The Pivotal Role of Causality in Local Quantum Physics

This is an expanded version of a talk given at the symposium entitled "New Insights in Quantum Mechanics-Fundamentals, Experimental Results, Theoretical Directions"
Goslar, Germany, September 1-3

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August 31, 1998

Abstract

Several aspects of the manifestation of the causality principle in LQP (local quantum physics) are reviewed or presented. Particular emphasis is given to those properties which are typical for LQP in the sense that they do go beyond the structure of general quantum theory and even escape the Lagrangian quantization methods of standard QFT. The most remarkable are those relating causality to the modular Tomita-Takesaki theory, since they bring in the basic concepts of antiparticles, charge superselections as well as internal and external (geometric and hidden) symmetries.

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1 Introduction

One of the most fundamental physical principles of this century, which has stood its grounds in the transition from classical into quantum physics, is relativistic causality as well as the closely related locality of quantum operators together with the localization of quantum states.

This principle entered physics through Einsteins 1905 special relativity, which in turn resulted from bringing the Galilei relativity principle of classical mechanics into tune with Maxwell's theory of electromagnetism.

The two different aspects of relativity, namely Poincaré covariance and the locally causal propagation of waves (in Minkowski space) were kept together in the classical setting. In LQP (local quantum physics[1]) on the other hand [1], it is appropriate to keep them (at least initially) apart in the form of positive energy representations of the Poincaré group (leading to Wigner's concept of particles) and Einstein causality of local observables (leading to local fields and local generalized "charges"). Here a synthesis is also possible, but it happens on a deeper level than in the classical setting and results in LQP as a new physical realm which is conceptually very different from both classical field theory and general QT (quantum theory). The elaboration of this last point constitutes one of the aims of these notes.

The most remarkable aspect of QFT in its more than 60 years existence is, in addition to its great success in perturbative QED and the (also not very recent) standard model, the perseverance of its causality principle. In addition to the experimental support through the validity of the Kramers-Kronig dispersion relations in high energy collisions up to the shortest accessible distances, the various unsuccessful theoretical attempts to construct viable nonlocal theories[2] testify to the strength of this principle. Nobody

[1] We use this terminology whenever we want to make clear that we relate the principles of QFT with a different formalism than that based on quantization through Lagrangian formalism.

[2] We do not mean extended charged objects in a theory of local observables (example

has succeeded to construct a viable *nonlocal* theory, despite the fact that, as a result of the apparent "bad" short distance behavior (stemming from perturbative causality down to arbitrary small distances) threatening the mathematical existence of many models, a lot of attention had been directed towards this point. Here "viable" is meant in the physical sense of conceptual completeness, namely that a theory is required to *contain its own physical interpretation* and that one does not have to invent or borrow formulas from outside this theory as it is done in e. g. phenomenological "effective" QFT (where many formulas linking the calculations with measurable quantities cannot be derived or justified and have to be taken from elsewhere). For example in LQP one can derive (LSZ) scattering formulas which constitute an important aspect of particle interpretation; there is no nonlocal Poincaré covariant scheme known, which guaranties the existence of a time dependent (or its stationary reformulation) scattering formalism. The importance of causality is highlighted by the failure of nonlocal modifications of which we will now briefly mention the more prominent ones.

Already in the 50^{ies} there were attempts to inject nonlocal aspects through extended interaction-vertices in Lorentz invariant Lagrangians. As mentioned before, this was motivated by the hope that a milder perturbative short distance behavior of correlation functions may be helpful for demonstrating the mathematical existence of the theory. It was soon realized, that if one pursues the effect of such modifications up to infinite order in perturbation theory, these nonlocal vertices would wreck even macrocausality (so that the theory looses its physical interpretation). A similar fate happened to the later proposal of Lee and Wick [2] to allow for complex (+ complex conjugate, in order to maintain hermiticity) poles in Feynman rules (it led to unacceptable time precursors [3]). Often the renormalization group ideas are used to justify a physical cutoff in order to do away with the threat of an apparent mathematical inconsistency. But physical principles should receive their limitation (as it always happened) from other more general principles and not from parameters into which one tries to dump ones lack of knowledge about the mathematical existence of the theory within the presently known principles. A successful phenomenological parameter by itself is not a substitute for a physical principle, even if it apparently improves the mathematics. In this context one should note that lattice theories define a different

string like anyons or plektons in d=1+2), but rather theories which have a fundamental cut-off or elementary length already in their algebra of observables.

(mathematically easier) framework which, if suitably restricted, shares with QFT that it is conceptually complete as far as the notion of particle excitations and their scattering theory (based on cluster properties) is concerned[3]. Despite some control of the extremely difficult scaling limits in certain (d=2 Ising like models), the relation between the two theories remains largely non understood.

Recently there was a more sophisticated attempt via noncommutative space time [4], based on spatial uncertainty relations following from a quasiclassical quantization interpretation of Einstein's field equation of general relativity and the assumed absence of very small black holes (similar uncertainty relation for the complete set of coordinates and momenta (i.e. for phase space) have been postulated on the basis of string theory [5]). These proposals, especially if they are backed up by uncertainty relations whose derivation is done in the spirit of Bohr-Rosenfeld as in [4], are not that easily dismissed as the two previous ones, partially because it is more difficult to physically interpret in such unusual frameworks. Whereas it is easy to agree that they serve as interesting tests for exploring the unknown territory beyond the reign of Einstein causality, they are not yet models for "Quantum Gravity" (if the latter exists as a viable physical theory) since they only replace the classical spacetime indexing of nets with a noncommutative one. However any step beyond the present causal framework must resolve Einstein causality as an limiting statement within some yet unknown new viable principle. Any concrete attempt in this direction was less than successful; although there have been a lot of promises on the basis of string theory. But unfortunately string theory has little to say (it hardly added anything) on conceptual problems because even if one ignores the lack of experimental motivation, on the theoretical side it is difficult to overlook the unfortunate preference of formalism over concepts. Whereas theories founded on principles as LQP allow an intrinsic characterization (e.g. in terms of correlation functions or observable nets) which is independent on the way they have been manufactured (e.g. Lagrangian quantization, bootstrap-formfactor method in d=1+1), this is not (yet?) the case with string theory.

Causality and locality are in a profound way related to the foundations of quantum theory in the spirit of von Neumann, which brings me a little closer

[3]In lattice theory one always has an easy start, but the conceptual and computational work in order to extract e.g. scattering theory of particles (which does make sense on the lattice!) becomes much more involved (instead of local causality one has to prove cluster properties from the range limitation of the lattice interaction).

to the topic of this symposium on "New Insights in Quantum Mechanics...". In von Neumann's formulation, observables are represented by selfadjoint operators and measurements are compatible if the operators commute. The totality of all measurements which are relatively compatible with a given set (i.e. noncommutativity within each set is allowed) generate a subalgebra: the commutant L' of the given set of operators L. Causality gives an a-priori information about the size of spacetime $\mathcal{O}$ affiliated operator algebras:

$$\mathcal{A}(\mathcal{O})' \supset \mathcal{A}(\mathcal{O}') \tag{1}$$

in words: the commutant of the totality of local observables localized in the spacetime region $\mathcal{O}$ contains the observables localized in the spacelike complement (disjoint) $\mathcal{O}'$. In fact in most of the cases the equality sign will hold in which case one calls this strengthened (maximal) form of causality "Haag duality" [1][6]:

$$\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \tag{2}$$

Whereas the Einstein causality (1) allows a traditional formulation in terms of pointlike fields $A(x)$ as

$$[A(x), A(y)] = 0, \quad (x-y)^2 < 0, \tag{3}$$

Haag duality can only be formulated in the algebraic net setting of LQP. This aspect is shared by many important properties and results presented in this article. LQP is much more than a Teutonic pastime.

If the vacuum net is Haag dual, then all associated "charged" nets share this property, unless the charges are nonabelian (in which case the deviation from Haag duality is measured by the Jones index of the above inclusion, or in physical terms the statistics- or quantum-dimension). If the vacuum representation violates Haag duality then this indicates spontaneous symmetry breaking [9]. In that case one can always enlarge (maximize) the algebra (without changing the Hilbert space) such that Haag duality is restored. This turns out to be related to the descend to the unbroken part of the symmetry which allows (since it is a subgroup) more invariants (more observables). Although these matters are good illustrations of the pivotal role of causality, we will concentrate on the closely related modular properties of causal nets which will make their appearance in the next section. Since QM does not know these concepts at all (trying to add them would mean leaving QM, since their realization requires infinite degrees of freedom), I am presenting in some sense a contrasting program to the main QT orientation of this symposium. But often one only penetrates the foundations of a framework, if one looks at a contrasting structure. In connection with this main theme of this symposium, it is interesting to ask if LQP could add something to our understanding of classical versus quantum reality (the ERP, Bell issue) or the measurement process. For the first issue I refer to [7]. Apart from some speculative remarks [8], there exists no investigation of the measurement process which takes into consideration the characteristic properties of the local algebras in LQP. I tend to believe that whereas most of the present scenarios on Schrödinger cats and their transition to von Neumann mixtures will remain, LQP could be expected to add a universal aspect to the issue of decoherence through environments. Contrary to QM where the environment is introduced by extending the system, localized systems in LQP are always subsystems for which the "causal disjoint" defines an a priori universal environment.

Historically the first conceptually clear definition of localization of relativistic wave function was given by Newton and Wigner [10] who adapted Born's x-space probability interpretation to the Wigner relativistic particle theory. The saying is that the result that there is no exact satisfactory relativistic localization (but only one sufficient for practical purposes), disappointed Wigner and made him distrustful of the usefulness of QFT in particle physics. Whereas we know that this distrust was unjustified, we should at the same time acknowledge his stubborn insistence in the importance of the locality concept in addition the positive energy property and irreducibility of the Wigner representations. Modular localization of algebras and modular localized subspaces in the Hilbert space of QFT on the other hand are not related to the Born probability interpretation. Rather modular localized state vectors preempt the existence of causally localized observables and may serve as a starting point for the construction of interacting nonperturbative LQP's [6][12][4]. It is worthwhile to emphasize that locality in LQP is not defined in terms of support properties of classical functions, although in many cases (CCR- or CAR-algebras, Wightman QFT) the algebraic formulation (1) can be reduced to this more classical concept.

Since the modular structure is in a deep way related to thermal behavior,

[4] In fact the good modular localization properties of positive energy properties, with the exception of Wigner's infinite component "continuous spin" representations, are guarantied. Only in the infinite component case it is not possible to come from the wedge localization to the spacelike cone localization which is the coarsest localization from which one can still obtain a Wigner particle interpretation.

it is not surprising that the latter is also related with localization. In fact there are two manifestations of thermality, the standard heat bath thermal behavior which is described by Gibbs formula or (after having performed the thermodynamic limit) by the KMS condition, and thermality caused by localization (with classical Killing-horizons as in black holes or in a purely quantum manner as the boundary of the Minkowski space wedge or double cone [11]). In the latter case the KMS state has no natural limiting description in terms of a Gibbs formula (which only applies to type I and II but not to type III von Neumann algebras), a fact which is also related to the fact that the hamiltonian (of the ground state problem) is bounded from below, whereas the e.g. Lorentz boost (the modular operator of the wedge) is not [12]. In [13] the reader also finds an discussion of localization in a heat bath thermal state. In these notes we will not enter these interesting thermal aspects.

2 Locality and Free Particles

The best way to make the pivotal nature of causality manifest is to access QFT via Wigner's group theoretical characterization of particles by restricting Wigner's theory to positive energy representations with good localization properties. It is well known that the Wigner wave functions ψ of massive spin s particles have 2s+1 components and (differently from covariant fields) transform in a manifestly unitary but p-dependent way:

$$(U(\Lambda)\psi_W)(p) = R(\Lambda, p) \cdot \psi_W(\Lambda^{-1}p) \tag{4}$$

The transition to covariant wave function and fields is done with the help of intertwiners $u(p, s_3)$ resp. the rectangular matrix $U(p)$ constructed from their 2s+1 column vectors of length $(2A+1)\cdot(2B+1)$

$$U(p)D^{(s)}(R()) = D^{(A,B)}(\Lambda)U(\Lambda^{-1}p) \tag{5}$$

i.e. within Wigner's Poincaré group positive energy representation theory one can intertwine the rotations (with the p-dependent Wigner R-matrix) with the (dotted and undotted) finite dimensional spinor representations $D^{(A,B)}$. Since the $D^{(s)}$ representation of the rotations is "pseudo-real", there exists another intertwiner matrix $V(p)$ which is "charge-conjugate" to $U(p)$. To each of the infinitely many intertwiner systems (the only restriction on A,B for given physical spin s is $|A-B| \leq s \leq |A+B|$) one has a local field obeying the spin-statistics connection:

$$\psi^{(A,B)}(x) = \frac{1}{(2\pi)^{\frac{3}{2}}} \int \left(e^{-ipx} \sum u(p,s_3) a(p,s_3) + e^{ipx} \sum v(p,s_3) b^*(p,s_3) \right) \frac{d^3p}{2p_0} \tag{6}$$

where a, b are the (creation) annihilation operators associated with the Fock space enlargement of the Wigner representation space and hence independent of the choice of intertwiners. All the different fields are describing the same (m, s) particle physics and live in the same Fock space. They constitute only the linear part of a huge (Borchers) equivalence class of fields. For free fields this equivalence class contains in addition all Wick-monomials which are useful for introducing interactions. The above different $\psi' s$ can be mutually solved:

$$\psi^{(A',B')}(x) = M_{(A,B)}^{(A',B')} \psi^{(A,B)}(x) \tag{7}$$

where $M_{(A,B)}^{(A',B')}$ is a rectangular matrix (matrix indices supressed) involving ∂_μ derivatives.

