Mechanisms in the Core of a Fee Game

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Abstract: A ***Fee Game*** is a cooperative game with incomplete information the *ex post* realizations of which show side payment character. The game appears in coalitional function form depending on the 'types' of the players which are randomly chosen and about which the players have private information. We specify incentive compatible mechanisms and show that with a natural condition the core of the game is not empty: it contains constant mechanisms.

1 Introduction

This paper deals with cooperative games and mechanism design. Incomplete information (or imperfect information) is a quite common subject within the framework of non-cooperative game theory. With cooperative game theory (the coalitional function) incomplete information was first introduced by Harsanyi-Selten (1972), however, they did not discuss incentive compatible mechanisms. Hence, in their framework, which is essentially the one of Nash bargaining, players would have an incentive to misrepresent their type. Myerson (1984) introduced Bayesian incentive compatible mechanisms within this framework; he also discussed a version of the Nash Bargaining solution and a version of the Shapley value in the context of a general coalitional form with incomplete information.

In Myerson's context players have finitely many decisions to agree upon but are permitted to randomize. As a consequence the coalitional function which is obtained by considering for each coalition the utilities available by joint (correlated) randomization is of NTU-type such that the values are polyhedral sets. Mechanisms, therefore, are mappings from types into joint distributions over the decisions. Again, if one considers the utilities available to a coalition by the application of such mechanisms, one obtains polyhedral sets.

By contrast the model we prefer admits for continuum of utility vectors to be available to each coalition, this is the familiar framework of NTU-games. However, the information available to players about the NTU-game at hand may be of private nature and governed by an *a priori* probability which is common knowledge to all the players.

Thus, the coalitional function depends on certain states of nature and the players observe different aspects of this state of nature. Following the tradition established by Harsanyi, we model the states of nature as a product of observations available to the different players. The relevant part of a state of nature which can be observed of a player is traditionally called his *type*. In order to introduce an example, let us think of a game with three players, two of which would like to cooperate in an economic enterprise in a foreign country. They would have to register a contract concerning this venture with a court. It is not uncommon

that the court will require a fee for the registration which may depend on the total worth of the contract. Also, each player may have expenses with respect to consulting experts on legal procedures and taxes in the foreign country; the actual amounts of these expenses are possibly not verifiable, hence part of this is private information of the players.However, they will specifically announce these expenses in the contract and the court will have to regard all these data, public and private information as well. We could introduce a third player representing a bank which has good connections to the foreign country or actually sustains a branch in that country. This player may have additional information which he may disclose to all parties involved including the court.

Obviously, the two players mentioned first will have to consider the benefits of cooperating with the banker, in view of the fact that everyone has private information which he may have an incentive to misrepresent. In addition the court which may be called upon to enforce the contract, should be aware of possible misrepresentation; most specifically the court should have a strong interest in registering incentive compatible mechanisms only such that players are induced to report the true type within the contract.

Methodically, this problem is basically one of cooperative game theory, but the introduction of mechanisms calls for incentive compatibility. Eventually the players will have to agree upon some contract; in cooperative game theory one should ask for a suitable definition of the *core.*

We will provide a definition of the core of a game with incomplete information and show that it is not empty given certain conditions, e.g. on every *ex post* cooperative game.

We would like to add a short hint concerning general equilibrium theory. In this context incomplete information is e.g. discussed by Vohra (1997)(see also Allen (1991)). As is frequently observed, it turns out that the core might be empty even if we have nice standard conditions concerning the exchange economy in question. It would be nice to somehow connect the two models since with complete information the core of the market game to be derived from the exchange economy is a close relative of the core of the economy.

We start out by specifying the model.

Definition 1.1 *An n-person **cooperative game** with incomplete information (a **C.I.I.game**) is a set of data*

$$\Gamma = (I, T; p; \underline{\mathbf{X}}; U) \tag{1}$$

including the following ingredients (that are interpreted accordingly).

