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ESSAY-WRITING IN MATHEMATICS (SECONDARY SCHOOL, 7TH TO 12TH GRADE)

Abstract:

Adequate *mathematisation* – the transformation of subject matters into mathematical structures – is a major difficulty in mathematics teaching: Students often think of mathematics as a *non-sense* science, or they see it as *MatheMagic*. Their paradigm for doing applied mathematics is the effortless and elegant celebration of prefabricated solutions. Mathematics education must address such *phenomena* in the broadest sense. In order to overcome such shortcomings, a didactic approach is here proposed: *essay-writing in mathematics*. It has been tested in two classes of 7th grade students of high achievement level. Empirical results show that, compared to solving traditional word-based number problems, *essay-writing in mathematics* facilitates a more subject-oriented and conceptual argumentation in the process of mathematisation. This effect becomes most evident among female students, while male students also show a similar tendency (HOLLENSTEIN 1996).

1. What made me tick?

A major difficulty in teaching applied mathematics is the mathematisation of problems in context (COHORS-FRESENBORG, 1996). Mathematics educators often refer to the so-called captain's syndrome (BARUK, 1989; FREUDENTHAL, 1984; STERN, 1992) which describes situations in mathematics courses whereby students tend to solve nonsense puzzles without any hesitation, as long as the given numbers can be manipulated according to some informal conventions. (*17 sheep and 11 goats are on board of a ship. How old is the captain?* A consistent number of students in a class will answer: *17+11=28. The captain is 28 years old.*)

Mathematics education has often discussed explanations for such strange behaviour.

(1) Many students think of mathematics as a non-sense science (BARUK, 1989; BURMESTER & BÖNIG, 1994; SCHOENFELD, 1988; STERN, 1992, 1994). Context problems are seen as „one-way-wrappings for calculations". By employing informal strategies, the hidden calculation can be unpacked, the wrapping disposed of and the calculation can then be executed.

(2) Problem solving is seldom modelled by mathematics teachers as an activity demanding endurance. Making mistakes or even ending the process without a valuable result is unthinkable in a traditional mathematics course – or even worse, it is considered a flaw in pedagogy. On the contrary, the teacher usually aims at an immediate, elegant and effortless problem solving technique. They tend to hide the actual process of mathematisation with all its inherent uncertainty or pain. Mathematisation looks like the *deus ex machina* in classical Greek drama, where a god appears – out of nowhere – transforming *catastrophe* into heavenly order. Traditional mathematics teaching is thus oriented towards results and answers, while neglecting the underlying thinking processes (BARUK, 1989; SCHOENFELD, 1988).

(3) Highly standardised procedures, intended as an aid to students in their tackling of context problems, in fact cripples their problem solving abilities (GALLIN & RUF, 1990; GAMPER, 1983; GINSBURG, 1977; HENGARTNER, 1992, 1994, 1995; WITTMANN, 1991; WITTMANN & MÜLLER, 1990, 1993, 1994, 1996). The ensemble of standardised problem solving procedures constitutes a psychological *framework* of the „mathematics class" (BAUERSFELD, 1983; RADATZ, 1983, 1986; SELTER, 1994), in which subject-oriented mathematical ar-

gumentation, based on insight and common sense, is unusual. There is „problem solving beyond the logic of things“ (REUSSER, 1986): Every facet of a given problem ought to have its hidden standard procedure. By analysing the *surface structure* of the problem text, students are merely trying to figure out which conventional formula to choose.

2. Research questions and hypotheses

In the field of applied mathematics, all the traditional patterns mentioned above concentrate on the *solving of word/number puzzles.* In extreme cases, the teacher and student may work with text based problems without ever getting involved in subject-oriented problem solving. They are guided by standardised formulations and compatible procedures. Therefore, solving the „word/number puzzles“ sometimes takes on the quality of ritual behaviour and does not consist of any kind of conceptual assimilation of „reality“. With this in mind, I have developed a method of teaching applied mathematics, that of *essay-writing in mathematics*.

