Fair-Negotiation Procedures

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Abstract

In this paper, we reformulate three fair-division algorithms as practical, cooperative negotiation procedures that can be applied to a more general class of multiple-issue negotiations. The procedures consist of two simple and intuitive steps: The first step ensures an efficient outcome, and the second step establishes "fairness" through a redistribution of gains. For all procedures we also compare the outcomes of 'issue-by-issue' negotiations and 'package deals.'

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JEL Classification: C71, C78, D63, D74

1. Introduction

In contrast to 'normative' mathematical models of bargaining or 'descriptive' analyses of actual negotiations, *negotiation analysis* is a practically oriented field of research that can be characterized as 'prescriptive,' meaning that the objective is to give procedural advice on how negotiators can reach a mutually beneficial outcome.[1] However, despite its theoretical orientation, one cannot claim that negotiation analysis is firmly rooted in bargaining or, more generally, game theory.[2]

The practical deficit of 'cooperative' game theory, on the one hand, is that the characterization of the game does not include the 'plan of play.' Axiomatic solution concepts focus on the desirable properties of bargaining solutions, but they do not specify the process that leads to them. This has spurred the development of 'non-cooperative' or strategic bargaining games, on the other hand, as an attempt to model bargaining procedures that lead to or approximate axiomatic solutions. The latter approach is debatable, though, because the characterization of a specific bargaining procedure makes it difficult to derive general implications. Moreover, the strictly non-cooperative behavior of the players must be viewed critically: Even when parties have strongly conflicting interests, it would be a mistake to automatically infer that the negotiators themselves behave non-cooperatively. Indeed, many descriptions of professional political or business negotiations seem to indicate a more cooperative attitude.[3]

According to Brams and Taylor (1996), bargaining theories have proved inapplicable to the settlement of real-life disputes because of their divorce from theories of fair-division. They show that, by viewing negotiation problems as problems of fair-division, one can apply intuitive procedures to a variety of complex conflicts.

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[1]For an overview of the wide range of topics belonging to negotiation analysis, see the book edited by Young (1991).

[2]Sebenius (1992), for example, even takes a strong stand for "a de-emphasis on game-theoretic solution concepts."

[3]In his recent 'lectures,' Raiffa (1996) devotes most of analysis to situations where negotiators are engaged in what he refers to as 'full open truthful exchange.'

The important feature of this approach, however, is not its view of negotiation as a fair-division problem. The relevant aspect is the applicability of fair-division procedures to conflicts in general.[4] A basic requirement is that they are simple enough to follow without a specific training and plausible enough to argue.

In this paper, instead of viewing negotiation problems as fair-division problems, we reformulate fair-division procedures as cooperative negotiation procedures, in order to apply them to general multiple-issue negotiations. A fair-division problem is a special type of negotiation problem, where the negotiated issues are the items to be divided. To force a multiple-issue negotiation problem into this tighter corset thus requires additional simplifying assumptions. As a consequence, a fair-division procedure generally does not acknowledge the full problem.

We focus here on bilateral problems where items or issues are valued by both parties in some common standard of value, e.g. a unit of money. If items can be valued in terms of money, then a fair-division procedure based on a bidding procedure seems intuitive. This concept was developed by Knaster (1946) and Steinhaus (1948). We refer to it here as the "Knaster procedure."[5] With this simple method, items are assigned to the players who value them most, and "fairness" is established through monetary transfers. The resulting division is 'efficient' and for two parties also 'envy-free,' meaning that neither player would want to trade shares with the other. However, due to its rigid transfer mechanism, the outcome is not 'equitable,' meaning that players do not enjoy equal relative gains.[6]

Brams and Taylor (1996) see a major advantage in their procedure "Adjusted Winner," which acknowledges this additional, and apparently relevant criterion of fairness. The algorithm is closely related to the Knaster procedure, but the transfer

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[4]Brams and Taylor (1999) present a variety of qualitatively different examples that illustrate this point.

[5]While Raiffa (1982) refers to the "Steinhaus procedure," Brams and Taylor (1996) argue that the procedure should, in fact, be attributed to Knaster (1946) who formulated it first.

[6]The contrast between 'efficient' and 'envy-free' resource allocations is emphasized by Foley (1967). Note, however, that there is a difference in terminology. Foley (1967) uses the term 'equitable' in order to characterize what we refer to here as an 'envy-free' allocation.

mechanism is specifically designed to establish equitability. In addition, the fair-division does not require money, since the transfer is achieved by shifting items. This feature is extremely useful for fair-division problems, where items cannot be valued in terms of money, or when parties do not have access to extra money for compensation.

However, when side payments are possible, Adjusted Winner may be too restrictive. We, therefore, take the 'best' of both procedures by combining the efficient side-payments of the Knaster procedure with the equitability condition of Adjusted Winner. We acknowledge its individual components by naming this new method of fair division "Adjusted Knaster." By imposing an equitable monetary transfer, Adjusted Knaster implements an equitable outcome that is always as least as good as that of Adjusted Winner.

We generalize the fair-division procedures in order to apply them to multiple-issue, multiple-option negotiations. We formulate the algorithms as intuitive two-step procedures. The first step implements an efficient outcome, and it is the same for all three procedures. They differ in the second step, though, which establishes "fairness" through a redistribution of gains.

We show that the adjustment mechanisms have different procedural implications, when they are used in 'issue-by-issue' negotiations. This form of negotiation has a high practical relevance, since it reduces the complexity and allows negotiators to follow an agenda. Although axiomatic solution concepts do not tell us how to implement a cooperative solution, we find them to be useful for designing cooperative implementation procedures that meet practical requirements.

Our approach in this paper is complementary to that of Raiffa (1996), who shows how the use of computer spread-sheet analysis can provide strong negotiation-analytic support. The important difference of our procedural view here is that it is focused on argumentative support and does not require computer assistance.

In order to illustrate the applicability of the three 'fair-negotiation' procedures, we choose an existing problem taken from Gupta (1989). In this marketing nego-tiation between a manufacturer of multiple products and a large retailer, players

negotiate over six different marketing plans for each of six different products. Thus, the problem features enough complexity in order to assess the potential of the procedures.