1. $I = \{1, ..., n\}$ *is the set of **players**;*

2. $T = T^1 \times ... \times T^n$ *is the cartesian product of finite sets T_i $(i \in I)$, T_i is the set of **types** of player i.*

3. $p > 0$ *is a **probability** on T.*

4. *Next*

$$\underline{\mathbf{X}} = \{x \in I\!\!R_+^n \mid \sum_{i=1}^n x_i \leq 1\} \tag{2}$$

is the set of possible **contracts**, x_i is the share of player i at a contract $x = (x_1, ..., x_n)$ (they could distribute less than the full value available). In particular, if a coalition

$$S \in \underline{\underline{P}} := \{T \subseteq \Omega\}$$

agrees to cooperate by contract, then they register some $x \in \underline{\mathbf{X}}_S$ with

$$\underline{\mathbf{X}}_S = \{x \in I\!R^n_{S+} \mid \sum_{i \in S} x_i \le 1\}.$$

We assume that, technically, $I\!R^n_S$ is imbedded in $I\!R^n$ by projection (though the zero coordinate assigned to players not in S are not interpreted). Moreover, T_S denotes the cartesian product $\prod_{i \in S} T_i$.

5. Finally, U represents the family of **utility functions.** In general, the utility of player i may depend on the coalition he is joining, thus, the family U may be written as

$$(U_i^{S,t})_{i \in I, S \in \underline{\underline{P}}, t \in T_S} \tag{3}$$

where for some $t \in T_S := \prod_{i \in S} T_i$, the utility function

$$U_i^{S,t} : \underline{\mathbf{X}}_S \longrightarrow I\!R$$

of player i is defined on contracts of S, i.e., contracts $x \in \overline{\mathbf{X}}_S$ that distribute a share of what can be achieved by the members of S.

This way we have finished the description of the general (N.T.U.) game with incomplete information.

In case of complete information or "ex post", i.e., if some $t \in T$ is known to the players, a C.I.I.-game Γ results in a traditional NTU game which is specified by (ex post) feasible sets of utilities for each coalition, i.e. for every realisation $t \in T$ of the types we have an *ex post* game given by

$$V^t(S) = \{(U_i^{S,t|S}(x))_{i \in S} \mid x \in \underline{\mathbf{X}}_S\} \quad (S \in \underline{\underline{P}}) \tag{4}$$

However, some geometric properties of the feasible sets (convexity, comprehensiveness) should be ensured by proper assumptions concerning the functions $U_i^{S,t}$. We would like to consider a class of functions which generates "side payment games", but also includes the definite influence of private information. The resulting game will be called a **fee game.**

To this end, we start out with the traditional side payment or T.U. concept of game theory, the characteristic or **coalitional function**. That is, we specify a mapping,

$$v : \underline{\underline{P}} \longrightarrow I\!R, \quad v(\emptyset) = 0 \tag{5}$$

to be interpreted as a 'side payment game'. Assuming a universally accepted utility scale *in the presence of full information*, $v(S)$ is to be seen as a monetary value coalition S can in principle obtain by cooperation. However, since players cannot

observe all random influence, the establishment and enforcement of contracts is a difficult procedure which is to be supervised by some powerful agency, the referee or rather the **court.**

Therefore, we assume that there is a system of fees or a taxation rule (a **fee schedule**) represented by a set of vectors

$$b = (b^{S,t})_{S \in \underline{\mathbf{P}}, t \in T_S} , \quad b^{S,t} \in I\!\!R_S^n. \tag{6}$$

The meaning is that, if a contract $x \in \underline{\mathbf{X}}$ of coalition S is registered with the court, then player $i \in S$ is (legally) required to pay a certain fee or an amount of taxes proportional to the total of this contract.

To be more precise, if we write $e = (1, ..., 1) \in I\!\!R_+^n$ and

$$e_S = (0, \ldots, 1, \ldots, 1, \ldots, 0, \ldots, 0) \in I\!\!R_S^n$$

then the total of the contract $x \in \overline{\mathbf{X}}_S$ is

$$ex = e_S x = \sum_{i \in S} x_i \tag{7}$$

and the proportionality factor is $b_i^{S,t}$ for player i if the types of the players are given by $t \in T$ and cooperation takes place in $S \in \underline{\mathbf{P}}$.

The (preliminary, naive) rules of the game are described as follows: a coalition $S \in \underline{\mathbf{P}}$ may register a contract $x \in \overline{\mathbf{X}}_S$. Then by cooperation (which can be enforced by the registering agency, the court), they may aquire a monetary value $v(S)$. However, player i is required to pay a fee towards the court, thus his utility resulting from x is

$$U_i^{S,t}(x) = v(S)(x_i - (e_S x) b_i^{S,t}). \tag{8}$$

Concentrating our above remarks we come up with

Definition 1.2 *A C.I.I. game Γ is said to be a **fee game** if there is a coalitional function v and a fee schedule b (see (5) and (6)) such that the utility functions collected in U are given by (8).*

In this context, $x \in \overline{\mathbf{X}}_S$ achieves a certain monetarian character, thus normalization may become questionable. But as the fee schedule is assumed to be linear, we may accept this concession for the moment. Note that the term 'fee' implicitly suggests that the players indeed have to pay something (hence $b_{\bullet}^{\bullet}$ is nonnegative) and that, for some fixed contract, the utility of a player is nonnegative (hence $b_{\bullet}^{\bullet}$ does not exceed 1.) That is, we will assume that

$$0 \leq b_{\bullet}^{\bullet} \leq 1 \tag{9}$$

holds true.