Explicit formulas can be found in the first volume of [14]. Among the infinitely many possibilities essentially only one is "Lagrangian" i.e. can be used in a quantization approach starting from a classical Hamiltonian principle. The other descriptions are physically as suitable as that derived from a bilinear Lagrangian since there is no "quantization" principle which enforces to do quantum physics through a classical parallelism.

For LQP, pointlike fields (6)are like coordinates in differential geometry: it may be sometimes convenient to use them but structural theorems on charge-carrying fields (classification of statistics, including braid group statistics for low dimensional charge carriers, TCP...) and internal symmetries (symmetries and their spontaneous breaking, the Schwinger-Higgs screening mechanism...) are best done in terms of the properties of the net:

$$\mathcal{O} \rightarrow \mathcal{A}(\mathcal{O}) \tag{8}$$

The causality and spectral properties of these nets constitute the physical backbone of LQP. The notion "local" is then extended to all Boson and Fermion fields, because they allow an unrestricted iterative application to the vacuum without encountering local annihilators. More general charge carrying fields which extend the above local (bosonic or fermionic) net are called "localizable". In particular plektonic (braid-group statistics) d=1+2 dimensional fields can never have a Fock space structure and always locally annihilate charge sectors when the operator domain does not match the range of the charge sector of the state vector. Although such fields (as some fields used in gauge theory) have necessarily a semi-infinite (spacelike) string-like extension, these charge carriers are associated with a local net of observables i.e. they do not bring in an aspect of elementary length or any other restriction of the causality principle. A genuinely nonlocal theory violates causality in its observable algebra.

It is important to note that the (Wigner) free fields have operator dimensions which increase with spin: $dim\psi_{(s=0)} = 1,\ dim\psi_{(s=\frac{1}{2})} = \frac{3}{2},\ dim\psi_{(s=1)} \geq 2$. This is the deeper reason why the incorporation of interacting theories into perturbative renormalization requires special cohomological tricks (BRS) for $s \geq 1$ (the LQP version of gauge theories, see next section).

The Wigner approach for $(m = 0, s \geq 1)$ leads to a more restricted class of intertwiners, since many representations (e.g. $D^{(\frac{1}{2},\frac{1}{2})}$), as a result of the different nature of the "little group, *cannot* be intertwined with the physical photon representations. There are two methods to overcome this restriction; one physical way of introducing a semiinfinite spacelike localized vectorpotential $A_\mu(x,n)$ depending on a spacelike string direction n into the Wigner photon space, or the well-known ghost (indefinite metric or different star-operation) formalism which keeps the formal Lorentz-covariance (together with the point-like nature) in the form of "pseudo-unitarity" representations). Whereas the first method is physically deeper and more promising, the second one is the only one which is compatible with the present mathematical formalism of perturbation theory. The remaining positive energy representations are Wigner's famous "continuous spin" representation which are infinite component (infinite dimensional representations of the massless "little group"). They are usually dismissed by saying that nature does not make use of them. Apart from the fact that a theoretician, should not argue in this way (and in fact he doesn't if it comes to supersymmetry), the dismissal is probably founded on the naive identification of irreducible positive energy representation with physical particles. This ignores that particles should be described by states which in addition to forming irreducible positive energy representations must also have good localization properties. The modular localization method below applied to the infinite component representation reveals that the localization is weaker than the compact localization of s=(semi)integer (Fermions)Bosons and even weaker than the semiinfinite string localization of (d=2+1) anyons and plektons. In fact it cannot be much sharper than a wedge (an intersection of two wedges is the borderline case) and it is definitely too weak in order to attribute a particle representation to those infinite component objects. This does not disqualify these objects from future use in particle physics; after all there are many useful objects or states which are not described by (m,s=semi-integer) Wigner representations as e.g. infraparticles (electron), ultraparticles, quarks...[15]).

If fields are analogous to coordinates, there should be a way to construct interaction free nets directly without ever using free fields. The idea behind this is to characterize wedge localized real subspaces in Wigner space with the help of modular operators. Assume integer spin (Bosons) and define a real subspace $H_R(W_{st})$ of H_{Wigner} as:

$$\begin{aligned} H_R(W_{st}) &= closure\ of\ \text{real}\ lin.comb.\{\psi \mid s\psi = \psi\} \\ s &\equiv j\delta^{\frac{1}{2}}, \quad s^2 = 1 \end{aligned} \tag{9}$$

The notation is as follows: $\delta^{i\tau} = U(\Lambda_{x,t}(2\pi\tau))$ is the Lorentz boost in the x-t direction associated to the standard x-t wedge and $j = \theta \cdot rot_x(\chi = \pi)$ is, apart from a π-rotation around the x-axis, the antiunitary TCP transformation θ acting on the Wigner one-particle space (which for non-selfconjugate particles consists of a direct sum of the particle and antiparticle space), and the unbounded $\delta^{\frac{1}{2}}$is defined by functional calculus from δ^{it} and has a domain consisting of boundary values of analytically continuable wave function which have the momentum space rapidity ($p_0 = m\cosh\theta$, $p_x = m\sinh\theta$) analyticity in the strip $0 < Im\theta < \pi$. The involutive property $s^2 = 1$ on this domain, is a consequence of this definition. Such unbounded (but yet involutive) operators do not occur in any other area of mathematical physics and therefore are not treated in books on mathematical methods. In fact they seem to be characteristic of the Tomita-Takesaki modular theory. It is precisely the unboundedness and involutiveness which is responsible for the emergence of localization and geometrical properties from domain properties of quantum physical operators. The real closed subspace may be used to define a dense wedge localization space[5] $H(W_{st}) \equiv H_R(W_{st}) + iH_R(W_{st})$ on which the operator s acts as:

$$s(h + ih) = h - ih \tag{10}$$

[5]A change of sign in the definition of $H_R(W)$ would not change the dense complex localization space (which is a Hibert space in the graph s-norm).

The natural localization topology is the graph norm of s. For a physicist it is somewhat unusual that the formula for s looks so universal and the differences in the localization for different wedges $W = \Lambda W_{st}$, $W = T(a)W_{st}$ is solely encoded in the domain of definition of s, which he usually considers to be a fine and somewhat irrelevant point. For positive energy representations the geometric inclusion $\hat{W} = T(a)W \subset W$ (translating wedges into themselves) implies the proper inclusion (D. Guido, private communication 1996) $H_R(\hat{W}) \subset H_R(W)$, in fact the geometric inclusion properties are equivalent to the positive spectrum condition.

Having constructed a net of wedge-localized real subspaces $H_R(W)$, one may move ahead and introduce compactly localized spaces $H_R(\mathcal{O})$ through intersections $\cap_{W\supset\mathcal{O}}W$

$$H_R(\mathcal{O}) = \cap_{W\supset\mathcal{O}}H_R(W) \tag{11}$$

In this way one obtains the net of double cones; a direct construction of the associated s-operator is more difficult because it behaves "geometric" (a diffeomorphism of Minkowski space) near the "horizon" (boundary of causal closure) of the region. Note that in order to define these localization spaces we did not use any intertwiners. If we had done this, the present intrinsic concept of localization would have agreed with certain x-space properties of covariant wave functions (or if we use free fields, with the support properties of smearing functions). But the covariant wave function or fields destroyed the unicity and the present way maintains it.

The last step to the nets consists in the application of the Weyl functor which maps real subspaces into the von Neumann subalgebras of a net:

$$H_R(\mathcal{O}) \stackrel{\mathcal{F}}{\rightarrow} \mathcal{A}(\mathcal{O}) \tag{12}$$

$$\begin{aligned} \mathcal{A}(\mathcal{O}) &= \mathrm{alg}\,\{W(f) \mid f \in H_R(\mathcal{O})\} \\ W(f) &= e^{i(a^*(f_1)+h.c.)+i(b^*(f_2)+h.c.)}, \;\; f = (f_1, f_2) \end{aligned}$$

where $b^{\#}, a^{\#}$ stand for (anti)particle Wigner creation and annihilation operators. The functor $\mathcal{F}$ is orthocomplemented i.e. the symplectic or real orthogonal complement of a real subspace is mapped into the von Neumann algebraic commutant. The images J, Δ^{it}, S of j, δ^{it}, s under $\mathcal{F}$ are the modular objects of the Tomita Takesaki modular theory for the special case of the pair $(A(W_{st}), \Omega)$ of wedge algebra and vacuum vector [6].

[6] A direct construction of the net without using modular localization methods can be found in [18].

The general theory says that for a von Neumann algebra $\mathcal{A}$ with a cyclic and separating vector Ω , the definition:

$$SA\Omega = A^{*}\Omega,\ A \in \mathcal{A} \tag{13}$$

introduces a closable operator, whose polar decomposition;

$$S = J\Delta^{\frac{1}{2}} \tag{14}$$

leads a unitary Δ^{it} and a antiunitary involution J which are of fundamental significance for the pair $(\mathcal{A}, \Omega)$. The operator Δ^{it} defines the "modular" automorphism σ_t of $\mathcal{A}$ (a kind of generalized hamiltonian) with respect to Ω and J the modular involution j (a kind of generalized TCP reflection):

$$\begin{aligned} \sigma_t(\mathcal{A}) &= \mathcal{A},\ \ \sigma_t(A) \equiv \Delta^{it} A \Delta^{-it} \\ j(\mathcal{A}) &= \mathcal{A}',\ \ j(A) \equiv JAJ \end{aligned} \tag{15}$$

This basic theorem was stated and proved by Tomita with significant improvements due to Takesaki [16]. In the physical context of thermal quantum physics it received an independent contribution in form of the KMS condition from Haag Hugenholz and Winnink. Its relevance for localization in QFT was first seen by Bisognano and Wichmann [19] and the thermal aspects of (wedge) localization (the Hawking-Unruh connection) were first stressed by Sewell.

Already in the very early development of algebraic QFT [17] the nature of the local von Neumann algebras became an interesting issue. Although it was fairly easy (and expected) to see that i.e. wedge- or double cone-localized algebras are von Neumann factors (in analogy to the tensor product factorization of standard QT under formation of subsystems, it took the ingenuity of Araki to realize that these factors were of type III (hyperfinite type III_1 as we know nowadays, thanks to the profound contributions of Connes and Haagerup), at that time still an exotic mathematical structure. Hyperfiniteness was expected from a physical point of view, since approximatability as limits of matrix algebras harmonizes very well with the idea of thermodynamic+scaling limits of lattice approximations. A surprise was the type III nature which implies the absence of pure states (in fact all projectors are Murray von Neumann equivalent to 1) on such algebras; this property in some way anticipated the thermal aspect (Hawking-Unruh) of localization. Overlooking this fact (which makes local algebras significantly different from QM), it is easy to make conceptual mistakes which could e.g. suggest an apparent breakdown of causal propagation (for the discussion of such kind of error see [20]). Especially if one simply mixes concepts of QM with the causality structure of LQP without deriving them in LQP (quantum mechanical tunnelling, structure of states), one runs the risk of wrong conclusions about e.g. the possibility of superluminal velocities.

A very interesting question is: what is the influence of the always present causally disjoint environment on the measurement process, given the fact that in the modern treatment the coupling to the environment and the associated time dependent decoherence are very important. Only certain aspects of classical versus quantum reality, as expressed in terms of Bell's inequalities, have been discussed in the causal context of LQP [7].

