and Swieca led to the notion of compactness. Later this notion was sharpened to the "nuclearity criterion" of Buchholz and Wichmann [3] which does not use a sharp cutoff in hamiltonian- resp. in the "modular"-energy but rather an exponential fall off. Contrary to the nonrelativistic case and to popular opinion, the relativistic localization concept (as opposed to the standard box quantization) together with the finite energy projection does not lead to a finite number of quantum states ("bits") but rather only to a compact (Haag-Swieca) or nuclear (Buchholz-Wichmann) set. A computation for free fields reveals that this behavior is optimal i.e. local quantum physics cannot reproduce the finite degrees of freedom behavior of quantum mechanics, but comes pretty close:

$$\begin{aligned} H-S &: \quad \{P_E A\Omega \mid A\in\mathcal{A}(\mathcal{O}), \|A\|\leq 1\} = compact\, set \\ B-W &: \quad e^{-\beta H}\mathcal{A}(\mathcal{O}) = nuclear\, set \end{aligned} \tag{10}$$

The interrelation between these slightly different forms of relativistic "local degree's of freedom counting" has been discussed in [3]. This property forbids infinite towers of particles (as they occur e.g. in genus ≤ 1 string perturbation) and an associated limiting Hagedorn temperatures. The most valuable consequence is a very profound interpretation of the ancient issue of Heisenberg-Weisskopf vacuum fluctuations: if a spatial volume is not interpreted as a quantization box, but rather as a region for localization of a partial charge via a conserved current (say inside e.g. an already defined Minkowski space free field theory), then the vacuum fluctuation near the boundary are infinitely large. In order to control them, it is necessary to allow a smooth transition to zero charge density inside a "collar" around the localization region. The "split property" of algebraic QFT [3], which is a consequence of the above "nuclearity" property of degree of freedom counting, just provides the mathematical precision for this intuitive idea[13]:

$$\exists\; type\, I\;\, factor\; \mathcal{N}\;\, s.t.\;\, \mathcal{A}(\mathcal{O}_1)\subset\mathcal{N}\subset\mathcal{A}(\mathcal{O}_2) \tag{11}$$

Here one should imagine two concentric double cones $\mathcal{O}_i$ with their associated hyperfinite III_1 factors. The type I factor $\mathcal{N}$ has a "fuzzy" localization inside

[13] The idea to define entropy with the help of the split property is due to Heide Narnhofer (this was pointed out to me by H-W. Wiesbrock) [17]. It constitutes one important element in a joint unfinished project of Wiesbrock with the present author.

the bigger double cone, and it is just this fuzziness which allows the definition of partial charges without infinite vacuum fluctuation and with a clear-cut split between the physics "inside and outside" [3]. Needless to add that the algebras underlying QM are always of type I, whereas the relativistic causality and associated localization structure always lead to hyperfinite III_1 factors at least if the regions allow for a nontrivial causal complement. So in order to find quantum mechanical structures inside local quantum physics, one needs type I factors inside local hyperfinite III_1 factors. The split property gives also a preferred candidate [13] for such an interpolating type I factor $\mathcal{N}$. The scenario for a definition of entropy from first principles is in terms of the modular group of $\mathcal{N}$. As a consequence of type I this modular group is inner, i.e. there exists a "hamiltonian" described by a hermitian operator K associated to $\mathcal{N}$ (this never happens for the $\mathcal{A}(\mathcal{O})$ factors).

The issue of entropy is then closely related to the problem of the modular hamiltonian K of $\mathcal{N}$ which in turn is determined in terms of the modular objects of the split data [3]:

$$\Delta_{\mathcal{O}_1}^{it}, \Delta_{\mathcal{O}_2}^{it}, \Delta_{\mathcal{O}_1' \cap \mathcal{O}_2}^{it}; J_{\mathcal{O}_1}, J_{\mathcal{O}_2}, J_{\mathcal{O}_1' \cap \mathcal{O}_2} \tag{12}$$