2.1 Theoretical Background

a) Mathematics teaching can be seen in the context of language and communication. Two pedagogically important ideas blend in my work:
   1) GALLIN & RUF‘s ideas (1990, 1993, 1995) about *mathematics and language* – oriented towards the practice of teaching, in which the main idea is that of journal writing in mathematics. Setting out from carefully chosen problems, the students first develop their own *individual mathematical world*. They search, on their own, for approaches to and ways of solving the given problem. These individual lines of thought are registered in the journals, entitled „diary of a journey“. In a second phase, the divergent ideas, approaches and methods which result are discussed in the group. The group tries to understand the different approaches in this divergent world of thought. It is in the third phase that the teacher places these ideas in the regular world of school mathematics.
   2) Since the early seventies cognitive constructivism has become the theoretical backbone of education in mathematics. Meanwhile social constructivism has also gained importance. Essential to my work are the concepts of shared cognition or co-operative learning (DUIT, 1997; HUBER et al., 1992; RESNICK, LEVINE & TEASLEY, 1991). The concept of socially mediated learning is also very helpful in the analysis of the processes involved in meaningful argumentation during problem solving (WERTSCH, 1991).
b) Shortcomings in teaching applying mathematics, as outlined above, cannot be resolved by the infinite refinement of the traditional „step-by-step“ instruction. By making the steps even smaller, the students will only increase their alienation from the genuine mathematical thinking and problem solving (HENGARTNER & RÖTHLISBERGER, 1995; KRAUTHAUSEN, 1997; WIELAND, 1996; WITTMANN et al., 1994). It is not instruction through small rigid steps that will enrich mathematical thinking, but rather learning through active discovery in well designed learning environments like open situations (EGGENBERG & HOLLENSTEIN, 1997). This kind of learning environment has to be substantial, representing important objectives, content and principles of mathematics. Their potential for mathematical activity must be extensive, they must be flexible and adaptable in the hands of the teachers and they should integrate mathematical, psychological and pedagogical aspects of learning and teaching (KRAUTHAUSEN, 1997). Substantial problems foster the potential for natural inner differentiation (WITTMANN et al., 1997) in a class or a

group of learners – the students define the level of difficulty by their subject-oriented choices and decisions.

c) With BAUERSFELD (1988) and WITTMANN (1995) the discipline of mathematical education is seen as a design science. In this view, it is not possible to gain insights into methods or teaching strategies merely by deduction from related disciplines such as mathematics, pedagogy and psychology. The fundamental question is how mathematics teaching can be designed to achieve their objectives. During the design processes, of course, concepts from related scientific fields are crucial. But „[...] the quality of these constructs depends on the theory-based constructive fantasy, the „ingenium“, of the designers, and on systematic evaluation, both typical of design sciences“ (WITTMANN, 1995, p. 13). CLARK and YINGER (1987) call teaching a design profession, and thus mathematics education can be seen as a design science. It may be compared to some aspects of medical science in which forms of therapy are designed, theoretically analyzed and empirically tested. Engineering science shows some parallels, too. It is a reflective practice of planning and design, followed by testing and evaluation. The aim of my work is to design a learning environment which enables students to construct mathematical models for problem solving in context, thereby overcoming the weaknesses of the traditional „doing word/number puzzles“.

d) Mathematisation of problems in context is not as if an „immediate enlightenment“ strikes like lightning. It is a process which demands time and effort, requiring strategies and planning. In this way the learner can analyze and influence the process itself. Metacognition and heuristic knowledge are essential (BAER et al., 1994c; FLAVELL, 1985). This metacognitive faculty is well established in the practice of traditional essay writing and can be easily transferred to mathematics teaching.

e) Teaching experience and empirical research has pointed out gender weighted differences in learning. In doing mathematics, female learners appeaer to act differently from male learners (JUNGWIRTH, 1991; DICK, 1992; Effe-STUMPF, 1992; HORSTKEMPER, 1992; RICHTER & BRÜGELMANN, 1994):

   1) Even high achieving girls show a tendency to attribute low success in mathematics to the „I‘m not gifted for maths“ syndrome, which runs contrary to language studies, where they attribute failure to statements such as „I‘m kind of gifted in this field but I didn‘t work hard enough“. Boys, on the other hand, do not show this tendency to contrast attribution for low achievement in mathematics to attributions for failure in branches like literature, human sciences or language studies (ECCLES, 1989). My hypothesis is that the self-confidence and trust in themselves shown by female students with regard to their own essay-writing abilities is also evident in essay-writing in mathematics.