Let me, at the end of this section mention two more structural properties, intimately linked to causality, which distinguish LQP rather sharply from QM. One is the Reeh-Schlieder property:

$$\begin{aligned} \overline{\mathcal{P}(\mathcal{O})\Omega} &= H, \ \textit{cyclicity of } \Omega \\ A \in \mathcal{P}(\mathcal{O}),\ A\Omega &= 0 \Longrightarrow A = 0 \ \textit{i.e. } \Omega \ \textit{separating} \end{aligned} \tag{16}$$

which either holds for the polynomial algebras of fields or for operator algebras $\mathcal{A}(\mathcal{O})$. The first property, namely the denseness of states created from the vacuum by operators from arbitrarily small localization regions (a state describing a particle behind the moon[7] and an antiparticle on the earth can be approximated inside a laboratory of arbitrary small size and duration) is totally unexpected from the global viewpoint of general QT. In the algebraic $\mathcal{A}(\mathcal{O})$ formulation this can be shown to be dual to the second one (in the sense of passing to the commutant), in which case the cyclicity passes to the separating property of Ω. The large enough commutant required by the latter property is guarantied by causality (the existence of a nontrivial $\mathcal{O}'$) and shows that causality is again responsible for the unexpected property. Later we will see that most of the very important physical and geometrical informations are encoded into features of dense domains in fact the aforementioned modular theory is explaining such relations. Almost every apparently generic exotic aspect of QFT has led to useful new concepts. For the case at hand

[7]This weird aspect should not be held against QFT but rather be taken as indicating that localization by a piece of hardware in a laboratory is also limited by an arbitrary large but finite energy, i.e. is a "phase space localization" (see subsequent discussion). In QM one obtains genuine localized subspaces without energy limitations.

the difficulty in defining localization subspaces (in view of the above density statement) have led to the discovery of the physical relevance of localization with respect to phase space in LQP, i.e.e the understanding of the size of degrees of freedom in the set:

$$\begin{aligned} P_E\mathcal{A}(\mathcal{O})\Omega \ &is\ compact \\ e^{-\beta\mathbf{H}}\mathcal{A}(\mathcal{O})\Omega \ is\ nuclear,\ \mathbf{H} \ &= \ \int EdP_E \end{aligned} \tag{17}$$

The first statement was derived way back by Haag and Swieca whereas the second statement (and similar nuclearity statements involving modular operators of local regions instead of the global hamiltonian) which is more informative and easier to use, is a later result of Buchholz and Wichmann [1]. This degree of freedom counting is very much related to on of the oldest of the "exotic" quantum problems of QFT: vacuum polarization. As discovered by Heisenberg and studied further by Weisskopf, the partial charge:

$$Q_V = \int_V j_0(x)d^3x = \infty \tag{18}$$

diverges as a result of uncontrolled vacuum fluctuations near the boundary. For the free field current it is easy to see that a more general definition (which takes into account the fact that the current is a 4-dim distribution and has no restriction to equal times) leads to a finite expression [41]. The algebraic counterpart is the so called "split property" namely the statement [1] that if one leaves between say the double cone observable algebra $\mathcal{A}(\mathcal{O})$ and its causal disjoint $\mathcal{A}(\mathcal{O}')$ a "collar" $\mathcal{O}_1 \cap \mathcal{O}$:

$$\mathcal{A}(\mathcal{O}) \subset \mathcal{A}(\mathcal{O}_1),\ \ \mathcal{O} \ll \mathcal{O}_1\,,\ properly \tag{19}$$

then it is possible to construct in a canonical way a type I tensor factor $\mathcal{N}$ which extends into the collar $\mathcal{A}(\mathcal{O})' \cap \mathcal{A}(\mathcal{O}_1)$ i.e. $\mathcal{A}(\mathcal{O}) \subset \mathcal{N} \subset \mathcal{A}(\mathcal{O}_1)$. With respect to $\mathcal{N}$ the vacuum state factorizes i.e. as in QM there are no vacuum fluctuations for the "smoothened" operators in N. The algebraic analogon of Heisenberg's smoothening of the boundary is the construction of a factorization of the vacuum with respect to a suitably constructed type I factor algebra which uses the collar extension of $\mathcal{A}(\mathcal{O})$. It turns out that there is a canonical (mathematically preferred) factorization which lends itself to define a natural "localizing map" Φ which has given valuable insight into an intrinsic LQP version of Noether's theorem [1], i.e. one which does not rely on any parallelism to classical structures as is the case with quantization.

There are also interesting "folklore theorems" i.e. statements which are mostly taken for granted but for which yet no rigorous argument exists (but also no counter-example). One is the statement of "nuclear democracy". In the context of LQP it states that an operator from a (without loss of generality) double cone algebra $A \in A(\mathcal{O})$ or a pointlike field couples to all states to which the superselection rules allow a nonvanishing matrixelement. In particular we expect:

$$\left\langle \varphi^{in} \left| A \right| \psi^{in} \right\rangle \neq 0 \tag{20}$$

if the (say incoming) multiparticle state vector φ^{in} lies in the same charge superselection sector as $A \left| \psi^{in} \right\rangle$. A special case is the phenomenon of vacuum polarization through interaction i.e. the idea that there may be no local operator $A \in \mathcal{A}(\mathcal{O})$ at all (even allowing other operators than Heisenberg's currents) such that $A\Omega$ is in the one-particle space without additional $p\bar{p}$-contributions. In order to suppress vacuum polarization in state vectors of interacting theories, one has to allow at least a semiinfinite localization region as the wedge region. For any compact region the infinite particle clouds and the field point of view take over. The polarization cloud content of $A\Omega$ is intimately related to the modular objects of $(\mathcal{A}(\mathcal{O}), \Omega)$. If one could back up these expectations (based on model observations) by rigorous theorems, one would have achieved an intrinsic understanding of interactions.

3 Renormalized Perturbation

Following Tomonaga, Feynman and Schwinger and the other pioneers of perturbative renormalization, interactions are traditionally introduced through one of the various forms of quantization (canonical, path integral,..).

The method which brings out the pivotal role of causality is however the so called "causal perturbation method" of Stuekelnberg, Bogoliubov Parasiuk and Shirkov [21] which was formulated as a finite iteration method within the principles of LQP without reference to quantization[8] (with some recent refinements notably related to curved space time and gauge theories) by [24][23] following the Epstein and Glaser Glaser method [22].

It is a conceptual weakness of any quantization approach that contrary to QM, where this can be given a rigorous meaning, quantization in field theory

[8]Also Weinberg's derivation of Feynman rules for arbitrary spin [14] is in this spirit.

remains more on the intuitive artistic side. Only for a so-called superrenormalizable interactions is the assumed canonical or functional Feynman-Kac representation structure reflected in the result, in all other cases it only serves as a vehicle or catalyzer of the human thought and does not survive the renormalization procedure. This "artistry" pervades the standard text book formulation of QFT. Such an approach is completely acceptable as long as one remains aware that (what I will summarily call) the Lagrangian quantization is nothing more than an efficient chain of formal manipulations and tricks which leads to the correct perturbative results, even if in most cases these renormalized results violate the initial assumptions (canonical [time-like or light cone quantization] commutation relations, functional integral representations,.. almost never hold, except for superrenormalizable interactions). In order to rescue these structures physicist sometimes resort to imagine the existence of physical cutoffs or regulators and uses the euphemism "cut-off canonical variables or cut-off functional representations".

Sometimes the infinities of the unrenormalized theory are attributed more physical significance than just indicating the necessity of repairing a slightly incorrect classical starting point (the classical Poincaré-Lorentz- instead of the Wigner-particle picture). These conceptual draw backs of the quantization artistry are however offset by the extreme efficiency of the method of renormalizing away infinities by Feynman rules. On the other hand, Schwinger's finite split point method for the nonlinear terms in field equations may be conceptually cleaner because one never meets an infinity (as long as one does not interchange short distance limits with the other operations), but is practically less efficient as Feynman's infinity (or ad hoc cut-off) method.

Different from quantization, LQP only uses physical assumptions which are also genuinely reflected in the results (causality, spectral properties, modular structure of local algebras,..) i.e. it uses the same principles as standard QFT but it does so in a more conscientious way. In such an approach the short distance properties of individual fields are, apart from perturbation theory (infinitesimal deformations around free fields), not so tightly connected with the existence of the model. We will come back to this important point in the nonperturbative section 5. In the following we will illustrate the strength of the LQP point of view in perturbation theory. The renormalized results are of course the same as in the functional approach, but the derivation and the guiding physical ideas differ in an interesting way.

In causal perturbation theory, the interaction is implemented by locally coupling the free fields (any choice possible, ψ does not have to be Lagrangian!) by an L-invariant sum over Wick monomials $W_i(x)$ and one defines the following formal transition operator in Fock space:

$$\begin{aligned} S(g,h) &= Te^{i\int\{W(x)+h(x)\psi(x)\}d^4x} \\ \tilde{C} &\subset \operatorname{supp} g \subset C \\ g_i &= const\ in\ \tilde{C} \end{aligned} \tag{21}$$

where $W(x) = \sum g_i W_i(x)$ and $C, \tilde{C}$ are large double cone regions. In the following we specialize to one field and one coupling (the notation for the general case with several fields and monomials we leave to the reader). Already without the time-ordering T, the exponential is a mathematically delicate object since the smeared Wick-powers beyond the second are not essentially selfadjoint on their natural domains. With the time ordering it is more serious: apart from certain W's with low operator dimensions (a situation which cannot occur in d=1+3 dimensions), there is no functional operator $S(g)$ in Fock space for which a mathematical control has been achieved. Therefore one proceeds along the following two lines:

- **Extraction of general causality properties for $S(g)$ and related operators (the "Bogoliubov axiomatics").** The basic causality in the time-ordered formalism is:

  $$\begin{aligned} T(\psi(x_1)...\psi(x_n)) &= T(\psi(x_1))..\psi(x_k))\cdot T(\psi(x_{k+1})..\psi(x_n)) \\ if\ x_j &\notin x_i + \bar{V}_+,\ i=1,...,k,\ j=k+1,...,n \end{aligned} \tag{22}$$

  For the purpose of (formally) extracting a causal net it is helpful to reformulate this property in terms of another transition operator:

  $$\begin{aligned} V(g,h) &\equiv S(g,h=0)^{-1}S(g,h) \\ causality &: \quad V(g,h_1+h_2) = V(g,h_1)V(g,h_2) \\ if\ \operatorname{supp} h_1 &\notin \operatorname{supp} h_2 + \bar{V}_+ \end{aligned} \tag{23}$$

  With the local algebras being now defined as (the notation alg includes the von Neumann closure):

  $$\mathcal{A}_g(\mathcal{O}) \equiv \operatorname{alg}\{V(g,h), \operatorname{supp} h \subset \mathcal{O}\} \tag{24}$$

In fact a change of the coupling strength g outside C (see 21) does not change the net $\mathcal{A}_g(\mathcal{O})$ for $\mathcal{O}$ inside $\tilde{C}$, except for a common unitary (the nets are isomorphic i.e. considered to be identical)

$$\begin{aligned} V(g+\delta g,h) &= AdU(g,\delta g)V(g,h) \\ &\quad \mathrm{supp}\,\delta g\, outside \;\; \tilde{C} \end{aligned} \tag{25}$$

With this, the transition from the BPS-EG to the LQP net formalism has been achieved [24]. The algebraic content has been constructed in an auxiliary Fock space whose particle content has nothing to do with the physical particle content and the adiabatic limit of the E-G approach (which would have forced the coalescence of the two) has been avoided.

- **Perturbation as a deformation of free fields.** Having no control over the objects in the Bogoliubov axiomatics, we satisfy ourselves with existence and properties of causal power series

$$S(g) = \sum \frac{i^n}{n!} \int g(x_1)....g(x_n)TW(x_1)....W(x_n) \tag{26}$$

which allows a iterative construction in n with W serving as the input. The main inductive step is the construction of the total diagonal part in n+1 order, assuming that the n^{th} order time ordered product has been fully (i.e. as an operator-valued distribution on all Schwarz test functions) constructed. Causality defines the n+1 order object on all test functions which vanish on totally coalescent diagonal point [24]. The (Hahn-Banach) extension problem allows for totally locally supported terms with a priori undetermined coefficient. These local terms are often (as "counter-terms") lumped together with the n=1 term. Mere perturbative locality and unitarity requirements do not fix this ambiguity (i.e. perturbatively one always operators in Hilbert space[9]). Rather the introduction of a suitable degree function allows to control this ambiguities in terms of a finite number of physical parameters, at least in the case of so-called renormalizable interactions W with $\dim W \leq 4 = d$. Perturbation is a deformation around known theories

[9]This is not necessarily so in other (e.g. functional integral) formulations, where the connection with operator aspects of QT may get lost (even the introduction of cut-offs or regularizations is no assurance for maintaining it).

which in the present case are free fields. It only explores an infinitesimal neighborhood around free fields and not suited for deciding questions about the mathematical existence. In fact beyond deformation theory it is not physically compelling to implement the idea of interactions by coupling free fields to $W's$ in Fock space. Rather this is the perturbative way of introducing interactions and not a general consequence of the general framework. Indeed the nonperturbative attempts based on modular theory use a different implementation of "interaction", as will be shown later. The causal perturbation theory leads to the same renormalized correlation functions as e. g. the one based on functional integrals. However, as shown in the sequel, the physical concepts and calculational rules are somewhat different. In particular all differential identities (as equations of motion) can be used freely in the "on shell" causal formulation whereas this is not the case in the off shell functional (euclidean) approach.