In zero mass conformal theories the double cone J and Δ^{it} relative to the massless vacuum are geometric transformations inside the full (including reflections) conformal group [3], in particular Δ^{it} transforms the r,t coordinates but not the two transversal angular variables of the double cone. On the other hand the massive double cone theory can be incorporated into the same Hilbert space or more precisely, the massless and the massive. This suggests that the modular object of the massive situation are nonlocal deformations of the conformal massless split situation. One expects that the K-hamiltonian is well enough in order to allow for the existence of the von Neumann entropy:

$$S = -trDlnD \tag{13}$$

where $D = \frac{1}{tre^{-K}}e^{-K}$ is the density matrix defined in terms of the modular hamiltonian K. This is a quantity which depends on the size of the collar ε and which diverges as $\varepsilon \to 0$ i.e. when the fuzzy type I factor becomes hyperfinite type III_1. If the result of the existing proposals [21] is compatible with this idea, we should expect a universal logarithmic divergence in the inverse size $\varepsilon^{-1}$of the collar which controls the vacuum fluctuations:

$$S = -trDlnD \sim Cln\varepsilon^{-1} \tag{14}$$

with C related to the longitudinal 2-dim. conformal theory which according to our previous discussion we expect to determine the geometric core of the fuzzy modular group of the double cone algebra $\mathcal{A}(\mathcal{O})$. Indeed the formula [21]

$$C \sim Area\sqrt{c} \tag{15}$$

where $Area$ denotes the area of the double cone and c the vacuum fluctuation strength of the energy momentum tensor. Although the limiting entropy is certainly infinite, we have not yet been able to confirm that this infinity is universal and behaves exactly as argued by Larsen and Wilszek.

In order to prove this one must do some new computations on modular data resp. on the "localizing map" which is the most convenient way to compute the distinguished type I factor [3]. The relevant degrees of freedom would "live", as we will argue later, inside the collar and the ratios of this "collar entropy" stay finite for vanishing collar size. This remains a fascinating program for the future.

Now we are able to formulate our two conjectures:

Conjecture 1 *The modular group of the (nonconformal) massive double cone algebra $\mathcal{A}(\mathcal{O})$ with respect to the massive vacuum vector (i.e. the physical vacuum state restricted to $\mathcal{A}(\mathcal{O})$ is cocycle-related to the known geometric modular group of the associated conformally invariant situation belonging to the pair $(A(O), \omega_{m=0})$ where $\omega_{m=0}$ denotes the conformal invariant vacuum state. For the equivalence of the massive with the massless algebra one may either invoke the construction of the double cone algebra by canonical quantization or the fact that local algebras are always hyperfinite III_1factors and the latter is unique modulo unitary equivalence. The cocycle accounts for the difference in the local propagation of massless (Huygens principle) and massive theories and its presence renders the action of the modular group "fuzzy". Only asymptotically near the horizon i.e. the boundary of the double cone, the fuzzyness decreases and the geometric conformal modular transformation reappears. Although a single algebra $\mathcal{A}(\mathcal{O})$ of the massive theory and its scale invariant limit may be identified, the two nets inside $\mathcal{O}$ remain different. However the conjecture that the difference is due to the different propagation suggests that the massive net inside $\mathcal{O}$ may be obtained from the massless $\mathcal{A}(\mathcal{O})$ by adjoining the action of the Poincaré covariances inside $\mathcal{O}$.*

Remark 1 *For a massive free Fermi field*[14] *in d=1+1 this can be shown. One notes that the restriction of such a free massive theory to the light rays which constitute the boundary of the d=1+1 double cone is simply the restriction of the corresponding massless theory and that by propagating the chiral conformal data on the one dimensional horizon inside with the massive propagator, one regains the massive free field net inside $\mathcal{O}$. A general proof of this reduction of a d=1+1 situation to its chiral conformal limit (+ possible covariance operators) would be extremely desirable because it would explain the association of the degree of freedoms of d=1+1 theories with the horizon.*

Conjecture 2 *The double cone algebras $\mathcal{A}(\mathcal{O})$ are identical to any of the two-dimensional double cone algebras $A(O^{(2)})$ obtained by cutting the double cone by a two-dimensional plane which contains the t-axis and one coordinate axis. The net inside $O^{(2)}$ may be obtained from the associated chiral conformal net on the one-dimensional horizon and a local representation of Poincaré covariances.*

