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Looking beyond the Thermal Horizon: Hidden Symmetries in Chiral Models

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Abstract

In thermal states of chiral theories, as recently investigated by H.-J. Borchers and J. Yngvason, there exists a rich group of hidden symmetries. Here we show that this leads to a radical converse of of the Hawking-Unruh observation in the following sense. The algebraic commutant of the algebra associated with a (heat bath) thermal chiral system can be used to reprocess the thermal system into a ground state system on a larger algebra with a larger localization space-time. This happens in such a way that the original system appears as a kind of generalized Unruh restriction of the ground state sytem and the thermal commutant as being transmutated into newly created "virgin space-time region" behind a horizon. The related concepts of a "chiral conformal core" and the possibility of a "blow-up" of the latter suggest interesting ideas on localization of degrees of freedom with possible repercussion on how to define quantum entropy of localized matter content in Local Quantum Physics.

1 Introduction

In a previous paper we defined and illustrated the concept of a "hidden symmetry" [S-W1]. We remind the reader that we use the terminology "hidden" for such symmetries of local quantum physics (LQP) which cannot be seen

and understood by (Lagrangian-, functional integral-) quantization. They simply do not exist on a classical level nor in a euclidean functional integral representation, and hence cannot be transposed into the noncommutative realm by e.g. adapting (via quantization[1]) Noether's theorem. One rather needs the power of the modular theory, which is totally characteristic of the noncommutative aspects of LQP (**L**ocal **Q**uantum **P**hysics), and which in addition to the classical space-time diffeomorphisms (Poincaré- or Conformal- automorphisms) of Minkowski space-time also reveals a host of non-Noetherian "Hidden Symmetries" of modular origin. The latter escape the quantization approach to LQP (see the previous footnote), whereas the modular approach to LQP shows both kinds of symmetries; in fact the hidden symmetries may be considered in some sense as being the generalization of the diffeomorphisms of chiral conformal theory to higher dimensions [S-W1].

There are two mechanisms which reveal such hidden structures: the modular theories of (multiply) localized double cone algebras together with suitably chosen states, and the symmetries constructed from modular inclusions and modular intersections. The first mechanism might lead to a "fuzzy" action of the modular group for which the double cone situation for the free massive theory is the simplest illustration. It is believed that such modular actions become asymptotically pointlike near the horizon (boundary) of the localization region [S-W1]. We will use the word hidden in this paper exclusively for the second inclusion/intersection mechanism which leads to (partially) geometric transformations of localization regions (i.e. of the net) which cannot be implemented as point transformations. For details see below and [S-W1].

Modular inclusions/intersections of von Neumann algebras are significantly different from the well-studied V. Jones inclusions (subfactors). Whereas by definition the latter have conditional expectations onto the smaller algebra ("shallow" inclusions), the former, as a result of their modular structure, cannot have conditional expectations ("deep" inclusions). In passing we mention that there is a third type of deep inclusion which also has no conditional expectation and is not modular; it is the so-called split inclusion which is

[1]If a quantization approach could lead to nonperturbative mathematically controllable real time local quantum physics, then these symmetries would show up in modular properties of the local algebras generated by the fields. But as everybody should know, we are (despite intense efforts for more than 40 years) still far away from such a goal, and it appears presently much more plausible that this will be achieved by modular methods [S-W2] instead of quantization.

related to a strong form of statistical independence in LQP [Su].

It is well-known that the V. Jones inclusions lead to vast generalizations of compact group symmetry. On the other hand modular inclusions/intersections have only been around for some years and very little is known about their consequences except that noncompact groups (as the two parametric translation/dilation group as well as SL(2,$\mathbf{R}$)) emerge already from the simplest examples. Although modular (hsm) standard inclusions are known to be in one-to-one correspondence to strongly additive chiral quantum fields (and thereby there are good reasons for expecting to be able to classify them), no results and even no efforts in this direction is known today. These groups act in a natural way on the two original algebras and their intersection and generate a space-time indexed net. The considerations in this paper lead to an increase in knowledge about modular inclusions/intersections.

In our previous work we commented among other things on a very revealing chiral illustration of such a situation which was found by Borchers and Yngvason [Bo/Yng]. In this note we use their example as a starting point for a furthergoing analysis of hidden symmetries in low-dimensional theories.