This formal counting argument, if strictly true, would rule out all massive higher spin s$\geq$ 1 fields as candidates to be used for interaction polynomials W since there are no intertwiners from the Wigner particle to covariant local representations ψ with dim$\psi < 2$. For example a massive s=1 object in the vectormeson description has operator dimension dim$A_\mu = 2$ (the use of different intertwiners does not improve this increase of quantum versus classical dimension). so that any trilinear interaction involving A_μ (and lower spin) has dim$W \geq 5$. Fortunately this barrier put by Wick-polynomials and Borchers classes of free fields has an interesting hole. It can be undermined by a cohomological trick which consists in the following observation. Find a cohomological representation of the e.g. (m, s=1) physical Wigner space:

$$H_{Wigner} = \frac{\ker s}{\operatorname{im} s}, \;\; s^2 = 0 \tag{27}$$

Here s acts on H_{ext} and the Poincaré group is still covariantly represented on H_{ext} (the pseudo-unitary nature of the boost representors however turns out to be unavoidable). The transversality of the covariant inner product of the vectorpotential (which was the origin of dim$A_\mu = 2$ instead of the classical dimension 1) only emerges in the cohomological descend from H_{ext} to H_{Wigner}. The minimal extension of the vectorpotential requires two ghost fields.

The simplest cohomological extension of the Wigner wave function space which allows a nilpotent operation s with $s^2 = 0$ such that the physical transversality condition $p^\mu A_\mu(p) = 0$ follows from the application of s needs besides two scalar ghosts wave functions ω and $\bar\omega$ another scalar field φ (often called the Stueckelnberg field):

$$\begin{aligned} (sA_\mu)(p) &= p_\mu \omega(p) \\ (s\omega)(p) &= 0 \\ (s\bar\omega)(p) &= p^\mu A_\mu(p) - im\varphi(p) \\ (s\varphi)(p) &= -im\omega(p) \end{aligned} \tag{28}$$

One immediately realizes that $s^2 = 0$ and that $s(\cdot) = 0$ enforces the vanishing of ω and relates φ to $p^\mu A_\mu$. At this point there is no grading in the formalism, i.e. the ω and φ are simply ungraded wave functions. However the functorial transition from Wigner theory to QFT requires the introduction of a grading with $\deg\omega = 1, \deg\bar\omega = -1$, and $\deg A_\mu = 0$, with s transferring degree 1. The reason is that only with this grading assignment [29] the s allows a natural tensor extension to multiparticle spaces with stable nilpotency which insures the commutativity of the above diagram representing the cohomological ascend and descend: $s(a \otimes b) = sa \otimes b + (-1)^{\deg a} a \otimes sb$.

This suggests to view the Fock space version δ of s as the image of a (pseudo) Weyl functor Γ as $\delta = \Gamma(s)$ and to write the δ in the spirit of a formal Noether symmetry charge Q associated with the free field (linear) version of the BRST charge [25] [26]:

$$Q = \int (\partial_\mu A^\mu(x) + m_a \phi(x)) \overleftrightarrow{\partial}_0 \omega(x) d^3x = Q^\dagger \tag{29}$$

The only way to convert this into the Fock space setting is to supply the multiparticle version of s with a Z-grading. In this form one obtains an object δ of a differential algebra with $\delta^2 = 0$ which acts on vectors and operators in H_{ext} similar to a Noether charge

$$\begin{aligned} \delta A = i\,[Q, A] = \delta A \equiv i\left\{QA - (-1)^{\deg A} AQ\right\} \\ Q\ in\ H_{ext}, \quad Q^2 = 0 \end{aligned} \tag{30}$$

Where the nilpotency together with the hermiticity prevents a positive inner product in *-representation of such algebras. This cohomological construction is well known from the work of Kugo-Ojima who formulated the BRS formalism in these setting [25]. There was a revival of interest due to some recent curious and relevant observations by Scharf and collaborators [27] in the context of "causal perturbation theory" of gauge theories (in the terminology of the present paper: to the problem of renormalizable spin s=1 interactions). They are usually applied to zero mass gauge theories, but here we use them in order to overcome the obstruction against renormalizability of higher spin couplings. The idea is that the unphysical theory in H_{ext}^{Fock} is renormalizable in the sense of $\dim W_{ext} \leq 4$ and that the physics can be constructed by cohomological descend $H_{ext}^{Fock} \to H_{phys}$ at the end. The basic simplification (thanks to scattering theory which is only applicable in the massive case without infrared problems) is the bilinear nature of Q. From the physically required commutation of S with Q one argues that since formally $S = \lim S(g)$ for $\mathrm{supp} g \to \mathbf{R}^4$, the $[S(g), Q]$ commutator is supported near the boundary $\mathrm{supp}\partial g \neq 0$ (21):

$$[Q, T_n(x_1, ..., x_n)] = \sum \frac{\partial}{\partial x_i^\mu} T_{n,i}^\mu(x_1, ..., x_n) \tag{31}$$

As was shown in [27], these consistency relations in H_{ext} are very restrictive and have powerful consequences.

Without going into any computation, let me verbally present some results which are based on the mentioned computations and my own interpretation and extrapolation (the University of Zurich group did not yet present a sufficiently systematic account of their approach). Suppose we are dealing with massive abelian s=1 vectormesons coupled to s=$\frac{1}{2}$ particles, then without extension the interaction density would have $\dim W = 5$, which is too high for renormalizability. There are two ways of overcoming this obstruction against renormalizability. One is the Stueckelnberg extension which, similar to the Gupta-Bleuler formalism for the massless case (but different from the previously presented Q-graded differential calculus) reduces short distance powers. In that theory the *spinor field is covariant and renormalizable* but, different from e.g. the spinor current, *not physical* (i.e. it changes in the cohomological descend). By an operator transformation involving the Stueckelnberg field (the divergence of the unphysical vectorpotential), one can turn the situation around and obtain a *physical but nonrenormalizable* (no polynomial bound which is independent of the perturbative order, i.e. nontempered distribution) covariant field. The second possibility is to apply the above cohomological Q-approach. Starting with a first order W which also accounts for the presence of the ghosts, one finds together with the next order, that the consistency relations (31) demand not only that the admissable couplings are precisely those which one would write down in the spirit of a classical gauge theory, but also that the ghosts from the extension of the Wigner theory are not enough in the next order and that one needs in addition a physical degree of freedom. The simplest possibility which works is a scalar field. This is the alias Higgs field, but this time without a vacuum expectation one point function (which would not be needed anyhow, since the vectormeson is massive from the start). In this theory the ψ-field is *physical and renormalizable.* As a matter of fact, the only nonphysical field is the vectormeson field itself. A posteriori one can of course define a physical A_μ with a smaller operator dimension $\dim A_\mu = 1$ by integrating up the field strength $F_{\mu\nu}$ along a semiinfinite spacelike string, but in that case one would trade the pointlike field against one with a semiinfinite stringlike localization. In other words the interaction-mediating vectorpotential exists as a renormalizable physical operator in H_{phys}, but its localization is not pointlike i.e. a nonlocal object in a local theory. A closer examination reveals that the vectormeson-charge (i.e. the analogous charge to the Maxwell-charge) is screened . Therefore as expected, the Schwinger charge screening property [30] is intrinsic, whereas the Higgs condensate picture is not. In order to give the latter at least a formal meaning, one has to read the theory as a quantized classical gauge theory with a photon with a mass coming from a Goldstonian "fattening" mechanism. Whereas this viewpoint is not forbidden, it is hardly natural from a physical point of view (things may look different if ones main aim is to study differential geometric aspects of QFT). It is an interesting question whether the required physical (scalar) degree of freedom ($\simeq$perturbative alias Higgs) also exists for consistency reasons in a nonperturbative version of massive vectormesons (related questions: does renormalizable beyond perturbation theory mean polynomial bounded? What does "cohomological representation" mean outside of perturbation theory?). In view of the experimental importance attributed to the Higgs particle, this somewhat different point of view of looking at massive $s = 1$ theories is not without interest.

Since the massive vectormeson theory is conceptually simpler (no infrared problems, no interaction terms in Q even for nonabelian vectormesons), it is very tempting to think of an off-shell zero mass limit which should logically link the decoupling of the alias Higgs with the conversion of the local physical ψ with the expected semiinfinite stringlike localized ψ_{ch} which carries the liberated Maxwell charge. The nature of the zero mass limit of the analogous problem in the case of the charged screened Schwinger model suggest that one must modify the ψ so that the infrared convergent string localization will emerge. The understanding of such a charge liberation mechanism (instead of the opposite Schwinger-Higgs screening mechanism) could enrich our insight into physical aspects of infrared problems which are outside the standard particle-scattering interpretation and which hence do not permit an incoming Fock space as a reference representation space.

This point of view is particularly interesting for selfcoupled (nonabelian) massive vectormesons. In that case, according to the best of our knowledge, there is only one way of maintaining renormalizability namely that via the cohomological representation (the Stueckelnberg formalism alone is not capable of maintaining consistency). With this surprising restriction on the interaction parameters from the cohomological representation and consistency, the word mass via "Higgs mechanism of gauge theories" becomes somewhat physically void, since in contrast to spin<1 models there is only one massive interacting theory for a given field content; even the Jacobi identities for the vectormeson couplings f_{abc} are a result of the above principles of causal perturbation [27]. Whereas it is not literally wrong to say that this theory is obtained by quantizing classical gauge theories, this terminology hides more than it reveals. It is really the LQP of $s = 1$ which *tells the classical field theory where to go* (for classical fields with more Lorentz indices the number of couplings increases and needs a gauge principle to select among the many possible couplings) rather than the other way around. The most important application of the gauge principle (really its historical origin) is the theory of quantum matter coupled to external (semiclassical) electromagnetic fields. From a fully QFT point of view this is of course nothing but the conceptual dominance of Bohr's correspondence principle (quantum→(semi)classical) i. e. (semi)"classifization" over its "quantization" opposite.

It is an interesting question whether the cohomological ideas are restricted to s=1, or if they can also lead to renormalizability for s>1 couplings. All this may seem strange to a physicist who grew up with the differential geometric fibre bundle method of gauge theories. But it really shouldn't, because he probably would be even more surprised if there would be other consistent renormalizable models of massive selfinteracting (i.e. contributing to their own source) spin s=1 particles besides the standard one (which he happens to calls "the Higgs mechanism in nonabelian gauge theories"). A theory however, which is already unique by the existing causal and spectral principles augmented by renormalization, does not require any additional principle (but perhaps an efficient cooking recipes to obtain the renormalized answer quickly). The present viewpoint is really quite old; it is that of

Lewellyn-Smith augmented by the magic of BRST-Kugo-Ojima cohomological extension. What is really physically behind this magic is the necessity of having stringlike localized vectorpotentials as mediators of the interaction together with the recognition that this cannot be implemented in standard perturbation theory (which requires the deformation of pointlike free fields), i.e. our interpretation of the cohomological trick is that it hides localization problems.

A direct causal perturbative approach to s=1 massless theories was recently formulated by Duetsch and Fredenhagen [23]. The necessity to avoid the (physically controversial) adiabatic limit requires the use of the full nonlinear BRS structure and to confront a situation in which (unlike as in the above case with bilinear Q) the position of the physical cohomology space keeps changing with the perturbative order. Lacking a fixed physical reference space (e.g. an incoming scattering space) the physical space only appears at the end as a representation space of a perturbative observable *-algebra. This construction was carried out in QED, but there is little doubt that with more work it also works for the nonabelian case.

We do of course not claim that the BRS-like cohomological construction (for the rescue of renormalizability in the face of higher spin) advocated in these notes is less mysterious then the quantization gauge principle. However it is a bit closer to the spirit of LQP and perhaps less so to quantization and differential geometry. It keeps the attention on the unsolved infrared problems[10] and the idea that the present day perturbative formalism is apparently not capable do deal directly with non-pointlike charge-carrying fields alive. It is a good investment into the future to keep deep physical problems open (and not to suffocate them in seemingly elegant differential geometry). Last not least it teaches us that LQP of s=1 particles with renormalizable interactions has no place for a new physical principle as the classical principle of gauge invariance. If by the terminology "gauge QFT" one wants to highlight the curious fact that the quantum causality and positive energy principle in the renormalizable setting is much more restrictive for s=1 than for s<1 and give a brief description of how to compute, than this is quite reasonable; if on the other hand one thinks of this terminology as introducing an additional physical principle this could cause misunderstandings (because as stated be-

[10] From a physical point of view the estetical lure of differential geometry of fibre bundles in gauge theories is a bit dangerous, because it takes one away from the harder but physically more important infrared phenomena of the LQP of s=1..

fore, it is not the LQP which needs this but rather the (semi)classical field theory.

