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ON THE SUBJECTIVE CONCEPTION OF NUMBERS IN ELEMENTARY SCHOOL CHILDREN

Abstract:

A study is presented with the intent to explore the subjective conception and opinion of numbers in elementary school children during their development in the first to fourth grades, and to show by means of examples how and if such a conception influences their learning in elementary school mathematics classes. The individual conversations held up to this point consist of speaking about the subjective conception and opinion of numbers in terms of their spatial distribution and the personification of numbers (favourite numbers, least favourite numbers, the subjective association of numbers with people or objects or events) and to the association between numbers and colours. The results available to date in this study are shown in an overview and analysed. General didactic conclusions for the consideration of subjective conception of numbers in mathematics classes in primary school are drawn as well.

Today it is widely accepted that learning is an individually determined constructive process, (compare GLASERSFELD 1995, COPP 1996, BAUER 1988, BAUERSFELD 1983, BUSSMANN 1997, SFARD 1996). Because of this theory, interest in subject-related methodologies in the subjective conceptualization of pupils about mathematical circumstances and elements is growing. Their subjective conception of numbers is of great relevance in this context, because numbers are used in many different areas of daily life and, at the same time, numbers are a fundamental component of mathematics lessons from first grade on.

Subjective conception has a determining influence on dealing with numbers and according to a constructive viewpoint it is formed by individual processes of learning. A wide range of psychological studies was carried out to develop patterns of subjective conception of numbers and their association (compare inter alia MC CLOSKEY 1992, SOKOL 1991, DEHAENE, BOSSINIE et al 1993, GRAY & TALL 1994).

DEHAENE assumes that there are three cardinal conceptions of numbers referring to the verbal representation (e.g. „three“) and to the figure („3“) as well as to the analogical conception of its value in which „numerical quantities are represented as inherently variable distributions of activation over an orientated analogical number line“ (DEHAENE & MEHLER 1992, S. 30). DEHAENE & COHEN (1995) also assume that the semantic code of numbers is associated with a pattern of a number line on which certain parts will be activated if a meaning of numbers is called up. Furthermore, the association of the order of magnitude with its subjective conception on a number line is described by RESTLE (1970). With regard to the above mentioned studies two aspects should be analysed critically. Firstly all patterns by DEHAENE et al are apparently concentrated on conception of numbers obtained by interviewing adults. In view of the fact that children are highly imaginative in dealing with numbers and that their experiences with numbers are to a larger extent connected to sensory impressions, one can assume that children develop a diversity of conceptions, not always based on mathematical criteria. Therefore it is questionable whether patterns referring to the subjective conception of numbers on a number line as well as to a written representation are suitable to reflect the diverse association of children to numbers. Secondly the patterns don’t take into consideration the arithmetical correlation between numbers or special meanings of numbers in mathematical structures. That’s why the patterns mentioned seem to be drawn too „close“ even from a clearly mathematical viewpoint of numbers.

Interesting results in the field of subjective conception of numbers obtained by interviewing adults were published by SERON, PESENTI et al (1992). In their studies they refer to cases of visual conception of numbers, which are partly of a synaesthetic character. Furthermore they noticed the arrangement of numbers in a mental structure partly in a linear arrangement in accordance with the pattern of a number line. Some of the experimentees arranged the numbers in a lattice structure. Moreover certain numbers were arranged with certain colours, individually varying (compare LIETZMANN 1919, p. 143). Hereby SERON, PESENTI et al show a relatively complex viewpoint on possible subjective conceptions of numbers. But it has not been settled yet what subjective conceptions of numbers are developed by children in particular and whether such a conception is already formed during the elementary school period. (The question rises because SERON, PESENTI et al ascertain in the results of a survey of adults, that most of the interviewed persons formed a subjective conception of numbers between the ages of 5 and 8, and that these opinions - in accordance with the assessment of the persons surveyed - were apparently formed during the elementary school period (compare SERON, PESENTI et al, 1992)).

Special studies of individual conception of numbers, the subjective conception of arithmetical correlations between numbers were carried out by GRAY & TALL (1994). The results of their investigations show a clear difference regarding the quality of perception and the registration of the meaning of concrete arithmetical correlations with elementary school children like for example in the formal expression „3 + 2“. Gray and Tall „hypothesize that the successful mathematical thinker uses a mental structure that is manifest in the ability to think proceptually. ... The less able are doing a more difficult form of mathematics, which eventually causes a divergence in performance between them and their more successful peers.“ (GRAY & TALL 1994, p. 116)

These studies point out that subjective conceptions of numbers can influence the quality of mathematical abilities. It hasn’t been explored so far in what way varied subjective conceptions of numbers can influence the children’s attitude to maths classes.

