PREDICTION INTERVALS FOR FUTURE ORDERED OBSERVATIONS IN DOUBLY TYPE-II CENSORED SAMPLES FROM A TWO PARAMETER EXPONENTIAL DISTRIBUTION

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von

Adel Mohamed Mahmoud Omar

aus

Kena, Ägypten

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D 7
Referent Prof. Dr. Krengel
Korreferent Prof. Dr. Denker
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CONTENTS

Page

Chapter three

Estimation for the tow-parameter exponential distribution under doubly type-II censoring

3-1 Estimation for tow-parameter exponential distribution 58
(i) The lower confidence bound for the parameter θ 58
(i) The lower confidence bound for the parameter μ 61

Chapter four

Prediction interval for future observations using a maximum likelihood estimator

4-1 Maximum likelihood estimate 63
4-2 Prediction using a maximum likelihood estimator 65
4-3 Prediction interval using a maximum likelihood estimator 68
4-4 Special cases 70

Chapter five

The Bayesian prediction

5-1 Prediction intervals for a future observation 74
5-2 Special cases 78
References 80

CHAPTER THREE

ESTIMATION FOR THE TWO-PARAMETER EXPONENTIAL DISTRIBUTION UNDER DOUBLY TYPE-II CENSORING

The exponential distribution has an important position in lifetime models.It is the first lifetime model for which statistical methods were extensively developed.Many authors have contributed to the study of this distribution.
Important works include (Epstein and Sobel,1954),(Epstein,1960)
,(Mann,Schafer and Singpurwalla ,1974),(Bain,1978),(Lawless ,1982),(Balakrishnan and Colen,1991)
,(Balasubramanian and Balakrishnan,1992) and (Kong,F. and Fei,H.1994)

3-1 Estimation for two-parameter exponential distribution under doubly type-II censoring

We assume that the survival times of certain items have a two-parameter exponential distribution with a density function

$$f(x,\mu,\theta)=\frac{1}{\theta}e^{-\left(\frac{x-\mu}{\theta}\right)},\ x\geq\mu,\ \theta>0 \tag{3.1.1}$$

This model is employed in situations where it is believed that death or failure can't occur before a certain time μ.
When μ is known it can be analyzed as for the one-parameter exponential distribution ,because the variable $X-\mu$ has a one-parameter exponential distribution.
When the model includes the parameter μ the data usually do not provide enough informations about μ .Including μ could cause rather special statistical problems especially for models such as the Weibull ,lognormal ,and gamma distributions .Fortunately for the exponential distribution the problems have been satisfactorily solved.
Suppose that $X_{(k)},X_{(k+1)},\cdots,X_{(r)}$ are order statistics in a doubly type-II censored sample from a sample of size n from a two-parameter exponential distribution $exp(\mu,\theta)$.
Then we shall obtain the lower confidence bounds and significance tests for the parameters μ and θ .

(i) The lower confidence bound for the parameter θ

Let $X_{(k)},X_{(k+1)},\cdots,X_{(r)}$ be the (r-k+1) order statistics from a sample of size n from a two-parameter exponential distribution $exp(\mu,\theta)$.
The following results have been obtained in the special case k=1,by lemma

1 ,and our arguments are similar.
The random variables

$$Z_i = \frac{(n-i+1)(X_{(i)} - X_{(i-1)})}{\theta} \quad , i = k+1, k+2, \cdots, r \tag{3.1.2}$$

are independent and identically distributed with exponential distribution exp(0,1).
Define

$$Q_k = \frac{(X_{(k)} - \mu)}{\theta} \tag{3.1.3}$$

and

$$Q_{r,k} = \sum_{i=k+1}^{r} \frac{X_{(i)} - X_{(i-1)}}{\theta}. \tag{3.1.4}$$

Then

$$Q_{r,k} = \sum_{i=k+1}^{r} \frac{Z_i}{n-i+1} \tag{3.1.5}$$

where $Z_1, Z_2, \cdots, Z_r$ are identically independent distributed from $exp(0,1)$.
The pivotal $Q_{r,k}$ do not contain the parameter μ .Therefore the confidence bound for the parameter θ can be constructed through $Q_{r,k}$.
$Q_{r,k}$ is a sum of (r-k) independent identically distributed random variables with exponential distribution.
Then $Q_{r,k}$ has a gamma distribution $G(\alpha, \beta)$,where $\mu_{r,k} = E(Q_{r,k}) = \alpha\beta$,and $V_{r,k} = Var(Q_{r,k}) = \alpha\beta^2$.
Then $\beta = \frac{V_{r,k}}{\mu_{r,k}}$,and $\alpha = \frac{\mu_{r,k}^2}{V_{r,k}}$,and the random variable $\frac{2Q_{r,k}}{\beta}$ has a $\chi^2(2\alpha)$-distribution.
Observe that

$$\frac{2\mu_{r,k}Q_{r,k}}{V_{r,k}} \sim \chi^2(\frac{2\mu_{r,k}^2}{V_{r,k}}) \quad see[38]. \tag{3.1.6}$$

The mean and the variance of $Q_{r,k}$ are given by

$$\mu_{r,k} = \sum_{i=k+1}^{r} \frac{1}{n-i+1} \tag{3.1.7}$$

and

$$V_{r,k} = \sum_{i=k+1}^{r} \frac{1}{(n-i+1)^2}. \tag{3.1.8}$$

Then the $(1-\alpha)\%$ lower confidence bound for the parameter θ is

$$\theta_l = \frac{2\mu_{r,k} \sum_{i=k+1}^{r} (X_{(i)} - X_{(i-1)})}{V_{r,k}\chi^2_{(1-\alpha)}(\frac{2\mu^2_{r,k}}{V_{r,k}})}. \tag{3.1.9}$$