   2) Female students care more about the social aspects of classroom life. They tend to support learning processes by fostering interaction and dialogue, whereas male students are often described by teachers and students as being „self-centred“ or even „disturbing“ in their classroom behaviour (HERZOG, 1989, 1995). An essential part of *essay-writing in mathematics* is the sharing of approaches, solutions and ideas, as opposed to the typical aftermath discussion of the traditional word/number problems which are largely restricted to „what‘s the answer?“ or „how do we do it correctly?“.

   3) HORSTKEMPER (1989a), NETH & VOIGT (1991) et al. found in both mathematics and natural science classes that there were gender related differences in the use of language: Female students strongly prefer informal rather than formal language. They prefer to

discuss the topics and the background in a broader sense, tending to avoid mathematical formalisations as long as possible.

2.2 Basic Thesis

My *basic thesis*, therefore, is that traditional essay-writing in school generates a meta-cognitive and heuristic way of thinking, as well as problem solving strategies and attitudes towards the time consuming composition of a text. This *framework* of conventions (FREUDENTHAL, 1984), which is fundamentally different from the traditional framework of „doing verbal puzzles", is transferable to mathematics courses. This spre-existing knowledge may and should be used to solve applied mathematics problems.

2.3 Essay-writing in mathematics – a didactical approach

Essay-writing in mathematics means essentially the solving of text-based[1] problems in the field of applied mathematics. The given problem – the topic of the essay to be written – is called a *torso*, which shows an *unfinished gestalt* (WERTHEIMER, 1945). Writing the essay means choosing an interesting aspect of the given semantic structure and striving for a *good gestalt*, completing the *torso's* structure in this very aspect (AEBLI, 1981, 1991). All the essays emerging from this process have a common core element, the *torso*. But they may be very different in their objectives, ways of dealing with the problems and methods of tackling the problem.

The learning environment of *essay-writing in mathematics* shows the following properties:

(1) The teacher introduces the new kind of text to be written by the students: the „mathematical essay". Mathematical essays may be composed of written language, calculations, sketches, diagrams, symbols or pictures. It is important to stress the *framework* of essay-writing[2] as it is known from language-oriented courses. The students ought to know the outlines of the actual procedure (time frame, working conditions, organisation and so on).

(2) The problem text is presented and discussed. This is to be the theme for the mathematical essay.

(3) Students look for interesting questions that may be – at least partly – answered by the calculations. They may also formulate statements to be supported or denied by meaningful calculations. The students choose their respective topics and start to write the essay.

(4) After finishing the writing process, essays are presented and discussed in class.

(5) The criteria of assessment and content of the discussion should not simply be „correct or not". What is essential is the type of topics raised by the student(s), the originality and line of argumentation, the adequacy and suitability of the calculations, elegance and economy of style ... the *quality* of the essay.

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[1]Beck & Maier (1994) introduce the concept of *text* into mathematics education. Going back to Ricoeur (1986), the *text* is defined in a broad sense as any represented and „readable" information. A similar concept is used by Weidenmann & Krapp (1986) and Doelker (1993). Written language, ikons, pictures, sketches, diagrams, symbols, are all *text*.

[2]In a pilot study, we asked some students to *invent word problems* related to a given torso. The outcome was stunning: They showed a strong tendency to look out first for number combinations they knew how to calculate in some way. Secondly, they invented „stories" to wrap up and hide the envisaged calculations. They produced caricatures of the *word/number puzzles* written by teachers and authors of textbooks. Perceptual subject oriented argumentation was almost entirely absent.