The intent of one of my studies, having been carried out since 1995, consists of the desire

- to explore the subjective conceptions of numbers made by elementary pupils in their development in the first to fourth grades by means of individual conversations,
- to study the stability of subjective conception during a certain period,
- to test whether the above mentioned results by SERON, PESENTI et al can also be applied to children
- to investigate by means of examples, how and if such subjective conceptions of numbers influence the learning of children in mathematics classes in elementary school.

In order to relate to the variety of subjective conceptions of numbers with children I decided to hold one-to-one conversation with the children. The investigation took into consideration results published by DEHAENE & MEHLER (1992), by DEHAENE & COHEN (1995) as well as aspects described by SERON, PESENTI et al (1992) GRAY & TALL (1994).

In all, the individual conversations with the children dealt with their subjective conception of

- the arrangement or geographical distribution of the numbers,
- the personification of numbers (favourite or least-favourite numbers, subjective assignment between numbers and people or objects) and
- any relationship between numbers and colours (see also LIETZMANN 1919, p. 142 - 143).

In the first investigation, 27 pupils just beginning school (class consisting of 10 girls, 17 boys) as well as 27 third- and fourth-graders (11 girls and 16 boys, who were taking part in advanced math classes)[1] were interviewed.
In order to make a comparison between the children's results possible the individual conversations were carried out in the same manner in each case. As an introduction, I mentioned to every child that the conversation would revolve around his or her personal relationship to numbers. Then each child was given the first assignment: *Try to imagine the numbers up to 10. Can you put them in a certain place? Where do you think the three should be? What about the number one? Six or seven? Here is a clean sheet of paper. If you were to assign the numbers a specific place, if you believe that the numbers belong in a certain order, then put them where you think they should be!"*

After a child had drawn his/her „number map" he/she was asked about a favourite number, a least-favourite or bad-luck number, and the subjective assignment of a number to a person, an object, an event or a colour. In this comparatively open conversation the reason for this link was always asked. The children's answers were either immediately written down or were recorded on tape. When analysing the children's answers one child's answers at various interviews days were compared to each other as well as to the answers of other children. In order to make a „subjective conception pattern" of the varied results, it seemed wise to group the approximately overlapping answers in each area together.
In addition, conversations were held with the classroom teachers and new teachers who were observing in math classes, in order to recognise influences on subjective number conception in the elementary school.

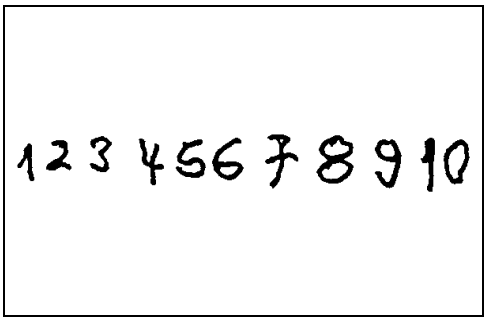
Present results:

1. Subjective conceptions of the spatial arrangement of the numbers up to 10
The following table gives a general overview of the subjective conception of the spatial distribution or, respectively, the arrangement, of the numbers between 0 and 10 during the first set of interviews in the 1995/96 school year.

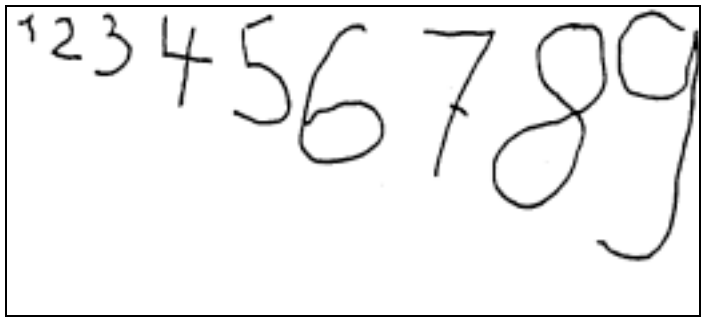
Principle of Distribution/ Arrangement	**School Beginners**	**Pupils in 3rd and 4th grades**
straight line arrangement, with respect to their value	19 children (6 girls, 13 boys) → ≈ 70 %	9 children (6 girls, 3 boys) → ≈ 33 %
wavy or zigzagged lines in the arrangement, in accordance with their value	4 children (2 girls, 2 boys) → ≈ 15 %	3 children (1 girl, 2 boys) → ≈ 11 %
unusual distribution or arrangement of the numbers according to different criteria	4 children (3 girls, 1 boy) → ≈ 15 %	15 children (3 girls, 12 boys) → ≈ 56 %