[14]We want to avoid the infrared problems of massless Bose fields

Remark 2 *The first part is actually a consequence of Haag duality and the fact that the causal completion of $\mathcal{O}^{(2)}$ gives $\mathcal{O}$:*

$$\mathcal{A}(\mathcal{O}^{(2)}) \equiv \cap_\delta A(\mathcal{O}_\delta) \tag{16}$$

Where $\mathcal{O}_\delta$ denotes the middle slice of thickness δ by cutting the double cone parallel to the t-axis. Each $\mathcal{O}_\delta$ has $\mathcal{O}$ as its causal completion and the property of "Haag Duality" demands the equality of $A(O_\delta)$ with the algebra of the causal completion $\mathcal{A}(\mathcal{O})$. The essential step in the holographic reduction is the appearance of chiral conformal degrees of freedom after removal of the angular degrees of freedom due to angular symmetry (substituting the transversal symmetry in the case of the wedge). The envisaged entropy is therefore not proportional to area$(\mathcal{O}^{(2)}) \times angular\,volume$ but rather to horizon-length$(\mathcal{O}^{(2)}) \times angular\,volume = volume\,of\ horizon(\mathcal{O})$. Actually such a situation would also suggest that there may be an infinite hidden nongeometric (fuzzy) symmetry algebras in the nonperurbative structure of any QFT[15]. Although they are local in the sense of keeping things inside say $\mathcal{O}$, their action within $\mathcal{O}$ is totally fuzzy. Such symmetries of nonperturbative local quantum physics would escape differential geometric methods. Note that the two conjectures cannot even be formulated in terms of properties of expectation values of fields; the use of the algebraic i.e. field coordinate independent concepts is indispensable for the formulation. If algebraic QFT did not already exist, one would have to invent it in order to understand the above thermal and entropic properties.

As we have seen, the thermal and entropic aspects which are erroneously attributed exclusively to black holes, are in fact a generic nonperturbative feature of the modular localization structure of QFT. They make their appearance e.g. in the formfactor bootstrap construction program (viz. the KMS origin of crossing symmetry as a generalization of TCP) and also show up in CST QFT for the same (localization) reason. The latter case is only distinguished by the fact that these concepts allow for a classical (thermodynamic) interpretation which is of course the reason why they were first noticed there. This begs the question of the algebraic point of view about "Quantum Gravity". This time we put these words in quotation mark in order to indicate their precarious physical status, especially in the algebraic approach. It is agreed upon by most physicists that Quantum Gravity, whatever it is, does not fit into the framework of local quantum physics as say another spin=2 QFT. Therefore one may ask the more general question of physically consistent theories outside the framework local quantum physics. Surely there have been several attempts to imagine such possibilities, e.g. the pure S-matrix approach, the modifications of Lagrangians by formfactors or structure functions, SO(4) invariant cutoffs in euclidean QFT as candidates of real time relativistic nonlocal theories after analytic continuation and the peratization program (pairs of complex conjugate poles in Feynman

[15] In principle every modular automorphism has the interpretation of a (localized but fuzzy) physical symmetry. Some of these are "semi-geometric" i.e. they act geometric on subnets. The previous modular intersection situation which led to the transversal Galilei-transformation is such a case of a "semi-hidden" symmetry [16].

rules) of A. Pais and T. D. Lee. If the proposal was not already defeated on the mathematical front, it turned out that the physical interpretation was either inconsistent (existence of precursors violating the indispensable macrocausality) or was not, as in local quantum physics part of the theory, but (as a consequence of the missing localization) had to be enforced from the outside. There is one recent proposal [22] which has survived recent years as a possible scenario (but its future survival is by no means guarantied) which roughly speaking consists in substituting the classical indexing of the algebras in a net by spacetime regions in Minkowski space by noncommutative versions in the spirit of noncommutative geometry. Algebraic QFT would favor a situation in which no a priori spacetime indexing (neither commutative or noncommutative) appears. Preferably one would like to have global algebra with an intrinsic substructure such that it would contain our physical world of localization and causality only in the germs of certain representations (states). Such an idea would be much more radical[16] than say string theory, because going from pointlike to string extension does not mean that one abandons localization altogether..