As was shown by those authors, it might happen that the modular group acts only geometrically on a certain subregion. We show that in their case this group acts geometrically in a much larger region! Moreover there is a much richer "hidden symmetry" behind. In fact it turns out that in addition to the hidden symmetry generated by the two-parametric translation/dilation group [Bo/Yng] (with the dilation part being hidden), there is a hidden SL(2,$\mathbf{R}$) Möbius symmetry with a differently localized net which is partially local relative to the old thermal net. In fact the thermal "shadow world" associated with a heat bath state becomes converted into a piece of real space-time which is to be localized behind the horizon! This is an inversion from the Hawking-Unruh situation in a two-fold sense: not only is the thermalization through localizing (Unruh) horizons undone and the thermal region returned as a piece of Minkowski space to the vacuum world (in analogy to the Unruh effect [S/S-V]), but even the "shadow world" (the apparently non-geometric thermal commutant) of a standard *heat-bath thermal situation* becomes converted into "virgin *space-time*" *behind the horizon*, thus giving additional weight to some speculative remarks of Jaekel [Ja]. Although we can presently illustrate this idea only in the chiral light ray setting, we consider the potential possibility of a space time interpretation of heat bath temperature, and a still unknown but expected ensuing conversion of a (still missing) notion of quantum entropy as sufficiently intriguing in order

to justify publication.

Using our recent ideas on the modular origin of the chiral diffeomorphism group which we illustrated with some computations on the Weyl-algebra [S-W1], it could be interesting to extend also these ideas about a modular origin of chiral diffeomorphism groups to the thermal setting. Leaving this for future investigations, we make some remarks on the existence of hidden SL(2,$\mathbf{R}$) symmetries in d=1+1 massive theories and we conclude with some comments on higher dimensional theories. In all cases the modular method is used to extend the hidden action of translation/dilation subgroup [S-W1] to a hidden SL(2,$\mathbf{R}$) action.

2 Beyond the Borchers-Yngvason Results

Consider a chiral quantum field theory given by a local net of observables. The Bisognano-Wichmann property holds for the net, as it was in general shown by [B/G/L, Fr/Ga]. For simplicity we also assume strong additivity or equivalently, Haag duality on the line, see [G/L/W]. This can always be achieved by passing to the dual net. With Borchers and Yngvason we take a thermal state on the net w.r.t. the lightlike translations. Let $\mathcal{N}$ denote the observable algebra to the half-line $[0,\infty[$, then Borchers and Yngvason showed that the modular group of $\mathcal{N}$ w.r.t. the thermal state transforms local algebras on the half-line geometrically (see below). The global thermal algebra is known to be of hyperfinite von Neumann type III_1 and hence inequivalent to a ground state representation. This is the prerequisite for an Unruh-like interpretation in terms of a ground state representation thermalized by local restriction.

We will show that this group also acts geometrically on the whole line and that even more, there is a "hidden" conformal group SU(2,$\mathbf{R}$)/Z_2 acting geometrically on intervals, but not pointwise in the sense of spatial automorphisms..

For describing our observation, let us first recall the setting of Borchers and Yngvason. Denote by $\mathcal{A}$ the quasilocal algebra of the chiral model, by Ω_+ the GNS-vector to the thermal state w.r.t. the unitary group $U(a)$ of lightlike translations and by $\widetilde{\mathcal{N}}$ the C*-algebra of observables localized in $[0,\infty[$. Let J_0 be the CPT-symmetry of our model. Then we easily have by strong additivity [Wie2]

$$J_0\widetilde{\mathcal{N}}\,J_0 \vee \widetilde{\mathcal{N}} = \mathcal{A}. \tag{1}$$

Let us pass to the weak closure of these algebras w.r.t. the thermal state, denoted by $\mathcal{M}$ and $\mathcal{N}$. Under these circumstances U is the modular group associated with $(\mathcal{M}, \Omega_+)$. By the very definitions [Wie1] we have $(\mathcal{N} \subset \mathcal{M}, \Omega_+)$ is a -hsm inclusion. Now $J_0 \widetilde{\mathcal{N}} J_0 \subset \mathcal{N}' \cap \mathcal{M}$, and it is easy to see that therefore $(\mathcal{N} \subset \mathcal{M}, \Omega_+)$ is a standard -hsm inclusion. But we even have $\mathcal{M} = \mathcal{N} \vee (\mathcal{N}' \cap \mathcal{M})$, see [G/L/W]. Now in general , we have due to Borchers result [Bo1][Wie1]

$$[J_0, U(a)] = 0, \tag{2}$$

which easily implies that

$$A \mapsto < \Omega_+, J_0 A^* J_0 \Omega_+ >, \ A \in \mathcal{A} \tag{3}$$

is again a thermal state of the chiral theory w.r.t. the lightlike translations. The absence of phase transitions in chiral theories implies that we can anti-unitarily implement the CPT-operator. Then we immediately get with $J_0 \widetilde{\mathcal{N}} J_0 = \mathcal{N}' \cap \mathcal{A}$

$$J_0 \mathcal{N} J_0 = \mathcal{N}' \cap \mathcal{M} \ . \tag{4}$$

This implies that the relative commutant of the observable algebra to a half-line is the algebra to the opposite half-line.