The linear arrangement of the numbers in accordance with their value was representative for large number of those children just starting school:

[1] Due to the selection process of the pupils, a comparison of the opinions of the beginning pupils is only possible in a very limited sense. In further studies it is planned that the development of the subjective number concept of the first graders who were interviewed is analysed up to the end of the fourth grade. It is planned that the interviews will take place twice each school year.

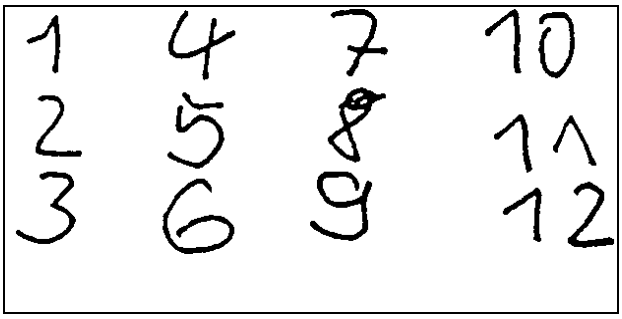


Clemens (6 years): *„If you put the numbers like this it is easier to count them.“*

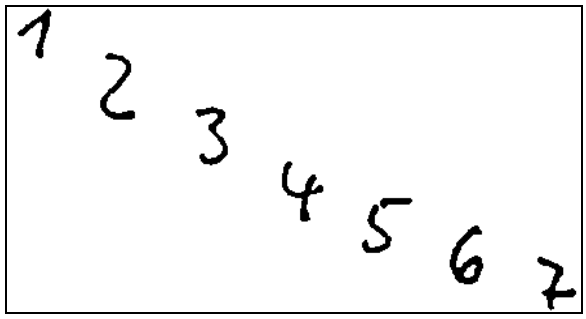


Karsten (7 years): *„The numbers belong in a line because that's how we're learned it. And they should really get bigger because their value keeps getting bigger.“*

A link to number patterns[2] or numerical connections from different aspects of life were clear with children who arranged the numbers in wavy or zigzagged lines:

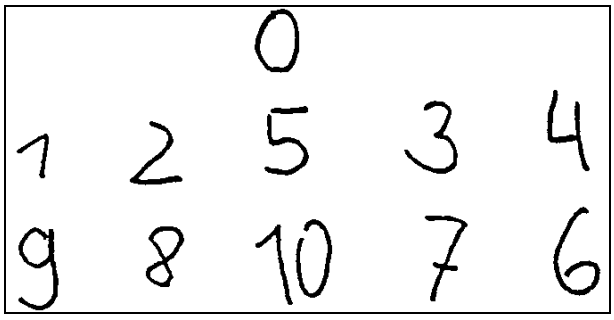


Benjamin (7 years): *„The numbers have to be written in columns. That's how they are in vending machines, calendars or the newspaper.“*



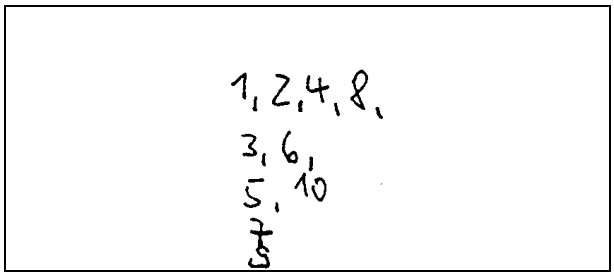
Fanny (9 years, 3rd gr.*): „I thought of stairs you can go down. Sometimes I count on the steps when I climb stairs“*

The following answers are characteristic of the numerous unusual arrangements of numbers in the advanced group of third and fourth graders:



Michael (9years, 3rd gr.): *„I think that when the numbers are all mixed up it looks more colourful and interesting. It is an imaginary arrangement. I think that every child has another arrangement. And I think that's good. I think it's boring that we only have one arrangement in school. Math is one of my favourite subjects. When I calculate an answer I use my imagination and add up the numbers differently. My teacher doesn't know. The exercises „Kreuz und Quer“ (like a magic square) and puzzles are my favourites.“*

[2] A special form of representation of patterns of numbers was the lattice structure mentioned by SERON, PESENTI et al which apparently shows relations to arrangements of numbers on calendars, vending machines or in a „hundred field“ - as Benjamin's subjective „number-picture“ expresses.