There is one intriguing property in chiral conformal QFT which has a certain quantum gravity "touch" to it. This is the fact that the "averaging over a Fuchsian group" of a chiral conformal QFT (assuming that the Poincaré series converge in some sense) converts the Moebius invariant vacuum expectation values into expectation values which loose this invariance but gain deformation parameters (generalizations of "compact temperatures"). Formally the positivity property holds as in Wightman theory, but the old localization region S^1 is now totally fuzzy. With other words there is no causal complement in the quantum sense. In such a scenario there would be no global concept (a priori knowledge of what is spacelike) of causality and hence of localization, and the place to find the lost net properties would be in certain states and even there they would only appear in their germs. Our second conjecture which led via modular theory to the speculative existence of a hidden "fuzzy" realization of the Moebius group would suggest that such a scenario may also be possible in d=1+3 theories.

Summing up our excursion on nonperturbative QFT we would like to stress again that the algebraic method allows for a completely intrinsic definition and understanding of QFT independent of its Lagrangian or non Lagrangian origin. Any quantum theory which fulfills the stability requirements of positive energy and allows for a net interpretation and the associated localization concepts is a QFT par excellence and enjoys all the general structural properties which feature in this article as TCP, spin &statistics, crossing symmetry & modular localization & thermality, wedge-localized fields without vacuum polarization, hidden modular symmetries, Haag duality (an abstract form of the 2-d Kramers-Wannier-Kadanoff Duality), nuclearity for the phase space degrees of freedom & the conjectured "Holographic Entropy" and all the other yet unraveled properties of nonperturbative local quantum physics. The main obstacle against

[16] Note that this is the only radicalism in the algebraic approach since all other deviations from the standard QFT approach, as uncommon and revolutionary as they may appear at first sight, are all in accordance with the physical principles of this century.

progress is not so much the novel mathematics which these new physical concepts require, but rather (as always in the past) prejudices. One prejudice is that field theory has to be "Lagrangian". In view of the many existing low-dimensional non-Lagrangian models and the fact that they hardly rocked the Lagrangian boat, this appears to be the mightiest prejudice.

3 The Bizarre Path from the Dual Model to M(atrix) Theory

In the following we briefly describe the development which led from S-matrix theory via the Veneziano model and string theory to the recent M(atrix) theory.

A good starting point is the dispersion theory which was the main nonperturbative attempt of the 50 and 60 to go beyond the Feynman approach. The main issue was to find sufficiently many "on shell" properties of QFT such that an S-matrix theory or at least a phenomenologically successful scheme could emerge. Besides the obvious properties like unitarity and certain analytic properties, the on shell property which apparently was most intimately and deeply related to the off shell causality principle of QFT was crossing symmetry[17]. Therefore it was considered a major achievement when Veneziano succeeded to construct a S-matrix model which fulfilled crossing symmetry exactly and allowed for a systematic unitarization which maintained crossing symmetry in each step and in some sense was reminiscent of the perturbative systematics, although it had little in common with ordinary perturbation theory. Later it was realized that the quantum mechanics of strings can successfully describe this model and its unitarization. The infinite tower of particles which, if they would remain stable under unitarization, would violate the principles of local quantum physics (the aforementioned degree of freedom behavior, leading to a Hagedorn temperature) could, as in Feynman's perturbation theory, become unstable particles i.e. poles in the second Riemann sheet of the S-matrix and in this way the model could be perfectly consistent with nonperturbative QFT. This was at least what I and many of my contemporary QFT colleagues thought when we got used to those nice pictures involving Regge trajectories.

But, as everybody knows, things happened differently. Instead we had to witness the "Bartholomew night massacre" (a bit poetic, but part of the story really happened in Paris [18]), also often referred to as the first string revolution, in which the old string theory, which served as a laboratory of certain aspects of nonperturbative QFT (notably strong interactions), was killed and the mathematical formalism (without the slightest change) was pushed upward in energy by more than 15 orders of magnitude and physically outed as "quantum gravity". Only later, after most quantum field theorist (who had never seen a semantic miracle of a proposed physical theory like this before) had left

[17]The complete nonperturbative understanding of crossing symmetry including the requires on shell analytic behaviour and the precise relation to TCP invariance and causality was never achieved; only partial results as the famous Lehmann ellipse were established.

the scene, string theory obtained the modern differential geometric wrapping, which partially expressed the increasing mathematical sophistication of theoretical physics community. In fact in this modern fashion it became an impressive source of mathematical innovations. It was precisely the distance from any kind of laboratory physics, which protected these developments from usual fate of theories whose relevant energy scales stays close to the experimentally accessible region. Because of the involved fantastic "scale sliding" and its innovative differential geometric content, it is often referred to as the "second string revolution".