In case of a -hsm standard inclusion we can always construct out of these data a chiral vacuum net [Wie2][G/L/W]. Therefore, within the above thermal situation we encounter a "hidden" vacuum theory. For this, let us sketch the general construction in the context of this particular case. One starts by associating the algebra $\mathcal{M}$ to the upper half-circle. Then exploiting the fact that the modular groups of $\mathcal{M}$, $\mathcal{N}$, and $\mathcal{M} \cap \mathcal{N}'$ generate a positive energy representation of the Möbius group, we implement the covariance of the algebra system by defining the various local algebras by mapping $\mathcal{M}$ according to the modular representation of the space-time symmetry. In this way we generate a Möbius-covariant net.

Let us now compare both theories from a more conceptual point of view. First, they have the weak closure $\mathcal{M}$ of the quasilocal algebra $\mathcal{A}$ in common. Further, the modular theory $U(a)$ to that algebra w.r.t. Ω_+ acts geometrically in both systems, namely as lightlike translations in the thermal case, and as half-line dilations in the vacuum case.

Secondly, they have the observable algebras $\mathcal{N}$ and $\mathcal{N}' \cap \mathcal{M}$ in common, which in the thermal case are interpreted as the algebras to the two half-lines beginning at origin, whereas in the vacuum case they are associated to $[1, \infty[$, and $[0, 1]$.

Now we notice that the physical symmetry in the thermal case is given by the lightlike translations, whereas in the vacuum theory we have a natural conformal symmetry $SL(2,R)/Z_2$ behind, where the lightlike translations of the former theory are equal to the dilations in the latter.

Let us exploit the common structures in order to compare the localizations in both theories. For this we denote the local algebras by $\mathcal{A}_{th}(a,b)$ if we refer to the thermal theory, $\mathcal{A}_0(a,b)$ in the vacuum case. So we can rephrase the above remark by

$$\mathcal{A}_{th}(0,\infty) = \mathcal{N} = \mathcal{A}_0(1,\infty), \qquad \mathcal{A}_{th}(-\infty,0) = \mathcal{N}' \cap \mathcal{M} = \mathcal{A}_0(0,1) \tag{5}$$

and $U(2\pi t) = \Delta_{\mathcal{M}}^{it}$ where we set the inverse temperature to be 2π. (The necessary modifications for the general case is left to the interested reader.). Then we have

$$\begin{aligned} \mathcal{A}_{th}(a,\infty) &= Ad\ U(a/2\pi)(\mathcal{A}_{th}(0,\infty)) = Ad\ U(a/2\pi)(\mathcal{N}) \\ &= \mathcal{A}_0(e^{2\pi a},\infty) \end{aligned} \tag{6}$$

and similarly

$$\mathcal{A}_{th}(-\infty,a) = \mathcal{A}_0(0,e^{2\pi a}). \tag{7}$$

Now, by the very definition we have

$$\mathcal{A}_0(e^{2\pi a},e^{2\pi b}) = \mathcal{A}_0(0,e^{2\pi b}) \cap \mathcal{A}_0(e^{2\pi a},\infty) \tag{8}$$

and similarly

$$\mathcal{A}_{th}(a,b) = \mathcal{A}_{th}(-\infty,b) \cap \mathcal{A}_{th}(a,\infty) \tag{9}$$

holds due to the strong additivity of the thermal net. Therefore we conclude

$$\begin{aligned} \mathcal{A}_{th}(a,b) &= \mathcal{A}_{th}(-\infty,b) \cap \mathcal{A}_{th}(a,\infty) \\ &= \mathcal{A}_0(0,e^{2\pi b}) \cap \mathcal{A}_0(e^{2\pi a},\infty) \\ &= \mathcal{A}_0(e^{2\pi a},e^{2\pi b}). \end{aligned} \tag{10}$$

But now we know, by the very definition, that the conformal group $Sl(2R)/Z_2$ acts geometrically on the vacuum theory. And due to the above relation we immediately get the action of that group on the thermal theory. Therefore, restricting on vacuum localization intervals in the half-line $[0,\infty[$ and to group actions mapping this interval into another one contained in the half-line, we can transfer this action onto a geometrical action in the thermal

theory. Moreover we see, that there is a natural space-time structure also on the shadow world i.e. on the thermal commutant to the quasilocal algebra on which this hidden symmetry naturally acts. In conclusion, we have encountered a rich hidden symmetry lying behind the tip of an iceberg, of which the tip was first seen by Borchers and Yngvason.