The paper is structured as follows: In section 2, we analyze a simple fair-division problem and compare the solutions of Adjusted Winner and the Knaster procedure. As an alternative we introduce "Adjusted Knaster." In section 3, we describe a multi-issue negotiation problem, where both players value options by the same standard of value, viz. money. In section 4, we characterize the three fair-negotiation procedures, and in section 5 we analyze their performance in issue-by-issue negotiations. Practical conclusions of our analysis are given in section 6.

2.   Fair-Division Procedures

Consider the following fair-division problem between two players, M and R: A mutual friend had moved out of town, leaving behind two items to be split between the two of them – a sizable collection of artistic photographs and an oil painting. Player M, an art freak, values both highly at 100 Dollars. Player R sees them mainly as souvenirs, and values the photo collection at 30 Dollars and the painting at 10 Dollars. Both players' subjective valuations of the estate are given in Table 1. The question is: What is a fair division?

	M	R
1. Photo Collection	100	30
2. Oil Painting	100	10
Estate Value	200	40

Table 1: Valuations of an Estate

Player M is indifferent between both items, but player R clearly prefers item 1. If one item is given to each player, then R will presumably ask for item 1, leaving item 2 for player M. In percentage terms, M receives 50% of the estate, while R can

reap 75% for herself.[7] There is no redistribution of items that makes both players better off, and no player alone will want to trade items either. The allocation is thus 'efficient' and 'envy free.'

The question arises, though, whether the distribution procedure should take into account that player M values the photo collection (item 1) more than three times as much as player R who, nevertheless, gets it. Indeed, if the two players were to bid for these items in an auction, both would go to player M.

The idea of a fair-division procedure based on an auction was developed by Knaster (1946) and Steinhaus (1948). We refer to it here as the "Knaster procedure." With the help of our example, we describe the simple algorithm – the individual steps are summarized in Table 2.

		M	R
1. Photo Collection		<u>100</u>	30
2. Oil Painting		<u>100</u>	10
Estate Value	$\hat{v}^x$	200	40
Envy-free Share	$\hat{v}^x/2$	100	20
Received Value	v_T^x	200	0
Excess	$E = 80 =$	100	–20
Share of Excess	$E/2$	40	40
Transfer	t^x	–60	+60
Total Value	v^x	140	60
Percent of Estate		70	150

Table 2: The Knaster Procedure

Players first submit (sealed) monetary bids for each individual item. The sum of each player's bids determines her subjective valuation of the estate, $\hat{v}^x$, $x = M, R$, and, thereby, her envy-free (or fair) share of 50%, i.e. $\hat{v}^x/2$. In our example, player

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[7]Note that it is the different valuation of items that lets both players' shares add up to more than 1.

M expects to receive a share of the estate worth at least 100, while player R would surely complain about a share worth less than 20.

Each item is assigned to the highest bidder. If players bid the same amount for an item, then a referee decides. Summing over all assigned items then yields each player's received value, v_T^x, $x = M, R$, and, thus, her excess over her fair share. In our example, player M receives both goods which gives her twice as much as her fair share. In contrast, player R receives nothing, which is why her excess is -20.

The Knaster procedure splits the total excess E, in our example 80, evenly among the players.[8] Since player M should receive a share of the excess worth only 40, she must pay $100 - 40 = 60$ to R. Player R is compensated for her loss of 20 and, in addition, receives her half of the excess, giving her a total of $20 + 40 = 60$. Hence, all transfer payments t^x cancel out, i.e. $t^M + t^R = 0$. Players' total values are then $v^x = (\hat{v}^x + E)/2$, $x = M, R$.

By introducing side-payments in terms of the standard of value, the Knaster procedure introduces a form of compensation that is more efficient than the redistribution of goods. In our example, M's valuation of item 1 is three times as high as R's in terms of money. Or, the other way around: R's valuation of money is three times as high as M's in terms of item 1. Consequently, M is better off compensating R with money rather than with item 1.

The outcome of the Knaster procedure is that M receives a total value of 140, while R gets 60. In percentage terms, M receives 70% of the estate and R even gets 150%. Compared with the previous envy-free distribution, both players now have significantly more. Nevertheless, player M may feel unsatisfied, because R got much more of what she wanted. Although the division is 'efficient' and 'envy-free,' it is not 'equitable.' In fact, R's share is higher than her valuation of the complete estate, which appears somewhat absurd.

Brams and Taylor (1996) introduce a fair-division algorithm called "Adjusted Winner," which implements an efficient, envy-free, and equitable outcome. We use our example to describe the individual steps, which are summarized in Table 3.

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[8] This shows that the procedure is easily extendible to n players.

	M	R
1. Photo Collection	50	<u>75</u>
2. Oil Painting	<u>50</u>	25
Estate Value	100	100
Received Value	50	75
$50 + \alpha 50 = (1 - \alpha)75$		
M's Share of Item 1: $\alpha = 0.2$		
Percent of Estate	60	60

Table 3: Adjusted Winner

As Table 3 shows, both players view the whole estate as 100 percent of the 'pie.' An important feature of Adjusted Winner is that it takes the whole estate as the standard of value, rather than some monetary unit. This makes the algorithm particularly interesting for problems of fair division, where the items cannot all be valued in terms of money. In that case, it may be much simpler for players to allocate 100 percentage points among the items to be divided.

In a first step, Adjusted Winner assigns each item to the player who values it most.[9] Item 1 now goes to player R and item 2 to player M. This allocation determines that R is the 'temporary winner' and M the 'temporary loser.' The main aspect of this procedure is that the outcome of step 1 is guaranteed to be efficient:[10] By giving each item to the player who values it higher, the sum of both players' utilities is maximized for each issue. Therefore, the sum of their utilities is also maximized over all issues together. Hence, the outcome is efficient, since there is no outcome that can give both players a higher utility.

In a second step, Adjusted Winner redistributes items from the temporary winner to the temporary loser at the lowest cost-gain ratio in order to maintain efficiency. In our simple example, only item 1 can be shifted from R to M. Since this would

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[9]If items are valued equally by both players, then a referee can decide.

[10]A proof is given by Brams and Taylor (1996) (Theorem 4.1).

give M 100% and leave R with nothing, equitability requires a division of item 1. If M receives 20% of the photo collection and R 80%, then each player, in her view, will end up with 60% of the whole estate (cf. Table 3).

Despite the equitability and the efficiency of the outcome, one may question, though, whether Adjusted Winner is the most plausible procedure for this fair-division problem. Note that players are both worse off than with the Knaster procedure. The reason, of course, is that equitability requires players to divide item 1 instead of money.