From the point of view of exhausting the scenarios offered by Lagrangian quantization, the sliding up the energy scale was very logical indeed. It is legitimate and even useful to stretch a framework (as the Lagrangian quantization, canonical or functional), which was so successful as Feynman's renormalized perturbation theory, to its physical limits set by the Planck scale. A successful formalism was always pushed to its limits. The exaggeration starts if, as a result of apparent lack of alternatives, one identifies its consequences as the "big desert" region beyond the Laboratory energies up to the Planck mass, or as the omnipotence of supersymmetry in its underlying differential geometric mathematical formalism with what should be expected from nature.

The string theoretic mode of thinking about particle physics is not limited to string theory proper. Observations as the Seiberg-Witten global e.m.-duality in the effective actions associated with certain supersymmetric gauge theories also belong to this formal extension of the quantization ideas behind renormalized perturbation theory. Problems as vacua degeneracy away from spontaneous symmetry breaking are presently too nonlocal in order to allow a quantum physical (not differential-geometric) understanding within either standard or algebraic QFT.

Very recently another, this time more quantum physical element has been added namely it was observed that the real time light cone description of QFT brings a unexpected simplification and unification. What makes this new proposal of "matrix theory" interesting even in QFT is that it offers a new controllable and systematic approximation scheme in terms of quantum mechanical concepts. It is precisely this new idea which makes it possible and interesting to compare this with those recent new ideas in algebraic QFT about modular localization a task we will turn to in the next section. A brief description of lightcone quantization follows. In analogy to canonical quantization but different from the discussion of the bifurcated wedge situation one distinguishes one x-t plane as the canonical quantization plane and the other for the definition of a propagating time. In the Lagrangian setting we have for a selfinteraction of a scalar field:

$$L = \int dx^- dx^\perp \left\{ \partial_+ \varphi \partial_- \varphi - \frac{1}{2} (\partial_\perp \varphi)^2 - \mathcal{L}_{int} \right\}$$

where $x^\pm = \frac{1}{\sqrt{2}}(t \pm x)$, $x^\perp$= transversal coordinates. Using longitudinal momenta k_-, the "hamiltonian" becomes:

$$H = \int dk_- dx^\perp \left\{ \frac{\partial_\perp \varphi^*(k_-, x^\perp) \partial_\perp \varphi(k_-, x^\perp)}{2k_-} + \mathcal{H}_{int} \right\}$$

The main difference to standard canonical quantization is the absence of vacuum polarization. The prize for this apparent simplification is a somewhat hidden nonlocal interpretation which went unnoticed by the authors of (M)atrix theory[18]. This, together with the existence of the 8-parametric "Galileian extension" $G^{(+)}(8)$ of the longitudinal modular group which made its appearance in the previous section in connection with the modular properties of the wedge algebra $\mathcal{A}(W)$, constitutes the starting point of the BFSS [19] light cone quantization framework. One obstacle against a quantum mechanical description, namely the presence of vacuum polarization, has been taken care of. Here the terminology QM should be understood in sufficient generality. Galilei-invariant field theories without vacuum polarization, but with rich channel couplings between different multiparticle sectors (as the T.D. Lee model [20], just to mention one) can a priori not be excluded. The analogy of the polarization cloud free state vectors $\varphi_{light-cone}\Omega$ with the wedge thermal space affiliated vectors $A_W\Omega$ is very startling. The wedge picture is more general since the existence of the semilocal A_W operators (in d=1+1 factorizing models related to the Zamolodchikov-Faddeev fields) suffers no restriction from possibly infinite wave function renormalization Z constants.

By compactification of the light cone time x^- one formally obtains a discretization of LCQ called DLCQ. As with lattice discretization, DLCQ allow a matrix approximation which, following BFSS, posses a natural interpretation in the modern string setting. In our attempt to translate these situations into the conceptual framework of algebraic QFT, we would draw the analogy to the compact double cone situation $\mathcal{A}(\mathcal{O})$ which is the modular substitute for the box quantization in QM. This is the spacetime localization by which one must substitute the wedge region if there is no LSZ scattering theory as a result of the presence of infrared photon clouds. In that case the electrically charged "infraparticles" have vanishing LSZ limits and one only can work with a Fock reference space for compact modular localization regions.