In fact using our previous result on the modular origin of the diffeomorphism for the special case of the Weyl-algebra [S-W1], it seems plausible that there is even a much larger infinite dimensional symmetry of modular origin hidden in this thermal situation.

Before we make some qualitative remarks about hidden SL(2,$\mathbf{R}$) symmetries in the higher dimensional case, let us separately look at d=1+1 massive theories. It is clear that in this case we should use the two modular inclusions which are obtained by sliding the (right hand) wedge inside itself ($\mathcal{M} \to \mathcal{M}_{a_\pm}$) by applying an upper/lower lightlike translation $a_\pm$ and forming the relative commutant (the size of $a_\pm$ is irrelevant)

$$\mathcal{M}(I_\pm) \equiv \mathcal{M}'_{a_\pm} \cap \mathcal{M} \tag{11}$$

where the notation indicates that the localization of $\mathcal{M}(I_\pm)$ is thought of as the piece of the upper/lower light ray between the origin and the endpoint of the $a_\pm$ lightlike translation. By viewing this relative commutant as a lightlike limiting case of a spacetlike shift of W into itself (and using Haag duality), on obtains the interval $I_\pm$ as a limit of a double cone. The net obtained by applying the modular transformation Δ_M^{it} to the $\mathcal{M}(I_\pm)$ via its ad action is a chiral net with total algebra

$$\widetilde{\mathcal{M}}_\pm \equiv \bigcup_t \Delta_W^{it} \left(\mathcal{M}(I_\pm)\right) \subset \mathcal{M}, \tag{12}$$

where we used the hsm standard inclusion ($\mathcal{M}(I_\pm) \subset \widetilde{\mathcal{M}}_\pm, \Omega$) for the construction, see the Appendix below. Since this net is chiral, it cannot create the full space from the vacuum. Rather the cyclically generated space is a genuine subspace with projector $P_\pm$. Since the modular group of $(\widetilde{\mathcal{M}}_\pm, \Omega)$ is obtained from restricting $\Delta_{\mathcal{M}}^{it}$ to $P_\pm \mathcal{H}$, the projection is associated with a conditional expectation of the algebra

$$E_\pm(\mathcal{M}) = P_\pm \mathcal{M} P_\pm = \widetilde{\mathcal{M}}_\pm \tag{13}$$

Although the two-dimensional conformal theory lives in the tensor-product space of the two chiral theories on the upper/lower light ray, the two chiral components constructed in the present way do not commute with each

other unless one also performs the massless limit m$\to$ 0. Indeed since the upper/lower light ray have a relative $\pm$time-like separation, one does not expect Huygens principle to become effective outside of $m = 0$. In fact a simple straightforward calculation on algebras generated by massive currents or energy-momentum tensor reveals that the relative commutator is proportional to a power of the mass but independent of the space-time coordinates. The mechanism can be illustrated in the simplest way by looking at the relative commutator of the 1-2 components of the Fermion field on the upper/lower light ray

$$\begin{aligned} \left\{\psi(x), \bar{\psi}(y)\right\} &= \left(i\gamma_\mu \partial^\mu - m\right) i\Delta(x-y) \qquad (14) \\ i.e.\ \{\psi_1(u)\psi_2^*(v)\} &\simeq \pm m \end{aligned}$$

where the last relation holds in the limit x,y$\to$upper/lower light ray. So it appears that one has to distinguish between the tensorproduct algebra $\mathcal{M}(I_+) \otimes \mathcal{M}(I_-)$ associated with the full d=1+1 conformal theory (the zero mass limit) and the combined $\mathcal{M}(I_+) \vee \mathcal{M}(I_-)$ subalgebra of $B(\mathcal{H})$. In fact, since one expects the vacuum to be cyclic in $\mathcal{H}$ with respect to $\mathcal{M}(I_+) \vee \mathcal{M}(I_-)$, from the shared modular group structure with $\mathcal{M}$ we will take $\mathcal{M} = \mathcal{M}(I_+) \vee \mathcal{M}(I_-)$. for granted So modulo a fine point concerning the difference to a tensor product, the two conformal symmetries $SL(2,\mathbf{R})/Z_2$ of the upper/lower light ray reductions are in fact inherited as hidden symmetries by the massive theory. The light ray reduction of the massive theory, although being a chiral conformal theory in its own right, can be transmuted into the original massive theory by adjoining the opposite light cone translation $U_-(a)$ i.e. by extending $M(I_+)$ to $alg\{M(I_+), U_-(a)\}$ [S-W2]. One would expect that the relation between the thermal version and the groundstate representation of the massive theory goes through the light ray reduction. In addition to the proper conformal transformation which are already hidden in the massive theory there should be a "thermal hiding" for all transformations except the translations.

