Basic Quantum Theory and Measurement from the Viewpoint of Local Quantum Physics

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Abstract

Several aspects of the manifestation of the causality principle in LQP (local quantum physics) are reviewed or presented. Particular emphasis is given to those properties which are typical for LQP in the sense that they do go beyond the structure of general quantum theory and even escape the Lagrangian quantization methods of standard QFT. The most remarkable are those relating causality to the modular Tomita-Takesaki theory, since they bring in the basic concepts of antiparticles, charge superselections as well as internal and external (geometric and hidden) symmetries.

1 LQP Principles and some Consequences

If one thinks about the fundamental physical principles of this century which have stood their grounds in the transition from classical into quantum physics, relativistic causality as well as the closely related locality of quantum operators (together with the localization of quantum states) will certainly be the most prominent one.

This principle entered physics through Einstein's 1905 special relativity, which in turn resulted from bringing the Galilei relativity principle of classical mechanics into tune with Maxwell's theory of electromagnetism. Therefore

it incorporated Faraday's "action at a neighborhood" principle which revolutionized 19^{th} century physics.

The two different aspects of Einstein's special relativity, namely Poincaré covariance and the locally causal propagation of waves (in Minkowski space) were kept together in the classical setting. In the adaptation of relativity to **LQP** (local quantum physics[1]) on the other hand [1], it is appropriate to keep them at least initially apart in the form of positive energy representations of the Poincaré group (leading to Wigner's concept of particles) and Einstein causality of local observables (leading to observable local fields and local generalized "charges"). Here a synthesis is also possible, but it happens on a deeper level than in the classical setting and results in LQP as a new physical realm which is conceptually very different from both classical field theory and general QT (quantum theory). The elaboration of some of these differences, in particular as they may be relevant with respect to the measurement process, constitutes one of the aims of these notes. For material which already entered textbooks or review articles, we have preferred to quote the latter. A more detailed account of the consequences of causality in a much broader context can be found in [2][3].

As a result of this added locality, LQP acquires a different framework than the kind of general quantum theory setting [5] in which the basics of quantum theory and measurement (including those ideas, which in the fashionable language of the day, are referred to as "quantum computation") are presented . Those concepts, which originate from the quantum adaptation of Einstein causality, lead in the presence of interactions to real particle creation (which artificially could be incorporated into a multichannel version quantum theory of particles) and, what has more importance within our presentation, to *virtual particle structure* (related to the phenomenon of vacuum polarization) which has no counterpart in global general quantum theory as quantum mechanics and cannot be incorporated into it at all. The latter remark preempts already the greater significance of superselected charges and their fusion, as opposed to particles and their quantum mechanical bound states. Thus the hierarchy of particles in QM is replaced by the hierarchy of charges and consequently we obtain "nuclear democracy" between particles. This is closely related to an almost anthropological principle which LQP realizes in a perfect way in laboratory particle physics: whenever energy-momentum and (generalized) charge conservation allow for particle creation channels to be opened, nature will maximally use this possibility. To be sure there are theoretical models of LQP (integrable/factorizing models in d=1+1 spacetime dimensions) which do not follow this dictum, but even in those cases at least its theoretical "virtual" version is realized: a vector state created by the application of an interacting field to the vacuum which has a one-particle component, is inexorably accompanied by a "polarization cloud" of particles/antiparticles (the hallmark of LQP). As already emphasized the *only exception are free bosonic/fermionic fields* and in a

[1]We use this terminology, whenever we want to emphasize that we relate the principles of QFT not with necessarily with the standard text-book formalism that is based on quantization through Lagrangian formalism.

somewhat pointed (against history), but nevertheless correct manner, one may say that this very exception is the reason why QM as a nonrelativistic limit of LQP has a physical reality at all. More general braid group statistics, as it can occur together with exotic spin in low dimensional QFT, requires these polarization clouds already in the "freest" realization of anyons/plektons and they are not fading away in the nonrelativistic limit because they are needed to uphold braid group statistics in that limit. This is the reason why the attempts of Leinaas- Myrheim, Wilscek and many others, which draw on the analogy with the Aharanov-Bohm quantum mechanics may catch some aspects of plektons but miss the spin-statistics connection which is their most important property (i.e. their LQP characterization).