2.4 Hypotheses

- *Essay-writing in mathematics* fosters natural inner differentiation. The problems generated by the students are different in terms of content and level of complexity.
- Compared to the processes involved in solving traditional word/number puzzles, *essay-writing in mathematics* facilitates mathematisation and problem solving based on subject-oriented conceptual argumentation. Mathematisation based on syntactical properties of the given problem text are less frequent in an experimental group writing mathematical essays than in a control group doing traditional word/number puzzles.
- Fostering subject-oriented argumentation by *essay-writing in mathematics* is more important among female than male students.

3. Empirical design

Two classes of high achieving students, 42 in all, 7th grade, ages from 12.5 to 15.0, collaborated on this project. To be accepted as a student by their high school, all of them recently passed a test in mathematics, German, and French as a foreign language. The experimental data was gathered two and three weeks after they started at their new school.
A week before the gathering of data, the two co-operating classes were contacted. Starting out from a *torso* („Dinosaurs"; ERICHSON, 1992), the students looked for feasible topics to write about. They were told to „ask different questions that may be answered by calculations or to formulate statements that could be upheld or denied by numbers and calculations". The ideas raised were discussed in the class, although no essays were written at that stage (one lesson). In a second lesson, the students divided into pairs for the subsequent co-operative work. The students were free to choose their partners and generally they paired up with the same sex. There was only one mixed pair in each class, the „leftovers" from the grouping process.
These dyades were placed either in an *experimental group* or a *control group*. These groups were balanced according to the following criteria: sex, interest in writing essays, interest in doing word/number puzzles, and factors of motivation (HERMANS, 1976).

3.1 The experimental work

The *experimental group* carried out the work in the learning environment of *essay-writing in mathematics*. Each dyade is confronted with a sheet of paper containing the torso „The Eagle" (ERICHSON, 1992):

Together you will write a short mathematical essay.

- *Read the text „The Eagle". It is the theme of your project.*
- *Search together for some feasible topics for your mathematical essay: Ask different questions that can be answered by calculations, or formulate statements that may be upheld or denied by numbers and calculations.*
- *Choose one of these topics for your mathematical essay.*
- *Start writing an outline for the essay. It should in the end contain sentences and calculations, and perhaps some drawings, sketches and diagrams.*

The Eagle (Historic drawing: Inauguration of the Railway Line from Nürnberg to Fürth, 1835)
On December 7th. 1835, the first railway line in Germany was inaugurated. It connected Nürnberg with Fürth and was a mere 6 km long. The engine towing the train was bought from

the factory owned by the famous George Stephenson. It was named „The Eagle“. At precisely 9 o‘clock, a cannon shot gave the signal for the start. Ten minutes later the train had already reached its destination. William Wilson, the engineer driving „The Eagle“, came from Britain, too (for safety reasons). He was paid like a star: 2250 marks was his yearly salary; the director of the railway company earned only 1360 marks. „The Eagle“ was a huge success. In the first year 448,950 tickets were sold, at 21 cents per ticket. The value of the mark was different from today‘s DM (Deutsche Mark).

	At that time	***Today***
Ticket Nürnberg – Fürth	*21 cents*	*2.80 DM*
1 kg of bred	*30 cents*	*3.- DM*
1 liter of milk	*12 cents*	*1.20 DM*
1 kg of meat (veal)	*1.20 Mark*	*24.- DM*
1 kg of sugar	*60 cents*	*2.40 DM*
1 egg	*4 cents*	*30 cents*

b) The dyades in the control group solved traditional-looking word puzzles, which were constructed by dividing the experimental text into numbered sections, and convergent questions were added.

Together you will solve the following problems:

The Eagle (Historic drawing: Inauguration of the Railway Line from Nürnberg to Fürth, 1835)

1. *On December 7th. 1835, the first railway line in Germany was inaugurated. It connected Nürnberg with Fürth and was a mere 6 km long. The engine towing the train was bought from the factory owned by the famous George Stephenson. It was named „The Eagle“. At precisely 9 o‘clock, a cannon shot gave the signal for the first start. Ten minutes later the train had already reached its destination.*

What was „The Eagle‘s“ average speed during its maiden voyage?

2. *William Wilson, the engineer driving „The Eagle“, came from Britain, too (for safety reasons). He was paid like a star: 2250 marks was his yearly salary.*

   How much did Mr Wilson earn in a month?