4 Modular Origin of Geometric and Hidden Symmetries

We have seen from the wedge localization in section 2 that the modular objects associated to a standard (vector Ω cyclic and separating) pair $(\mathcal{A}(\mathcal{O}), \Omega)$ has, under certain circumstances a geometrical significance, e.g. for the wedge in a massive (Poincaré-invariant) or the double cone in a massless (conformally-invariant) theory. This suggests the question whether all spacetime symmetries can be viewed as having a modular algebraic origin, i.e. if they originate from the relative positions of individual algebras in a net. *This would elevate spacetime from its role of indexing individual algebras in the net to a structure which is more intimately related with the physical aspects of LQP and in this way emphasize structural properties which seem to be closer to the elusive "Quantum Gravity".* It turns out that in chiral conformal theories the Moebius group together with the net on which it acts can be easily constructed from two properly positioned algebras which give rise to two "halfsided modular inclusions" (see below). In this conformal setting the Haag duality is automatic and there is no spontaneous symmetry breaking. The analogue in the higher dimensional case is to assume wedge duality (always achievable, as previously mentioned, by maximalization) and to prove the equality of the modular group with the Lorentz-boost without assuming (as Bisognano and Wichmann did) that the algebras are generated by local fields. Presently this cannot be done without making a mild technical assumption [33]. Again one succeeds to build up the whole Poincaré group as well as the net from a finite number of algebras in appropriate modular positions (using modular inclusions and modular intersections).

Since modular groups exist for each space time region one may ask about their physical interpretation. Let us start with posing the opposite question in a context where there are geometric candidates without obvious modular origin. In chiral conformal theories one has a rich supply of diffeomorphisms of the circle. The way these mathematical structures were discovered by physicist is somewhat bizarre and confusing (as it sometimes happens with ideas which are in the air). Let me use the opportunity in order to recall some aspects. The Witt algebra (infinitesimal diffeomorphisms without the central extension) was first observed in the Veneziano dual S-matrix model by Virasoro [31]. Parallel to this, but without interrelation, there were detailed field theoretic investigations of the representation of conformal generators in terms of the energy momentum tensor T and their action e.g. on the Thirring fields [34] (which was the only structure needed to unravel the conformal representation theory of these models) without the T-T commutation relation being written down . Shortly after Ramond [35] obtained the central structure for the free Fermi situation within the dual model setting, but with wrong coefficients (corrected in some later paper). In QFT, the full general structure of the chiral energy momentum algebra was first seen (according to the best of my knowledge) as a structural consequence of translational covariance and causality imposed on the energy-momentum tensor in chiral theories [36]. Again the motivation on the QFT-side was quite different from that of the dual model. My main motivation was to find the first nontrivial "Lie field" (see remarks in the conclusions).

Let us now look at a special subgroup whose Lie-algebra is isomorphic to that of the Moebius group. Its action on the circle is

$$z \rightarrow \sqrt{\frac{a+bz^2}{c+dz^2}},\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SU(1,1) \tag{32}$$

where the cuts connecting both poles and zeros are chosen outside the unit circle. In fact this defines a two-fold covering of the Moebius group. Given an interval, its square root (inverse image of $z \rightarrow z^2$) consists of two disjoint intervals which are separately left invariant under the above transformation group. The obvious conjecture is of course that (as for the case of a single interval) the covering dilation subgroup is the modular group of the pair $(\mathcal{A}(I_1 \cup I_2), \Omega)$. But this cannot be because this action restricted to one interval is the same as that of the dilation in the Moebius group but this, according to a theorem by Takesaki this is not possible if the vacuum state fulfills the Reeh-Schlieder property of being cyclic and separating for only one interval. Since it never happens that two disjoint square root intervals are contained in one interval of another such pair, there will be no contradiction with the lack of the Reeh-Schlieder property for one interval. A state on the Weyl algebra (which we take as an illustration of a simple conformal model) which is invariant under the above covering transformation [33] is easily found in terms of the following scalar product:

$$\langle f, g \rangle = \int \frac{f(x)g(y)}{\left[(x-y)(1+xy)+i\varepsilon\right]^2}(1+x^2)(1+y^2)dxdy \tag{33}$$

where we used the linear presentation instead of the circular one ($SL(2,R)$ instead of $SU(1,1)$). This is to be compared with the standard inner product belonging to the vacuum representation

$$\langle f,g\rangle_0 = \int \frac{f(x)g(y)}{(x-y+i0)^2}dxdy \tag{34}$$

One easily checks that this inner product belongs to the same symplectic form as the standard one namely

$$\omega(f,g) = \mathrm{Im}\,\langle f,g\rangle = \int fg'dx = \omega_0(f,g) = \mathrm{Im}\,\langle f,g\rangle_0 \tag{35}$$

As for the standard case the criterium for a Fock representation is that the inner product can be represented in terms of ω with the help of a complex structure $I_0, I_0^2=-1$, with

$$\begin{aligned} \langle f,g\rangle_0 &= \omega(I_0f,g) = -\omega(f,I_0g) \\ (I_0f)(x) &\equiv \int \frac{-1}{(x-y+i\varepsilon)}f(y)dy \end{aligned} \tag{36}$$

the analogous statement holds for $\langle f,g\rangle$ with I_0 replaced by I

$$\begin{aligned} I &= \Gamma^{-1}\circ I_0\circ\Gamma \\ (\Gamma f)(x) &\equiv \int f(\frac{x}{2}+sign(x)\sqrt{1+\left(\frac{x}{2}\right)^2}) \\ \langle\Gamma f,\Gamma g\rangle_0 &= \langle f,g\rangle \end{aligned} \tag{37}$$

The changed inner product defines a changed quasifree state on the Weyl algebra. The proof that the covering dilation

$$\begin{aligned} U(\lambda) &= \Gamma^{-1}\circ V(\lambda)\circ\Gamma \\ (V(\lambda)f)(x) &= f(\lambda x) \end{aligned} \tag{38}$$

is indeed the modular group for the algebra of the disjoint intervals $[-\infty,-1]\cup[0,1]$ in this quasifree state, we only have to check the appropriate KMS condition. From:

$$\begin{aligned} \lim_{\theta\uparrow 2\pi} &< U(\lambda)f,g>_1 = \lim_{\theta\uparrow 2\pi}< V(\lambda)\circ\Gamma(f),\Gamma(g)> = \langle\Gamma(g),\Gamma(f)\rangle = \\ &= \langle g,f\rangle_1 \end{aligned} \tag{39}$$

one sees that the $U(\lambda)$ fulfils the KMS condition if both f and g are from one of the two intervals since Γ transforms the space of $[0,1]$ localized functions into $[-\infty,-1]$ localized ones and vice versa.

This situation is very interesting, since although the chiral diffeomorphisms allows no geometric generalization to diffeomorphisms in higher dimensional LQP, the disconnected (and multiply connected) algebras have modular groups which act in a non-pointlike manner inside these disconnected local regions[11]. This is what we mean by "hidden symmetries". In the following we will look at two more illustrations.

As a reference wedge we may take the wedge $W(l_1, l_2)$ spanned by the light like vectors $l_{1,2} = e_\pm = (1,0,0,\pm 1)$, in which case we call z,t the longitudinal and x,y the transversal coordinates (the light like characterization of wedges is convenient for the following). This situation suggests to decompose the Poincaré group generators into longitudinal, transversal and mixed generators

$$P_\pm = \frac{1}{\sqrt{2}}(P_0 \pm P_z),\ M_{0z};\ M_{12},\ P_i;\ G_i^{(\pm)} \equiv \frac{1}{\sqrt{2}}(M_{i0} \pm M_{iz}),\ i = 1,2 \quad (40)$$

The generators $G_i^{(\pm)}$ are precisely the "translational" pieces of the euclidean stability groups $E^{(\pm)}(2)$ of the two light vectors $e_\pm$ which appeared in Wigner's representation theory for zero mass particles. More recently these "translations" inside the homogenous Lorentz group appeared in the structural analysis of "Modular Intersections" of two wedges [37][38]. Apart from the absence of the positive spectrum condition, its role is analogous to that of the true translations $P_\pm$ with respect to halfsided "Modular Inclusions" [38].

As one reads off from the commutation relations, $P_i, G_i^{(+)}, P_\pm$ have the interpretation of a central extension of a transversal "Galilei group"[12] with the two "translations" $G_i^{(+)}$ representing the Galilei generators, P_+ the central "mass" and P_- the "nonrelativistic Hamiltonian". The longitudinal boost M_{0z} scales the Galilei generators $G_i^{(+)}$ and the "mass" P_+. Geometrically the $G_i^{(+)}$ change the standard wedge (it tilts the logitudinal plane) and the corresponding finite transformations generate a family of wedges whose envelope is the halfspace $x_- \geq 0$. The Galilei group together with the boost M_{0z} gen-

[11] Observable algebras in disconnected regions have also played a role as indicators of the presence of charge sectors [32][49][41].

[12] This G's are only Galileian in the transverse sense; they tilt the wedge so that one of the light like directions is maintained but the longitudinal plane changes.

erate an 8-parametric subgroup $G^{(+)}(8)$ inside the 10-parametric Poincaré group[13]:

$$G^{(+)}(8): \ P_{\pm}, \ M_{0z}; \ M_{12}, \ P_i; \ G_i^{(+)} \tag{41}$$

The modular reflection J transforms this group into an isomorphic $G^{(-)}(8)$.

The Galileian group is usually introduced as a "contraction" of the Poincaré group. But as the present discussion, the wedge (or rather as in the following remarks, two wedges in a special modular intersection position) shows , it also appears as a genuine subgroup of the Poincaré group. The latter fact seems to be less known.

All observation have interesting generalizations to the conformal group in massless theories in which case the associated natural space-time region is the double cone.

This subgroup $G^{(+)}(8)$ is intimately related to the notion of modular intersection see [37][38]. Let l_1, l_2 and l_3 be 3 linear independent light like vectors and consider two wedges $W(l_1,l_2), W(l_1,l_3)$ with Λ_{12} and Λ_{13} the associated Lorentz boosts. As a result of this common l_1 the algebras $\mathcal{N} = \mathcal{A}(W(l_1,l_2)), \mathcal{M} = \mathcal{A}(W(l_1,l_3))$ have a modular intersection with respect to the vector Ω. Then $(\mathcal{N} \cap \mathcal{M}) \subset \mathcal{M}, \Omega)$ is a so-called modular inclusion [38][39]. Identifying $W(l_1,l_2)$ with the above standard wedge, we notice that the longitudinal generators $P_{\pm}, \ M_{0z}$ are related to the inclusion of the standard wedge algebra into the full algebra $\mathcal{B}(H)$, whereas the Galilei generators $G_i^{(+)}$ are the "translational" part of the stability group of the common light vector l_1 (i.e. of the Wigner light-like little group).

To simplify the situation let us take d=1+2 with $\mathcal{G}(4)$, in which case there is only one Galilei generator G. In addition to the "visible" geometric subgroup of the Poincaré group, the modular theory produces a "hidden" symmetry transformation $U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(a)$ which belongs to a region which is a intersection of two wedges:

$$U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(a) := \exp(\frac{ia}{2\pi}(\ln\Delta_{N\cap M} - \ln\Delta_M)) \tag{42}$$

is a unitary group with positive generator. Moreover one has:

$$U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(1-e^{-2\pi t}) = \Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{N}\frown\mathcal{M}}^{-it} \tag{43}$$

[13] The Galileian group is usually introduced as a "contraction" of the Poicaré group. But as the present discussion about the wedge (or rather the following remarks about two wedges in a special modular intersection position) shows , it also appears as a genuine subgroup of the Poincare group. The latter fact seems to be less known.

$$U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(e^{-2\pi t}a) = \Delta_{\mathcal{M}}^{it} U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(a)\Delta_{\mathcal{M}}^{-it} \tag{44}$$

$$AdU_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(-1)(\mathcal{M}) = \mathcal{N}\cap\mathcal{M} \tag{45}$$

and

$$J_{\mathcal{M}} U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(a) J_{\mathcal{M}} = U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(-a). \tag{46}$$

Similar results hold for $\mathcal{N}$ replacing $\mathcal{M}$. Due to the intersection property we finally have the commutation relation

$$[U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(a), U_{\mathcal{N}\frown\mathcal{M},\mathcal{N}}(b)] = 0 \tag{47}$$

which enables one to define the unitary group

$$U_{\mathcal{N}\frown\mathcal{M}}(a) = U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(-a) U_{\mathcal{N}\frown\mathcal{M},\mathcal{N}}(a). \tag{48}$$

This latter group can be rewritten as

$$U_{\mathcal{N}\frown\mathcal{M}}(1-e^{-2\pi t}) = \Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{N}}^{-it} \tag{49}$$

and thereby recognized to be in our physical application the 1-parameter Galilean subgroup G (41) in the above remarks.