3 Qualitative Remarks about Higher Dimensions

In a previous paper we have shown how for $d \geq 1+2$ one may use the situation of modular intersections to come to hidden symmetries [S-W1].

In higher dimensions one expects that in general a conversion of thermal nets into spatially extended ground state nets, if possible at all, will involve $SL(2,R)/Z_2$ symmetries of modular origin with positive energy translation operators i.e.symmetries which (since a massive theory does not possess such pointwise acting symmetries) have hidden pieces already in their ground state realization.

For this purpose we introduce the notion of a "conformal core" with the "blow-up" property. Let us start with a d=1+1 theory defined in terms of algebraic net data i.e. a coherent map of space-time regions into von Neumann algebras. Such theories have two light-like translations $U_{\pm}(a)$ with positive generators. We use the right upper light ray translation U_+ in order to construct a modular inclusion by sliding the standard wedge W at the origin into itself $W \to W_+ \subset W$ using $U_+(a=1)$ [S-W2]. Consider the modular inclusion defined in terms of the relative commutant of $\mathcal{M}_+ \equiv U(1)\mathcal{A}(W)U^*(1)$ in $\mathcal{M} = \mathcal{A}(W)$

$$(\mathcal{M}'_+ \cap \mathcal{M} \subset \mathcal{M}, \Omega) \tag{15}$$

This inclusion leads to a full-fledged chiral conformal theory on the line (the light ray reduction) which generates from the vacuum its own Hilbert space which is a genuine subspace of the original space

$$\begin{aligned} \mathcal{A}^{>}_{\pm} &\equiv \bigcup_t \Delta_W^{it} \left(\mathcal{M}'_{\pm} \cap \mathcal{M}\right) \subset \mathcal{M} \equiv \mathcal{A}(W) \\ \overline{\mathcal{A}^{>}_{\pm}\Omega} &= H_{\pm} \subset H = \overline{\mathcal{M}\Omega} \\ \mathcal{A}_{\pm} &= \mathcal{A}^{>}_{\pm} \vee J\mathcal{A}^{>}_{\pm}J \end{aligned} \tag{16}$$

Note that the full conformal invariance is only realized on the reduced space $H_{\pm}$

Since the modular theories of the combined upper/lower light ray reductions $\mathcal{A}^{>}_{+} \vee \mathcal{A}^{>}_{-}$ defines a 2-dim. net which lives on a bigger Hilbert space than the reduced one and since this combined algebra has the same modular theory and the same light-like translations as $\mathcal{M}$ there exists according to Takesaki a conditional expectation from $\mathcal{M}$ to $\mathcal{A}^{>}_{+} \vee \mathcal{A}^{>}_{-}$ and the two algebras are either identical (iff the cyclically generated Hilbert spaces are the same) or the $\mathcal{A}^{>}_{+} \vee \mathcal{A}^{>}_{-}$ -algebra is obtain from $\mathcal{M}$ as a fixed point algebra under the action of an internal symmetry. A more efficient way to reconstruct the original massive net from its light ray reduction is the following "blow-up"

of a light ray reduction, say $\mathcal{A}_+$, by use of the opposite positive ($a > 0$) light-like translation

$$\mathcal{B}_+ \equiv \bigvee_{a>0} alg\,\{A_+, U_-(a)\} \tag{17}$$

Intuitively one expects that $\mathcal{M}$, $\mathcal{A}_+^> \vee \mathcal{A}_-^>$ and $\mathcal{B}_+, \mathcal{B}_-$ are all identical, but without further detailed investigations, which go beyond the identity of their modular structure, we cannot decide whether there are differences due to different internal symmetries.

This light ray reduction supplemented by the blow-up idea can be generalized to higher dimensions, where however it becomes more tricky. Let us explain the situation for d=2+1, using the notation of [S-W1]:

$U_{12,13}$ for the Galilean "translations" inside the Lorentz group, which in d=2+1 exhausts the isotropy ("little") group to l_1.