This aspect of virtuality, which at first sight seems to complicate life since it activates the coupling between infinitely many degrees of freedom/channels, is counterbalanced by some very desirable and useful features: whereas general quantum theory needs an outside interpretative support, LQP carries this already within itself. It was emphasized already at the end of the 50^{ies} (notably by Rudolf Haag [1]), that e.g. for a particle interpretation one does not need to resolve the distinction between the various local observables which are localized in the same space-time region (laboratory extension and time duration of measurement), the knowledge of the space-time affiliation of a generic observable from a region $\mathcal{O}$ is enough. The experimenter does not know more than the geometric spacetime placement of his counters and their sensitivity; the latter he usually has to determine by monitoring experiments. The basic nature of locality in interpreting the particle aspect of a theory is underlined by the fact that despite intense efforts nobody has succeeded to construct a viable *nonlocal* theory. Here "viable" is meant in the sense of conceptual completeness, namely that a theory is required to *contain its own physical interpretation* i.e. that one does not have to invent or impose formulas from outside this theory.

Although physical reality may unfold itself like an onion or an infinite Russian "matrushka" with infinitely many layers of ever more general physical principles towards higher energies (smaller distances), it should still continue to be possible to have a mathematically consistent theory in each layer which is faithful to the principles valid in that layer. This has been fully achieved for quantum mechanics, but this goal was not yet reached in QFT. As a result of lack of nontrivial d=1+3 models or structural arguments which could demonstrate that the physical locality and spectral requirements allow for nontrivial solutions, the theory is still far from conceptual maturity, despite its impressive perturbation successes in QED, the Standard Model and in the area of Statistical Mechanics/Condensed Matter physics.

Causality and locality are in a profound way related to the foundations of quantum theory in the spirit of von Neumann, which brings me a little closer to the topic of this symposium. In von Neumann's formulation, observables are represented by selfadjoint operators and measurements are compatible if the operators commute. The totality of all measurements which are relatively compatible with a given set (i.e. noncommutativity within each set is allowed) generate a subalgebra: the commutant L' of the given set of operators L. In

particular in LQP, a conceptual framework which was not yet available to von Neumann, one is dealing with an isotonic "net" of subalgebras (in most physically interesting cases von Neumann factors, i.e. weakly closed operator algebras with a trivial center) $\mathcal{O} \to \mathcal{A}(\mathcal{O})$. Therefore unlike quantum mechanics, the spatial localization and the time duration of observables becomes an integral part of the formalism. *Causality gives an a-priori information about the size of spacetime $\mathcal{O}$ -affiliated operator (von Neumann) algebras:*

$$\mathcal{A}(\mathcal{O})' \supset \mathcal{A}(\mathcal{O}') \tag{1}$$

in words: the commutant $\mathcal{A}(\mathcal{O})'$ of the totality of local observables $\mathcal{A}(\mathcal{O})$ localized in the spacetime region $\mathcal{O}$ contains the observables localized in its spacelike complement (disjoint) $\mathcal{O}'$. In fact in most of the cases the equality sign will hold in which case one calls this strengthened (maximal) form of causality "Haag duality" [1]:

$$\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}') \tag{2}$$

In words, the spacelike localized measurements are not only commensurable with the given observables in $\mathcal{O}$, but every measurement which is commensurable with all observables in $\mathcal{O}$, is necessarily localized in the causal complement $\mathcal{O}'$. Here we extended for algebraic convenience von Neumann's notion of observables to the whole complex von Neumann algebra generated by hermitian operators localized in $\mathcal{O}$. If one starts the theory from a net indexed by compact regions $\mathcal{O}$ as double cones, then algebras associated with unbounded regions $\mathcal{O}'$ are defined as the von Neumann algebra generated by all $\mathcal{A}(\mathcal{O}_1)$ if $\mathcal{O}_1$ ranges over all net indices $\mathcal{O}_1 \subset \mathcal{O}'$.