If $(\frac{2\mu^2_{r,k}}{V_{r,k}})$ is an integer, then the distribution values and the quantiles of $\chi^2(\frac{2\mu^2_{r,k}}{V_{r,k}})$ can be found directly in a regular $\sim \chi^2$ table.
In general if $\chi^2(\nu)$ is a χ^2 random variable with ν degrees of freedom ,then approximately for large ν

$$[(\frac{\chi^2(\nu)}{\nu})^{1/3} + \frac{2}{9\nu} - 1](\frac{9\nu}{2})^{1/2} \sim N(0,1) \quad see[20] \tag{3.1.10).}$$

If $z_{1-\alpha}$ is the $(1-\alpha)\%$ quantiles of N(0,1) ,then correspondingly

$$[(\frac{\chi^2_{1-\alpha}(\nu)}{\nu})^{1/3} + \frac{2}{9\nu} - 1](\frac{9\nu}{2})^{1/2} \approx z_{1-\alpha}, \tag{3.1.11}$$

that is

$$\chi^2_{1-\alpha}(\nu) \approx \nu[1 - \frac{2}{9\nu} + \frac{1}{3}(\frac{2}{\nu})^{1/2} z_{1-\alpha}]^3. \tag{3.1.12}$$

In our case $\nu = \frac{2\mu^2_{r,k}}{V_{r,k}}$ so

$$\chi^2_{1-\alpha}(\frac{2\mu^2_{r,k}}{V_{r,k}}) \approx \frac{2\mu^2_{r,k}}{V_{r,k}}[1 - \frac{V_{r,k}}{(3\mu_{r,k})^2} + \frac{V^{1/2}_{r,k}}{3\mu_{r,k}} z_{1-\alpha}]^3. \tag{3.1.13}$$

Then

$$\theta_l = \frac{2\mu_{r,k} \sum_{i=k+1}^{r} (X_{(i)} - X_{(i-1)})}{V_{r,k}\frac{2\mu^2_{r,k}}{V_{r,k}}[1 - \frac{V_{r,k}}{(3\mu_{r,k})^2} + \frac{V^{1/2}_{r,k}}{3\mu_{r,k}} z_{1-\alpha}]^3}. \tag{3.1.14}$$

Then we have proved:

Theorem 3:

The lower confidence bound for the parameter θ in doubly type-2 censored samples from a two-parameter exponential distribution is given by

$$\theta_l = \frac{2\mu_{r,k}\sum_{i=k+1}^{r}(X_{(i)} - X_{(i-1)})}{V_{r,k}\frac{2\mu_{r,k}^2}{V_{r,k}}[1 - \frac{V_{r,k}}{(3\mu_{r,k})^2} + \frac{V_{r,k}^{1/2}}{3\mu_{r,k}}z_{1-\alpha}]^3}. \tag{3.1.15}$$

(ii)The lower confidence bound for the parameter μ

The confidence bound for the parameter μ can be constructed through the pivotal quantity

$$Z = \left(\frac{2Q_k}{\frac{2\mu_{r,k}Q_{r,k}}{V_{r,k}}}\right) = \left(\frac{\frac{2(X_{(k)} - \mu)}{\theta}}{\frac{2\mu_{r,k}}{V_{r,k}}\sum_{i=k+1}^{r}\frac{(X_{(i)} - X_{(i-1)})}{\theta}}\right) \tag{3.1.16}$$

Z has the form of the ratio of two chi-square random variables.

The distributions of Q_k *and* $Q_{r,k}$ have been given above, therefore the distribution of Z is easy to derive. Note that,

$$2Q_k \sim \chi^2(m_1) \tag{3.1.17}$$

and

$$\frac{2\mu_{r,k}Q_{r,k}}{V_{r,k}} \sim \chi^2(m_2) \tag{3.1.18}$$

where $m_1 = 2$ and $m_2 = \frac{2\mu_{r,k}^2}{V_{r,k}}$.

Then

$$Z = \frac{\chi^2(m_1)}{\chi^2(m_2)} \tag{3.1.19}$$

$$\left(\frac{Q_k}{\frac{2\mu_{r,k}Q_{r,k}}{V_{r,k}m_2}}\right) \sim f_{(2,m_2,1-\alpha)} \tag{3.1.20}$$

If $f_{(2,m_2,1-\alpha)}$ is the $(1-\alpha)\%$ percentile point of the F-distribution ,then the $(1-\alpha)\%$ lower confidence bound for μ is

$$\mu_l = X_{(k)} - \frac{1}{\mu_{r,k}} f_{(2,m_2,1-\alpha)} \sum_{i=k+1}^{r} (X_{(i)} - X_{(i-1)}). \tag{3.1.21}$$

We have proved:
Theorem 4:
The lower confidence bound for the parameter μ in doubly type-2 censored samples from a two-parameter exponential distribution is given by

$$\mu_l = X_{(k)} - \frac{1}{\mu_{r,k}} f_{(2,m_2,1-\alpha)} \sum_{i=k+1}^{r} (X_{(i)} - X_{(i-1)}). \tag{3.1.22}$$

When m_1 and m_2 are not integers ,interpolation in f-tables for integral degrees of freedom have to be used to determine $f_{(m_1,m_2,1-\alpha)}$.
Observe that in life-time models negative values of μ do not make sense. Therefore if the estimation of μ gives a negative value, we can simply replace it by zero.

