Integrating the Nash Program into Mechanism Theory

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February 1999

Abstract

The present paper provides a method by which the Nash Program may be embedded into mechanism theory. It is shown that any result stating the support of any solution of a cooperative game in coalitional form by a noncooperative equilibrium of some suitable game in strategic form can be used to derive a mechanism theoretic implementation in this equilibrium of that solution.

1 Introduction

The Nash Program aims to support solutions of cooperative coalitional NTU-games by equilibria of non-cooperative games in strategic or extensive form. This program is purely game theoretic as it neither relies nor depends on any underlying social or economic model. Using the terminology of social choice theory the Nash Program "operates in a purely welfaristic framework". There is no arbiter or social planner involved in the analysis. Therefore the question as to such an actor's information about the players' characteristics is meaningless. Also it is taken for granted in this context that there is common knowledge among the players about the data of the games, both the cooperative one and non-cooperative one. The Nash Program does not specify a formal model which would allow to derive information about one of the games and its solution resp. equilibria from the other one. Nevertheless any contribution to the Nash Program is considered to

*I thank for helpful comments from participants at seminars at UCLA, UC Berkeley, CalTech, Duke University, UC Irvine, USC, VirginiaTech, Washington University, Brown University and Harvard/MIT and, in particular, Anke Gerber. The hospitality of the University of California, Los Angeles and of the University of Arizona at Tucson as well as financial support through a sabbatical grant of the Volkswagen Foundation are gratefully acknowledged.

provide insights into some aspects of the cooperative game and its solution, that are not obvious from the underlying axioms, or of the non-cooperative game and its equilibria, which are not directly deducible from the strategic interaction. In Nash's own words, in the context of bargaining games, this program relates

" ...two approaches to the problem, via the negotiation model or via the axioms [which] are complementary; each helps justify and clarify the other."

[Nash(1953,p. 129)]

Any attempt to look at the Nash Program as a specific part of implementation theory must rely on an alteration of the framework discussed above. A designer or social planner has to be added to the model as well as a set of social states in which he is interested. The solution of the cooperative game is to be interpreted as the set of socially desirable states. The designer's goal is to invent rules for games which would force any population of players to establish one of the desirable social states by playing an equilibrium of a game according to those rules. So while an additional agent, the designer, enters the scene when the analysis moves from game theory to mechanism theory, the players leave it. They are replaced by a class of potential player populations, for each of whom the designed mechanism is intended to be effective.

The important question now arises, namely what exactly the designer knows about the player populations, the actions of the players and what he is able to enforce. The very idea of implementing a socially desired state via strategic interaction of the members of a society under certain rules implicitly restricts the power and the knowledge of the designer, who otherwise could act as a dictator and establish the desired state by fiat. Therefore the question of what a reasonable mechanism could be should be posed only for well specified information and enforcement contexts of the designer.

The attempt to use results from the Nash Program for a planner's design problem requires that the common knowledge of players about their mutual characteristics is taken for granted for any potential feasible population. This clearly excludes the framework of asymmetric information among players. Also one should only consider a scenario where the planner is unable to enforce the desired state by fiat and where he, because of the design of the mechanism for a whole class of player populations, is also unable to know the players' preference profiles. It is assumed implicitly that the players' participation in the game can be taken for granted. Given this fact, we may assume that an equilibrium will be established, at least when it is unique, because of the idea that it is selfenforcing. This whole aspect of an appropriate enforcement is quite delicate but is excluded from the formal modelling. For an inspiring discussion of this point we refer to Hurwicz (1994, pp. 11-12).

As soon as refinements of the Nash equilibrium are considered there arises an additional problem, namely that of the agreement among all players in all potential populations on the same refinement.