Christian (9 years, 3rd gr.): *„I started with 1, then doubled the number. Then I started over - with 3, because that was the first number that was missing. Then I doubled it, etc. until I had all of the numbers. I like doubling. It's easy. When I do a math problem I usually just think about the problem, but sometimes I think about my number arrangement.“*

The examples lead us to assume that the arrangement of the numbers in mathematically gifted children are apparently based on meaningful mathematical structures. This assumption, however, is only partially supported by our results to date. Some third and fourth graders sort the numbers according to even and odd numbers, or other children form „number bundles“, in which it is easier to add the numbers (for example, 2 + 1+ 7= 10). But there were also gifted pupils who sorted the numbers - like other children with otherwise average mathematical abilities - according to the form of the number or the subjective popularity of the numbers, or according to certain configurations (for example, the „five-dot dice form“). All in all we noted a wider variety of subjective conception of numbers and the distribution or arrangement of the numbers. It was especially interesting that many mathematically gifted children wrote down unusual arrangements of numbers only partly expressing meaningful mathematical structures.

2. Subjective conception regarding the personification of numbers

The following table shows the number of those pupils of both age groups where personification of numbers existed.

	School Beginners	**pupils from grade 3 and 4**
favourite number	27 children (10 girls, 17 boys) → ≈ 100 %	25 children (11 girls, 14 boys) → ≈ 93 %
least favourite number, bad luck number	26 children (10 girls, 16 boys) → ≈ 96 %	18 children (6 girls, 12 boys) → ≈ 67 %
subjective association of a number with a certain person or object	2 children (1 girl, 1 boy) → ≈ 7 %	13 children (6 girls, 7 boys) → ≈ 48 %

From the table can be seen that all or almost all of the school beginners questioned had at least one favourite number and one least favourite number. In comparison to that almost all of the „gifted“ third grade math pupils had a favourite number, but „only“ two-thirds of the children had a least favourite number. The clear difference with regard to a least favourite number might be linked to the pupils' different attitude to mathematics. The fun that many of the mathematically advanced children had with puzzles and „playing“ with numbers, and also the expanded experience of these gifted pupils in the third and fourth grades could be an important reason for the fact that many more of the advanced children show a subjective association of a number and a person or object when they are just beginning school. More indepth studies are needed in order to justify these possible and primarily speculative relationships. The fol-

lowing examples illustrate the astounding variety of content for the personification of numbers mentioned by the children.

Examples for favourite numbers:
Maria (7 years): *My favourite number is 1, because it is easy to write and because it is the best mark you can get.“*
Robert (10 years): *I like the number ‘2’ best because it is an even number and it’s easy to do calculations with.“*

Examples for least favourite numbers:
Daniel (7 years): *„ I don’t like ‘0’ because it doesn’t accomplish anything, although there are two zeros in 100* (his favourite number)*. Zero is so dumb, it means so little. It is bad because it has no value.“*
Anne (7 years): *„My least favourite number is 10. It looks so cheeky and difficult. When I write it I find it scribbly.“*
Sven (9 years): *„I don’t like the number 6 because it is the worst mark the teacher can give you. In my opinion we don’t need the six at all.“*

Examples of the association of a number and a person:
Daniel (7 years): *„The number 12 is like a ghost, because at midnight the clock strikes twelve times, and then the ghosts come.“*
Mandy (10 years): *„ When I go into my room then I always think of the number 5, because the posters on my wall all have my favourite group with 5 singers.“*

3. Subjective conceptions of numbers regarding the association of a number with a certain colour

The first interview conducted in the 1995/96 school year yielded the following statistics on subjective conception, regarding the association of a number with a colour:

	School Beginners	**Pupils from grade 3 and 4**
Presence of a subjective association of a number and a colour	13 children (5 girls, 8 boys) $\rightarrow \approx 48\ \%$	13 children (7 girls, 6 boys) $\rightarrow \approx 48\ \%$