Let us again shortly consider the "ex post" situation, i.e., the state of the world in which some $t \in T$ is commonly known.

In this case, the N.T.U. game (ex post) suggested by (4) has obviously side payment character (each feasible set is bounded by a hyperplane), hence we may as well introduce the "ex post T.U. game" given for fixed $t \in T$ by

$$v^t : \underline{\mathbf{P}} \longrightarrow I\!\!R, \quad v^t(S) = v(S)(1 - b^{S,t|_S}(S)) \quad (S \in \underline{\mathbf{P}}). \tag{10}$$

Clearly, $v^t(S) = e_S U^{S,t}(x)$ whenever x is Pareto efficient in $v^t(S)$.

2 Incentive Compatible Mechanisms

Let us now change the story so as to incorporate incomplete information. In this scenario, the types are chosen at random, the distribution is given by p. Player i observes the realisation of his own type t_i only. However, p and all other data are common knowledge; thus it makes sense that player i computes conditional expectations of his data given he observes his type. As a consequence, we have to enhance the set-up by the introducing an abstract probability space, say $(\Omega, \underline{\mathbf{F}}, P)$ together with a random variable $\tau : \Omega \longrightarrow T$ which selects types at random. The distribution of τ is given by p, i.e., we have

$$p(\cdot) = \tau P(\cdot) = P \circ \tau^{-1}(\cdot), \quad \text{i.e.,} \quad P(\tau = t) = p(\{t\}) =: p_t \quad (t \in T). \tag{11}$$

As in most probabilistic models in Statistics or Economics it is easily seen that all data depending on chance can be computed by means of p. That is, it suffices to have knowledge of the distribution of τ in order to compute (conditional) expectations or to decide (later on) whether a mechanism is incentive compatible (Definition 2.2).

In view of this set up, coalition S may bargain about some contract $x \in \underline{\mathbf{X}}_S$, we assume that this is done "ex ante", i.e., before the types are being observed. The general agreement the coalition finally wants to register will include the disclosure of types, thus it will be contingent with types $t \in T_S = \prod_{i \in S} T_i$. Such kind of agreement causes the players to behave strategically. Generally, the contingent set of contracts is called a ***mechanism***.

Definition 2.1 *A **mechanism** for coalition S is a mapping $\mu^S : T_S \longrightarrow \underline{\mathbf{X}}_S$.*

The interpretation is clearly that player $i \in S$, having observed his "type" $t_i \in T_i$, announces his observation, upon which $\mu^S(t_S) = \mu^{S,t_S}$ with $t_S = (t_i)_{i \in S}$ is executed by the court.

In view of this, the players may compute their expected utility conditioned on their observation, this quantity for $i \in S$ is computed by

$$\hat{U}_i^S(\mu, t_i) = E\left(U_i^{S,\tau|_S} \circ \mu^{S,\tau|_S} \;\middle|\; \tau_i = t_i \right) \tag{12}$$

If Γ is a fee game, then (12) may be specified to

$$\hat{U}_i^S(\mu, t_i) = v(S) E\left(\mu_i^{S,\tau|_S} - (e_S \mu^{S,\tau|_S}) b_i^{S,\tau|_S} \;\middle|\; \tau_i = t_i \right) \tag{13}$$

Now, in view of an enforcable mechanism μ or μ^S, player i develops strategic behavior, he may have an incentive to misrepresent his type. If he observes $t_i \in T_i$ but announces $s_i \in T_i$, then his utility is given by

$$\hat{U}_i^S(\mu, t_i; s_i) = E\left(U^{S,(\tau^{-i}|_S, t_i)} \circ \mu^{S,(\tau^{-i}|_S, s_i)} \;\middle|\; \tau_i = t_i \right). \tag{14}$$

The notation $t = (t^{-i}, t_i)$ for splitting up a vector for all players in $i's$ coordinate and the coordinates of all other players will be used throughout. This way we are let to introduce

Definition 2.2 *Let μ^S be a mechanism for coalition $S \in \underline{\mathbf{P}}$. μ^S is **incentive compatible** (for short **IC**) if, for all $i \in I$ and all $s_i, t_i \in T_i$, it follows that*

$$\hat{U}_i^S(\mu, t_i) \geq \hat{U}_i^S(\mu, t_i; s_i) \tag{15}$$

holds true. That is, given that all other players represent their types truthfully, player i cannot improve his payoff by misrepresenting his type.