CHAPTER FOUR

PREDICTION INTERVAL FOR FUTURE OBSERVATIONS USING A MAXIMUM LIKELIHOOD ESTIMATOR

Our principal application is to the prediction of higher order statistic from lower ones in type-II censored random samples.
Let $X_1, X_2, \cdots, X_n$ be independent real valued random variables.Assume that the distribution of the random variables is identical and that it has a density function $f(x, \beta), \beta = (\mu, \theta)$.
let $X_{(1)}, X_{(2)}, \cdots, X_{(n)}$ be the order statistics for these variables .
Let the random vector $X = (X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ be observed ,and let Y be the unobserved (future) random variable $X_{(s)}$,where $1 \leq k < r < s \leq n$.
We proceed as follows :
We find $\beta^\star$ which is the maximum likelihood estimate of β .For this value $\beta^\star$ we take the maximum likelihood predictor $x^\star_{(s)}$ which is the value maximizing the conditional likelihood function of $X_{(s)}$ given $X_{(k)} = x_{(k)}, \cdots, X_{(r)} = x_{(r)}$,$L(x_{(s)}|X_{(k)} = x_{(k)}, \cdots, X_{(r)} = x_{(r)})$

4-1 Maximum likelihood estimate

Let $X_{(k)}, X_{(k+1)}, \cdots, X_{(r)}$ denote (r-k+1) order statistics of a random sample of size n from a two-parameter exponential distribution having a density function

$$f_X(x, \theta, \mu) = \frac{1}{\theta} e^{-(\frac{x-\mu}{\theta})}, \ x \geq \mu, \mu > 0, \ \theta > 0 \tag{4.1.1}$$

and

$$F_X(x) = 1 - e^{-(\frac{x-\mu}{\theta})}, \ x \geq \mu, \ \theta > 0 \tag{4.1.2}$$

To derive the maximum likelihood estimate ,we consider the likelihood function of the variables $Z_{k+1}, Z_{k+2}, \cdots, Z_r$ and θ ,where ,from lemma 1 ,the variables $Z_i = (n - i + 1)(X_{(i)} - X_{(i-1)}), i = k + 1, \cdots, r$,are independent and identically distributed random variables with exponential distribution.
Then the value of $\theta^\star(X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ which is a maximum likelihood estimate can be obtained from a likelihood function of $Z_{k+1}, Z_{k+2}, \cdots, Z_r$ and θ.

$$L(z_{k+1}, \cdots, z_r, \theta) = \prod_{i=k+1}^{r} \left(\frac{1}{\theta} e^{-(\frac{z_i}{\theta})} \right) \tag{4.1.3}$$

$$L(z_{k+1},\cdots,z_r,\theta)=\left(\frac{1}{\theta^{r-k}}e^{-\sum_{i=k+1}^{r}(\frac{z_i}{\theta})}\right). \tag{4.1.4}$$

Then the value $\theta^\star$, which maximizes L satisfies

$$\frac{\partial logL}{\partial\theta}=0 \tag{4.1.5}$$

$$logL=-(r-k)log\theta-\frac{\sum_{i=k+1}^{r}z_{(i)}}{\theta} \tag{4.1.6}$$

From (2.2.25),and (2.2.26),we obtain

$$T=\sum_{i=k+1}^{r}X_{(i)}+(n-r)X_{(r)}-(n-k)X_{(k)}=\sum_{i=k+1}^{r}Z_i$$

$$\frac{\partial logL}{\partial\theta}=-\frac{(r-k)}{\theta}+\frac{T}{\theta^2}=0 \tag{4.1.6}$$

Then

$$\theta^\star=\frac{T}{r-k} \tag{4.1.7}$$

We have proved:

Theorem 5:

The maximum likelihood estimate for the parameter θ from the observation of $X_{(k)},X_{(k+1)},\cdots,X_{(r)}$ is given by

$$\theta^\star=\frac{\sum_{i=k+1}^{r}X_{(i)}+(n-r)X_{(r)}-(n-k)X_{(k)}}{r-k}. \tag{4.1.8}$$

4-2 Prediction using a maximumu likelihood estimator

Let the random vector $X = (X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ be observed.For the value $\theta^{\star}$ we take the maximum likelihood predictor $x^{\star}_{(s)}$ which is the value maximizing the conditional likelihood function of $X_{(s)}$ given $X_{(k)} = x_{(k)}, \cdots, X_{(r)} = x_{(r)}$

$$(4.2.1) \quad L(X_{(s)}|X_{(k)} = x_{(k)}, \cdots, X_{(r)} = x_{(r)}) = \frac{L(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)}, x_{(s)})}{L(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)})}$$

(4.2.2)

$$L(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)}, x_{(s)}) = \frac{n!}{(k-1)!(s-r-1)!(n-s)!} \prod_{i=k}^{r} f_X(x_{(i)})$$

$$\times \left(F_X(x_{(k)})\right)^{k-1} \left(F_X(x_{(s)} - F_X(x_{(r)}))\right)^{s-r-1} \left(1 - F_X(x_{(s)})\right)^{n-s} f_X(x_{(s)})$$

and

$$(4.2.3) \qquad L(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)}) = \frac{n!}{(k-1)!(n-r)!} \prod_{i=k}^{r} f_X(x_{(i)})$$

$$\times \left(F_X(x_{(k)})\right)^{k-1} \left(1 - F_X(x_{(r)})\right)^{n-r}.$$

Then

$$(4.2.4) \qquad L(X_{(s)}|X_{(k)} = x_{(k)}, \cdots, X_{(r)} = x_{(r)}) = \frac{(n-r)!}{(s-r-1)!(n-s)!}$$

$$\times \left(F_X(x_{(s)} - F_X(x_{(r)}))\right)^{s-r-1} \left(1 - F_X(x_{(s)})\right)^{n-s} f_X(x_{(s)}) \left(1 - F_X(x_{(r)})\right)^{-(n-r)}.$$

$$(4.2.5) \qquad L = \frac{(n-r)!}{(s-r-1)!(n-s)!\theta} \left(e^{-(\frac{x_{(r)}-\mu}{\theta})} - e^{-(\frac{x_{(s)}-\mu}{\theta})}\right)^{s-r-1}$$

$$\times \left(e^{-(\frac{x_{(s)}-\mu}{\theta})}\right)^{n-s+1} \left(e^{-(\frac{x_{(r)}-\mu}{\theta})}\right)^{-(n-r)},$$

$$L = \frac{(n-r)!}{(s-r-1)!(n-s)!\theta}\left(1-e^{-(\frac{x_{(s)}-x_{(r)}}{\theta})}\right)^{s-r-1} \tag{4.2.6}$$

$$\times\left(e^{-(\frac{x_{(s)}-\mu}{\theta})}\right)^{n-s+1}\left(e^{-(\frac{x_{(r)}-\mu}{\theta})}\right)^{-(n-s+1)}.$$