Intuitively, one can expect to obtain a better outcome by marrying the efficient adjustment method of the Knaster procedure with the equitability condition of Adjusted Winner. By doing this, we construct a method of fair division which we call "Adjusted Knaster." As Table 4 shows, the procedure is just as simple as its 'parents.'

		M	R
1. Photo Collection		<u>100</u>	30
2. Oil Painting		<u>100</u>	10
Estate Value	$\hat{v}^x$	200	40
Envy-free Share	$\hat{v}^x/2$	100	20
Received Value	v_T^x	200	0
Excess	$E = 80 =$	100	–20
Equitability: $\dfrac{\alpha E}{200} = \dfrac{(1-\alpha)E}{40}$			
$\alpha = \dfrac{200}{200+40} = \dfrac{5}{6}$			
Share of Excess		66.67	13.33
Transfer	t^x	–33.33	+33.33
Total Value	v^x	166.67	33.33
Percent of Estate		83.33	83.33

Table 4: Adjusted Knaster

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Adjusted Knaster first follows the Knaster procedure by giving both items to player M, so that the total excess is again equal to $E = 100 - 20 = 80$. Now, however, the excess is not split evenly. Since both players value the estate differently, their valuation of the monetary excess in terms of the estate is not the same. Therefore, in the second step, Adjusted Knaster equalizes the players' shares of the excess in relation to their valuation of the estate. Instead of 1/2, player M now receives $\alpha = 5/6$ of the excess, which she values at 1/3 of the estate, the same as player R values her share of $1 - \alpha = 1/6$ of the excess. As a consequence, the required transfer to R, $t^R = -t^M$, reduces to 33.33. Since this leaves both players with 83.33% of the estate, the outcome of Adjusted Knaster is equitable. We formalize this result in the following theorem.

Theorem 1 *Assume that the 'excess' value, defined as the difference between the total value of received items and the sum of the envy-free (50%) shares, is distributed through monetary transfers. If players receive an equitable share of the excess, then they will also receive an equitable share of the estate. Moreover, their share of the total value of received items is equal to their share of the excess.*

Proof: Denote players' valuation of the estate by $\hat{v}^M$ and $\hat{v}^R$, respectively. With $\alpha \in [0, 1]$ denoting player M's share of the excess E, equitability implies that

$$\frac{\alpha E}{\hat{v}^M} = \frac{(1 - \alpha)E}{\hat{v}^R} \ ,$$

yielding a share of $\alpha = \hat{v}^M / (\hat{v}^M + \hat{v}^R)$. By adding 1/2 to both sides of the above equation, one obtains the equivalent condition

$$\frac{\alpha E + \frac{1}{2}\hat{v}^M}{\hat{v}^M} = \frac{(1 - \alpha)E + \frac{1}{2}\hat{v}^R}{\hat{v}^R} \ .$$

The denominators on both sides of this equation are equal to the values of the final outcomes, v^M and v^R, such that

$$\frac{v^M}{\hat{v}^M} = \frac{v^R}{\hat{v}^R} \ ,$$

which proves the first part of the theorem. In order to show the second part, note that the final outcomes are simply a redistribution of the total value of received

items. With $v^M + v^R = v_T^M + v_T^R$, equitability implies that

$$\frac{v^M}{\hat{v}^M} = \frac{(v_T^M + v_T^R) - v^M}{\hat{v}^R} ,$$

which can be rearranged to

$$v^M = \frac{\hat{v}^M}{\hat{v}^M + \hat{v}^R} \left(v_T^M + v_T^R\right) = \alpha \left(v_T^M + v_T^R\right) . \qquad \Box$$

The second part of the theorem shows that Adjusted Knaster is even simpler than Table 4 suggests. In order to determine the transfer payment, one only needs players' valuation of the estate and the total value of received items. There is no need to calculate the excess.

Compared with the outcome of Adjusted Winner, the improvement through Adjusted Knaster is substantial. Two critical aspects need to be mentioned, though. First, Adjusted Knaster requires a common standard of value (e.g. money) for each item. This restricts the procedure's applicability, since there are many fair-division problems, where a valuation of all items in terms of money is not possible or may even be considered as immoral. Second, a side payment requires money, which the paying party simply may not have. Brams and Taylor (1996) emphasize that Adjusted Winner is immune to both of these weaknesses, since it is a pure point-allocation procedure.

On the other hand, Adjusted Winner generally requires that one of the items is divided. Fortunately, the division involves at most one item, but it is not certain that this particular item is divisible and how players value fractions of it. Indeed, dividing an item can be just as problematic as assigning to it a monetary value. Adjusted Knaster requires no division of goods whatsoever, since it establishes equitability through a monetary transfer. Hence, Adjusted Knaster may be seen as the natural extension of Adjusted Winner, when side payments are possible.

Problems of fair division concerning multiple items are special cases of multiple issue negotiations. In order to see this, we interpret our example as a negotiation problem over two issues (i.e. the items), where for each issue there are two discrete options – option 1: M gets the item and R gets nothing; and option 2: R gets the item and M gets nothing. With two issues, there are $2^2 = 4$ discrete outcomes. If

the items are divisible – which Adjusted Winner requires – one can also consider convex combinations of options. Formally, we assume that players' preferences are characterized by utility functions $v^x : [0,1]^2 \to \mathbb{R}$, $x = M, R$, where v^x is assumed to be linear on the pair of divisions. In particular, this implies that players' utilities are additively separable across issues. Figure 1 shows the 4 discrete outcomes, denoted by "o". The solid line indicates the efficient boundary of the bargaining set.
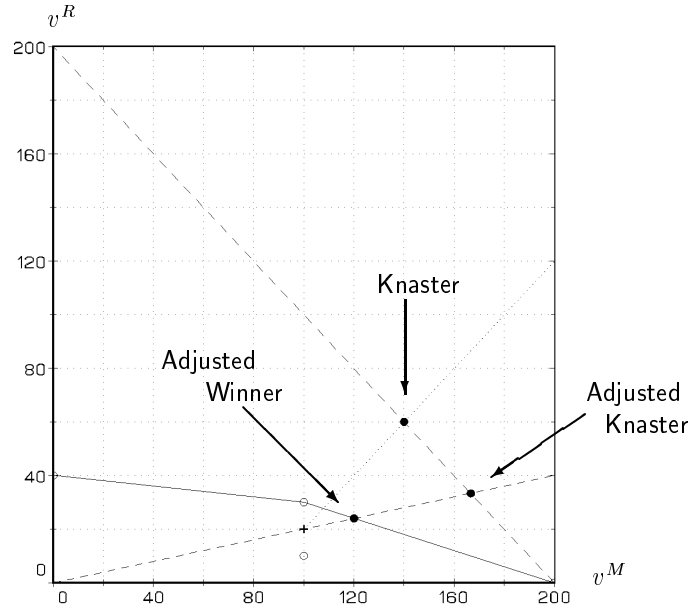