At this instant a familiar remark is in order. Given a mechanism $\mu^S : T_S \longrightarrow \underline{\mathbf{X}}_S$, players will behave strategically and ponder about the announcement of their type once they observe its true value. That is, there arises a noncooperative game, say $\Gamma^{\mu,S}$. Within this game, the strategies for player i are 'observation dependent announcements', i.e., mappings $\sigma_i : T_i \longrightarrow T_i$, $(i \in S)$ resulting in $S-$tupels $\sigma_S : T_S \longrightarrow T_S$ and in payoffs given by

$$EU_i^{S,\tau|_S} \circ \mu^{S,\sigma_S \circ \tau_S}$$

For μ^S to be incentive compatible means that the strategy 'telling the truth' (i.e., the identity in T_i) for each player $i \in I$ constitutes a Nash eauilibrium in $\Gamma^{\mu,S}$. Now let

$$\mathcal{M}_S = \left\{ \mu^S \mid \mu^S : T_S \longrightarrow \overline{\mathbf{X}}_S, \mu \; is IC \right\} \tag{16}$$

the set of IC mechanisms feasible for coalition S. When S commences bargaining *ex ante* , then the utily vectors available from the formation of agreement concerning IC mechanisms are given by

$$V^{\mathcal{M}}(S) = \left(\left(E\left(U_i^{S,\tau|_S} \circ \mu^{S,\tau|_S} \right) \right)_{i \in S} \mid \mu \in \mathcal{M}_S \right). \tag{17}$$

This way we have constructed a mapping

$$V^{\mathcal{M}} : \underline{\mathbf{P}} \longrightarrow I\!\!R_S^n \tag{18}$$

which apparently has the character of a cooperative game. However, should we assume that Γ is a fee game and hence represents side-payment character, then nevertheless in general $V^{\mathcal{M}}$ will be an *NTU game*. This is an important observation: in view of incomplete information and the necessary passage to IC mechanism the TU property of a cooperative game is subdued.

The geometric shape of the utility sets involved in $V^{\mathcal{M}}$ is cleared by the following theorem, which is traditional in other contexts (see Myersom (1984)and also Rosenmüller (1992)) :

Theorem 2.3 *Let Γ be a fee game. Then $\mathcal{M}_S$ and $V^{\mathcal{M}}$ are nonempty, closed polyhedra. If one restricts the definitions to nonnegative mechanisms ($\mathcal{M}_{S+}$), then the corresponding quantities are nonempty, compact polyhedra.*

Proof: Since constant mechanisms are IC, the quantities involved are nonempty sets. The natural linear structure of the set $\{\mu \mid \mu : T_S \longrightarrow \overline{\mathbf{X}}\} = \overline{\mathbf{X}}_S^{T_S}$ is the one of $I\!\!R^{T_S}$. With regards to this structure, the inequalities listed in equations (15)

are linear ones as the coordinates of μ appear only linearly. This is a consequence of the fact that within (15) we just form conditional expectations. Therefore, $\mathcal{M}_S$ is the set of solutions of a System of linear inequalities, hence convex and closed.

Next, if we write $U^{S,t}(x) = (U_i^{S,t}(x))_{i \in S} = (v(S)(x - (e_S x)b_i^{S,t}))_{i \in S}$, then the linear nature of U ensures that, for $\mu, \nu \in \mathcal{M}_S$, we have

$$\frac{1}{2} E \left(U^{S,\tau} \circ \mu^{S,\tau} \right) + \frac{1}{2} E \left(U^{S,\tau} \circ \nu^{S,\tau} \right) = E \left(U^{S,\tau} \circ \frac{\nu^{S,\tau} + \mu^{S,\tau}}{2} \right), \quad (19)$$

and $\frac{\nu^{S,\bullet} + \mu^{S,\bullet}}{2}$ is an element of $\mathcal{M}_S$ in view of the first step of our proof. Similarly, the fact that $V^{\mathcal{M}}$ is closed followes from the continuity of the mappings involved. **qed.**

3   The Core of a Fee-Game

Within this section we speak about fee games only, nevertheless some definitions apply for a more general class as well.