Put $Z = X_{(s)} - X_{(r)}$.
The predictive likelihood function (PLF) of Z and $\theta^\star$ is given

$$L(\theta^\star, z) = \frac{(n-r)!}{(s-r-1)!(n-s)!\theta^\star}\left(e^{-(\frac{z}{\theta^\star})}\right)^{n-s+1}\left(1-e^{-(\frac{z}{\theta^\star})}\right)^{s-r-1} \tag{4.2.7}$$

$$L(\theta^\star, z) = \frac{1}{\theta^\star\beta(s-r, n-s+1)}\left(e^{-(\frac{z}{\theta^\star})}\right)^{n-s+1}\left(1-e^{-(\frac{z}{\theta^\star})}\right)^{s-r-1}. \tag{4.2.8}$$

The value $z^\star$ which maximizes the likelihood function $L(\theta^\star, z)$ satisfies

$$\frac{\partial logL(\theta^\star, z)}{\partial z} = 0 \tag{4.2.9}$$

$$logL(\theta^\star, z) = logC - log\theta^\star - \frac{(n-s+1)z}{\theta^\star} + (s-r-1)log\left(1-e^{-\frac{z}{\theta^\star}}\right) \tag{4.2.10}$$

where $C = \dfrac{1}{\beta(s-r, n-s+1)}$.
Then the maximum likelihood predictor $z^\star$ which is the value maximizing $L(\theta^\star, z)$ is given by

$$\frac{\partial logL(\theta^\star, z)}{\partial z} = \frac{(s-r-1)e^{-\left(\frac{z}{\theta^\star}\right)}}{\theta^\star\left(1-e^{-\left(\frac{z}{\theta^\star}\right)}\right)} - \frac{n-s+1}{\theta^\star} = 0 \tag{4.2.11}$$

$$\frac{e^{-\left(\frac{z}{\theta^\star}\right)}}{\left(1-e^{-\left(\frac{z}{\theta^\star}\right)}\right)} = \frac{n-s+1}{s-r-1} \tag{4.2.12}$$

$$(n-r)e^{-\left(\frac{z}{\theta^\star}\right)} = n-s+1 \tag{4.2.13}$$

$$Z^\star = \theta^\star \log \frac{n-r}{n-s+1} \tag{4.2.14}$$

We have proved:
Theorem 6:
The maximum likelihood predictor for the future order statistic $X_{(s)}$ under the parameter value $\theta^\star$ is given by

$$X^\star_{(s)} = X_{(r)} + \frac{T}{r-k} log(\frac{n-r}{n-s+1}). \tag{4.2.15}$$

From (2.3.3) it can be shown that

$$\frac{2T}{\theta} \sim \chi^2(2(r-k)). \tag{4.2.16}$$

Thus an approximate $100\%(1-\alpha)$ confidence interval for the mean lifetime θ is

$$\frac{2T}{\chi^2(2(r-k), 1-\frac{\alpha}{2})} < \theta < \frac{2T}{\chi^2(2(r-k), \frac{\alpha}{2})} \tag{4.2.17}$$

where $\chi^2(2(r-k), \alpha)$ denotes the quantile of the χ^2-distribution .
To find a confidence interval for the parameter μ ,if we define from (3.1.3)

$$Q_{(k)} = (\frac{X_{(k)} - \mu}{\theta}). \tag{4.2.18}$$

From lemma (1) and (3.1.17) ,(2.3.3) $2Q_{(k)}$ and $\frac{2T}{\theta}$ are independent ,and have chi-square distributions with 2 and 2(r-k) degrees of freedom respectively. Thus the ratio

$$Y = \frac{(\frac{2Q_k}{2})}{(\frac{2T}{2(r-k)\theta})} = \frac{(r-k)(X_{(k)} - \mu)}{T} \tag{4.2.19}$$

follows the f-distribution with 2 and 2(r-k) degrees of freedom.
Let $f_{(2,2(r-k),1-\alpha)}$ be the $100\%(1-\alpha)$ percent points of the f-distribution, that is

$$P(Y \geq f_{(2,2(r-k),1-\alpha)}) = 1-\alpha \tag{4.2.20}$$

We have proved:
Theorem 7:
The $100\%(1-\alpha)$ percent confidence interval for μ from the observation of $X_{(k)}, X_{(k+1)}, \cdots, X_{(r)}$, and T is given by

$$X_{(k)} - \frac{T}{(r-k)} f_{(2,2(r-k),1-\alpha)} < \mu < X_{(k)}. \tag{4.2.21}$$

4-3 Prediction interval using a maximum likelihood estimator
Let $\theta^\star$ be a maximum likelihood estimate of θ ,and let $f(y|\theta)$ be the probability density function of a random variable Y which is an unobserved (future) observation, and let $f(y|\theta^\star)$ be the estimated probability density function of Y.
Then if θ is known ,a $100\%(1-\alpha)$ prediction interval for Y ,is $[U_1(X,\theta), U_2(X,\theta)]$,and ,if θ is unkown ,then a reasonable approximate $100\%(1-\alpha)$ prediction interval for Y is $[U_1(X,\theta^\star), U_2(X,\theta^\star)]$,where $U_1(X,\theta^\star)$ and $U_2(X,\theta^\star)$ are the solution of these integral equations

$$\int_{-\infty}^{U_1} f(y|\theta^\star)dy = \frac{\alpha}{2} \quad see[2] \tag{4.3.1}$$

and

$$\int_{-\infty}^{U_2} f(y|\theta^\star)dy = 1 - \frac{\alpha}{2} \tag{4.3.2}$$

Let a random vector $X = (X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ be observed from an exponential distribution with a density function in (4.1.1).
Let $\theta^\star(X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ be a maximum likelihood estimator of θ. Using $\theta^\star$ we want to find an approximate $100\%(1-\alpha)$ prediction interval for a future observation $Y = X_{(s)} - X_{(r)}$.
From (4.2.8), the density function of the random variable Y is

$$f(y|\theta) = \frac{1}{\theta\beta(s-r, n-s+1)} \left(e^{-(\frac{y}{\theta})}\right)^{n-s+1} \left(1 - e^{-(\frac{y}{\theta})}\right)^{s-r-1} \tag{4.3.3}$$

and

$$f(y|\theta^\star) = \frac{1}{\theta^\star\beta(s-r, n-s+1)} \left(e^{-(\frac{y}{\theta^\star})}\right)^{n-s+1} \left(1 - e^{-(\frac{y}{\theta^\star})}\right)^{s-r-1}. \tag{4.3.4}$$