Now we notice that for $a<0$

$$\begin{aligned} AdU_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(a)(\mathcal{M}) &= Ad\Delta_{\mathcal{M}}^{-i(\frac{1}{2\pi}\ln -a)} U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(-1)(\mathcal{M}) \\ &= Ad\Delta_{\mathcal{M}}^{-i(\frac{1}{2\pi}\ln -a)}(\mathcal{N}\cap\mathcal{M}) \end{aligned} \tag{50}$$

Because $\Delta_{\mathcal{M}}^{it}$ acts geometrically as Lorentz boosts, we have full knowledge of the geometrical action of $U_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(a)$ on $\mathcal{M}$ for $a<0$. For $a>0$ we notice

$$\begin{aligned} AdU_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(1)(\mathcal{M}) &= AdU_{\mathcal{N}\frown\mathcal{M},\mathcal{M}}(2)(\mathcal{M}\cap\mathcal{N}) = AdJ_{\mathcal{M}}J_{\mathcal{N}\frown\mathcal{M}}(\mathcal{M}\cap\mathcal{M}) \\ &= AdJ_{\mathcal{M}}(\mathcal{M}'\cup\mathcal{N}') \end{aligned} \tag{51}$$

and again, due to the geometrical action of J_M we have a geometrical action on $\mathcal{M}$ for $a>0$.

$$AdU_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(a)(\mathcal{M}) = Ad\Delta_{\mathcal{M}}^{-i(\frac{1}{2\pi}\ln a)} J_{\mathcal{M}}(\mathcal{M}'\cup\mathcal{N}') \tag{52}$$

From these observations and with $U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(1-e^{-2\pi t}) = \Delta_{\mathcal{M}}^{it}\Delta_{\mathcal{M}\cap\mathcal{N}}^{-it}$ we get for $t<0$:

$$Ad\Delta_{\mathcal{N}\cap\mathcal{M}}^{it}(\mathcal{M}) = Ad\Delta_{\mathcal{M}}^{(-\frac{i}{2\pi}\ln(e^{-2\pi t}-1))} J_{\mathcal{M}}(\mathcal{M}'\cup\mathcal{N}') \tag{53}$$

and in case of $t > 0$:

$$Ad\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}(\mathcal{M}) = Ad\Delta_{\mathcal{M}}^{(-\frac{i}{2\pi}\ln(1-e^{-2\pi t}))}(\mathcal{N}\cap\mathcal{M}). \tag{54}$$

Similar results hold for $\mathcal{N}$ replacing $\mathcal{M}$. With the same methods we get:

$$\begin{aligned} Ad\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}\Delta^{is}_{\mathcal{N}}(\mathcal{M}) &= Ad\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}\Delta^{is}_{\mathcal{N}}\Delta^{-is}_{\mathcal{M}}(\mathcal{M}) \\ &= Ad\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}U_{\mathcal{M}\cap\mathcal{N}}(e^{-2\pi s}-1)(\mathcal{M}) \end{aligned} \tag{55}$$

where $U_{\mathcal{N}\cap\mathcal{M}}$ is the 1-parameter Lorentz subgroup (the Galilei subgroup G in (41) associated with the modular intersection. This gives:

$$\begin{aligned} Ad\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}\Delta^{is}_{\mathcal{N}}(\mathcal{M}) &= AdU_{\mathcal{M}\cap\mathcal{N}}(e^{-2\pi t}(e^{-2\pi s}-1))\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}(\mathcal{M}) \\ &= AdU_{\mathcal{M}\cap\mathcal{N}}(e^{-2\pi t}(e^{-2\pi s}-1))\Delta_{\mathcal{M}}^{-\frac{1}{2\pi}\ln(1-e^{-2\pi t})}(\mathcal{M}\cap\mathcal{N}), \end{aligned} \tag{56}$$

if $t > 0$ and similar for $t < 0$.Therefore we get a geometrical action of $\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}$ on $Ad\Delta^{is}_{\mathcal{N}}(\mathcal{M})$.

A look at the proof shows that the essential ingredients are the special commutation relations. Due to

$$\Delta^{it}_{\mathcal{M}\cap\mathcal{N}} = \Delta^{it}_{\mathcal{M}}U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(1-e^{-2\pi t}) = \Delta^{it}_{\mathcal{M}}J_{\mathcal{M}}U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(e^{-2\pi t}-1)J_{\mathcal{M}} \tag{57}$$

and the well established geometrical action of $\Delta^{it}_{\mathcal{M}}$ and $J_{\mathcal{M}}$, it is enough to consider the action of $U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}$ or similarly $U_{\mathcal{N}\cap\mathcal{M},\mathcal{N}}$. For these groups we easily get

$$AdU_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(a)\Delta^{is}_{\mathcal{N}}\Delta^{-it}_{\mathcal{M}}(\mathcal{N}) = Ad\Delta^{is}_{\mathcal{N}}\Delta^{it}_{\mathcal{M}}U_{\mathcal{N}\cap\mathcal{M},\mathcal{M}}(e^{-2\pi(s+t)}a)(\mathcal{N}) \tag{58}$$

and due to the above remarks the geometrical action of $\Delta^{it}_{\mathcal{N}\cap\mathcal{M}}$ on the algebras of the type $Ad\Delta^{is}_{\mathcal{N}}\Delta^{-it}_{\mathcal{M}}(\mathcal{M})$.

Now, the lightlike translations $U_{transl_1}(a)$ in l_1 direction fulfill the positive spectrum condition and map $\mathcal{N}\cap\mathcal{M}$ into itself for $a > 0$. Therefore we have the Borchers commutator relations with $\Delta^{it}_{\mathcal{M}\cap\mathcal{N}}$ and get

$$Ad\Delta^{it}_{\mathcal{N}\frown\mathcal{M}}U_{transl_1}(a)(\mathcal{M}) = AdU_{transl_1}(e^{-2\pi t}a)\Delta^{it}_{\mathcal{N}\frown\mathcal{M}}(\mathcal{M}) \tag{59}$$

The additivity of the net tells us that taking unions of the algebra corresponds to the causal unions of localization regions. The assumed duality allows us to pass to causal complements and thereby to intersections of the underlying localization regions. Therefore the algebraic properties above transfer to unions, causal complements and intersections of regions. We finally get [33]:

Theorem 1 *Let $\mathcal{R}$ be the set of regions in $\mathbf{R}^{1,2}$ containing the wedges $W[l_1, l_2]$, $W[l_1, l_3]$ and which is closed under:*
a) Lorentz boosting with $\Lambda_{12}(t), \Lambda_{13}(s)$,
b) intersection
c) (causal) union
d) translation in l_1 direction
e) causal complement
Then $\Delta^{it}_{W[l_1,l_2]\cap W[l_1,l_3]}$ maps sets in $\mathcal{R}$ onto sets in $\mathcal{R}$ in a well computable way and extends the subgroup (41) by a "hidden symmetry".

Similarly we can look at a (1+3)-dim. quantum field theory. Then we get the same results as above for the modular theory to the region $W[l_1, l_2] \cap W[l_1, l_3] \cap W[l_1, l_4]$, where l_i are 4 linear independent lightlike vectors in $\mathbf{R}^{1,3}$. Moreover in this case the set $\mathcal{R}$ contain $W[l_1, l_2], W[l_1, l_3]$ and $W[l_1, l_4]$ and is closed under boosting with $\Lambda_{12}(t), \Lambda_{13}(s), \Lambda_{14}(r)$.

The arguments are based on the Borchers commutation relation and modular theory and apply also if we replace modular intersection by modular inclusion. One recovers in this way easily the results of Borchers and Yngvason, [40] who found an illustration of hidden symmetries in thermal chiral conformal QFT (Note that in thermal situations we have no simple geometrical interpretation for the commutants as the algebra to causal complements. Therefore in these cases we have to drop e) in the above theorem.).

The final upshot of this section is to show that there might be a well defined meaning of a geometrical action of modular groups by restricting on certain subsystems.

For conformal LQP in any dimension, one obtains a generalization of the previous situation. In particular the modular group with respect to the vacuum of the double cone algebra is geometric[1]. Consider now a double cone algebra $\mathcal{A}(\mathcal{O})$ generated by a free massless field (for s=0 take the infrared convergent derivative). Then according to the previous remark, the modular objects of $(\mathcal{A}(\mathcal{O}), \Omega)_{m=0}$ are well-known . In particular the modular group is a one parametric subgroup of the proper conformal group. The massive double cone algebra together with the (wrong) massless vacuum has the same modular group σ_t however its action on smaller massive subalgebras inside the original one is not describable in terms of the previous subgroup. In fact the geometrical aspect of the action is wrecked by the breakdown of Huygens principle, which leads to a nonlocal reshuffling inside $\mathcal{O}$ but still is local in the sense of keeping the inside and its causal complement apart. This mechanism can be shown to lead to a pseudo-differential operator for the infinitesimal generator of σ_t whose's highest term still agrees with conformal zero mass differential operator. We are however interested in the modular group of $(\mathcal{A}(\mathcal{O}),\Omega)_m$ with the massive vacuum which is different from the that of the wrong vacuum by a Connes cocycle. We believe that this modular cocycle will not wreck the pseudo-differential nature and that as a consequence the geometric nature of the conformal situation will still be asymptotically true near the horizon of the double cone, however we were presently not able to show this. This modular aspect of the horizon could be linked with what people think should be the quantum version of the Bekenstein-Hawking classical entropy considerations, in particular the ideas about "holographic properties". To be more precise, we expect that even for double cones in Minkowski space (i.e. without a classical Killing vector as for black holes) there will be a finite relative quantum entropy as long as one allows for a "collar" between the double cone and its spacelike complement and that with vanishing size of this collar these entropies will diverge in such a way that ratios (e.g. for differently sized double cones) will stay finite and be determined by the conformal limits. In this way one could hope to prove that e.g. the speculations about entropy, holography and the occurrence of the central terms in the energy momentum commutation relations are nonperturbative generic properties of ordinary LQP [6]. For the thermal aspects this is of course well known..

The modular group structure also promise to clarify some points concerning the physics of the Wightman domain properties [12]. In fact these groups act linearly on the "field space" i. e. the space generated by applying a local field on the vacuum. Therefore this space, which is highly reducible under the Poincaré group, may according to a conjecture of Fredenhagen (based on the results in [42]) in fact carry an irreducible representation of the union of all modular groups (an infinite dimensional group $\mathcal{G}_{mod}$ which contains in particular all local spacetime symmetries). The equivalence of fields with carriers of irreducible representations of an universal $\mathcal{G}_{mod}$ would add a significant conceptual element to LQP and give the notion of quantum fields a deep role which goes much beyond that of being simply generators of local algebras. Our arguments suggest that in chiral conformal QFT $\mathcal{G}_{mod}$ includes all local diffeomorphism.

A related group theoretical approach to LQP which uses both modular groups and modular involutions in order to formulate a new selection principle for states ("The Condition of Geometric Modular Action") was proposed in [43]. In addition to the modular groups which leave the defining local algebras invariant, these authors obtain a discrete group (from the conjugations) which transform the (spacetime) index set. All these true QFT properties remain invisible in any quantization approach. Combining modular theory with scattering theory, the actual J together with the incoming J^{in} can be used to obtain a new framework for nonperturbative interactions [12]. This last topic will be presented in the following section; more details will be deferred to a separate paper together with H.-W. Wiesbrock [56].

5 Constructive Modular Approach to Interactions

The starting observation for relating the modular structure of LQP nets to interactions is that the latter is solely contained in those anti-unitary reflections of the full Poincaré group which contain the time reversal. The continuous part is, thanks to the fact that scattering (Haag-Ruelle, LSZ) theory is a consequence of LQP, the same for the free incoming particles as for the interacting net [41]:

$$\begin{aligned} U(\Lambda, a) &= U(\Lambda, a)^{in} && (60) \\ J &= S_{sc} J^{in} \\ S &= J \Delta^{\frac{1}{2}} && (61) \end{aligned}$$

Here S_{sc} is the scattering matrix (the subscript distinguishes it from the Tomita involution S) and the $J's$ are the Tomita reflections for interacting (incoming) wedge algebras. The standard point of view, where the interaction is introduced in terms of a pair of Hamiltonians (Lagrangians) H, H_0, accounts for the interaction in a different (more perturbative) way which uses different states.

The most promising candidates for a modular construction are obviously massive theories (in order to be able to use S-matrix scattering theory which leads to a reference Fock space) with a known S-Matrix i.e. models which permit a bootstrap construction of S on its own, without using the off-shell fields or local operators. For such S-matrix integrable models, there already exists a constructive formfactor program which goes back to Karowski and Weisz and has been significantly extended by Smirnov [44]. It uses suggestive prescriptions and assumptions within the dispersion theoretical LSZ framework.

Since the bulk of the LSZ formalism is a consequence of the more basic algebraic QFT, it is reasonable to ask if our modular localization framework is capable to shed additional light on this program; in particular whether it can be understood as a special (analytically simple) illustration of a more general nonperturbative construction without the restriction to d=1+1 factorizing theories. Preliminary investigations indicate that this indeed the case [12][6].