4 A Comparison of LCQ and Modular Localization

The most striking analogy of the real time BFSS proposal is the similarity of the generating on shell (absence of vacuum polarization) degrees of freedom of modular wedge localization constructed with the help of the modular Møller operator. These degrees of freedom agree with a thermal realization of the Zamolodchikov algebra and reveal the physical concepts behind the d=1+1

[18]The LCQ field variables are very nonlocal with respect to the usual local canonical fields. Only in this way these degrees of freedom manage to surpress vacuum fluctuations.

bootstrap formfactor program. Further striking similarities are the (less firmly established) holographic relation of the double cone algebras with their longitudinal zero mass projections. Although this property is most useful in black hole physics (in fact there it was conjectured by G. 't Hooft see [2]), its physical origin is modular localization and as such it is a generic property of local quantum physics. With a better mathematical understanding and physical interpretation of the modular Tomita-Takesaki theory for double cone operator algebras and a deepening of the notion of "Holography" in the spirit of Matrix-theory, one should expect a more profound understanding of these analogies. Related to this is the hope to obtain an appropriate fundamental (i.e. without counting bits in theories whose degrees of freedom fulfill "nuclearity") definition of entropy via the use of the mathematical notion of relative entropy and the split property of algebraic QFT as indicated in section 2.

These points of coincidence would not be so interesting, if it would not be for the many startling differences and antinomies. Let us note some of them.

1. In the algebraic approach to QFT one does not set out to study extended objects for their own sake. Rather one only excepts them if they are necessary to enlarge the possibilities of realizations of a physical principle. The principle looks for the objects in which it can be realized. The main aim of the algebraic approach is to classify nets of observable algebras and finite energy states (or better representations) on them. A reasonably clear picture exists if one assumes (as often done also in condensed matter physics) the existence of a spectral gap. In that case one can proof the existence of "charged" operators in an extended Hilbert space which create those finite energy particle states by acting on the vacuum. These operators have a localization which is semiinfinite string like and carry "topological charges" (in the sense of algebraic QFT). The compactly localized (i.e. around a point) operators which carry the ordinary charges (i.e. those belonging to e.g. global symmetries of the Lagrangian approach) constitute a special subclass. In d=1+2 the topological charges lead to braid group statistics whereas without the string like localization of the charge carriers one falls back into the realm of standard (Fermions and Bosons) particles. In higher dimensions the difference between the two charges is less pronounced and leads to qualitative differences in the analytic behavior of the S-matrix (threshold, crossing etc.)

2. The algebraic framework has much more problems with gauge theories than the various differential geometric quantization approaches. The reason can be traced back to the Wigner representation theory of zero mass particles. The algebraic approach treats the BRS formalism as correct for the purpose of studying deformations of free fields, but would not support the many sweeping generalizations outside the perturbative use[19] in the second section. It rather considers this formalism as a preliminary but perturbatively successful method which hides a lot of ill understood

[19] I am happy that this critical attitude has the support of Raymond Stora.

physics (in particular a weakened localization property which are at odds with the standard Bogoliubov-Shirkov-Epstein-Glaser causal deformation approach (which is based on *formal* locality and covariance independent of its physical content).