Since $V^{\mathcal{M}}$ has the character of an NTU game the **core** of the game, written $\mathcal{C}(V^{\mathcal{M}})$ is well defined. We may, however, discuss matters in terms of mechanisms instead of utilities, for this case we present a formal definition:

Definition 3.1   *1. We shall say that a mechanism $\mu \in \mathcal{M} = \mathcal{M}_I$ is **dominated** by a mechanism $\mu^S \in \mathcal{M}_S$ and write $\mu^S \; dom_S \; \mu$, if*

$$E \left(U_i^{S,\tau|_S} \circ \mu^{S,\tau|_S} \right) \geq E \left(U_i^{S,\tau} \circ \mu^\tau \right) \qquad (20)$$

holds true for all $i \in S$ with a strict inequality for at least one $i \in S$.

*2. The **core** of a game Γ is given by*

$$\mathcal{C}(\Gamma) := \left\{ \mu \in \mathcal{M} \mid \nexists S \in \underline{\mathbf{P}}, \mu^S \in \mathcal{M}_S \; : \; \mu^S \; dom_S \; \mu. \right\} \qquad (21)$$

A well known condition for the core (of $V^{\mathcal{M}}$) to be nonempty is that the game is balanced (cf. Shapley (1973)). But this is an *ad hoc* condition as far as our present context is concerned, since we do not have the slightest idea concerning such properties of $V^{\mathcal{M}}$.

The general discussion of the core might be a formidable task, presently we shall restrict ourselves to the question as to whether there exist *constant* mechanisms in the core. Since there is so much linearity in the model it may then be useful to consider the 'expected' games (TU and NTU) which the players are facing *ex ante* when restricted to constant mechanisms. The TU version is given by

$$\begin{aligned} \bar{v} : \; &= \; Ev^\tau : \; \underline{\mathbf{P}} \longrightarrow I\!R \\ \bar{v}(S) \; &= \; v(S)(1 - Eb^{S,\tau|_S}(S)) \;\; (S \in \underline{\mathbf{P}}). \end{aligned} \qquad (22)$$

This game may or may not have a nonempty core according to whether it is balanced or not. Naturally, we call this the **ex ante core** (of Γ or of $\bar{v}$,) the definition is given by

$$\mathcal{C}(\bar{v}) := \{u \in I\!\!R^n \mid u(S) \geq \bar{v}(S) (S \in \underline{\mathbf{P}}.)\} \tag{23}$$

In any case the existence of a nonempty core for $\bar{v}$ is established by familiar conditions. As a first and most simple step we should, therefore, discuss the question as to whether from this we can derive some first clue concerning the existence of a nonempty core of Γ.

Now, those utility vectors $\bar{u} \in I\!\!R^n$ which are feasible distributions of utility for the grand coalition with respect to $\bar{v}(I)$ my be generated by *constant* mechanisms $\bar{x} \in \overline{\mathbf{X}} = \overline{\mathbf{X}}_I$ via

$$\bar{u} = v(I)(\bar{x} - Eb_\bullet^{I,\tau}) \tag{24}$$

On the other hand, if we are given a utility distribution $\bar{u}$ we may *construct* a candidate for a contract by putting

$$\bar{x} := \frac{\bar{u}}{v(I)} + Eb_\bullet^{I,\tau}. \tag{25}$$

Now we have

Theorem 3.2 *Let $\bar{u} \in \mathcal{C}(\bar{v})$ be an element of the ex ante core and let $\bar{x}$ be generated by equation (25). Then $\bar{x}$ constitutes an IC–contract for the grand coalition and*

$$\bar{x} \in \mathcal{C}(\Gamma) \tag{26}$$

holds true.

Proof: First of all, $\bar{x}$ is immediately seen to be a contract for the grand coalition by summing over the cordinates, using (24), and observing that $\bar{u}(I) = \bar{v}(I)$ follows from $\bar{u} \in \mathcal{C}(\bar{v})$. Incentive compatibility is clear since we are dealing with a constant mechanism. It remains to be shown that the mechanism is located within the core of Γ.