Whereas the Einstein causality (1) allows a traditional formulation in terms of pointlike fields $A(x)$ as

$$[A(x), A(y)] = 0, \;\; (x-y)^2 < 0, \tag{3}$$

Haag duality can only be formulated in the algebraic net setting of LQP, since it is not a property which can be expressed in terms of individual operators. This aspect is shared by many other important properties and results [1].

One can prove that Haag duality always holds after a suitable extension of the net to the so-called dual net $\mathcal{A}(\mathcal{O})^d$. The latter may be defined independent of locality in terms of relative commutation properties as

$$\mathcal{A}(\mathcal{O})^d := \bigcap_{\mathcal{O}_1, \mathcal{O}_1' \subset \mathcal{O}} \mathcal{A}(\mathcal{O}_1)' \tag{4}$$

The relative commutance with respect to the observables is called (algebraic) "localizability". These considerations show that causality, locality and localization in LQP have a natural and deep relation to the notion of compatibility of measurements. In addition there are subtle modifications with respect to the basic quantum structure with possible changes of environmental and other aspects of quantum measuring. The fundamental reason for all such modifications in the interpretation of LQP versus QM is the different structure of local

algebras: the vacuum is not a pure state with respect to any algebra which is equal to or contained in an $\mathcal{A}(\mathcal{O})$ with $\mathcal{O}'$ nonempty, and the sharply localized algebras $\mathcal{A}(\mathcal{O})$ themselves do not admit pure states at all[2]! They possess an algebraic structure which has not been taken into account in the present day presentation of quantum basics including quantum computation. Since these fine points can only be appreciated with some more preparation, I will postpone their presentation.

If the vacuum net (i.e. the vacuum representation of the observable net) is Haag dual, then all associated "charged" nets share this property, unless the charges are nonabelian (in which case the deviation from Haag duality is measured by the Jones index of the above inclusion, or in physical terms the statistics- or quantum-dimension [13]). If on the other hand even the vacuum representation of the observable net violates Haag duality, then this indicates spontaneous symmetry breaking [6] i.e. not all internal symmetry algebraic automorphisms are spatially implementable. As already mentioned, in that case one can always maximize the algebra without destroying causality and without changing the Hilbert space, such that Haag duality is restored. This turns out to be related to the descend to the unbroken part of the symmetry which allows (since it is a subgroup) more invariants i.e. more observables.

Since QM and what is usually referred to as the basics of quantum theory do not know these concepts at all, I am presenting in some sense a contrasting program to the (global) QT orientation of this symposium. But often one only penetrates the foundations of a framework more profoundly, if one looks at a contrasting structure even if the difference is (presently) not measurable. For an analogy we may refer to the Hawking effect which has attracted ever increasing attention as a matter of principle, even though there is hardly any experimental chance.

In connection with this main theme of this symposium, it is interesting to ask if LQP could add something to our understanding of classical versus quantum reality (the ERP, Bell issue) or the measurement process i.e. production of "Schrödinger cat states" and observation of their subsequent decoherence. For the first issue I refer to [4]. Apart from some speculative remarks [5], there exists no investigation of the measurement process which takes into consideration the characteristic properties of the local algebras in LQP. I tend to believe that, whereas most of the present ideas on coherent states of Schrödinger cats and their transition to von Neumann mixtures will remain or at least not suffer measurable quantitative modifications, LQP could be expected to lead to significant conceptual changes. Certainly it will add a universal aspect to the issue of decoherence through environments. Contrary to QM where the environment is introduced by extending the system, localized systems in LQP are always open subsystems for which the "causal disjoint" defines a kind of universal environ-

[2]In order to find local algebras which are anywhere near quantum mechanical algebras and admit pure states and tensor products with entanglement similar to the inside/outside quantization box situation in Schrödinger theory, one has to allow for a "fuzzy" transition "collar" between a double cone and its causal disjoint outside, in more precise terms one has to consider a so-called split inclusion [1].

ment which is build into its formalism.

Another structurally significant deviation which was already alluded to results from the fact that the vacuum becomes a thermal state with respect to the local algebras $\mathcal{A}(\mathcal{O})$. There are two different mechanisms to generate thermal states: the standard coupling with a heat bath and the thermal aspect through restriction or localization and the creation of horizons [8][9]. The latter is in one class with the Hawking-Unruh mechanism; the difference being that in the localization situation the horizon is not classical i.e. is not defined in terms of a differential geometric Killing generator of a symmetry transformation of the metric.