Figure 1: Outcomes of Fair-Division Procedures

Figure 1 also shows the outcomes of the three fair-division procedures. The outcomes of the Knaster procedure and Adjusted Knaster are located on the negatively sloped 45°-line passing through the allocation (200,0), since both involve monetary transfers. The Knaster outcome is at the intersection with the positively sloped 45°-line passing through the 50%-allocation, denoted by "+" at (100,20). Adjusted Knaster has its outcome at the intersection with the equitability line, which indicates equal percentage gains for both players; its slope is given by the ratio of players' valuations of the estate. Adjusted Winner implements an outcome on the Pareto frontier of the bargaining set at the intersection with the equitability line. Figure 1 clearly illustrates how Adjusted Knaster extends Adjusted Winner. Under Adjusted Knaster, players always do at least as well as under Adjusted Winner.

It is important to note that the algorithm underlying Adjusted Winner does not require the allocation of 100 points, but can also be conducted in payoff space. As Figure 1 shows, the outcome of Adjusted Winner corresponds to the axiomatic solution of Kalai and Smorodinsky (1975). An important feature of the Kalai-Smorodinsky solution is its scale invariance. Since the point allocation is a normalization method, it rescales payoffs at a constant factor. This affects the individual steps of Adjusted Winner, but not its final outcome. With the valuations given in Table 1, the first step gives both items to player M, analogous to the other two procedures. The second step then transfers items from M to R at the most efficient rate. The equitable allocation involves the same division of item 1.

3.   A Multiple-Issue Multiple-Option Negotiation

Consider now a negotiation between a manufacturer (M) and a retailer (R) over six different marketing plans (A–F) for each of six different products (1–6). We borrow the example of Gupta (1989), which features the structure and complexity of marketing negotiations that have been studied in the literature. Table 5 shows both negotiators' gains for each of the marketing plans in units of one mill. Dollars.

The six different products are the issues of this negotiation. An option is a marketing plan A–F for each of the products 1–6. An agreement consists of one plan for each product. If no option is divided, then there are $6^6 = 46,656$ possible agreements to this negotiation. Figure 2 shows players' total gains, v^M and v^R, for each discrete outcome. As the picture shows, the complexity of this negotiation far exceeds that of a fair-division problem consisting of six items, which would feature only $2^6 = 64$ discrete outcomes.

Players value gains in terms of Dollars, so we can view these as a characterization of their individual utilities. Alternatively, the standard of value could also be the aggregate over all 6 issues. For player M, the complete pie is worth $\hat{v}^M = 24$ mill. Dollars, while for R, it has a total value of $\hat{v}^R = 20$ mill. Dollars. By using these two values to normalize each player's Dollar gains, one obtains their percentage gains. The maximum percentage gain that a player can achieve or lose on a specific issue

OPTIONS

		A	B	C	D	E	F	Max
1	M	0.00	1.65	3.00	3.66	5.00	6.00	6.00
	R	2.00	1.65	1.40	1.22	1.00	0.00	2.00
2	M	0.00	0.72	1.20	1.50	1.70	2.00	2.00
	R	2.00	1.50	1.20	1.00	0.60	0.00	2.00
3	M	2.00	1.70	1.50	1.20	0.72	0.00	2.00
	R	0.00	0.60	1.00	1.20	1.50	2.00	2.00
4	M	0.00	1.00	1.25	1.50	1.80	2.00	2.00
	R	8.00	6.67	5.00	4.00	1.72	0.00	8.00
5	M	6.00	5.00	3.66	3.00	1.65	0.00	6.00
	R	0.00	1.00	1.22	1.40	1.65	2.00	2.00
6	M	0.00	1.40	2.80	3.75	5.00	6.00	6.00
	R	4.00	3.40	2.80	2.50	2.00	0.00	4.00

(left margin label: ISSUES)

Table 5: Gains of players M and R from plans A–F for products 1–6

can be seen as her interest in this issue. For each player, the sum of interests is equal to 100 percent. The two characterizations of utilities are the same as in the preceding section, but since it is more natural to assume that a negotiation between a manufacturer and a retailer is conducted in terms of Dollars, we maintain this characterization in the following analysis.

For a direct application of the fair-division procedures of the preceding section, we need to assume that players view issues as items that can be divided between them. With linear, additively separable utility functions, players' preferences are characterized by their interests in complete issues, i.e. by the values of their best options. In Table 5, this is captured in the right column.

Applying a fair-division procedure to the negotiation problem of Table 5, however, will generally lead to an inefficient outcome. This is because the linearization of preferences regards only convex combinations between the best and the worst options and, thereby, blends out efficient intermediate agreements. This is shown in Figure 3, where we have plotted players' gains for each of the six issues. By choosing the same scale for each graphic, one can see the relative importance of the individual
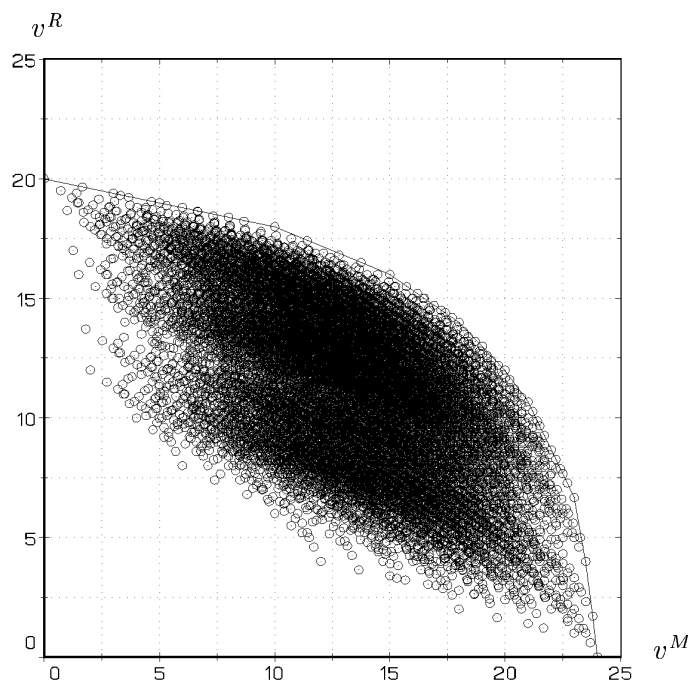
Figure 2: Total gains for 46,656 possible discrete agreements

issues for both players. The six hollow points in each panel denote the individual options. The dashed lines illustrate how the assumption of linear preferences leads to a loss of efficiency.