This table shows that half of the children in each group of pupils felt an association of a number with a certain colour. This is apparently independent of the age, gender and mathematical ability of the children. The following examples illustrate the variety of the children’s reactions to numbers and forms:
Christoph (7 years): *„The colour red belongs to the number 5. Yellow is 2, blue is 3. That’s at least what pops into my mind most of the time. Somehow I got that stuck in my head.“*
Julia (9 years): *„ Yes, I do make such connections. For example, I think of white and 19. Nineteen is my favourite number, and white is my favourite colour. Sometimes we play like that, too: my favourite number, my favourite colour, my favourite animal. Dark blue and the number 45 belong together, too.“*
Lars (9 years): *„White -1, yellow - 2,... the colours get a little bit darker as the numbers get bigger. That’s just what I think, I got that in my head somehow.“*

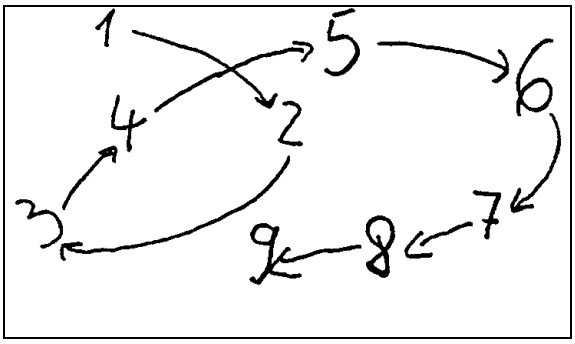
4. Changes in the subjective conception of numbers during elementary school

Three months after the first conversations, I interviewed the same pupils about their subjective conception of numbers once more. There were no noticeable changes in the third and fourth graders, but in the first graders, one third of the children had changed their number arrangement, and 70 % of the children had new favourite or least favourite numbers. The subjective association of numbers and people or objects as well as colours remained the same in most cases. A similar tendency was shown in the interviews to date with the 27 new pupils in the school year 1995/96 in the course of their second school year.

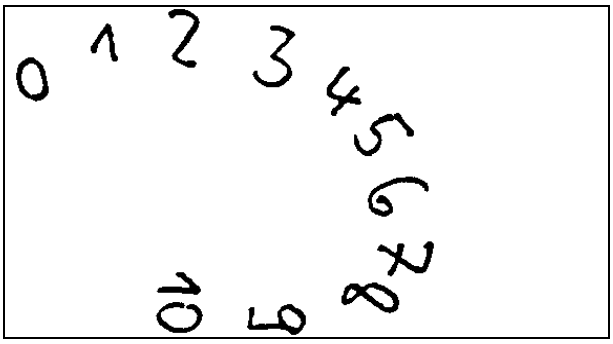
In detail, this showed that the „number map“ remained relatively constant especially with those children who had a linear method of arranging the numbers. (The overlapping of the subjective number arrangement with the thematic „official“ arrangement seemed to have more firmly cemented their number map.) In contrast, the pictures of children with unusual patterns changed in so far as their individual structures became clearer.

For example, Christian (6 years) first wrote the numbers 0, 5 and 10 directly under one another, because of a „feeling“, whereas he wrote the other numbers all mixed up and distributed all over the paper. After three months his individual number structure was easily recognisable; in the middle there stood the „threesome“ of 0, 5, and 10, and the other numbers formed „twosomes“ which all added up to 10 (for example 1,9).

The following example from Sarah also shows that her subjective arrangement of numbers „developed“, that the fundamental structure is more easily recognisable:



„I wanted the numbers to be distributed this way and not just one after the other“



„It is a circle. The numbers must form a circle. I like it this way. That's how it is on pullovers. I think it's nice.“

With respect to the comparatively noticeable change in the subjective conception of the connection between a number and a person, object or event, the empirical evidence shows that such conceptions are „area specific“ to the individual child's experiences (compare BAUERSFELD 1983). Everyday experiences, to which school certainly belongs, cause children to repeatedly change their favourite and least favourite numbers and new associations between numbers and people or events to occur. First graders, for example, often based their favourite number on „how difficultly“, „how easily“ or „how beautifully“ they could write the number, whereas among the second, third and fourth graders a greater influence from the daily life of the children (birthdays, highest number of goals shot while playing soccer, numbers on textiles) played a bigger role.