The fact that algebras of the type $\mathcal{A}(\mathcal{O})$ have no pure states is related to the different behavior of the pair inside/outside with respect to factorization: whereas in QM the boxed system factorizes with the system outside the box, the total algebra $B(H)$ in LQP is generated by $A(O)$ and its commutant $B(H) = A(O) \vee A(O)'$, but *it is not the tensor product* of the two factor algebras $\mathcal{A}(\mathcal{O})$ and $\mathcal{A}(\mathcal{O})' = \mathcal{A}(\mathcal{O}')$. In order to get back to a tensor product situation and be able to apply the concepts of entanglement and entropy, one has to do a sophisticated split which is only possible if one allows for a "collar" (see later) between $\mathcal{O}$ and $\mathcal{O}'$ [1].

Since the thermal aspects of localization are analogous to black holes[3], there is no chance to directly measure such tiny effects. However in conceptual problems, e.g. the question if and how not only classical relativistic field theory, but also QFT excludes superluminal velocities, these subtle differences play a crucial role. Because of an unusual property of the vacuum in QFT (the later mentioned Reeh-Schlieder property), the exclusion of superluminal velocities requires more conceptual and mathematical understanding than in the classical case. Imposing the usual algebraic structure of QM (i.e. assuming tacitly that the local observables allow pure states) onto the local photon observables will lead to nonsensical results. Most sensational theoretical observations on causality violations which entered the press and in one case even Phys. Rev. Letters, suffer from incorrect tacit assumptions (if they are not already caused by a misunderstanding of the classical theory). We urge the reader to look at the fascinating reference [12] and the conceptually wrong preceding article.

Historically the first conceptually clear definition of localization of relativistic wave function was given by Newton and Wigner [7] who adapted Born's x-space probability interpretation to the Wigner relativistic particle theory. Apparently the result that there is no exact satisfactory relativistic localization (but only one sufficient for all practical purposes) disappointed Wigner so much, that he became distrustful of the usefulness of QFT in particle physics altogether (private communication by R. Haag). Whereas we know that this distrust was unjustified, we should at the same time acknowledge his stubborn insistence in

[3]The analogy is especially tight for the wedge localization since the boundary of wedges define bifurcated classical "Killing horizons" (Unruh), whereas the boundary of e.g. a double cone in a massive theory defines a "quantum horizon". This concept has a cood meaning with respect to the nongeometrically acting modular group associated with the latter situation, and it has no classical analogon (it is in fact a "hidden symmetry").

the importance of the locality concept which he thought of as an indispensable requirement in addition the positive energy property and irreducibility of the Wigner representations. Without explanation we state that modular localization of state vectors is different from the Born probability interpretation. Rather subspaces of modular localized wave functions preempt the existence of causally localized observables already on the level of the Hilbert space of relativistic wave functons and have no counterpart at all in N-particle quantum mechanics. As will be explained later, modular localization may serve as a starting point for the construction of interacting nonperturbative LQP's [13][4]. It is worthwhile to emphasize that sharper localization of local algebras in LQP is not defined in terms of support properties of classical smearing functions but via the rather unusual formation of intersection of localized algebras; although in some cases as CCR- or CAR-algebras (or more generally Wightman fields) the algebraic formulation (1) can be reduced to this more classical concept.

Since the modular structure is related to the so-called KMS property [1], it is not surprising that the modular localization has thermal aspects. In fact as mentioned before, there are two manifestations of thermality, the standard heat bath thermal behavior which is described by Gibbs formula or, after having performed the thermodynamic limit, by the KMS condition, and thermality caused by localization either with classical bifurcated Killing-horizons as in black holes [8][9] curved spacetime and (Rindler, Unruh, Bisognano-Wichmann) wedge regions, or in a purely quantum manner as the boundary of the Minkowski space double cones. In the latter case the KMS state has no natural limiting description in terms of a Gibbs formula (which only applies to type I and II, but not to type III von Neumann algebras), a fact which is also related to the boundedness from below of the Hamiltonian, whereas the e.g. Lorentz boost (the modular operator of the wedge) does not share this property. In [10] the reader also finds an discussion of localization and cluster properties in a heat bath thermal state. Although in these notes we will not enter these interesting thermal aspects, it should be emphasized that thermality (similar to the concept of virtual particle clouds) is an inexorable aspect of localization in LQP and does not need the Hawking type of Killing vector horizons. The close relation of particle and thermal physics (KMS thermal property$\simeq$crossing symmetry of S-matrix and formfactors [13]) is a generic property of LQP and should not be counted as a characteristic success of string theory.