Instead of interpreting a negotiation as a fair-division problem, we wish to reformulate our fair-division procedures in order to fit the negotiation problem. We, therefore, assume that players have utility functions $v^x : [0,1]^6 \to \mathbb{R}$, $x = M, R$, with $v^x \equiv v_1^x + v_2^x + \cdots + v_6^x$, where the subutility functions, v_i^x, $i = 1, \ldots, 6$, are now assumed to be piecewise linear and concave. The solid lines in the six panels of Figure 3 indicate the efficient boundaries for the individual issues. Note that not all options are efficient if convex combinations are possible. The solid line in Figure 2 shows the efficient boundary of the negotiation problem over all six issues together. In order to adapt the fair-division procedures of section 2 to this more general negotiation problem, we do not change their individual steps, but only phrase them more generally.
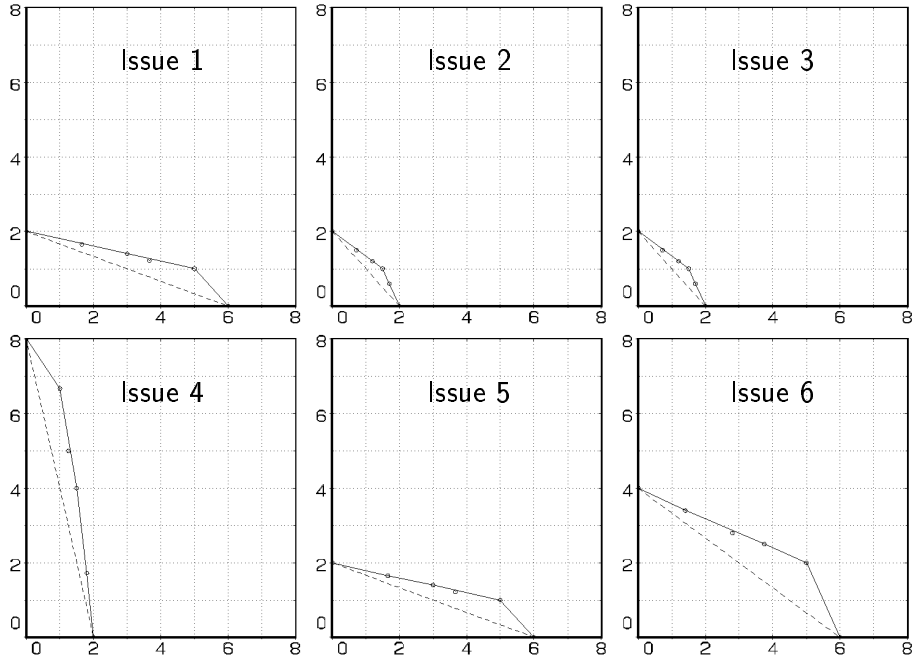
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Figure 3: Players' Gains in a Multiple-Issue Multiple-Option Negotiation

4.   Fair-Negotiation Procedures

An important aspect to be learned from the fair-division procedures is that the issue-option characterization in Table 5 contains more structural information than the joint payoffs of package agreements shown in Figure 2. This additional information allows us to construct procedures that do not require computer support.

In payoff space, all three fair-division procedures first assign the individual issues to the player who values them most. The reason why this induces an efficient outcome is because, with linear preferences, the sum of both players payoffs is maximized. With this goal in mind, we are able to formulate the first step of our fair-negotiation procedures.

Step 1: *For each issue i choose the 'temporary options' iT that maximize the sum of players' gains, i.e. $iT \in \arg\max_T \{v_{iT}^M + v_{iT}^R\}$. If more than one option satisfies this criterion, then let a referee decide which option to choose from this set. All temporary options, together, constitute the 'temporary agreement.' The sum of each player's temporary gains, $v_T^x = \sum_i v_{iT}^x$, $x = M, R$, determines the 'temporary winner'*

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W and the 'temporary loser' L through the condition

$$\frac{v_T^W}{\hat{v}^W} \geq \frac{v_T^L}{\hat{v}^L} \, . \qquad\qquad\qquad \diamond$$

We illustrate step 1 with the help of our example. For issue 1, the sum of payoffs is maximized with options E and F, which both yield a total gain of 6. Either can be chosen as the temporary option. Presumably, players would select the less extreme option E. For issue 2, the temporary option is D, for issue 3, it is option C, and for issue 4, the joint maximum is with option A. Issue 5 offers a choice again between options A and B, where we assume that players would pick B. For issue 6, option E maximizes joint gains. The 'temporary agreement' is then given by the 6-tuple of options [EDCABE], which yields total payoffs of $v_T^M = 18$ and $v_T^R = 14$. In Figure 4, the temporary outcome is denoted by T.
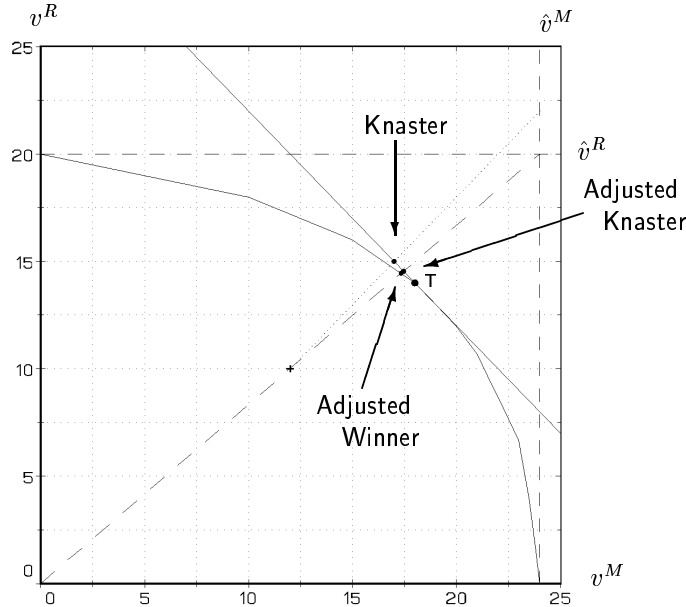

Figure 4: Pareto optimal Gains for six issues with 6 options each

Since step 1 maximizes the sum of payoffs for each individual issue, it also maximizes the sum of payoffs over all issues together. Hence, the outcome is efficient, because there is no agreement which yields a higher gain for both players. The outcome of step 1 matches that of the *utilitarian* solution characterized by Myerson (1981).