Apart from the different sets of acting agents a crucial difference between the purely game theoretic framework of the Nash Program and the framework of implementation theory is the explicit occurrence of an outcome space in the latter. Formally, the games, including specifications of the players' preferences, are replaced by game forms containing all the information about the game except that information concerning players' preferences or payoff functions. A non-trivial factorization of players' payoff functions into an outcome function, common to all players in all populations, and individual utility functions defined on the outcome space enables us to distinguish between observability of the result of players' actions as opposed to their resulting payoffs. The question as to what extent the possibility of observation is crucial for enforcement depends on the degree of selfenforcement power of the considered equilibrium concept. For instance, a dominant strategy equilibrium could be considered a perfect substitute for observability whenever the players' participation can be taken for granted. In this case the rationality that is assumed for all players guarantees the desired result. A similar claim could be made, even if with weaker justification, for a unique Nash equilibrium.

The very fact that the presence of an outcome space is an additional ingredient in mechanism theory as compared to the Nash Program indicates that these two can hardly be considered "equivalent", as claimed, for instance, in Dagan and Serrano (1997, Abstract). Also the objectives of both theories are quite different. While the explicit judgement of the adequateness of a certain game form in view of the enforcement and informational restrictions of the planner has no meaning in the Nash Program a certain result in the Nash Program might provide the theorist with some valuable insight but could still be useless for a planner in a specific context. Also a formal result in the Nash program could possibly lack any informational value for the game theorist and simultaneously, for a planner amount to the enforcement of the desired state by fiat. Let us take, for example, the following modification of Nash's simple demand game: The two players choose real numbers, which, when suitably coordinated, result in a feasible bargaining outcome. If both choose their respective coordinate of the Nash solution they receive these as their payoffs. Otherwise both receive zero. This game obviously supports the Nash solution of the bargaining problem by the dominant strategy equilibrium of the game. However,it provides no valuable information about the specifics of the Nash solution, because any other solution can be supported in an analogous way. For the planner in the derived implementation theoretic context this is tantamount to his direct forcing any population of any bargaining problem to agree on the Nash solution.

Yet, though the objectives of both theories are different, it appears natural to exploit results yielded in the Nash Program for the purpose of mechanism design, or as Serrano (1996) has put it, "to adapt the mechanisms in the Nash program as standard game forms of the theory of implementation". In fact, in his paper Serrano provides such an adaptation. But for this purpose he uses the explicit introduction of a specific underlying production economy. In his framework Serrano then establishes several impossibility results for the implementability in Nash equilibrium of cooperative solution concepts, among them the Shapley value as well as the Kalai-Smorodinsky and the Nash bargaining solutions. The basis for these results is the lack of Maskin-monotonicity of these solutions interpreted as social choice rules.

In a similar way Dagan and Serrano (1997) add a set of physical outcomes to the structure of a cooperative game in order to adapt the Nash Program into the theory of implementation. From their analysis in a specific context they derive the general conclusion that "major solution concepts in coalitional games (e.g. the Nash bargaining solution, the NTU-Shapley value) can be derived strategically only by considering the possibility of random outcomes: either chance moves, mixed strategies or pure strategy equilibrium refinements based on trembles must appear in the analysis".

A critical discussion and analysis of the possibility to Nash-implement the Nash bargaining solution will be performed elsewhere. In the present paper I propose a general procedure of embedding the Nash program into the theory of implementation. That procedure enables us in our framework to transform any supporting result from the Nash Program into an implementation result in mechanism theory. In particular, our method enables us to implement the Nash and the Kalai-Smorodinsky solutions in dominant strategy equilibria (cf. Trockel (1999) and Haake (1998), respectively).

The adoption of results from the Nash Program to implementation theory, as discussed above, is really problematic only for games in strategic form. For extensive form games there does not arise the problem of a non-trivial factorization of the payoff functions. There, the terminal nodes of the game tree build the endogenously given canonical outcome space. The game is represented already as a pair consisting of a game form, namely the tree, and the profile of payoff functions. The game tree is more than just a representation of players' strategic options. It also represents the rules of the game. Therefore to take the set of terminal nodes of a game tree as the outcome space is fundamentally different from taking the product of strategy sets as the outcome space in games in strategic (or normal) form. The latter would only result in a trivial factorization of the payoff functions into their compositions with the identity map on the strategy space.