U_{l_1} for the lightlike translations along l_1, $\mathcal{A}([l_1, l_2])$ for the wedge algebra to $W[l_1, l_2]$

With this notational matter out of our way, we present now two interesting proposals for the definition of candidates for a "chiral conformal core" associated to a lightlike direction. The first one leads to a construction which by fiat is independent of the choice of the wedge and only depends on the light ray l_1. Start from

$$Ad\ U_{l_1}(1)(\mathcal{A}([l_1, l_2]) \subset \mathcal{A}([l_1, l_2]). \tag{18}$$

This defines a conformal theory, see the appendix. The resulting translations, namely U_1, commutes with the Galilean "translations". We define:

$$\cap_{\lambda\in R}\, Ad\ U_{l_1 l_2, l_1 l_3}(\lambda)(Ad\ U_{l_1}(1)(\mathcal{A}([l_1, l_2])) \subset \mathcal{A}([l_1, l_2]). \tag{19}$$

This gives a modular standard inclusion in a canonical way, see appendix. This proposal has the advantage to be covariant under the action of the isotropy:

$$\begin{aligned} & U_{l_1 l_2, l_1 l_3}(\mu) \cap_{\lambda\in R} Ad\ U_{l_1 l_2, l_1 l_3}(\lambda)(Ad\ U_{l_1}(1)(\mathcal{A}([l_1, l_2])) \\ = \;& Ad\ U_{l_1}(1)(\cap_{\lambda\in R} Ad\ U_{l_1 l_2, l_1 l_3}(\lambda) U_{l_1 l_2, l_1 l_3}(\mu)((\mathcal{A}([l_1, l_2])))) \\ \subset \;& U_{l_1 l_2, l_1 l_3}(\mu)(\mathcal{A}([l_1, l_2])). \end{aligned} \tag{20}$$

where the Galilean translation $U_{l_1 l_2, l_1 l_3}(\mu)$ simply turns $W[l_1, l_2]$ into a $W[l_1, l_2']$.

The drawback of this construction could be that it is empty, as we will argue in the sequel.

An equivalent construction based on the same intuition starts with the modular inclusion

$$\mathcal{A}([l_1,l_2]) \cap \mathcal{A}([l_1,l_3])) \subset \mathcal{A}([l_1,l_2]) \tag{21}$$

and suggests to represent the algebra (19) as a limiting intersection algebra:

$$\lim_{\lambda\to\infty} Ad\ U_{l_1l_2,l_1l_3}(\lambda)(\mathcal{A}([l_1,l_2]) \cap \mathcal{A}([l_1,l_3])) \tag{22}$$

and this seems to be $\mathbf{C}\cdot 1$, a multiple of the identity.

Let us therefore mention another possible construction which starts with the modular inclusion:

$$Ad\ \ U_{l_1}(1)(\mathcal{A}([l_1,l_2]) \cap \mathcal{A}([l_1,l_3])) \subset \mathcal{A}([l_1,l_2]) \tag{23}$$

Again intuitively this might be interpreted as a conformal core to the light ray l_1. This definition depends on the lightlike directions, but it does so in a covariant way since $U_{l_1l_2,l_1l_3}(\lambda)$ commutes with U_{l_1} and therefore the action of the isotropy group is computable. Moreover this modular inclusion is only invariant under the lighlike translations $U_{l_1}$and not under the transversal translations.

In a generic curved space-time situation with a bifurcated Killing horizon and in the absence of additional symmetries, one still can apply the methods of LQP [S/S-V] and associate a chiral conformal theory as was shown recently in a remarkable paper by Guido, Longo, Roberts and Verch [G/L/R/V]. In that case the conformal theory is not associated to a particular light ray, but rather is induced via a restriction of the theory onto the horizon.

Before we apply the achieved results to our main theme, namely how to convert heat bath temperature into Unruh temperature (or how to pass from the heat bath "shadow world" of the thermal commutant into new space-time behind a horizon), we cannot resist to make some comments about a fascinating but speculative connection of the blow-up picture of the chiral core with another speculative problem which presently is attracting the attention of many theoreticians: the problem of quantum entropy of algebras with a horizon or (in the context of LQP) a "quantum localization entropy". The blow-up picture tell us that in the case of the wedge geometry we can represent the original wedge algebra $\mathcal{M}$ by the algebra generated by the chiral conformal algebra augmented with certain symmetry generators which do the blowing up (principally longitudinal and transverse translations). One would