3. *The director of the railway company earned only 1360 marks.*

   How much more did Mr Wilson earn in a year than the director did?

4. *„The Eagle“ was a huge success. In the first year 448,950 tickets were sold, at 21 cents per ticket.*

   How much was the intake for the first year?

5. *The value of the mark was different from today‘s DM (Deutsche Mark).*

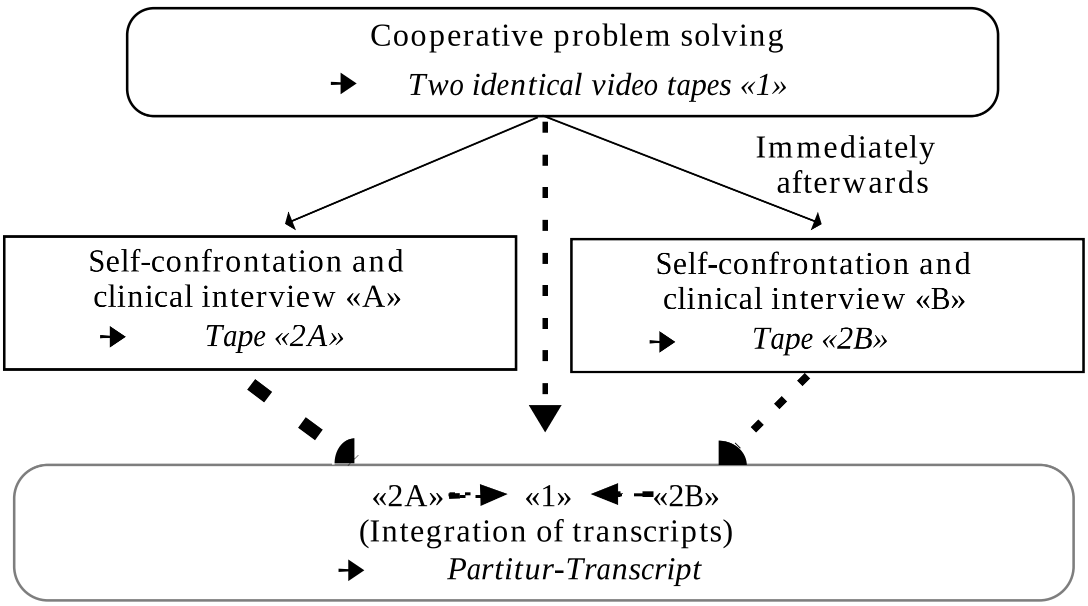
	At that time	***Today***	*How many times?*
Ticket Nürnberg–Fürth	*21 cents*	*2.80 DM*	*approx. 13 times*
1 kg of bred	*30 cents*	*3.- DM*	
1 liter of milk	*12 cents*	*1.20 DM*	
1 kg of meat (veal)	*1.20 Mark*	*24.- DM*	
1 kg of sugar	*60 cents*	*2.40 DM*	
1 egg	*4 cents*	*30 cents*	

How much would Mr Wilson‘s salary be worth today, in Swiss Francs? (exchange rate: 83.5)

Before the data gathering session, the students got used to watching and hearing themselves on video. During the co-operative pair-work stage, each dyade worked alone in a room, with the video camera in front of them. After 25 minutes, they were interrupted and two clinical interviews, based on self-confrontation, were carried out in two separate rooms. This second stage of data gathering was based on the *Re-Interview Technique* (WILD, 1994), a form of *self-confrontation.* Viewing their own performance on video, the students were interviewed separately and each student was asked to comment and interpret the dyade‘s process of defining and solving problems.

There are, therefore, three different recordings for each dyade: First, the video documentary of the co-operative pair-work session. Second, the two recordings of the clinical interviews (each student explaining the dyade‘s behaviour), which were transcribed and directly added as commentary to the pair-work transcripts. This was by no means easy, as the two interviewers were free to stop the tape, to repeat interesting scenes, or to wind backwards and/or forwards. The resulting *„partitur“-transcript* followed the chronology of the pair-work, but broke up the chronology of the clinical interviews.