2 Definitions

The integration of the Nash Program into implementation theory requires two steps. First, we have to derive a social choice rule from the solution concept under consideration. Any solution in the welfaristic context is represented for any game in its domain by the utility allocations to the players. In contrast, a choice rule maps preference profiles on the outcome space to outcomes. Consequently, the first step involves the choice of an appropriate outcome space. Secondly, we have to relate the non-cooperative game theoretic support of that solution to the implementation of the derived social choice rule.

This step involves the factorization of certain functions, a formal method quite prominent in mechanism theory. The implementation of a social choice rule by equilibria of games relies on the factorization of payoff functions, i.e. their representation as compositions of utility functions with an outcome function. The famous Revelation Principle is based on the factorization of a social choice function into the composition of a map from the space of preference profiles into the strategy space with the outcome function of the given mechanism. The factorization we shall employ is somewhat more complex. We need simultaneous factorizations of two different functions sharing one common factor, namely the profile of utility functions on the outcome space.

We begin by providing the basic notions from implementation theory and game theory. We shall deal with games and game forms for $n > 1$ players. So $N = \{1, ..., n\}$ is the set of $players'positions$.

$S^i \neq \emptyset$ denotes the *strategy set* for any player in position $i \in N$. The *joint strategy space* $S_1 \times ... \times S_n$ is denoted S. A *player* is characterized by his *payoff function* $\pi^i : S \to \mathbb{R}$, where the superscript i indicates the player's position $i \in N$. The tuple $\Gamma := (S, \pi_1, ..., \pi_n)$ is a *strategic* n-*person game*. Removing the population of players while keeping the rules of the game is achieved by replacing the payoff functions by an outcome function. A *mechanism* (or *game form*) is defined by a pair (S, h), where $h : S \to Z$ is an *outcome function* associating with any strategy profile in S a state in the *outcome space* $Z \neq \emptyset$.

Now any population of n individuals with preferences on Z, representable by utility functions u_i: $Z \to \mathbb{R}$, $i = 1, ..., n$, transforms the game form (S, h) into the game $(S, \pi_1, ..., \pi_n)$ via the compositions $\pi_i := u_i \circ h$, $i = 1, ..., n$. Denote $(u_1 \circ h, ..., u_n \circ h)$ by $u \circ h$.

Depending on the population specified by the utility functions on the outcome space Z one may distinguish certain states as desirable. This idea leads to a *social choice rule*, or in Hurwicz's (1994) terminology, a *desirability correspondence*, F, which associates with any feasible profile of utility functions on Z a subset of Z. The determination of which profiles of utility functions on Z are considered feasible is crucial for the derivation of implementation results.

The implementation of a social choice rule in equilibrium relates for any feasible $u = (u_1, ..., u_n)$ the set $F(u)$ of desirable outcomes to the set $E(S, u \circ h)$ of equilibria of the game $(S, u \circ h)$.Here E is either the Nash equilibrium or some of its refinements. We say that the mechanism (S, h) *weakly* $E-implements$ the social choice rule F over the class $U \subset (\mathbb{R}^N)^Z$ of (feasible) utility profiles, if for all $u \in U : E(S, u \circ h) \neq \emptyset$ and $h(E(S, u \circ h)) \subset F(u)$.

Weak E-implementation becomes *full* E-implementation if $h(E(S, u \circ h)) = F(u)$ for all $u \in U$.

Next, we need the concept of a (cooperative) game in *characteristic* or *coalitional form.*

Such a game is a pair (N, V), where the correspondence V associates with any non-empty subset S of N a subset $V(S) \subset \mathbb{R}^S$. These sets $V(S)$, $S \subset N$ are interpreted as the sets of feasible payoffs of players playing in positions $i \in S$ when acting or cooperating in a coalition.