Already in the very early development of algebraic QFT [11] the nature of the local von Neumann algebras became an interesting issue. Although it was fairly easy (and expected) to see that i.e. wedge- or double cone- localized algebras are von Neumann factors (in analogy to the tensor product factorization of standard QT under formation of subsystems, it took the ingenuity of Araki to realize that these factors were of type III (more precisely hyperfinite type

[4]In fact the good modular localization properties are guarantied in finite component positive energy representations, with the Wigner infinite component "continuous spin" representations being the only exception.. In this infinite component finite energy representation it is not possible to come from the wedge localization down to the spacelike cone localization which is the coarsest localization which one needs for a particle interpretation.

III_1, as we know nowadays thanks to the profound contributions of Connes and Haagerup), at that time still an exotic mathematical structure. Hyperfiniteness was expected from a physical point of view, since approximatability as limits of finite systems (matrix algebras) harmonizes very well with the idea of thermodynamic+scaling limits of lattice approximations. A surprise was the type III_1 nature which,as already mentioned, implies the absence of pure states (in fact all projectors are Murray von Neumann equivalent to 1) on such algebras; this property in some way anticipated the thermal aspect (Hawking-Unruh) of localization. Overlooking this fact (which makes local algebras significantly different from QM), it is easy to make conceptual mistakes which could e.g. suggest an apparent breakdown of causal propagation [12] as already mentioned before. If one simply grafts concepts of QM onto the causality structure of LQP (e.g. quantum mechanical tunnelling, structure of states) without deriving them in LQP , one runs the risk of wrong conclusions about e.g. the possibility of superluminal velocities.

A very interesting question is: what is the influence of the always present causally disjoint environment on the measurement process, given the fact that in the modern treatment the coupling to the environment and the associated decoherence relaxation are very important. Only certain aspects of classical versus quantum reality, as expressed in terms of Bell's inequalities, have been discussed in the causal context of LQP [4]. In the following we will sketch some more properties which set apart QM from LQP and whose conceptual impacts on decoherence of Schrödinger cats, entanglement etc. still is in need of understanding.

Let me mention two more structural properties, intimately linked to causality, which distinguish LQP rather sharply from QM. One is the Reeh-Schlieder property:

$$\begin{aligned} \overline{\mathcal{P}(\mathcal{O})\Omega} &= H,\ \textit{i.e. cyclicity of}\ \Omega \\ A \in \mathcal{P}(\mathcal{O}),\ A\Omega &= 0 \Longrightarrow A = 0\ \textit{i.e.}\ \Omega\ \textit{separating} \end{aligned} \tag{5}$$

which either holds for the polynomial algebras of fields or for operator algebras $\mathcal{A}(\mathcal{O})$. The first property, namely the denseness of states created from the vacuum by operators from arbitrarily small localization regions (a state describing a particle behind the moon[5] and an antiparticle on the earth can be approximated inside a laboratory of arbitrary small size and duration) is totally unexpected from the global viewpoint of general QT and has even attracted the interest of philosophers of natural sciences. If the naive interpretation of cyclicity/separability in the Reeh-Schlieder theorem leaves us with a feeling of science fiction, the way out is to ask: which among the dense set of localized states can be really produced with a controllable expenditure (of energy)? In QM to ask this question is not necessary since, as already mentioned, the localization at a given time via support properties of wave functions leads to a tensor

[5]This weird aspect should not be held against QFT but rather be taken as indicating that localization by a piece of hardware in a laboratory is also limited by an arbitrary large but finite energy, i.e. is a "phase space localization" (see subsequent discussion). In QM one obtains genuine localized subspaces without energy limitations.