expect that for the counting of degrees of freedom the Poincaré generators do not contribute. If we now use our physical imagination for an attempt to understand this picture beyond the wedge also for the rotationally symmetric double cone (in Minkowski space), and take notice that if the higher dimensional theory would be conformally invariant (zero mass), then there is a well-known transformation which carries the previous wedge situation into the double cone, then we obtain indeed a very attractive picture. The more convincing part is the blow-up representation of the higher dimensional conformal algebra in terms of a chiral core algebra on the surface (plus symmetry generators of the higher dimensional theory which do not participate in the degree of freedom counting). The second, less rigorous step, is the idea that the unknown "fuzzy" modular theory of the massive double cone algebra is asymptotically (near the horizon) equal to the geometric modular situation of the conformal double cone theory. However to convert this "holographic degree of freedom picture" into a Bekenstein-Hawking-like formula for entropy expressed in terms of data of the chiral conformal core theory (e.g. the structure constants of W-algebras) more needs to be done. There is finally the problem of defining what we mean by localization entropy in the chiral theory. The local algebras, whether chiral or higher dimensional, are known to be of von Neumann hyperfinite type III_1 which would lead to diverging (undefined) von Neumann entropies. As physicists we would try to regularize the situation. Indeed there is a natural way to do this, which is related to the phase space behavior of QFT. The latter is different from the behavior obtained by the box quantization at fixed time in the sense that the density based on the correct localization concept is bigger ("nuclear") than for the box case (finite) [Ha], a fact overlooked in most relativistic entropy discussions. Closely related to the nuclearity property is the so-called split property which suggests to define a kind of fuzzy localized type I algebra. This algebra has its support in the double cone plus a "collar" around it, so that there is a bigger double cone containing both which the given double cone and the collar [Ha]. A type I algebra, unlike hyperfinite type III_1, has no principle obstruction against the existence of a von Neumann entropy. Such an entropy takes into account the quantum "entaglement" between the inside of the smaller and the ouside of the bigger double cone across the collar. With vanishing size of the collar the entropy diverges which corresponds to the infinite (but still nuclear) correct degree of freedom counting. Filling the double cone with different kind of matter, the strength of divergence is expected to be the same but the leading coefficients vary. The missing

and difficult part of the discussion is the use of the vacuum state, because the modular simplicity of the doubly localized situation with the collar has a geometrically simple modular theory only with respect to another state [S-W1] whose Connes cocycle relation to the vacuum has nit been studied. An educated guess suggests that the divergence is logarithmic; also the claims which appeared in the literature that for minimal models the coefficient is proportional to the to the central charge constant c, are certainly not in contradiction with our blow-up picture of chiral cores and very suggestive indeed if one looks at the aforementioned relation of the two states and the role of the central term of the diffeomorphism group in the construction of the geometric modular state. Note that these arguments, if they withstand furthergoing detailed analysis, would interpret the holographic behavior as a *generic property of nonperturbative local quantum physics* and not as a particular behavior of special global properties[2] (topological field theories, gauge theories etc.). This yet speculative scenario build on modular theory has a some resemblance with Wald's recent more conservative ideas on entropy [Wa] and less so with string theory; although the results (but certainly not the physical concepts) may be similar.

The reader may find an earlier account of this picture on holographic reduction of degrees of freedom from the viewpoint of LQP in section of a book manuscript draft by one of the authors [Schr1]; the present blow-up mechanism of chiral light ray reduction lends considerably more credibility to those earlier remarks. There is a history and a long list of references on light cone holography in black hole physics by a variety of methods which seems to be based on quite different ideas than those of this work [Ca]. We hope to return with more results to this interesting and conceptually important entropy problem.

Returning now to the thermal theme of this paper, the qualitative idea of generalizing our discussion of the Borchers-Yngvason model to higher di-

[2]The present authors are aware of surprizing recent observations on entropy within the setting of string theories. Either string theories have locality properties (not known), in which case they are special LQP's; or they are something nonlocal in which case, as a result of lack of physical interpretability which for all important properties (scattering theory, superselected charges, statistics...) totally hinges on locality, they are remarkable mathematical (but physically nonunderstood) observations. The fruitful conceptual curiosity from the first half of this century which was crucial to resolve such such interesting situatios unfortunately seems to have been lost with the invention of the very string theory which led to those observations.

mensions is quite simple: do the construction of the previous section on a conformal core and then use the blow-up construction using the longitudinal and transversal translations once in the ground state and once in the thermal representation. Whether this really works in higher dimensional model cases, still remains to be seen.

Finally we comment on the problem of spontaneously broken symmetries. It is well-known that the change of temperature is often accompanied by a transition of phase related to a change in global symmetry. The possibility of a modular transmutation of a thermal into a ground state theory (with spatial extension behind a horizon) creates all kinds of curious consistency problems.

Unfortunately the better understood chiral conformal theories do not allow for spontaneous symmetry breaking. The only exceptional case is the thermal collapse of supersymmetry [B-O]. Since the "collapse mechanism" originates from the impossibility of annihilating a faithful state (any thermal state is faithful) on a C^*-algebra by an antiderivation, it is independent of space-time dimensions and holds in particular in chiral theories. Although it does not serve as a an illustration of spontaneous symmetry breaking, its thermal aspects are interesting in their own right. Assuming that our transmutation mechanism also works for general chiral models including supersymmetric ones, and confronting it with the collapse mechanism one finds a somewhat curious situation whose only resolution seems to be that the (Unruh) thermal restriction wrecks the action of the antiderivative on only one side of the horizon and in this way "explains" the violent collapse. Whether this results allow for a deeper understanding of supersymmetry or only adds to the growing suspicion that this symmetry is of an accidental nature [Schr2], remains to be seen.