Denote by $\mathcal{G}$ a set of n-person coalitional games. A *solution* on $\mathcal{G}$ is a correspondence L associating with any coalitional game $V \in \mathcal{G}$ a non-empty subset $L(V)$ of $\mathbb{R}^N$, such that for any $x \in L(V)$ there is a partition $\{T_1^x, ..., T_{m_x}^x\}$ of N such that $\forall\ j_x \in \{1, ..., m_x\}$: $(x_i)_{i \in T_{j_x}^x} \in V(T_{j_x}^x)$. This formula expresses the feasibility of the utility allocation x for the game V. We denote the set of feasible utility allocations for V by $F(V)$.

For any solution L on $\mathcal{G}$ we denote by $\mathfrak{S}_L$ the set of selections of L, i.e. of functions $f_L : \mathcal{G} \longrightarrow \mathbb{R}^N$ such that $\forall V \in \mathcal{G} : f_L(V) \in L(V)$.

We do not distinguish between a singleton-valued solution and its unique selection. Notice, that then any selection of a solution is itself a solution.

Let A denote the set of singleton-valued solutions l on $\mathcal{G}$.

3 The Implementation of Solutions

A crucial step for the integration of the Nash Program into implementation theory is the derivation of a social choice rule that represents the cooperative solution concept under consideration. A very important ingredient of this is the choice of the outcome space Z. Several authors [cf. Bergin and Duggan (1996), Howard (1992), Serrano (1997)] make general statements about the impossibility of Nash-implementation of certain solutions which are proved only for specific choices of outcome spaces. The impossibility is proved then by establishing the lack of Maskin-monotonicity in that specific context (for instance, if Z is a space of lotteries).

A main problem with the implementation of some solutions lies in the fact that these are defined in a purely welfaristic context without any underlying physical social or economic model. So without any concrete population of players at hand there seems to be no way to look at outcomes and distinguish which of them are desirable. This problem led Dagan and Serrano (1997) to the enrichment of the pure game structure by an underlying physical structure. The specific way this was done influenced the (im)possibility of implementation of some solution concepts. Our present approach is based on a quite abstract outcome space. Even in a welfaristic framework, where a utility payoff according to a certain solution cannot be defined unless players are present, the abstract solution concept as a rule can still be defined. These rules, i.e. the solutions themselves, build our coutcome space. Our approach is therefore somewhat related in spirit to Border and Segal (1998) and to van Damme (1986), where players have preferences over solution concepts.

Suppose that for a given solution L^* on a class $\mathcal{G}$ of coalitional games and for an equilibrium concept E for n-person strategic games we have a support result in the context of the Nash program asserting that for every $V \in \mathcal{G}$ there is a strategic game $\Gamma^V = (S, \pi^V)$ such that $\pi^V(E(\Gamma^V)) = L^*(V)$. So we may have several equilibria but all result in payoffs according to the solution L^*. Notice, that we required S to be the same independently of the specification of V. This should not be a real restriction in many cases as normalizations, embeddings or identifications might be used to establish that assumption.

We define a mechanism as follows: The outcome space Z is assumed to be the set A of solutions. The outcome function h is defined as

$$h(\sigma)(V) := \begin{cases} \pi^V(\sigma) & \text{if } \pi^V(\sigma) \in F(V) \\ d_V \in F(V) \setminus \pi^V(E(\Gamma^V)) & \text{otherwise} \end{cases}$$

Here d_V is an arbitrarily chosen point in $F(V)$ that is not an equilibrium payoff vector for Γ^V. Notice, that implicitly we assumed that $L^*(V)$ is a proper subset of $F(V)$ for all $V \in \mathcal{G}$.

Next we define our set of feasible profiles of utility functions on the outcome space A. For each $V \in \mathcal{G}$ we define $u_i^V : A \to \mathbb{R}$ by $u_i^V(l) := (l(V))_i, i \in N$. As for two different games $V, V' \in \mathcal{G}$ we must have $l_i(V) = l_i(V')$ for some $i \in N$ and some $l \in A$ this definition allows us to consider the set $\mathcal{G}$ of coalitional games as a subset of all utility profiles on A.