Empirical design



3.4 Interpretation of data

Tracking down the underlying processes meant interpreting these partitur transcripts as well as the corresponding mathematical essays written by each dyade. Analytical entities were to be isolated in the flow of the description and commenting processes. Avoiding a micro level of analysis, the work concentrated on *topics worked on by the pairs of students.* Each of these units of analysis – each one a problem tackled by a dyade – had to be interpreted and assessed. Two independent people carried out the interpretation of the data, one an experienced mathematics teacher, the other a researcher. Therefore two stages were involved: (1) units of analysis were defined in a co-operation process – single problems elaborated by the dyades were isolated; (2) these units were assessed by the two researchers separately, according to established categories. Differences in assessing a unit were discussed afterwards. In cases where no consent was reached, no assessment was given (HOLLENSTEIN, 1996; pp. 188–190).

The essays and their corresponding partitur-transcripts were interpreted from these perspectives:

a) What is the quality of the „corpus“ of elaborated problems raised by the writing of the mathematical essay? Of special interest are the richness and the level of differentiation.
b) To what extent is a subject-oriented and conceptual argumentation used to treat the chosen topic? Or how frequently do some syntactical properties of the given text trigger arithmetical operations (properties like the kind of given numbers, keywords used as clues for certain calculations, the quality of results emerging from trial and error use of the calculator ...)?

These assessments of strategy are based on the following categorisation:

- The dyade shows purely mechanical-associative behaviour. Problem solving is completely based on syntactical properties of the given problem text (*torso*), which triggers the arithmetical activities.
- Some logical steps taken in the problem-solving process are based on subject-oriented conceptual argumentation, some steps in the process show merely mechanical-associative behaviour.
- Every logical important step taken in the process is backed by a subject-oriented and conceptual argumentation. (There may be phases of mechanical-associative behaviour, but the logical steps concerned are discussed elsewhere, or they are of no logical importance in the line of thinking, somewhat like side actions, „playing“ with the calculator, teasing, etc.)

4. Results – discussion

4.1 The quality of elaborated problems – richness and level of differentiation

The types of problems generated and tackled by the *students writing an essay* are shown in the following table:

Type of problem (experimental group)	*Male dyades (5)*	*Female dyades(4)*
Speed (The Eagle, The Eagle compared to today‘s high speed trains, to a good middle-range sprinter, etc.)	7	3
Financial aspects of the railway company (Income in several years; dynamic systems based on annual rises in business volume; income vs. supposed costs)	14	8
Organisational aspects (scheduling manpower, trains, etc.)	1	4
Purchasing power, then and today	10	5
Misc.	1	1
Sum	33	21

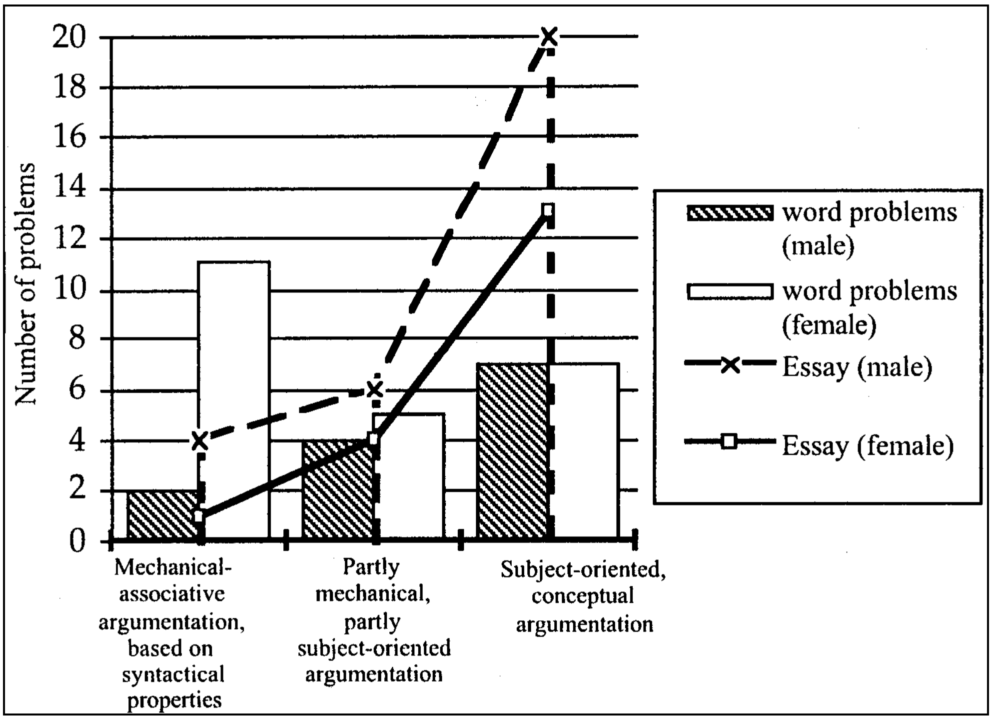
The content analysis supports the first hypothesis: *essay-writing in mathematics* seems to foster natural internal differentiation (KRAUTHAUSEN, 1997; WITTMANN et al., 1994). The problems worked on in the experimental groups are different from each other in terms of content and level of complexity. The variety is even astonishing. The corpus of issues raised by the experimental dyades in writing essays contains also openly formulated and divergent problems, i.e. similar topics are mathematised differently.
Meanwhile, the control group, of course, worked just at the given questions.