From (4.3.1) and (4.3.2) we obtain, an approximate $100\%(1-\alpha)$ prediction interval for a future observation Y is given by

$$\int_0^{U_1} \frac{1}{\theta^\star \beta(s-r, n-s+1)} \left(e^{-(\frac{y}{\theta^\star})}\right)^{n-s+1} \left(1-e^{-(\frac{y}{\theta^\star})}\right)^{s-r-1} dy = \frac{\alpha}{2} \tag{4.3.5}$$

and

$$\int_0^{U_2} \frac{1}{\theta^\star \beta(s-r, n-s+1)} \left(e^{-(\frac{y}{\theta^\star})}\right)^{n-s+1} \left(1-e^{-(\frac{y}{\theta^\star})}\right)^{s-r-1} dy = 1-\frac{\alpha}{2}. \tag{4.3.6}$$

Then

$$\int_0^{U_1} \frac{1}{\theta^\star \beta(s-r, n-s+1)} \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix} e^{-(\frac{(n-s+i+1)y}{\theta^\star})} dy = \frac{\alpha}{2}. \tag{4.3.7}$$

Then

$$\frac{1}{\beta(s-r, n-s+1)} \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix} \frac{1}{(n-s+i+1)} \times \left(1-e^{-\left(\frac{(n-s+i+1)U_1}{\theta^\star}\right)}\right) = \frac{\alpha}{2} \tag{4.3.8}$$

and

$$\frac{1}{\beta(s-r, n-s+1)} \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix} \frac{1}{(n-s+i+1)} \times \left(1-e^{-\left(\frac{(n-s+i+1)U_2}{\theta^\star}\right)}\right) = 1-\frac{\alpha}{2}. \tag{4.3.9}$$

Then for given values of k,r,s,n and α we can compute the values of $U_1(X, \theta^\star)$ and $U_2(X, \theta^\star)$.
We have proved:
Theorem 8:
An approximate prediction interval for a future observation $Y = (X_{(s)} - X_{(r)})$ under the parameter value $\theta^\star$ is given by $(U_1(X, \theta^\star, U_2(X, \theta^\star))$, where U_1 and U_2 are given from (4.3.8), and (4.3.9).

4-4 Special cases

(i) Prediction interval for a smallest future observation

In this case s=r+1,from (4.3.8) and (4.3.9) we get

$$\frac{1}{(n-r)\beta(1,n-r)}\left(1-e^{-\left(\frac{(n-r)U_1}{\theta^\star}\right)}\right)=\frac{\alpha}{2}, \tag{4.4.1}$$

$$\left(1-e^{-\left(\frac{(n-r)U_1}{\theta^\star}\right)}\right)=\frac{\alpha}{2} \tag{4.4.2}$$

$$e^{-\left(\frac{(n-r)U_1}{\theta^\star}\right)}=1-\frac{\alpha}{2}. \tag{4.4.3}$$

Then

$$U_1=\frac{\theta^\star}{n-r}log(\frac{1}{1-\frac{\alpha}{2}}) \tag{4.4.4}$$

and

$$\frac{1}{(n-r)\beta(1,n-r)}\left(1-e^{-\left(\frac{(n-r)U_2}{\theta^\star}\right)}\right)=1-\frac{\alpha}{2} \tag{4.4.5}$$

$$\left(1-e^{-\left(\frac{(n-r)U_2}{\theta^\star}\right)}\right)=1-\frac{\alpha}{2} \tag{4.4.6}$$

$$e^{-\left(\frac{(n-r)U_2}{\theta^\star}\right)}=\frac{\alpha}{2} \tag{4.4.7}$$

Then

$$U_2=\frac{\theta^\star}{n-r}log(\frac{2}{\alpha}) \tag{4.4.8}$$

Hence an approximate $100\%(1-\alpha)$ prediction interval for a smallest future observation is $(U_1(X,\theta^\star),U_2(X,\theta^\star))$.

(ii) Prediction interval for a largest future observation

In this case s=n ,from (4.3.8) and (4.3.9) we get

$$\frac{1}{\beta(n-r,1)}\sum_{i=0}^{n-r-1}(-1)^i\binom{n-r-1}{i}\frac{1}{(i+1)} \times\left(1-e^{-\left(\frac{(i+1)U_1}{\theta^\star}\right)}\right)=\frac{\alpha}{2}. \tag{4.4.9}$$

We can rewrite (4.4.9) as

$$1-\sum_{i=0}^{n-r}(-1)^i\binom{n-r}{i}\left(1-e^{-\left(\frac{iU_1}{\theta^\star}\right)}\right)=\frac{\alpha}{2} \tag{4.4.10}$$

$$\sum_{i=0}^{n-r}(-1)^i\binom{n-r}{i}\left(1-e^{-\left(\frac{iU_1}{\theta^\star}\right)}\right)=1-\frac{\alpha}{2} \tag{4.4.11}$$

and

$$=\frac{1}{\beta(n-r,1)}\sum_{i=0}^{n-r-1}(-1)^i\binom{n-r-1}{i}\frac{1}{(i+1)} \times\left(1-e^{-\left(\frac{(i+1)U_2}{\theta^\star}\right)}\right)=1-\frac{\alpha}{2}. \tag{4.4.12}$$

$$1-\sum_{i=0}^{n-r}(-1)^i\binom{n-r}{i}\left(1-e^{-\left(\frac{iU_2}{\theta^\star}\right)}\right)=1-\frac{\alpha}{2} \tag{4.4.13}$$

$$\sum_{i=0}^{n-r}(-1)^i\binom{n-r}{i}\left(1-e^{-\left(\frac{iU_2}{\theta^\star}\right)}\right)=\frac{\alpha}{2} \tag{4.4.14}$$