Finally, we define the social choice rule $\mathfrak{L}^*$ associated with a solution L^* on $\mathcal{G}$. For any solution $l \in A$ let $[l]_V$ be the V-equivalence class of those solutions $l' \in A$ which induce the same utility allocation for the game V as l does, formally: $[l]_V := \{l' \in A | l'(V) = l(V)\}$, $V \in \mathcal{G}$. We define the social choice rule $\mathfrak{L}^*$ associated with the solution L^* on $\mathcal{G}$ by $\mathfrak{L}^*(V) = \bigcup_{l \in \mathfrak{S}_{L^*}} [l]_V$.

This social choice rule reflects the idea that any population of n players as characterized by V evaluates a solution concept only on the basis of what that solution allocates to them in the game V. This population does not care about what a solution might give to other populations' players characterized by some $V' \neq V$.

Notice that, although it is meaningful to choose the set $\mathcal{G}$ as the set of feasible utility profiles, this choice represents a seriously restricted domain.

We consider now for any coalitional game $V \in \mathcal{G}$ the game in strategic form $\Gamma^V := (S, \pi^V)$. Let E be an equilibrium concept, i.e. some non-specified refinement of the Nash equilibrium. We can state the following

Lemma: Assume that for every $V \in \mathcal{G}$ we have $\pi^V(E(\Gamma^V)) = L^*(V) \neq \emptyset$. Then the mechanism (S, h) weakly E-implements $\mathfrak{L}^*$. Moreover, for every $V \in \mathcal{G}$ we have:
$u^V \circ \mathfrak{L}^*(V) = L^*(V)$.

Proof:

Let $\hat{\sigma}^V \in E(\Gamma^V)$ be any equilibrium of Γ^V. By the definition of the outcome function h we get:

$h(\hat{\sigma}^V)(V) = \pi^V(\hat{\sigma}^V)$. By assumption this is an element of $L^*(V)$. Therefore, there exists a selection $l^* \in \mathfrak{S}_{L^*}$ such that

$$l^*(V) = \pi^V(\hat{\sigma}^V) = h(\hat{\sigma}^V)(V) = u^V \circ h(\hat{\sigma}^V)$$

which implies

$$h(\hat{\sigma}^V) \in [l^*]_V \subset \mathfrak{L}^*(V)$$

and thus

$$h(E(S, \pi^V)) \subset \mathfrak{L}^*(V).$$

This establishes the weak E-implementation of $\mathfrak{L}^*$.

The second claim of the Lemma results from the following chain of inequalities:

$u^V(\mathfrak{L}^*(V)) = \{u^V([l]_V)|l \in \mathfrak{S}_{L^*}\} = \{u^V(l)|l \in \mathfrak{S}_{L^*}\} = \{l(V)|l \in \mathfrak{S}_{L^*}\} = L^*(V).$

■

The proof makes the two factorizations explicit which I discussed in section 2.

Mathematically, the social choice rule $\mathfrak{L}^*$ is a *lifting*. It lifts the solution, a correspondence from $\mathcal{G}$ to $\mathbb{R}^N$, to the utility vector function u^V, interpreted as a map from the quotient space $A/\sim_V$ into $\mathbb{R}^N$. Here $\sim_V$ denotes the V-equivalence on A defined above, i.e. $l \sim_V l' \Longleftrightarrow l' \in [l]_V$.

Notice, that it is the "solution correspondence" $\mathfrak{L}^*$ rather than the solution L^* itself that plays the role of the social choice rule in our model.