4.2 Extent of subject-oriented conceptual argumentation

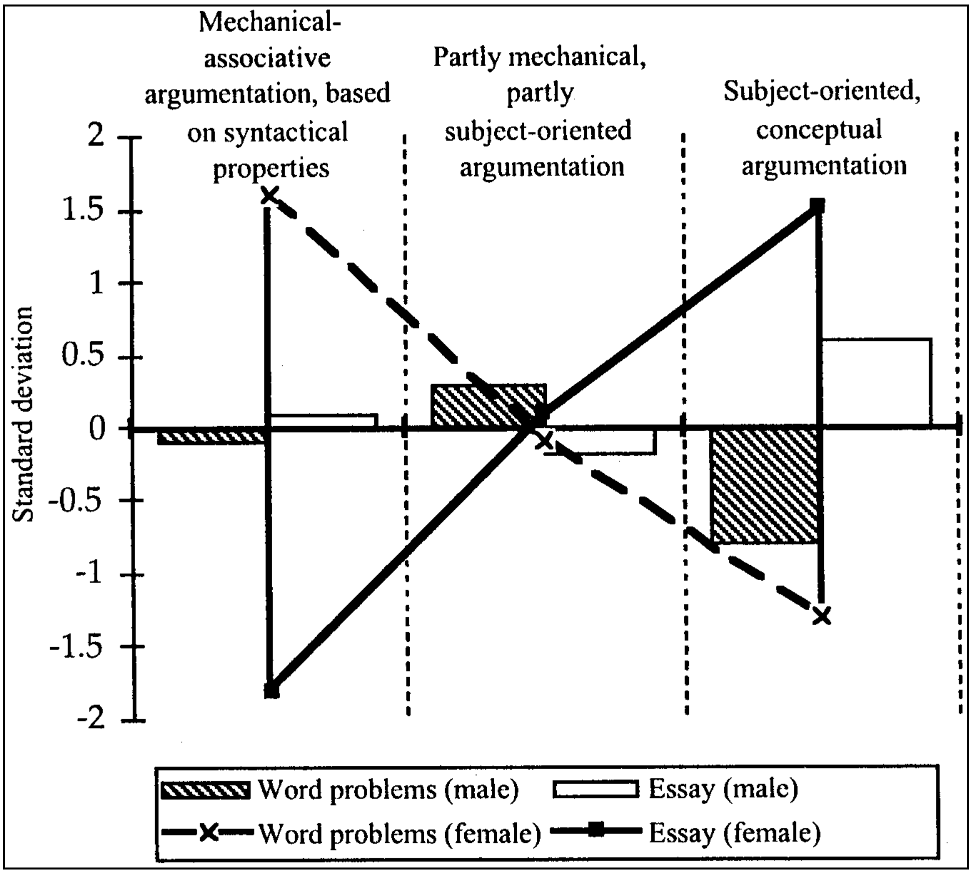
The numbers in the following table result from the interpretation and assessment process sketched above (HOLLENSTEIN, 1996, pp. 191–219).