Example

Suppose that in a laboratory experiment n=10 mice are exposed to carcinogens ,the experimenter decides to start the study after two of the mice are dead and to observe the other at that time the survival times of the expired are 62,84,106 and 144 hours.

Let the lifetimes of all n items be distributed according to the two-parameter exponential distribution with unknown parameter μ and θ.
We wish to find a 95% prediction interval for $X_{(7)}$,and a confidence interval for the mean lifetime and a maximum likelihood predictor (MLP) for $X_{(8)}$.
Solution
We first look for a maximum likelihood predictor $X_{(8)}$. In this case n=10 ,r=6 ,k=3 and s=8, and

$$T = \sum_{i=k+1}^{r} X_{(i)} + (n-r)X_{(r)} - (n-k)X_{(k)}$$

$$T = \sum_{i=4}^{6} X_{(i)} + 4X_{(6)} - 7X_{(3)} = 476.$$

The maximum likelihood predictor $X_{(8)}$ is

$$X_{(8)} = 144 + 158.7log(\frac{4}{3}) = 144 + 158.7(0.176) = 171.9 \ hours.$$

A 95% prediction interval for $X_{(7)}$ is obtained from (4.4.3) and (4.4.4):

$$U_1 = \frac{476}{12}log(\frac{1}{0.975}) = 39.6(0.01) = 0.396.$$

$$U_2 = \frac{476}{12}log(\frac{2}{0.05}) = 39.6(1.6) = 63.36.$$

Hence a 95% prediction interval for $X_{(7)}$ is (144.396,207.36).
A 95% confidence interval for the mean lifetime θ is

$$\frac{2T}{\chi^2(2(r-k), 1-\frac{\alpha}{2})} < \theta < \frac{2T}{\chi^2(2(r-k), \frac{\alpha}{2})}.$$

For $\alpha = 0.05$ the confidence interval is

$$\frac{952}{\chi^2(6, 0.975)} < \theta < \frac{952}{\chi^2(6, 0.025)}.$$

Using the tables of $\chi^2(.,.)$ we find the interval is

$$\frac{952}{14.45} < \theta < \frac{952}{1.23} = (65.88, 767.74).$$

CHAPTER FIVE
THE BAYESIAN PREDICTION

The idea behind prediction is to provide some estimate either point or interval for future observations of an experiment F based on the results obtained from an informative experiment E.
Let $X_1, X_2, \cdots X_n$ be independent real valued random variables with a density function $f(x|\theta),\ x \in X,\ \theta \in \Theta$.
Let $X_{(1)}, X_{(2)}, \cdots X_{(n)}$ be the order statistics for these variables.
Let the random vector $X = (X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$ be observed ,and let Y be the random variable $(X_{(s)} - X_{(r)})$, where $1 \leq k < r < s \leq n$.
We assume that the distributions of X and Y are indexed by the same parameter θ,our uncertainty about the true value of θ is measured by a prior density function $g(\theta)$.
Then the posterior density for the parameter θ is given by

$$h(\theta|X_{(k)} = x_{(k)}, X_{(k+1)} = x_{(k+1)}, \cdots, X_{(r)} = x_{(r)}) \tag{5.1}$$

$$= \frac{f(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)}|\theta)g(\theta)}{\int_\Theta f(x_{(k)}, x_{(k+1)}, \cdots, x_{(r)}|\theta)g(\theta)d\theta}.$$

If the information in E can be summarized by a sufficient statistic $T(X_{(k)}, X_{(k+1)}, \cdots X_{(r)})$,then we can write a posterior density for a parameter θ

$$h(\theta|T = t) = \frac{f_T(t|\theta)g(\theta)}{\int_\Theta f_T(t|\theta)g(\theta)d\theta} \quad see[32]. \tag{5.2}$$

Suppose we wish to predict a future statistic Y. If T and Y are independent ,and Y has a density function $f(y|\theta)$,then the predictive density function for Y is given by

$$p(y|T = t) = \int_\Theta f(y|\theta)h(\theta|T = t)d\theta \ see[7]. \tag{5.3}$$

A $(1 - \gamma)\%$ Bayesian prediction interval for a future observation Y is then defined as an interval A such that

$$P(Y \in A|T = t) = \int_A p(y|T = t)dy = 1 - \gamma \quad see[7]. \tag{5.4}$$

5-1 Prediction intervals for a future observation

Let $X_1, X_2, \cdots X_n$ be an independent random sample from a two-parameter exponential distribution having a density function

$$f_X(x|\theta,\mu) = \frac{1}{\theta} e^{-\left(\frac{x-\mu}{\theta}\right)},\ x \geq \mu,\ \theta > 0 \tag{5.1.1}$$

and a cumulative distribution function

$$F_X(x) = 1 - e^{-\left(\frac{x-\mu}{\theta}\right)},\ x \geq \mu,\ \theta > 0. \tag{5.1.2}$$

Suppose that the informative experiment E has a doubly type-II censored sample and it can be summarized by a sufficient statistic
$T(X_{(k)}, X_{(k+1)}, \cdots, X_{(r)})$.

$$T = \sum_{i=k+1}^{r} X_{(i)} + (n-r)X_{(r)} - (n-k)X_{(k)} \tag{5.1.3}$$

is a sufficient statistic for the parameter θ ,and the distribution of T from (2.2.27) is a gammma distribution.