4 Concluding Remarks

Although the fully pointwise geometric symmetries which are well-known from the quantization approach to relativistic QFT are, with the exception of chiral conformal QFT, restricted to finite dimensional automorphisms as Poincaré symmetry and (only for zero mass) conformal symmetry, the modular structure of LQP opens the gates for a vast variety of infinite dimensional groups. One may either vary the states and consider the modular groups generated by wedge- and double cone-algebras or one may investigate the mod-

ular inclusions/intersections generated from wedges with respect to natural reference states od curved sace-time or the vacuum state in case of Minkowski space-time. As we argued in [S-W1], we expect in the first case to obtain an (infinite dimensional) "hidden" analogue of the chiral diffeomorphism group. This is based on the fact that the chiral diffeomorphism group can be build up from infinitely many "Möbius layers" by lifting the vacuum Möbius group with a covering transformation and by realizing that each such nth layer lifted Möbius group[3] is the modular group of a pair $(\mathcal{A}(I_1) \vee ... \vee \mathcal{A}(I_n), \Omega_n)$. The von Neumann algebra n this case is n-fold localized and the "lifted vacua" Ω_n are suitably chosen states such that the endpoints of the I_k, $k = 1...n$ are fixed points of the nth layer Möbius transformation and Ω_n is its unique invariant state [S-W1]. The higher dimensional analogues of this construction beyond the Poincaré resp. conformal group are expected to be fuzzy (nonlocal) independent of the chosen state; in fact according to a conjecture of Fredenhagen their infinitesimal action on test functions should be described by pseudo-differential operators which are at most asymptotically geometric near the horizon i.e. the space-time border of the localization region). In the present paper we have studied the partially hidden symmetry groups associated with thermal modular inclusions in chiral models. Even though in this case we do find geometrical aspects, these symmetries are not implemented by pointlike transformations in the underlying space-time and hence are hidden in the quantization approach. In addition the concepts of "chiral core" and its "blow-up property" which led us to a holographic degrees of freedom picture and gave some nice ideas about quantum entropy is obtained by the extremely noncommutative modular theory. It is a generic property of local quantum theory and cannot be seen by quantization procedures (and therefore is e.g. not related to differential geometric properties of Chern-Simons actions nor to gauge theory). The modular method based on real time noncommutative LQP seems to develop into a viable alternative to the euclidean functional integral approach to QFT [S-W2].

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Appendix

Canonical construction which underlies the calculation of the chiral core

[3]These lifted Moebius transformations $Moeb_n$ are called "quasisymmetric" of order n in the mathematical literature.

Let $(\mathcal{N} \subset \mathcal{M}, \Omega)$ hsm with non trivial relative commutant. Then look at the subspace $\overline{(\mathcal{N}' \cap \mathcal{M})\Omega} \subset H$. The modular groups to $\mathcal{N}$ and $\mathcal{M}$ leave invariant this subspace: $\Delta_{\mathcal{M}}^{it}$ maps $\mathcal{N}' \cap \mathcal{M}$ into itself by hsm for say positive t. Look at the orthogonal complement of $\overline{(\mathcal{N}' \cap \mathcal{M})\Omega} \subset H$. This orthogonal complement is mapped into itself by $\Delta_{\mathcal{M}}^{it}$ for positive t. Let ψ be in that subspace, then

$$\left\langle \psi, \Delta_{\mathcal{M}}^{it}(\mathcal{N}' \cap \mathcal{M})\Omega \right\rangle = 0 \ for \ t > 0. \tag{24}$$

Analyticity in t then gives the vanishing for all t.

Due to Takesaki theorem we can restrict $\mathcal{M}$ to $\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}$ using a conditional expectation to this subspace. Then

$$(\mathcal{N}' \cap \mathcal{M}) \subset \mathcal{M}|_{\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}}$$

is a hsm on the subspace defined above. $\mathcal{N}$ also restricts to that subspace and this restriction is obviously in the relative commutant of $(\mathcal{N}' \cap \mathcal{M}) \subset \mathcal{M}|_{\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}}$. Moreover using arguments as above it is easy to see that the restriction is cyclic w.r.t.Ω on this subspace. Therefore we arrive at a hsm standard inclusion

$$(\mathcal{N}|_{\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}} \subset \mathcal{M}|_{\overline{(\mathcal{N}' \cap \mathcal{M})\Omega}}, \Omega) \tag{25}$$

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