3. The algebraic framework considers supersymmetry as an "accidental symmetry" and is therefore unable to attribute to it the same fundamental significance as string theory or the Seiberg-Witten duality theory does. Here "accidental" means that in order to understand the physical content of a model, it is not necessary (but also not incorrect) to use supersymmetry. This is borne out by all soluble (necessary low dimensional) models starting with the tricritical conformal invariant Ising model and continuing wit the various supersymmetric integrable models. Support for this point of view also comes from the Doplicher-Roberts theory which explains the physical origin of the internal group symmetry as a consequence of the causality principle and the ensuing particle (field) statistics. In 4-dimensional physics there is the claim that e.g. N=4 supersymmetric gauge theory is a nontrivial conformal theory in 4 dimensions and that in this case SUSY is essential for its understanding. This seems to contradict our view. But no gauge invariant lowest order physical correlation function has been computed during the last 15 years, ever since the invention of the model. I do not know the reason (but I am not an expert) for this state of affairs; one probably encounters nonlinear realizations of supersymmetry which are somewhat at odds with standard renormalization theory even within the regularization independent cohomological BRS formalism. As long as these problems remain open, it is difficult to decide if SUSY is essential for the understanding of this model or whether it is a true, but accidental property of a model which allows (similar to the mentioned low dimensional cases) a characterization in terms of different concepts. The most serious criticism of the standard view and a forceful support of ours comes from a recent paper of Buchholz and Ojima [23]. There it is shown that, contrary to popular opinion, the supersymmetry of the T=0 theory suffers a collapse of SUSY in the KMS states for T$\neq$ 0 instead of the expected spontaneous symmetry breaking (which the zero grading Lorentz generators suffer). This is of course expected on the ground of instabilities of accidental symmetries under thermal fluctuations. This thermal instability could of course be studied explicitly in the aforementioned models. A closer examination may very well reveal that this instability phenomenon is a quite general feature of noncausal currents (the fermionic current of SUSY is extremely noncausal) as e.g. the nonlocal conserved axial current which is the projection of the gauge dependent conserved current into the physical factor space (the thermal aspects of $\theta-$angles has apparently not been discussed in the literature).

4. The string proposal of generalizing (for what physical reasons? what are the new principles?) QFT is a post Veneziano dual model idea and, whereas the first step of constructing a crossing symmetric S-matrix had

a clear-cut physical aim (and bears similarities to the gauge theory like Polyakov strings and Wilson loops), the spirit of modern string theory is very alien to the algebraic approach. In particular the claim of having a quantum theory of gravitation without saying anything about a substitute for the principle of Einstein causality raises serious doubts (see end of section 2). For example the term "stringyness" has never received a conceptual intrinsic definition (see the remarks in the third section), but a useful critical dialogue requires that one can compares results and is not forced to look at the particular way in which one imagines them to be manufactured. Whereas the principles of locality require semiinfinite stringlike objects (they exist in a fully local theory of observables as the best localized carriers of e.g. plektonic charges) in order to e.g. realize braid group statistics in d=1+2 theories, there does not seem to be any discernible physical principle behind string theory except the Diracian pleasure of mathematical (differential-geometric) esthetics (a working principle which was very successful indeed during his time). The algebraic approach starts from causal observables and introduces those charge carrying extended operators which are convenient for the description of linking the higher charge sectors with the vacuum sector. If one is only interested in particle-like finite energy states with a mass gap, there is no reason to go beyond semiinfinite string like localization for objects which generate the so called topological charges (in the sense of algebraic QFT and not of differential geometry!). This does not preclude the possibility that for some not yet known physical reason one may not one day use possibly infinite energy states with still weaker (semiinfinite membrane like) localization properties. But even those, like the string like localized operators, do not carry an elementary lateral size, inasmuch as there was no elementary length in the causal observable algebra.

5. The thermal notions of temperature and entropy in this article depend only on the concept of modular localization and should not be considered as being peculiar attributes of black holes only. The only role of the black hole in the context of this discussion is that it allows a quasiclassical (and hence differential geometric) understanding because it is related to isometries (Killing vectors) in the metric and one does not have to study the more noncommutative setting of isometries in quantum space[20]. Similarly the topological field theories and the appearance of invariants of 3-manifolds are primarily attributes of Minkowski real time QFT and not of curved spacetime QFT [4]. The standard methods (functional integrals) are sending out the wrong physical messages as if the issue would be related to the "living space" (i.e. the localization) of $d = 1 + 2$ local QFT. Algebraic QFT suggest that these new invariants could show up in the unknown perturbation theory of the scattering of plektons (even the "free"

[20] In this aspect the present explanation in terms of nonperturbative new structure in local quantum physics should not be confused with proposals to explain M-theory in terms of special differential gemetric models involving higher Chern-Simons structure [24].

version of these particles and their fields has not yet been constructed).