First we will set the notation [46] by writing some useful formulas[14] for the case without interactions. The following integration path C notation is self-explanatory.. It is convenient for the rapidity representation of the wedge localized free fields. With $\hat{f}(x)$ having support in the wedge, we get for the generating fields of the wedge algebra:

$$\begin{aligned} A(\hat{f}) &= \int \hat{f}(x)A(x)d^2x = \int f(\theta)a^*(\theta) + \int f(\theta - i\pi)a^*(\theta + i\pi) \quad (62) \\ &\equiv \int_C f(\theta)a(\theta) \\ & \quad a(\theta) \equiv a^*(\theta + i\pi), \quad f(\theta + i\pi) = \bar{f}(\theta) \end{aligned}$$

C consists of the real $\theta-$axis and the parallel path shifted up by $i\pi$ and it is only the function f which is analytic in the strip $0 < Im\theta < \pi$ and not the operators. This analyticity is equivalent to the localization property. In the application to the vacuum, only the creation contribution from the lower rim of the strip survives. It is easy to see that the vectors obtained by applying the $A(\hat{f})$ n times fulfill the modular localization equation:

$$\begin{gathered} S\left|\psi^{(n)}\right\rangle = \left|\psi^{(n)}\right\rangle = A_{\psi^{(n)}}\Omega \qquad (63) \\ A_{\psi^{(n)}} = \int_C ... \int_C \psi(\theta_n, ...\theta_1) : a^*(\theta_1)...a^*(\theta_n) \quad : \\ S = J\Delta^{\frac{1}{2}},\ J = TCP,\ \Delta^{it} = U(\Lambda(2\pi t)) \end{gathered}$$

The modular localization subspaces are defined as

$$\begin{aligned} H_R(W) &= \oplus_n H_R^{(n)}(W) \qquad (64) \\ \psi^{(n)} \in & \ H_R^{(n)}(W) = \overline{\left\{\psi \in H_{Fock}^{(n)} \mid S\psi = \psi\right\}} \end{aligned}$$

[14] In our previous functorial construction the application of the Weyl functor gave directly the algebraic net. Such a simple functorial approach does not exist in the presence of interactions and we are forced to understand first the localized states in Fock space before we look at the local algebras.

The antilinear unbounded Tomita involution S consists (for Bosons) of the TCP reflection J and the analytically continued (by functional calculus) wedge affiliated Lorentz boost $U(\Lambda(\chi))$ and the analytical strip properties guaranties that the localized vectors $\left|\psi^{(n)}\right\rangle$ are in the domain of $\Delta^{\frac{1}{2}}$ and hence of S. The action of $\Delta^{\frac{1}{2}}$ corresponds to the continuation to the lower rim and the action of J is just complex conjugation in momentum space (for selfconjugate situations):

$$\psi^{(n)}(\theta_n, \theta_{n-1}, ...\theta_1) \overset{S}{\rightarrow} \overline{\psi^{(n)}(\theta_n - i\pi, \theta_{n-1} - i\pi, ..., \theta_1 - i\pi)} \quad (65)$$

$$or\ \psi^{(n)}(\theta_n + i\pi, \theta_{n-1} + i\pi, ...\theta_1 + i\pi) = \overline{\psi^{(n)}(\theta_n, \theta_{n-1}, ...\theta_1)} \quad (66)$$

so that modular localization equation (63) states that the value of the wave function on the $-i\pi$-shifted boundary equals the complex conjugate of the upper boundary. Note that the hermiticity of $A_{\psi^{(n)}}$ implies the reality condition $\overline{\psi^{(n)}(\theta_n - i\pi, \theta_{n-1} - i\pi, ...\theta_1 - i\pi)} = \psi^{(n)}(\theta_1, ...\theta_{n-1}, \theta_n)$ without assuming analytic properties. For Bosons the wave functions are of course symmetric. In case of Fermions it is well known that the Tomita reflection J has a Klein Twist in addition to the TCP [41].

The closure $H_R^{(n)}$of the real subspace of solutions of (63) contains the spatial part of modular wedge localization. Forming the complex combination $H_R^{(n)} + iH_R^{(n)}$, one obtains a dense set of localized vectors which turns out to be the n-particle component of the well known Reeh-Schlieder set of vectors obtained by applying an arbitrary number of fields restricted to the wedge region to the vacuum. This dense space becomes a Hilbert space in the graph norm $\langle\psi_2, \psi_1\rangle \equiv (\psi_2, \psi_1) + (S\psi_1, S\psi_2)$

The above relation of the J-reflections (for the modular theory of the wedge algebra in the vacuum state) with the S-matrix suggests to use the known S-matrix of these d=1+1 models in order to construct the wedge algebra $\mathcal{A}(W)$ and then to compute the net of double cone algebras by intersections. But even for these simplest interacting field theories, it is not possible to determine the wedge algebra directly (apart from cases with an rapidity-independent S-matrix as the Ising field theory or the Federbush model [12]). Rather the first step consists in computing the localized real subspaces $H_R^{(n)}$ of the Fock-space. The domain property of S or $\Delta^{\frac{1}{2}}$ tells us that we are looking for functions ψ which are analytic in the $i\pi$-strip of the n-fold rapidity plane and whose value at the upper rim is given by the formula:

$$\psi(\theta_1 + i\pi, ..., \theta_n + i\pi) = \prod_{i<j} S^{(2)}(\theta_i - \theta_j)\overline{\psi(\theta_n, ..., \theta_1)} \tag{67}$$

The space of solutions is most conveniently described in terms of a basis which is different from the standard $a^{\#}$-basis by applying so called Zamolodchikov-Faddeev operators $Z^*(\theta)$ to the vacuum. In the simplest case of a theory with only one kind of selfconjugate particle (which we choose for pedagogical reasons), this algebra has the form:

$$\begin{aligned} Z^{\#}(\theta_1)Z^{\#}(\theta_2) &= S_{sc}(\theta_1 - \theta_2)Z^{\#}(\theta_2)Z^{\#}(\theta_1), \; Z^{\#} \equiv Z \; or \; Z^* \\ Z(\theta_1)Z^*(\theta_2) &= S_{sc}^{-1}(\theta_1 - \theta_2)Z^*(\theta_2)Z(\theta_1) + \delta(\theta_1 - \theta_2) \end{aligned} \tag{68}$$

Fourier transformation to x space gives the following on-shell (they obey the free field equation) nonlocal operators:

$$\begin{aligned} Z(x) &= \frac{1}{2\pi}\int_C e^{-ipx} Z^*(\theta), \quad p = m(ch\theta, sh\theta) \\ Z(\theta) &\equiv Z^*(\theta + i\pi) \end{aligned} \tag{69}$$

The Wick-ordering for such fields is done in the same way as in the standard case; the only difference is the occurrence of θ-dependent c-numbers under commutation. We expect that these operators simplify the characterization of interacting localized states. Indeed vectors of the form:

$$\begin{aligned} \left|\psi^{(n)}\right\rangle &= A_{\psi^{(n)}}\Omega \\ A_{\psi^{(n)}} &\equiv \int_C \psi^{(n)}(\theta_n, \theta_{n-1}, ...\theta_1) : Z^*(\theta_1)...Z^*(\theta_n) : \end{aligned} \tag{70}$$

If we chose $\psi^{(n)}$ according to (67), we have solved the equation for the interacting wedge localized subspace $H_R^{(n)}(W)$:

$$S\left|\psi^{(n)}\right\rangle = \left|\psi^{(n)}\right\rangle, \; S = J\Delta^{\frac{1}{2}}, \; J = S_{sc}^* J_0 \tag{71}$$

The general solution can be written as:

$$\psi^{(n)}(\theta_n, ..., \theta_1) = F(\theta_n, ..., \theta_1)\psi_0^{(n)}(\theta_n, ..., \theta_1) \tag{72}$$

where F is a special solution and ψ_0 a general "free" solution with $S_{sc} = 1$. In accordance with the two-particle factorization nature of S_{sc} we may use the so-called minimal formfactor[15] and write:

$$F(\theta_n, ..., \theta_1) = \Pi_{i>j} F^{(2)}(\theta_i - \theta_j) \tag{73}$$

with $F^{(2)}$ a "minimal" solution of:

$$S_{sc}^{(2)*} F^{(2)}(-\theta) = F^{(2)}(\theta) \tag{74}$$

i.e. a meromorphic solution which is free of zeros and poles inside the strip. These $F_{\rm min}$-building blocks are identical to those of the formfactor program (but the determination of the localization spaces is much weaker than that construction program). They insure the "exchange relation":

$$\psi^{(n)}(\theta_n, ..\theta_j, \theta_{j+1}, ..\theta_1) = S(\theta_j - \theta_{j-1})\psi^{(n)}(\theta_n, ..\theta_{j+1}, \theta_j, ..\theta_1) \tag{75}$$

and may be easily calculated for every factorizing model [44].

Note that the spatial property alone already insures the existence of a strip analytic function $\varphi(z)$ fulfilling the KMS like boundary equation

$$\varphi(t) = \left\langle \psi_2 \mid \Delta^{it}\psi_1 \right\rangle, \ \ \varphi(t-i) = \left\langle \Delta^{it}\psi_1 \mid \psi_2 \right\rangle \tag{76}$$

or more general for matrix elements of localized fields $A \in \mathcal{A}(W)$ between localized vectors:

$$\varphi_A(t) = \left\langle \psi_2 \mid A\Delta^{it}\psi_1 \right\rangle, \ \ \varphi_A(t-i) = \left\langle \Delta^{it}\psi_1 \mid A^*\psi_2 \right\rangle \tag{77}$$

This thermal property is closely related to the so called cyclicity property (which in turn connects to the famous crossing symmetry property of particle physics) [45][41].

The next step, namely the passing from localized states to localized algebras, is only simple if the scattering matrix does not depend on the magnitude of θ, as it is the case for the Ising algebra or the Federbush model [41]. In that case the nonlocal field $Z(x)$ is already "wedge-local" i.e. fulfills

$$\left[Z^J(\hat{f}), Z(\hat{g})\right] = 0, \tag{78}$$

$$Z^J(\hat{f}) \equiv JZ(\hat{f})J \tag{79}$$

$$supp\hat{g} \in W, \ supp\hat{f} \in W^{opposite} \tag{80}$$

[15] For the present purpose one may also take the "trivial" F which results from "symmetrizing" with the $S(\theta_i - \theta_j)$ factors.

since the only dangerous term of creation-annihilation contribution remains a c-number equal to the free one.

For θ-dependent S-matrices, the commutator (78) does not vanish since Z^J and Z only commute under the Wick-ordering, whereas the Wick contraction contribution is a non vanishing particle conserving operator). In fact it is easy to see that no finite polynomial in the Wick-ordered $Z^{\#}$can fulfill this relation. Rather the requirement can only be fulfilled for infinite series with coefficient functions $\psi^{(n)}$ which for different n are recursively related:

$$A = \sum_n \int \psi_A^{(n)}(\theta_n, \theta_{n-1}, ...\theta_1) : Z^*(\theta_1)...Z^*(\theta_{n-1})Z^*(\theta_n) :, \;\; A \in \mathcal{A}(\mathcal{W}) \quad (81)$$

A closer examination of the relation:

$$\left[A^J, B\right] = 0, \;\; A, B \in \mathcal{A}(\mathcal{W}) \quad (82)$$

reveals that the coefficient functions cannot break off (i.e. unlike the free case, Wick-polynomials are not solutions) and that the strip-analytic coefficient functions ψ must have pole singularities on the boundary whose residua are recursively related to the lower $\psi's$. These relations are similar to the causality discussion in Smirnov's work and (in a manner even more closely related to the present approach) in [46][47]. I expect that the improvement of understanding in the modular objects for double cone algebras will lead do improvements in the reconstruction of nets of factorizable theories from the known S-matrix.

All the previous constructions of interacting wedge algebras only applied to situations with no real particle productions but nontrivial vacuum polarization (virtual particle nonconservation). No interacting d=1+3 theory can be of this kind. Even if the analytic part is too complicated in order to be manageable, we would still like to know to what degree the modular structure is able to uniquely characterize a QFT; in particular if the S-matrix (even if it does not determine individual fields) determines at most one net i.e. the answer to *the inverse problem of LQP*.