Geometrically, the temporary outcome T is a tangency point of the negatively sloped 45°-line with the efficiency frontier. As Figure 4 shows, the temporary outcome is not unique. This is because step 1 offers a choice of alternatives for issues 1 and 5. At point T, players outcomes yield

$$\frac{v_T^M}{\hat{v}^M} = \frac{18}{24} = 0.75 > 0.70 = \frac{14}{20} = \frac{v_T^R}{\hat{v}^R} \ .$$

According to the condition of step 1 above, player M is the temporary winner and player R the temporary loser. For all three of our fair-negotiation procedures, we take outcome T as the starting point of step 2. The final outcomes of the three algorithms are illustrated in Figure 4.

4.1. Adjusted Winner

Adjusted Winner redistributes gains by switching options with the aim of achieving equitability. Since step 1 already has an efficient outcome, the adjustment rule must be designed in order to keep players on the efficiency frontier. We describe the complete adjustment process in the following algorithm.

Step 2 (AW): *For each issue i and each temporary option iT, consider each efficient alternative option iO_T that gives the temporary loser a higher gain than with the temporary option iT. Calculate the rate of substitution*

$$\mathrm{RS}_{i:TO_T} := \frac{v_{iT}^W - v_{iO_T}^W}{v_{iO_T}^L - v_{iT}^L} \ .$$

Select the alternative options iO_T^* that yield the lowest cost-gain ratio, i.e. $iO_T^* \in \arg\min_O \mathrm{RS}_{i:TO_T}$. If more than one option satisfies this criterion, then let a referee choose from this set. Now determine the issues i^* with the lowest cost-gain ratio, i.e. $i^* \in \arg\min_i \mathrm{RS}_{i:TO_T^*}$. If there is no unique issue, then let a referee decide. Calculate players' overall utilities $v_{O_T^*}^W$ and $v_{O_T^*}^L$ by replacing only option i^*T with its efficient alternative $i^*O_T^*$. If

$$\frac{v_{O_T^*}^W}{\hat{v}^W} > \frac{v_{O_T^*}^L}{\hat{v}^L} \ ,$$

then make $i^*O_T^*$ the new temporary option of issue i^* and repeat step 2. Else, calculate the convex combination of options i^*T and $i^*O_T^*$ that equalizes the relative gains between the winner W and loser L.   ◇

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In order for the adjustment process to maintain efficiency, the transfer from the temporary winner to the temporary loser must be achieved at a minimum cost-gain ratio. With a piecewise linear Pareto frontier connecting discrete agreements, a movement along the efficient boundary requires switching only one option at a time. In order to find the efficient option switch, one must only find the best alternative within each issue, and then select the best alternative among all issues.[11]

For issue 1 in our example, the temporary option is E. Among the alternatives D, C, B, and A, options C and A both allow an efficient shift from player M to R, with $RS_{1:EC} = RS_{1:EA} = 5$. For issue 2, there are also two efficient alternatives, since $RS_{2:DC} = RS_{2:DA} = 1.5$. The same is true for issue 3, where $RS_{3:CD} = RS_{3:CF} = 1.5$. Issue 4 offers no possibility of substitution, since player M is at her minimum here. Issue 5 has options D and F as the efficient alternatives to B, with $RS_{5:BD} = RS_{5:BF} = 5$. Finally, for issue 6 the efficient alternatives to option E are D and A, with $RS_{6:ED} = RS_{6:EA} = 2.5$. The cheapest possibility of substitution from player M to R is thus given by issues 2 and 3. If players select issue 2 and begin with the smaller step of shifting to option 2C, the new agreement yields

$$\frac{v_{O^*}^M}{\hat{v}^M} = \frac{17.70}{24} = 0.7375 > 0.71 = \frac{14.20}{20} = \frac{v_{O^*}^R}{\hat{v}^R} \ .$$

According to the algorithm of Adjusted Winner, they must repeat step 2. They now have the choice of taking a larger step from option 2C to 2A or a smaller step from 3C to 3D, since both have the same substitution rates. The latter brings them to the agreement [ECDABE]. With

$$\frac{v_{O^*}^M}{\hat{v}^M} = \frac{17.40}{24} = 0.725 > 0.72 = \frac{14.40}{20} = \frac{v_{O^*}^R}{\hat{v}^R} \ ,$$

equitability is almost achieved. Indeed, if players now shift from 3D to 3F, the outcome implies

$$\frac{v_{O^*}^M}{\hat{v}^M} = \frac{16.20}{24} = 0.675 < 0.76 = \frac{15.20}{20} = \frac{v_{O^*}^R}{\hat{v}^R} \ .$$

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[11]Since Step 2 (AW) describes the adjustment of gains along the Pareto frontier, it can also be used to implement alternative bargaining solutions. For example, Raith (1998) adapts the algorithm to bargaining problems with reference points.

Equitability thus requires a convex combination of options 3D and 3F. Denoting the relative weight of option 3D by α, the equitability condition becomes

$$\frac{v^M}{\hat{v}^M} = \frac{16.20 + [\alpha 1.20 + (1-\alpha)0]}{24} = \frac{13.20 + [\alpha 1.20 + (1-\alpha)2]}{20} = \frac{v^R}{\hat{v}^R} \; ,$$

implying a value of $\alpha = 0.9\overline{4}$. The equitable agreement [EC(0.9$\overline{4}$D+0.0$\overline{5}$F)ABE] yields the payoffs $v^M = 17.\overline{33}$ and $v^R = 14.\overline{44}$, giving each player a percentage increase of $72.\overline{22}\%$.