4 Maskin-Monotonicity

The monotonicity of social choice rules or functions, first introduced by Maskin (1977), is a necessary but not sufficient condition for their Nash implementability. This holds true for various versions of weak, full or strong Nash implementation [cf. Osborne and Rubinstein (1994), Mas-Colell, Whinston and Green (1955)]. The use of the qualification "weak" in the literature refers to weakenings of equality to inclusions in either direction. While the weak Nash implementation of a social choice function used in Mas-Colell, Whinston and Green does not require its monotonicity, the weak Nash implementation of a social choice rule as defined by Hurwicz (1994) and used in the present paper, when applied to social choice functions becomes strong Nash implementability in the sense of Mas-Colell, Whinston and Green (1995).

In our present framework standard arguments easily show that Nash implementability of $\mathfrak{L}^*$ implies its Maskin-monotonicity. On the other hand, as Maskin monotonicity is not a sufficient condition for Nash implementability, there might exist solutions L^* where the derived social choice rule $\mathfrak{L}^*$ is Maskin-monotonic but nevertheless not Nash impementable. Clearly, because of our Lemma then the underlying solution L^* is not Nash supportable either. As the lack of Maskin-monotonicity of the Kalai-Smorodinsky and the Nash bargaining solutions has been used in the literature [cf. Bergin and Duggan (1996), Howard (1992), Dagan and Serrano (1997), and Serrano (1997)] to establish the impossibility of their Nash implementation it may be useful to notice that in our framework these solutions are Maskin-monotonic as is any Pareto efficient singleton-valued solution l^*.

Claim:

If $l^* \in A$ is Pareto efficient, then $\mathfrak{L}^*$ is Maskin-monotonic.

Proof:

We have to show that for any V, $V' \in \mathcal{G}$ the following holds true:

$l \in \mathfrak{L}^*(V), l \notin \mathfrak{L}^*(V') \Longrightarrow \exists i \in N, l' \in A$: ${u_i}^V(l) \geq {u_i}^V(l')$ and ${u_i}^{V'}(l') > {u_i}^{V'}(l)$.

We choose $l' := l^*$. It remains to show that $\exists i \in N$ such that

1) $u_i^V(l) \geq u_i^V(l^*)$
2) ${u_i}^{V'}(l') > {u_i}^{V'}(l)$.

1) follows (as equality) from $l, l' \in \mathfrak{L}^*(V)$.
2) follows from the Pareto-efficiency of l^*. Indeed, $l \notin \mathfrak{L}^*(V')$ implies: $\exists i \in N : {u_i}^{V'}(l) \neq {u_i}^{V'}(l^*)$. Pareto-efficiency of l^* excludes that for all $i \in N$: ${u_i}^{V'}(l) \geq {u_i}^{V'}(l^*)$ with strict inequality for at least one $i \in N$. Hence, there must be some $j \in N$ with ${u_j}^{V'}(l^*) = {u_j}^{V'}(l') > {u_j}^{V'}(l)$, which implies Maskin-monotonicity.

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5 Concluding Remarks

The present paper demonstrates a possible way of embedding the Nash Program into implementation theory. Thereby it provides the basis for some implementation results [cf. Trockel (1999), Haake (1998) and Naeve (1999)] which contrast several impossibility results in the literature. This paper does not claim, however, that looking at the Nash Program as a part of mechanism theory is particularly natural. As argued in Section 1, the goals of the Nash Program and of implementation theory are very different. Also, the informational context of the Nash Program relates only to decentralization aspects rather than to information eliciting aspects of implementation theory.

This paper shows, however, that even under the heroic assumption of a purely welfaristic framework for game theoretic analysis we are not automatically condemned to deprive ourselves of potential applications of mechanism theory.

It may also be questioned whether the outcome space and the mechanism employed for our above embedding lemma are very reasonable from a practical point of view. Such considerations, however, lead us immediately back to the question to what extent the presently established modelling of implementation via game forms is an adequate one.

Again, to answer that question we would have to deal with the problem of what "reasonable" mechanisms are. Our understanding of this problem is still in the beginning [cf. Jackson (1992), Jackson, Palfrey and Srivastava (1994)]. At least, our method is not dependent on integer or modulo games as it applies to every game that supports a certain solution.

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