	Solving traditional word problems		*Writing essays (experimental group)*	
Categories of strategy	*male*	*female*	*male*	*female*
Mechanical-associative, based on syntactical properties	2	11	4	1
Partly mechanical, partly based on subject-oriented conceptual argumentation	4	5	6	4
Based on *subject-oriented conceptual argumentation*	7	7	20	13
Not assessed	5	5	3	3
Total	18 *male*	28 *female*	33 *male*	21 *female*

Table: Strategy of problem solving – differentiated according to sex and didactic setting



Count of problems: Strategies of problem solving – differentiated according to sex and didactic setting



Standard deviation for Strategy of problem solving - differential according to sex and didactic setting

The significance of the gender-related differences is astonishing. Although I was fully aware of the research literature concerning this topic during the preparation stage of this experiment,

I was taken by surprise. The main quantitative hypothesis, which assumed a strong general effect, became subordinate to the gender-related one. The consequent supposition was the important one: All quantitative effects seen in this experiment are very clear as regards female students, whereas the effects for male students are minor.
Compared to the traditional word/number puzzles, the learning environment of *essay-writing in mathematics* seems quite clearly to facilitate mathematisation and problem solving based on subject-oriented conceptual argumentation. Solutions based on purely syntactic properties of the given problem text are less frequent in the work of the experimental group than in the control group doing traditional word/number puzzles.
There is also indirect evidence: The count of adequate mathematisation *as well as* the number of completely inadequate approaches to the problems concerned are significantly higher in the dyades doing traditional problem solving than in the experimental group. On the contrary, partly adequate mathematisation is more important among the essay-writing dyades. This effect allows for a conclusion regarding the underlying processes of problem solving: Among the pairs doing tradition word/number puzzles, mechanical-associative problem solving is widespread, which means that the count of „hits“ or „miss-hits“ is statistically higher than expected. But a subject-oriented argumentation is likely to result in partly adequate mathematisations.

Last but not least: A surprise result of the experiment is the massive *effect of mechanised thinking* (LUCHINS, 1971) in the control group. Parallel to the experiment, the mathematics class in school had been dealing with *proportions*. When the students came to calculating the „average speed of The Eagle“, almost all of them tried to use the quite complicated „official system“ of calculating proportions. They tried to calculate the would-be distance of an hour‘s cruising, but most of them failed (The Eagle takes 10 minutes to cover 6 km ... and these are high achieving students!). On the other hand, some dyades who were doing their essay ended up solving the same problem. None of them used the officially taught system and none of them failed! (HOLLENSTEIN, 1996; pp. 210–271)

5. What could this project mean for everyday school life and mathematics education in particular?

The proposed learning environment of *essay-writing in mathematics* is simple and efficient enough to be applied in everyday teaching. This is not to lay claim to some kind of „fundamental revolution“ but simply to represent a step forward in professional development.

The required work-load on the part of the teacher in setting up the learning environment for *essay-writing in mathematics* is moderate and it can be handled without any risky or far-reaching decisions for or against a new methodological approach. If the idea works, the practice of essay writing could be extended, even to other traditional didactic settings.

Essay-writing in mathematics clearly underlines the recent trends in interdisciplinary education, bringing together, for instance, the mastery of language, competence in formulating concepts and ability to articulate ideas systematically in almost any subject. Furthermore, this kind of mathematical activity stresses one of the most important *formative values* in mathematics.

In addition, female students seem to benefit enormously from this kind of approach, while male students show no negative reactions, and this may well be a way in future of encouraging female students to beat the pathway to mathematics and the sciences in general.

The students taking part in this project were by no means experienced writers and yet they showed significant stimulation. I can imagine that further experience would deepen the personal benefits to be gained not only in applied mathematics but also in the mastery of language.

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