It is important to note that, at each 'referee decision' of step 2, players could take an alternative route leading to a different agreement, but with the same payoff. For example, players could remain at option 2D and concentrate their adjustment process on issue 3. The final agreement could then be of the form [ED(βD+(1 − β)F)ABE], with $\beta = 0.69\overline{4}$. Hence, each referee decision gives negotiators more room to maneuver, without increasing the conflict, since their payoffs are unaffected.

Finally, any efficient agreement requires the convex combination of at most two options of only one single issue. This aspect is of great practical importance, because negotiators, attempting to reduce complexity, often follow the procedure of seeking a compromise on each individual issue. We deal with this aspect in section 5.

4.2. Knaster

If negotiators value options with the same standard of value (in our example money), they might also have the possibility of redistributing gains through transfers of this common standard. In section 2, we have seen that the Knaster fair-division procedure includes a simple transfer mechanism that leaves both negotiators with the same monetary excess over their envy-free share of 50%. The application of this procedure to a more general negotiation problem is straight-forward. As for Adjusted Winner, we formulate the adjustment process in the form of an algorithm. For our example, the individual steps of the procedure are given in Table 6.

Step 2 (KN): *Make the temporary agreement the final agreement. Determine each player's 'transfer' t^x as the difference between her share (50%) of the total excess*

and her individual excess or, equivalently,

$$t^x = \frac{1}{2}\left\{\left(v_T^y - \frac{\hat{v}^y}{2}\right) - \left(v_T^x - \frac{\hat{v}^x}{2}\right)\right\} , \qquad x, y \in \{M, R\},\ x \neq y .$$

Players' total payoffs are given by $v^x = v_T^x + t^x$. ◇

		M	R
Value of all issues	$\hat{v}^x$	24	20
Envy-free Share	$\hat{v}^x/2$	12	10
Received Value	v_T^x	18	14
Excess $\quad E = 10 =$		6	4
Share of Excess	$E/2$	5	5
Transfer	t^x	–1	+1
Total Value	v^x	17	15
Percentage Gain		70.8$\overline{3}$	75.00

Table 6: The Knaster fair-negotiation procedure

The simplicity of the Knaster procedure is remarkable: Players M and R agree on the combination of options [EDCABE], and player M pays R a compensation of 1 mill. Dollars. Moreover, the sum of players' total payoffs $(17 + 15 = 32)$ is higher than with Adjusted Winner $(17.\overline{3} + 14.\overline{4} = 31.\overline{7})$. In Figure 4, the outcome of Adjusted Winner is below the negatively sloped 45°-line. As before, the only disturbing feature of the Knaster procedure is that the outcome is generally not equitable.

4.3. Adjusted Knaster

In order to obtain an equitable outcome, we can formulate the algorithm of Adjusted Knaster analogously to the Knaster procedure. The only difference is that players receive an equitable rather than an equal share of the excess. However, the procedure becomes even simpler if we make use of Theorem 1.

Step 2 (AK): *Make the temporary agreement the final agreement. Determine each player's 'transfer' t^x as the difference between her equitable share of the aggregate payoffs and her individual payoff:*

$$t^x = \frac{\hat{v}^x}{\hat{v}^x + \hat{v}^y}(v_T^x + v_T^y) - v_T^x , \qquad x, y \in \{M, R\}, \ x \neq y .$$

Players' total payoffs are given by $v^x = v_T^x + t^x$. ◇

In our example, the temporary agreement [EDCABE] yields an aggregate payoff of $18 + 14 = 32$. Player M's equitable share of this value is $v^M = 24/(24+20) \times 32 = 6/11 \times 32 = 17.\overline{45}$, implying a transfer of $t^M = 17.\overline{45} - 18 = -0.\overline{54}$. Player R's share is $v^R = 20/(24+20) \times 32 = 5/11 \times 32 = 14.\overline{54}$, which results from the transfer $t^R = 14.\overline{54} - 14 = 0.\overline{54}$ from player M. With this adjustment, both players realize a gain of $72.\overline{72}\%$. Hence, with a transfer of only 0.55 mill. Dollars, player M can improve her outcome by 0.12 mill. Dollars and R by 0.10 mill. Dollars, if both choose Adjusted Knaster rather than Adjusted Winner.

5. Issue-by-Issue Negotiations vs. Package Deals

Our derivation of fair-negotiation procedures focused on agreements consisting of 6-tuples of options, i.e. package deals. Although the algorithms involve only minor computational effort, it may be difficult for negotiators to manage all issues at once. Indeed, issue-by-issue negotiations are common practice, especially if there is an agenda to follow. Given the remarkable performance of all three procedures for package deals, it is interesting to see how they work for issue-by-issue negotiations.

Theorem 2 *If Adjusted Winner is followed on an issue-by-issue basis, then the final outcome will, in general, be neither efficient nor equitable.*

Proof: For each issue, step 1 of Adjusted Winner maximizes the sum of players' payoffs. Step 2 then establishes equitability issue-wise, by switching options efficiently within each issue. However, since there is no minimization of substitution rates over all issues, the individual adjustments are not necessarily efficient for the aggregate of all issues.

Next consider equitability on an issue-by-issue basis. This implies $v_i^M \hat{v}_i^R = v_i^R \hat{v}_i^M$ for each issue i. On the other hand, equitability over all issues implies

$$v^M \hat{v}^R = v^R \hat{v}^M$$

$$\Leftrightarrow \quad \sum_i v_i^M \sum_i \hat{v}_i^R = \sum_i v_i^R \sum_i \hat{v}_i^M$$

$$\Leftrightarrow \quad \sum_i v_i^M \hat{v}_i^R + \sum_i \sum_{j \neq i} v_i^M \hat{v}_j^R = \sum_i v_i^R \hat{v}_i^M + \sum_i \sum_{j \neq i} v_i^R \hat{v}_j^M \ .$$

Issue-by-issue equitability thus implies overall equitability, only if

$$\sum_i \sum_{j \neq i} v_i^M \hat{v}_j^R = \sum_i \sum_{j \neq i} v_i^R \hat{v}_j^M \ .$$

If issues are valued differently, this condition does not hold, in general. □

Consider our example of Table 5. As one can verify with the help of Figure 3, the individual options of the agreement [DCDCCD] are equitable and (almost) efficient on an issue-by-issue basis.[12] The total payoffs of this agreement are $v^M = 14.67$ and $v^R = 12.34$. Although the inequitability of this outcome is negligible, its inefficiency is significant. Compared with the outcome of Adjusted Winner for the package deal, player M loses $17.33 - 14.67 = 2.66$ mill. Dollars, and player R loses $14.44 - 12.34 = 2.1$ mill. Dollars. Intuitively, Adjusted Winner, on an issue-by-issue basis, neglects mutually beneficial trade-offs when players value issues differently.