The idea of a more general relation between free and interacting wedge algebra comes from the unitary equivalence of the interacting and the free hyperfinite type III_1 algebras:

$$\mathcal{A}(W) = U\mathcal{A}^{in}(W)U^* \quad (83)$$

together with the postulated U-invariance of the vacuum $U\Omega = \Omega$. In the above cases of factorizing models such an U could be of the form (72):

$$(U\psi_0)^{(n)}(\theta_n, ..., \theta_1) = \frac{\Pi_{i>j} F^{(2)}(\theta_i - \theta_j)}{\sqrt{\Pi_{i>j} F^{(2)}(\theta_i - \theta_j) S(\theta_i - \theta_j) F^{(2)}(\theta_j - \theta_i)}} \psi_0^{(n)}(\theta_n, ..., \theta_1) \tag{84}$$

Here the first factor is unitary for real $\theta' s$ and analytic in the strip. The required pole structure on the boundary is supposed to be in the $\psi_0' s$. Although is there presently no proof, this gives a certain amount of plausibility in favor of such a "Møller operator" in the sense of a "square root" of the scattering operator S_{sc}. The physical idea behind the U-invariance of the vacuum and the one-particle state is that one expects that, whereas these states cannot be resolved from the rest of the energy momentum spectrum within the subspace of compactly localized states, the semiinfinite wedge region, which is left invariant by the associated Lorentz-boosts, does allow such a resolution.

The crucial question is whether the wedge localized vector states can be described in terms of auxilary operators like the Z-F operators for factorizable models. These would be nonlocal noncovariant[16] operators whose one-fold application to the vacuum create a one particle state without the additional vacuum polarization clouds. One expects that covariant wedge localized states without vacuum polarization clouds are characteristic for free systems. Formally the light cone quantization of interacting systems leads to vacuum polarization free (and hence necessarily nonlocal[17]) fields. In fact since lightcone quantization for strictly renormalizable theories is mathematically ill-defined, the wedge affiliated auxiliary operators of the above type may very well be its correct substitute. Whereas in the formal light cone quantization the relation of such nonlocal operators to the physical local fields is unknown, auxiliary wedge operators of the above kind would have a well defined relation to the local observables which is given by modular theory and hence not damage the interpretation. The fact that the Galilei extended subgroup $G(8)$ of the previous section does not only play a deep role in the modular analysis, but also appears in light cone quantization may be taken as an additional indication that the wedge localization approach and the more formal light cone quantization are in some way deeply related.

[16]In case of the Z-F the unitary parity transformation property does not hold.

[17]For light cone quantizations of free fields as well as for the free string, Dimock [50] has shown the locality. The fact that light cone fields immediatly become nonlocal in the presence of an interaction suggests that the same is true for strings.

Note that the unitary equivalence between free and interacting systems in the wedge situation is very different from the corresponding problem of equivalence of the canonical equal time commutation relations. A unitary equivalence in the latter case (of an algebra belonging to a region with trivial spacelike complement) would be forbidden by Haag's theorem [1] on the nonexistence of the interaction picture in QFT. The above characterization of U may be replaced by a slightly more convenient one[18] in terms of an intertwining property between modular operators:

$$US_0 = SU \tag{85}$$

Such a "Møller operator" cannot commute with the Poincaré group apart from the mentioned boosts. The spacetime symmetries serves to define a family of W-affiliated $U_W's$ from the standard wedge. In terms of localized spaces, the U_W has the property:

$$U_W \mathcal{H}_R^{in}(W) = \mathcal{H}_R(W) \tag{86}$$

In the (non-parity invariant) Federbush model which describes two species of particles with $S_{I,II} = e^{i\pi g \int \varepsilon(\theta_1-\theta_2)\rho_I(\theta_1)\rho_{II}(\theta_2)d\theta_1 d\theta_2}$($\rho_{I,II}(\theta)$ are the charge densities in rapidity space)[55][56] the U can be given in closed form and is simply $U = e^{\frac{1}{2}i\pi g \int \varepsilon(\theta_1-\theta_2)\rho_I(\theta_1)\rho_{II}(\theta_2)d\theta_1 d\theta_2}$.

One beneficial side result of this constructive net approach is that one of the cornerstones of QFT folklore, namely that the *existence of field theoretical models is directly linked to the short distance behavior of fields*, seems to be limited to perturbation theory and is less convincing in the nonperturbative setting of LQP (even in the d=1+1 existing formfactor-bootstrap approach it does not appear). In the context of the net approach to QFT this folklore would mean that the existence of nets depend on whether there exist good "good field coordinates" (i.e. generators with mild short distance properties), a restriction that is not very plausible. In fact among the models constructed with the formfactor-bootstrap formalism there are examples with higher powers than those allowed by renormalizability and whose coupling is discrete i.e. cannot be continuously linked to the free theory by a varying coupling constant. This folklore became a central part of QFT credo's ever since Pauli expressed his hope that quantum gravity may be necessary in order to soften light cone singularities.

[18] I am indebted to Hans-Werner Wiesbrock for emphasizing this intertwining property as the most convenient definition of U.

It may be helpful to separate the uniqueness problem of LQP namely the relation

$$S_{sc} \to \mathcal{A}(\mathcal{W})\, net \to \mathcal{A}(\mathcal{C})\, net \to A(x)'s \tag{87}$$

(where $\mathcal{A}(\mathcal{C})$ *net* stands for the net of double cone localized algebras and the last symbol denotes the pointlike fields which are the affiliated carriers of the modular group $\mathcal{G}_{\mathrm{mod}}$) from the actual construction. The uniqueness of this inverse problem of QFT is part of the more general "inverse problem of modular theory".

Mathematically the inverse problem of modular theory [51] is the construction of a von Neumann algebra $\mathcal{M}$ in standard position i.e. pairs $(\mathcal{M}, \Omega)$ such that a given test pair of modular operators (Δ_0, J_0) of $(\mathcal{M}_0, \Omega_0)$, with $\mathcal{M}$ isomorphic to $\mathcal{M}_0$, are also the modular operators of $(\mathcal{M}, \Omega_0)$. The apparently weaker (more general) version in which one only demands that the new algebra $(\mathcal{M}, \Omega_0)$ shares the same Δ_0, turns out to be reducible to the previous problem and is a type of inverse problem close to the physical one described below. There are other inverse problems which have been solved, the most prominent being Connes inverse problem of reconstructing von Neumann algebras from the position of their natural cones in Hilbert space which (as the below physical problem) has a unique solution.

The above isomorphy classes $IC(M_0, \Omega_0)$ have many elements. For example $M = VM_0V^*$ with a unitary V commuting with Δ_0, J_0 and obeying $V\Omega_0 = \Omega_0$ gives a family of von Neumann algebras. This construction does not exhaust the possibilities. Already for matrices i.e. I_n factors one can construct yet another family of inverse problem classes by choosing V as $V = V_1U$ with V_1having the same commutation properties as above but $V_1\Omega_1 = \Omega_0$. Here Ω_1 is such that (Δ_0^{-1}, J_0) are the modular objects of (M_0, Ω_1) and U commutes with J_0 and obeys $\Delta_0^{-1} = U^*\Delta_0U, U\Omega = \Omega, U\Omega_1 = \Omega_1$. This construction only works for I_n factors because only there Δ_0^{-1} can be the modular operator of a pair (M_0, Ω_1). For type I factors one has a rather complete solution of the inverse problem. For type III on the other hand, extremely little is known.

In the present formulation, the uniqueness of the physical problem (87) is equivalent to that U. This can be reduces (by playing two different $U's$ against each other) to the uniqueness problem of (87) for $S_{sc} = 1$. This reduced problem has a positive answer if crossing symmetry for formfactors also holds for the hypothetical second QFT. I hope to return to these issues in a planned joined paper [56].

6 Concluding Remarks

Whereas principles as causality and locality used to play an important role in the past (the LSZ framework, the Kramers-Kronig relations in high energy physics and their experimental check in high energy nucleon scattering) it is hard to detect them among recent inventions. Even though there is a weak historical connection e.g. of string theory with QFT, it goes entirely through formalism (notably functional integral representations) and not through the underlying principles. Whereas the Veneziano dual model was still based on the idea that the underlying crossing symmetry is a deep on-shell manifestation of causality, string theory has apparently abandoned any kind of conceptual thinking, with the result that the status of locality within interacting string theory is unknown (the answer you get depends on the person you ask[19]). A similar situation one finds with respect to the on-shell versus off-shell nature of strings as well as with most other concepts of QFT. This loss of physical conceptual interest and abilities at the end of this century (which were so gloriously represented at its beginning by Einstein, Bohr, Heisenberg,...) is certainly the most striking phenomenon in contemporary particle physics.

Somewhat to my surprise however, the locality principle was recently upheld as a unifying point of view in the context of Kac-Moody affine and W-algebras in the physical setting of chiral conformal QFT by the mathematician Victor Kac [52] who was inspired by some lectures of Goddard [53]. I refer to a lecture he gave at the same place two years ago under the title "the idea of locality". This is especially interesting for me, because it makes me recall the very early nonperturbative non-Lagrangian constructive attempts of some of my colleagues, after Wightman proposed his general framework. The first such attempt led to so-called generalized free fields. They turned out to be rather uninteresting (also mathematically) because they violated the time-slice property (often called "primitive causality") and did not contain interactions. The second attempt (which, if I remember correctly, goes back to W. Greenberg) in the middle 60^{ies} was that of "Lie-fields" i.e. the search for systems of nontrivial relatively causal fields which close in spacetime (not just equal time) commutators with nontrivial light and timelike terms. At the end of 60^{ies} one had already examples of d=1+1 abelian current algebras,

[19]Part of the problem may originate from the fact that quantum causality and locality is often confused with support or geometrical properties of Lagrangians, one of the devastating side effects of euclidean field theory.

but the main interest was in what is nowadays called W-algebras i.e. commutator closing causal field systems without group theoretic multiplicities. The difficulty was their construction; the idea of affine algebras was not known in the physics community (not to mention the modular methods for local nets). Therefore it is not surprising that the examples of Lie fields in John Lowenstein's thesis were rather artificial. In fact when I investigated the structure of the chiral energy momentum tensor (using causality and the translation covariance) in 1973 [36], I looked upon this result primarily as the construction of a Lie field. The first investigations of nonabelian chiral algebras in connection with nontrivial fixed points (still without knowledge of the Kac-Moody concepts) were already available at that time [57]. I mention these historical facts, because younger physicist often think that the differential geometric methods of Witten are the starting point for these developments.

Victor Kac's interest in the causality principle is in some sense complementary to the one in this article. He wants to start from the rich supply of affine algebras and use causality in the chiral conformal setting to aim at the most general conceptual basis for the extension of ideas which originate from Lie groups. In these notes, on the other hand, I illustrated a program which translates the causality principle with the help of the very general and powerful modular theory (within the setting of operator algebras) into physical and geometrical (or "hidden") symmetries and (from antiunitary modular reflections) nonperturbative construction ideas. The former approach leads in a very direct and easy way to global properties as modular (in the analytic sense) relations between affine characters resp. Verlinde identities between L_0- rotational thermal correlation functions (generalizations of the old Nelson-Symanzik duality between space and $i \cdot time$). The T-T modular approach on the other hand, as a result of its intimate relation to von Neumann commutants, Jones subfactors, causality and Haag duality, offers easy access to more local properties as e.g. the classification of fusions of localized charges, physically admissable statistics in low dim. theories etc. To put it more concretely: for the approach based on Verma modules of abstract algebras one uses the Kac determinant technique together with the Friedan-Qiu-Shenker theorem (supplemented by Fock space reductions of Olive et al.) in order to obtain concrete quantum operators in Hilbert space; the main idea in LQP in the chiral conformal setting is to classify admissable charges and their statistics (using the DHR-Jones-Wenzl method of Markov traces on the statistics algebras) and then to construct the space-time carriers of these localized charges. It is expected that the energy-momentum tensor which belongs to such charged fields will automatically fulfill the Kac-FQS quantization as a consequence of the (physically more fundamental, not restricted to d=1+1 conformal theories) DHR-Jones Wenzl statistics quantization[20]. In connection with Victor Kac's quest for the most fruitful generalization of finite dimensional Lie-group theory, the present approach offers the infinite dimensional universal modular group $\mathcal{G}_{mod}$ as a candidate. It is generated by the one-dimensional modular groups of all spacetime localized von Neumann algebras $(\mathcal{A}(\mathcal{O}),\Omega)$ of a LQP net. Although the existence of this object is guarantied, its present understanding and explicit construction is still in its infancy. In chiral conformal QFT it agrees with the geometric diffeomorphism group. Although it originates from causality in LQP (via modular theory), its action on fields is nonlocal in the sense of a non-pointlike extension of the Poincaré or conformal group. The advantage for a physicist is that it transcends low dimensional QFT.

Even though there already exist attempts [48][49] in this direction, to cross from one to the other framework is still difficult I sincerely hope that the knowledge of operator algebras will, at least in the next generation, be combined with the mathematical ideas in the work of Victor Kac (i.e. from T-T modularity to the modular identities of characters and thermal (torus) chiral correlations!).

For me the biggest recent surprise was to see that an old theory as QFT, which by many physicist was already considered as old-fashioned and frozen, suddenly permits a totally new conceptual framework with a new mathematical formalism. The hope that this may be a new start appears justified.

Acknowledgment: I am indebted to H. W. Wiesbrock for discussions and for critical reading of the manuscript.

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