$$f_T(t|\theta) = \frac{1}{\Gamma(r-k)\theta^{r-k}} t^{r-k-1} e^{-\frac{t}{\theta}}. \tag{5.1.4}$$

We assume a two-parameter gamma prior density function for θ

$$g(\theta|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} (\frac{1}{\theta})^{\alpha+1} e^{-\frac{\beta}{\theta}},\ \theta > 0,\ ,\ \alpha > 0,\ \beta > 0. \tag{5.1.5}$$

Then the posterior density for θ is given by

$$h(\theta|T=t) = \frac{f_T(t|\theta)g(\theta|\alpha,\beta)}{\int_0^\infty f_T(t|\theta)g(\theta|\alpha,\beta)d\theta} \tag{5.1.6}$$

$$h(\theta|T=t) = \frac{\frac{1}{\Gamma(r-k)\theta^{r-k}} t^{r-k-1} e^{-\frac{t}{\theta}} \frac{\beta^\alpha}{\Gamma(\alpha)} (\frac{1}{\theta})^{\alpha+1} e^{-\frac{\beta}{\theta}}}{\int_0^\infty \frac{1}{\Gamma(r-k)\theta^{r-k}} t^{r-k-1} e^{-\frac{t}{\theta}} \frac{\beta^\alpha}{\Gamma(\alpha)} (\frac{1}{\theta})^{\alpha+1} e^{-\frac{\beta}{\theta}} d\theta}. \tag{5.1.7}$$

Then to obtain the integral

$$\int_0^\infty \frac{\beta^\alpha}{\Gamma(r-k)\Gamma(\alpha)}(\frac{1}{\theta})^{\alpha+r-k+1}t^{r-k-1}e^{-(\frac{t+\beta}{\theta})}d\theta \tag{5.1.8}$$

put $U = \frac{T+\beta}{\theta}$.Then we obtain that the integral is

$$= \frac{\beta^\alpha t^{r-k-1}}{\Gamma(r-k)\Gamma(\alpha)}\int_0^\infty \left(\frac{u}{t+\beta}\right)^{\alpha+r-k+1} e^{-u}(\frac{t+\beta}{u^2})du \tag{5.1.9}$$

$$= \frac{\beta^\alpha t^{r-k-1}}{\Gamma(r-k)\Gamma(\alpha)(t+\beta)^{\alpha+r-k}}\int_0^\infty u^{\alpha+r-k-1}e^{-u}du \tag{5.1.10}$$

$$= \frac{\beta^\alpha t^{r-k-1}\Gamma(\alpha+r-k)}{\Gamma(r-k-1)(t+\beta)^{\alpha+r-k}\Gamma(\alpha)}.$$

Then the posterior density function is given by

$$h(\theta|T=t) = \frac{(t+\beta)^{\alpha+r-k}}{\Gamma(\alpha+r-k)}(\frac{1}{\theta})^{\alpha+r-k+1}e^{-(\frac{t+\beta}{\theta})} \tag{5.1.11}$$

$$h(\theta|T=t) = \frac{K^R}{\Gamma(R)}\left(\frac{1}{\theta}\right)^{R+1} e^{-\frac{K}{\theta}} \tag{5.1.12}$$

If the condition is T=t, then

$$R = \alpha + r - k, \quad K = T + \beta. \tag{5.1.13}$$

Let $Y = X_{(s)} - X_{(r)}$ .Then Y and T are independent and the predictive density function for Y is given by

$$p(y|T=t) = \int_0^\infty f(y|\theta)h(\theta|T=t)d\theta. \tag{5.1.14}$$

The density function of the future observation Y given from (2.2.14) is

$$f(y|\theta) = \frac{1}{\beta(s-r, n-s+1)\theta}\left(e^{-(\frac{y}{\theta})}\right)^{n-s+1}\left(1-e^{-(\frac{y}{\theta})}\right)^{s-r-1}. \tag{5.1.15}$$

Then the predictive density function of a future observation Y is given by

$$p(y|T=t) = \left(\frac{K^R}{\Gamma(R)\beta(s-r,n-s+1)}\right)\int_0^\infty \left(\frac{1}{\theta}\right)^{R+1} e^{-\frac{K}{\theta}} \tag{5.1.16}$$

$$\times\frac{1}{\theta}\left(e^{-(\frac{y}{\theta})}\right)^{n-s+1}\left(1-e^{-(\frac{y}{\theta})}\right)^{s-r-1}d\theta$$

(5.1.17)

$$p(y|T=t) = \left(\frac{K^R}{\Gamma(R)\beta(s-r,n-s+1)}\right)\int_0^\infty \sum_{i=0}^{s-r-1}(-1)^i\begin{pmatrix} s-r-1 \\ i \end{pmatrix}$$

$$\times\left(\frac{1}{\theta}\right)^{R+2} e^{-\left(\frac{(n-s+1+i)y+K)}{\theta}\right)}d\theta.$$

$$p(y|T=t) = \left(\frac{RK^R}{\beta(s-r,n-s+1)}\right)\sum_{i=0}^{s-r-1}(-1)^i\begin{pmatrix} s-r-1 \\ i \end{pmatrix} \tag{5.1.18}$$

$$\times\frac{1}{((n-s+i+1)y+K)^{R+1}}.$$

Then a $(1-\gamma)\%$ Bayesian prediction interval for a future observation Y is then defined as an interval A such that

$$P(Y\in A|T=t) = \int_A p(y|T=t)dy = 1-\gamma \tag{5.1.19}$$

where A is an interval (a,b) .
If we consider equal tail limits, then

$$P(Y\leq a|T=t) = \frac{\gamma}{2} \tag{5.1.20}$$

and

$$P(Y\geq b|T=t) = \frac{\gamma}{2}. \tag{5.1.21}$$

Then the prediction interval for Y is the interval (a,b),which a is determined by