Define the utility for player i *in mediis* , when $\bar{x}$ is applied, to be

$$\begin{aligned} \bar{u}_i^{t_i} : &= v(I)E(\bar{x}_i - b_i^{I,\tau} \mid \tau_i = t_i) \\ &= v(I)(\bar{x}_i - Eb_i^{I,\tau} \mid \tau_i = t_i). \end{aligned} \tag{27}$$

Assume now *per absurdum* that $\bar{x} \in \mathcal{C}(v)$ does *not* hold true. Then there is a coalition S and a mechanism μ suitable for S such that

$$\bar{u}_i^{t_i} \leq \tilde{u}_i^{t_i} := v(S)E\left(\mu_i^{S,\tau|S} - (e_S\mu_i^{S,\tau|S})b_i^{S,\tau|S} \mid \tau_i = t_i\right) (i \in S) \tag{28}$$

holds true with the additional understanding, that for at least one $i \in S$, there is a strict inequality in (28). Define now $(\tilde{x}^s)_{s \in T_S}$ to be a set of contracts feasible for S such that the following two conditions are satisfied:

$$\tilde{x}^s \text{ is Pareto efficient for } S \text{ for every} s \in T_S, \tag{29}$$

$$\mu^{S,s} - (e_S \mu^{S,s}) b^{S,s} \leq \tilde{x}^s - b^{S,s} \quad (s \in T_S), \tag{30}$$

this is possible for every $s \in T_S$ in view of our assumption ensuring that $b^{S,s} \leq e_S$ holds true (see (9)) Of course, the set $(\tilde{x}^s)_{s \in T_S}$ does not necessarily constitute an IC–mechanism, however it could be called a mechanism. Consider now the expected utility *in mediis* for player $i \in S$ with respect to this mechanism, given that he observes some $t_i \in T_i$, this is given by

$$\bar{\bar{u}}_i^{t_i} := v(S) E \left(\tilde{x}_i^{\tau|S} - (e_S \tilde{x}^{\tau|S}) b_i^{S,\tau|S} \mid \tau_i = t_i \right) \geq \tilde{u}_i^{t_i} \geq \bar{u}_i^{t_i}, \tag{31}$$

where again at least one inequality is strict. We may, therefore, derive the following chain of equations and inequalities:

$$
\begin{aligned}
\bar{u}(S) &= \sum_{i \in S} \bar{u}_i \\
&= \sum_{i \in S} \sum_{t_i \in T_i} \bar{u}_i^{t_i} P(\tau_i = t_i) \\
&< \sum_{i \in S} \sum_{t_i \in T_i} \bar{\bar{u}}_i^{t_i} P(\tau_i = t_i) \\
&= v(S) \sum_{i \in S} \sum_{t_i \in T_i} E \left(\tilde{x}_i^{\tau|S} - (e_S \tilde{x}^{\tau|S}) b_i^{S,\tau|S} \mid \tau_i = t_i \right) P(\tau_i = t_i) \\
&= v(S) \sum_{i \in S} E \left(\tilde{x}_i^{\tau|S} - (e_S \tilde{x}^{\tau|S}) b_i^{S,\tau|S} \right) \\
&= v(S) \sum_{i \in S} \left(E \left(\tilde{x}_i^{\tau|S} \right) - 1 \, E \left(b_i^{S,\tau|S} \right) \right) \\
&= v(S) \left(1 - E \left(b^{S,\tau|S}(S) \right) \right) \\
&= \bar{v}(S).
\end{aligned}
\tag{32}
$$

This however is a contradiction to our assumption according to which $\bar{u}$ is an element of the core of $\bar{v}$, **qed.**

Corollary 3.3 *Let Γ be a fee game. Suppose that every ex post game v^t (cf. (10)) has a nonempty core. Then the core of Γ is nonempty.*

The **Proof** is easy since, with a slight abuse of notation one can verify at once that

$$\mathcal{C}(\bar{v}) = \mathcal{C}(E(v^\tau)) = E(\mathcal{C}(v^\tau)) \neq \emptyset \tag{33}$$

holds true.

The condition of Corollary 3.3 is not an unnatural one and cannot be called *ad hoc*. The *ex post* game v^t as defined in (10) reflects the impact of the taxation enforcement induced by the court, as compared to the situation with full information and no fee being required which is represented by the coalitional function (5). Since taxation decreases the payoff to the players, the existence of a nonempty core may be destroyed, this is exactly seen by inspecting the two games in question.

Nevertheless, the more difficult question (and the more rewarding one) will arise from studying nonconstant mechanisms in the core, computing their extreme points and relating these results to exchange economies.

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