The counting of degrees of freedom and the entropy issue clearly transcends the concept of pointlike fields and points to the relevance of algebras, since these concepts cannot be related to individual field coordinates. To obtain all physics from the position of algebras is the main theme of algebraic QFT. For the time being, string theory has no information on equivalent "string coordinates"

6. A less serious point of disagreement is the fact that, different from the algebraic approach (where the obstruction to Haags theorem are avoided by using regions which admit a nontrivial spacelike complement), hyperplanes at $t = 0$ or $x^{-} = 0$ of the LCQ have no spacelike complement and therefore, although the principle mechanism for Haag's obstruction and vacuum polarizations is absent (e.g. no infinite volume divergencies resulting from vacuum polarization), the formulation may suffer from some finer infrared problems which are causing the unitary inequivalence of the canonical representation theory to a Fock representation. In addition there is the well-known short distance problem of the renormalized canonical commutation relation. Both problems together are usually handled by giving up the original operator theory and passing to ultraviolet cut-off box quantization correlation functions. The final operator formalism is then reconstructed from the renormalized ultraviolet cut-off independent correlation functions by passing to the thermodynamic limit V$\rightarrow \infty$. Some more work is required in order to see more explicitly how the undoubtedly attractive feature of absence of vacuum polarization of LCQ can be used for obtaining a good operator formulation Modular localization avoids these obstructions and provide a good vacuum polarization free operator formalism whose existence is not tied to the "bad" short distance behavior of specific "field coordinates", but it is only applicable to wedge type regions and not to hyperplanes which do not posses a nontrivial spacelike complement. Note that the compactified version DLCQ would be formally better in this respect, but be aware of the difference in the BFSS compactification (related to box-quantization and its associated noncausal localization) and say the modular double cone localization. My general suggestion to this kind of problem would be not to overestimate the power of physical intuition in an area which is so notoriously nonunderstood (although most particle physicist think that they understand it) as local quantum physics. Usually an intuitive physical idea in a not well understood area is compatible with several formal implementations and one has to rely on mathematical information (including no-go theorems) in order to select.

7. Algebraic compactification usually enlarges the algebra (degree of freedoms) instead of making it smaller. An interesting illustration is the compactification of the chiral conformal theory on a line to that on a circle. The universality construction (related to Voiculescu's freeness con-

cept) yields an algebra which has a nontrivial center consisting of central charges. The physical origin of the enlargement is that in e.g. a circular or toroidal spacetime the substitute for shifting charges out to infinity is to "transport them around" which creates the central charge "measurers". In the presence of charge carriers (endomorphisms of the algebra) they create a rich global "symmetry algebra" which contains the entrances of mapping class group representation matrices for any genus (even though we never left the circle!). This enormous enlargement of the noncompact algebra through compactification is not seen in the differential geometric (fibre bundle) approach which would tend to distribute this additional structure over infinitely many theories associated with different Riemann surfaces. Again the last sentence in the previous discussion would apply in this situation.

8. Most criticism of string theorist of QFT is clearly the result of misunderstandings by equating perturbative notions of QFT with QFT away from perturbation. Since nonperturbative QFT is in its infancy (viz. the impressive but few results on the d=1+1 bootstrap formfactor program) the possibilities for comparisons are very limited. In fact most string theorist who comment on QFT are totally unaware of these few results.

Let us add some concluding remarks. Although a direct comparison of nonperturbative QFT with string theory is presently not possible as a result of the lack of an intrinsic physical definition, we tried an indirect evaluation based on what may be called circumstantial evidence. If we take the findings of [2] serious and the title of [25] literal, then the success of nonperturbative field theoretic methods in string theory which are based on spacetime concepts (light cone, spacelike distance, light cone quantization, black hole entropy,...) suggests that the physical aim of string theory is an enrichment of nonperturbative aspects but by geometric and quasiclassical means. As one needs black hole horizons in order to obtain a quasiclassical understanding of thermality and the holographic aspect of entropy which in terms of true quantum concepts (no Killing vectors) are really generic properties of nonperturbative QFT, string theory and its M-theoretic extension appears as that part of the regime of (high dimensional,supersymmetric) local quantum physics which can be conquered by existing geometrical (commutative or noncommutative) methods without the use of new local quantum ("noncommutative" in order to use a fashionable, but in this case superfluous word) concepts. This is at least what I think an unflinching continuation and deepening of the ideas in [2] will lead to.

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