(5.1.22)

$$P(Y \leq a|T=t) = \int_0^a \left(\frac{RK^R}{\beta(s-r,n-s+1)}\right) \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix}$$

$$\times \frac{1}{((n-s+i+1)y+K)^{R+1}} dy = \frac{\gamma}{2}$$

(5.1.23)

$$P(Y \leq a|T=t) = \frac{K^R}{\beta(s-r,n-s+1)} \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix}$$

$$\times \left(\frac{1}{(n-s+i+1)K^R} - \frac{1}{(n-s+i+1)\left((n-s+i+1)(a+K)^R\right.}\right) = \frac{\gamma}{2}.$$

b is determined by

(5.1.24)

$$P(Y \geq b|T=t) = \int_b^\infty \left(\frac{RK^R}{\beta(s-r,n-s+1)}\right) \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix}$$

$$\times \frac{1}{((n-s+i+1)y+K)^{R+1}} dy = \frac{\gamma}{2}$$

(5.1.25)

$$P(Y \geq b|T=t) = \frac{K^R}{\beta(s-r,n-s+1)} \sum_{i=0}^{s-r-1} (-1)^i \begin{pmatrix} s-r-1 \\ i \end{pmatrix}$$

$$\times \frac{1}{(n-s+i+1)\left((n-s+i+1)b+K\right)^R} = \frac{\gamma}{2}.$$

Then for given values of k,r,s,n,α,β and γ the probabilities

$$P(Y \leq a|T=t) = \frac{\gamma}{2}$$

and

$$P(Y \geq b|T=t) = \frac{\gamma}{2}$$

can be evulated using a Newton-Raphson method.
We have proved:
Theorem 9:
The Bayes prediction interval for a future observation $Y = (X_{(s)} - X_{(r)})$ under a complete sufficient statistic of observed observations T, and a given prior density of the parameter is a two-parameter gamma distribution, the prediction interval is given by A=(a,b) ,where a and b are determined from (5.1.23) and (5.1.25)

5-2 Special cases

(i) Prediction intervals for a smallest future observation

In this case s=r+1. From (5.1.27) and (5.1.28) we obtain
(5.2.1)

$$P(Y \leq a|T = t) = \frac{K^R}{\beta(1, n-r)} \left(\frac{1}{(n-r)K^R} - \frac{1}{(n-r)\left((n-r)a + K\right)^R} \right) = \frac{\gamma}{2}$$

$$P(Y \leq a|T = t) = \left(1 - \frac{K^R}{\left((n-r)a + K\right)^R} \right) = \frac{\gamma}{2} \tag{5.2.2}$$

$$\left(\frac{K}{(n-r)a + K} \right)^R = 1 - \frac{\gamma}{2} \tag{5.2.3}$$

$$\left(\frac{K}{(n-r)a + K} \right) = \left(1 - \frac{\gamma}{2} \right)^{\frac{1}{R}} \tag{5.2.4}$$

$$1 + \frac{(n-r)a}{K} = \left(1 - \frac{\gamma}{2} \right)^{-\frac{1}{R}}. \tag{5.2.5}$$

Then

$$a = \frac{K}{n-r} \left((1 - \frac{\gamma}{2})^{-\frac{1}{R}} - 1 \right). \tag{5.2.6}$$

And

$$P(Y \geq b|T = t) = \frac{K^R}{\beta(1, n-r)(n-r)\left((n-r)b + K\right)^R} = \frac{\gamma}{2} \tag{5.2.7}$$

$$\frac{K^R}{((n-r)b+K)^R} = \frac{\gamma}{2} \tag{5.2.8}$$

$$\left(\frac{K}{(n-r)b+K}\right)^R = \frac{\gamma}{2} \tag{5.2.9}$$

$$\frac{K}{(n-r)b+K} = (\frac{\gamma}{2})^{\frac{1}{R}} \tag{5.2.10}$$

Then

$$b = \frac{K}{n-r}\left((\frac{\gamma}{2})^{-\frac{1}{R}} - 1\right) \tag{5.2.11}$$

(ii) Prediction intervals for the largest future observation

In this case s=n ,we wish to give a prediction intervals for a largest future observation Y from (5.1.27) and (5.1.28) we find

$$P(Y \leq a) = \frac{K^R}{\beta(n-r,1)} \sum_{i=0}^{n-r-1} (-1)^i \begin{pmatrix} n-r-1 \\ i \end{pmatrix} \tag{5.2.12}$$

$$\times \left(\frac{1}{(i+1)K^R} - \frac{1}{(i+1)\left((i+1)a+K\right)^R}\right) = \frac{\gamma}{2}$$

and

$$P(Y \geq b) = \frac{K^R}{\beta(n-r,1)} \sum_{i=0}^{n-r-1} (-1)^i \begin{pmatrix} n-r-1 \\ i \end{pmatrix} \tag{5.2.13}$$

$$\times \frac{1}{(i+1)\left((i+1)b+K\right)^R} = \frac{\gamma}{2}$$

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Lebenslauf

Am 31.Dezember 1963 wurde ich in Al-Shaghab, Kena, Ägypten geboren. Am Oktober 1970 besuchte ich in der Al-Shaghab Grunschule. Am Oktober 1980 besuchte ich in der Luxor Gymnasium.Ich begann das Studium der Mathematik am Oktober 1982 in faculty of Science ,Department of Mathematics, South valley University in Kena. Am Mai 1986 habe ich Bachelor of Science in Mathematics erworben. Seit dem März 1988 bin ich Wissenschaflicher Assistant in Department of Mathematics in faculty of Science in Kena. Am Mai 1994 habe ich Master degree of Science in Mathematics erworben. Seit dem Oktober 1996 arbeite ich Promotionsarbeit unter Anleitung von Herrn Prof. Krengel im Institut für Mathematische Stochastik der Universität Göttingen. Meine Staatsangehörigkeit ist ägyptisch.

Adel M.M.Omar