product factorization of inside/outside so that the ground state factorizes and the application of the inside observables never leads to a dense set in the whole space. It turns out that most of the very important physical and geometrical informations are encoded into features of dense domains and in fact the aforementioned modular theory is explaining such relations. For the case at hand, the reconciliation of the Reeh-Schlieder theorem with common sense has led to the discovery of the physical relevance of *localization with respect to phase space in LQP*, i.e. the understanding of the size of degrees of freedom in the set:

$$\begin{aligned} P_E\mathcal{A}(\mathcal{O})\Omega \ &is\ compact \\ e^{-\beta\mathbf{H}}\mathcal{A}(\mathcal{O})\Omega \ &is\ nuclear, \quad \mathbf{H} \ = \ \int EdP_E \end{aligned} \tag{6}$$

The first property was introduces way back by Haag and Swieca [1], whereas the second statement (and similar nuclearity statements involving modular operators of local regions instead of the global Hamiltonian) which is more informative and easier to use, is a later result of Buchholz and Wichmann [1]. It should be emphasized that the LQP degrees of freedom counting of Haag-Swieca, which gives an infinite (but still nuclear) number of localized states is different from the finiteness in QM, a fact often overlooked in present day's string theoretic degree of freedom counting. The difference to the case of QM decreases if one uses instead of a strict energy cutoff a Gibbs damping factor $e^{-\beta H}$. In this case the map $\mathcal{A}(\mathcal{O}) \to e^{-\beta H}\mathcal{A}(\mathcal{O})\Omega$ is "nuclear" if the degrees of freedom are not too much accumulative in order to prevent the existence of a maximal (Hagedorn) temperature. The nuclearity assures that a QFT, which was given in terms of its vacuum representation, also exists in a thermal state. An associated nuclearity index turns out to be the counterpart of the quantum mechanical Gibbs partition function [1] and behaves in an entirely analogous way.

The peculiarities of the above Haag-Swieca degrees of freedom counting are very much related to one of the oldest "exotic" and at the same time characteristic aspects of QFT: vacuum polarization. As discovered by Heisenberg, the partial charge:

$$Q_V = \int_V j_0(x)d^3x = \infty \tag{7}$$

diverges as a result of uncontrolled vacuum fluctuations near the boundary. For the free field current it is easy to see that a better definition involving test functions, which takes into account the fact that the current is a 4-dim distribution and has no restriction to equal times, leads to a finite expression. The algebraic counterpart is the already mentioned so called "split property" namely [1] that if one leaves between say the double cone ("relativistic box") observable algebra $\mathcal{A}(\mathcal{O})$ and its causal disjoint $\mathcal{A}(\mathcal{O}')$ a "collar" region, then it is possible to construct in a canonical way a type I tensor factor $\mathcal{N}$ which extends into the collar and one obtains inside/outside factorization if one leaves out the collar region (a fuzzy box). This is then the algebraic analog of Heisenberg's smoothening of the boundary to control vacuum fluctuations. It is this "split inclusion" which allows to bring back some of the familiar structure of QM, since

type I factors allow for pure states, tensor product factorization, entanglement and all the other properties at the heart of quantum theory and the measurement process. Although there is no time to explain this, let us nevertheless mention that the most adequate formalism for LQP which substitutes quantization and is most characteristic of LQP in contradistinction to QT, is the formalism of modular localization related to the Tomita modular theory of von Neumann algebras. The interaction enters through wedge algebras, thus giving wedges a similar fundamental role as they already had in the Unruh illustration of the thermal aspects of the Hawking effect. Modular localization also leads to a vast enlargement of the symmetry concepts in QFT [14][15] beyond those geometric symmetries which enter the theory through quantized Noether currents.

If by these remarks I have created the impression that local quantum physics is one of the conceptually most fertile and spiritually (not historically) young areas of future basic research with relevance to the basics of measurement and quantum computation, I have accomplished the purpose of these notes. Indeed I know of no other framework which brings together such seemingly different ideas as Spin & Statistics, TCP and crossing symmetry of particle physics on the one hand together with thermal and entropical aspects of (modular) localization & black hole physics on the other hand.

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