Theorem 3 *Adjusted Knaster in issue-by-issue negotiations yields the same aggregate payoff as Adjusted Knaster in a package deal. The outcome, however, is generally not equitable.*

Proof: According to Theorem 1, Adjusted Knaster yields

$$v_i^x = \frac{\hat{v}_i^x}{\hat{v}_i^x + \hat{v}_i^y}(v_{iT}^x + v_{iT}^y) \ , \qquad x = M, R \ ,$$

for each issue i. Hence, $v_i^x + v_i^y = v_{iT}^x + v_{iT}^y$. By summing over all issues, one then obtains $\sum_i (v_i^x + v_i^y) = \sum_i (v_{iT}^x + v_{iT}^y) = v_T^x + v_T^y$, which proves the first part of the theorem. The second part of the proof is analogous to that of Theorem 2. □

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[12]Remember that this example is taken from Gupta (1989) without any modification.

Adjusted Knaster on an issue-by-issue basis yields the outcomes $v^M = 17.3$, or 72.08%, and $v^R = 14.7$, or 73.5%. There is no efficiency loss, because step 1 of Adjusted Knaster contains an issue-by-issue maximization of aggregate payoffs. The monetary transfer thus redistributes the aggregate payoff induced by the utilitarian solution, which is not affected by the issue-by-issue approach. Only equitability is sacrificed, but this disturbance is only small if players' valuations over all issues together do not differ substantially, as in our example.

Theorem 4 *The Knaster procedure in issue-by-issue negotiations yields the same outcome as the Knaster procedure for package deals.*

Proof: The equality of aggregate payoffs under issue-by-issue negotiations and package deals follows from the proof of Theorem 3. What remains to be shown is that the total payoff for each individual player is also the same under both negotiation procedures. For each issue i, the payoff of player $x = M, R$ is $v_i^x = (\hat{v}_i^x + E_i)/2$, where the excess of issue i is given by $E_i = (v_{iT}^M + v_{iT}^R) - 1/2(\hat{v}_i^M + \hat{v}_i^R)$. By summing over all issues, one then obtains

$$
\begin{aligned}
v^x = \sum_i v_i^x &= \frac{1}{2}\sum_i \hat{v}_i^x + \frac{1}{2}\sum_i E_i \\
&= \frac{1}{2}\hat{v}^x + \frac{1}{2}\left[(v_T^M + v_T^R) - \frac{1}{2}(\hat{v}^M + \hat{v}^R)\right] \\
&= \frac{1}{2}(\hat{v}^x + E) .
\end{aligned}
$$
□

The robustness of the Knaster procedure with respect to issue-by-issue negotiation is due to the fact that it distributes the excess at a fixed fraction of $1/2$. Indeed, any fixed fraction will lead to the same result. We use this feature for the following modification of the Knaster procedure.

Corollary 1 *Let player $x = M, R$ receive a fixed fraction $\hat{v}^x/(\hat{v}^M + \hat{v}^R)$ of the excess of each issue. The Knaster procedure for issue-by-issue negotiations then yields the same outcome as Adjusted Knaster for package deals.*

Proof: The proof is analogous to that of Theorem 4, except that $E_i/2$ must now be replaced by $E_i\hat{v}^x/(\hat{v}^M + \hat{v}^R)$. □

Intuitively, the fixed fraction, characterizing the equitable package deal, introduces the necessary 'farsightedness' into the issue-by-issue procedure in order to establish the final equitable outcome.

6.   Conclusions

Rather than characterizing cooperative solutions through their properties, we motivate them as the outcome of a cooperative process. In contrast to an axiomatization, a fair-negotiation procedure provides the argumentative basis needed to implement cooperative solutions in actual negotiations.

Converting the properties of a solution into procedural arguments not only helps negotiators to understand the solution but also the process leading to its implementation. Moreover, the individual steps allow negotiators to be part of the process, i.e. they are actually able to 'play the game.' These aspects may explain why practitioners are often reluctant to use negotiation support tools that offer a solution at the push of a button.

The power of a fair-negotiation procedure depends on its plausibility and practicability. For multiple-issue negotiations, we derived three closely related algorithms. The most general approach is Adjusted Winner which, in its basic form, implements the Kalai-Smorodinsky solution, but it can be adapted to alternative solution concepts as well.[13] The application of the procedure does not require players to value issues with a common monetary standard, nor does it impose any monetary transfer.

When monetary transfers are possible, though, Adjusted Knaster dominates Adjusted Winner. The higher the equitable transfer is, the greater the difference between these two procedures will be. This is because the algorithms choose the same temporary outcome in the first step.

When issues are negotiated separately, due to their complexity or the fact that negotiators have difficulty with full cooperation, the Knaster procedure is the most robust one.   Since it simply implies a redistribution of the utilitarian outcome,

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[13]Raith and Welzel (1998) show how Adjusted Winner can be modified in order to implement the Nash bargaining solution.

efficiency is not affected by an issue-by-issue approach. The total monetary transfer is the same, because players receive the same share of the excess at each issue. The outcome, however, is not equitable.

Adjusted Knaster on an issue-by-issue basis guarantees overall efficiency and equitability on an issue-by-issue basis. But it also cannot ensure overall equitability, although it generally comes closer than the Knaster procedure. Indeed, we showed that both procedures can be combined to implement an overall efficient equitable outcome on an issue-by-issue basis.

An important insight from the procedural approach to cooperative negotiations is that different solution concepts are recommended for different steps of the procedure. Hence, in the first step, players can easily attain efficiency by focusing on the utilitarian solution which ignores the distribution of gains. In the second step, they then implement a distributive norm through an appropriate adjustment process. In contrast, by keeping the distribution of gains fixed throughout the process, negotiators would maintain a non-cooperative element, making it more difficult for them to achieve an efficient outcome.

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