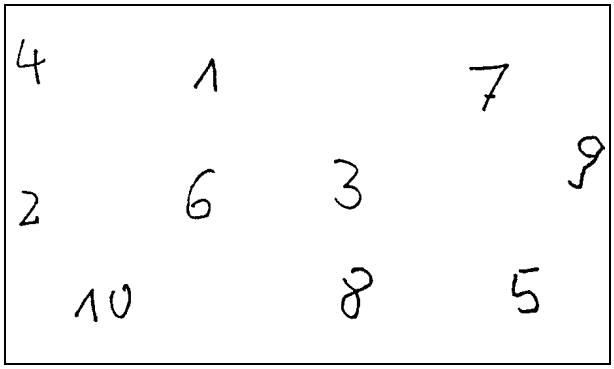
5. The influences of subjective conception of numbers on learning in elementary school mathematics classes

Assuming that learning is a subjectively viewed and an active-constructive process in which opinions, emotions, moods, etc. play a role, the question emerges to what extent the – pre-existing without any doubtfully - subjective conceptions of numbers of elementary school children in math classes are influenced. Answer:

Lisa is in the first grade. She usually likes learning. In math classes she is especially happy when her favourite number, 6, appears. She is then very active and tries hard to always write „her" number correctly. She already knows a lot of math exercises involving the number. However, when 0 is the topic, she is uninterested and inactive. She doesn't like this number. *„Zero is a bad number,"* she thinks *„because it doesn't have any value, and adding with it is strange."* Lisa has contrasting emotions to the numbers 0 and 6. This seems to have the effect that depending on her feelings towards the numbers, her motivation and activity in class either rises or falls. The teacher hadn't yet noticed this process. Lisa is one of the better students in the class. She usually understands the facts handled in math class quickly and she can use the techniques learned on the ample problems. Apparently her subjective relationship to numbers is not a problem for her work in class. Lisa herself explained to me that she never really thought about her conception of numbers and never talked to anyone about it.

„It's just the case that I think about the numbers. But when I have to calculate something it doesn't bother me.," she says. But the popularity of a number does affect her participation in math class. In addition, the question remains open as to whether abilities remain unused because of their subjective sensory construction for the triggering of additional learning activities. Thus Lisa's opinion of the „0" gives us the opportunity to ask what she means by *„Zero has no value"* or to find out why it is *„strange"* to work with.

Luisa, (9 years 3rd grade) doesn't like the *„strict mathematical order"* of the numbers. She says, *„It's always 0,1,2,3... that's boring. I'd rather have the numbers all mixed up. When they're all in a row they look like soldiers."* According to her subjective feelings, the numbers should be distributed as follows:



She commented on her numerical arrangement as follows: *„This way the numbers are all spread out in a room, like the things in my room."* Luisa apparently compares the numbers with the toys in her room and thinks the order of the toys can be transferred to the order of the numbers. Since the first grade, she has thought her subjective arrangement to be the *„best"*. But up to now she has never spoke to anyone about this order. The teacher says that Luisa is about average within her class. She often needs a certain amount of practice before she really understands a new process or has internalised the answers. For example, she was still confusing the symbols „ > „ and „ < " in third grade. The teacher thinks this is due to a lack of con-

centration. But I believe that this could be traced back to Luisa's unusual relationship to numbers. Luisa is forced to differentiate between the „officially correct" order of the numbers and her own personal order, and she doesn't always succeed. Especially in stressful situations, such as progress checks, she mixes up the order of numbers she has learned with great effort with her own subjective order, although this is only partly a conscious problem. Her results are then a „mistake" (see also LORENZ 1992). We also discovered that the problem appears in a similar manner in her calculating of problems. She invents her own ways of solving math problems. Because these methods are sometimes complicated or incorrect, and she doesn't spend enough time thinking about this difference (or can't differentiate), she has no choice but to adopt the misunderstood, yet „officially correct" procedure. In order to integrate and use the correct order, Luisa certainly needs „practice time". Luisa's example shows that it can be problematic if a child is left to his or her own imagination, and doesn't have the possibility of exchanging ideas with others. Thus there is the danger of a misunderstood „mechanical" acceptance of the techniques and procedures (see also SCHÜTTE 1995). An additional problem, as Luisa confirmed, is the lack of interest in mathematical exercises.
In contrast to Luisa, Michael, a very strong math pupil in the third grade, gets along brilliantly with his own subjective conception of numbers. He nurtures his playful dealing with his own arrangement of numbers and uses it to his advantage when computing problems as well as to help him work out original solutions. For example, Michael's favourite order of the numbers from 1 to 10 puts them in pairs that add up to 10, such as 1 and 9, 2 and 8, etc. This pairing of numbers to make „tens" is something he has developed for himself and which he consciously uses when adding. I observed that he first quickly formed bundles with the numbers of the same place value, then computed „subtotals" and was quickly able to solve problems when „chain" adding. The frequent and successful use of his method apparently cements the subjective conception of numbers and furthers its specific development.

These three examples clarify how the subjective conception of numbers can certainly influence the learning of mathematics in elementary school children. According to the results to date in this study, in the very least the motivation of many children is influenced by such subjective conceptions and opinions of numbers and thus their work regarding mathematical topics is affected (compare SCHÜTTE 1995). In addition their conceptions can influence their understanding processes. This can have different outcomes. While Luisa's subjective conception of numbers made her learning process more difficult, Michael's sensory construction set the stage for more creative and flexible thinking.

6. Preliminary (carefully formulated) tendencies in assessments regarding the conception of numbers in elementary school pupils and didactic conclusions with respect to their consideration in teaching-learning processes in elementary school mathematics classes

Because of the limited number and selection of the children interviewed, as well as the hitherto scope of the interviews we cannot make any generalisations. But there do appear to be indications for the following assumptions or tendencies:

1. Apparently a great number of children already have a subjective conception of numbers when starting school; a conception that develops individually in elementary school. Subjective opinions of the arrangement of numbers seem to form regardless of the illustrative means, such as the stream of numbers. In the course of the elementary years this continues to develop without any drastic changes, and becomes more fixed which is in accordance

with the results of SERON, PESENTI et al. A further development appears to be that the structures (for example bundles of numbers according to certain criteria, circle or serpentine order of numbers) become clearer. The representations of these arrangements of numbers are apparently subconsciously activated in the children, without the teacher knowing about such a device, and thus without considering this within the lessons. Subjective conceptions of numbers in terms of favourite or least favourite numbers, as well as the relationship between numbers and people, objects or colours are apparently subject to comparatively great changes. They are often dependent on specific childhood experiences (compare BAUERSFELD 1983). On the one hand, they are related to classroom material, (for example, with first graders this is often related to the ease in writing the number) but on the other hand they are related to the numerical representation and the numerical relationship to things important to the lives of the children (for example, the numbers on textiles or photos, or birthdays).

2. There do not appear to be any gender-related tendencies and only partial connections between the subjective conception of numbers and the mathematical ability. There are, however, clear hints for the different quality in the individual dealing with this subjective conception of numbers. For the children with learning disabilities these subjective opinions, which usually deviate from the „normal“ order of numbers in the classroom, create an extra hurdle for the learning of mathematics (compare also LORENZ 1992). In comparison, the third and fourth graders interviewed could usually differentiate between the „officially correct“ arrangement of numbers in math class and their own individual ideas. In fact, they often play with their individual „number maps“. In addition they use their subjective ideas for advantageous adding, or use this as a springboard for original ideas for solutions (*creativity-enhancing function of subjective conceptions and ideas*). In accordance with the test results published by GRAY & TALL the tendencies show that strong math elementary school children know how to deal flexibly with their subjective conceptions of numbers whereas less „gifted“ children do not. But I only partly agree with the opinion of GRAY & TALL (1994) that mathematically „gifted“ children unlike mathematically less talented children arrange numbers on specific criteria and see typical mathematical structures and circumstances. On the contrary. Our studies prove that strong math school children also have „mathematically untypical“ conceptions.
3. Further, the studies performed up to now show that the subjective conception of numbers is an important starting point and an inherent component of the mathematical learning process in children. Thus teachers should know about the possibility of subjective conception of numbers in children, and give the children room to develop these ideas, instead of trying to overwhelm them step by step with strange assignments. It is possible that the subjective conceptions could be used successfully in solving and organising learning processes in math class. Here is also the place that the subjective ideas of the children can occasionally be addressed (for example, a project such as „my favourite number“). Our experiences to date show that addressing such topics work as a *stimulus to motivation*. It is important for a child to experience that his or her subjective conception of numbers is taken seriously by the teacher and the classmates and that he or she recognizes that other children also have such individual relationships to numbers. For teachers, such a lesson plan offers very good possibilities for differentiated pupil analysis (see also KÄPNICK 1997).

7. Conclusion

The results available up to now in this study apparently show that elementary school children have a diverse subjective conception of numbers. According to this conception of numbers, connections to mathematical aspects are to be seen as well as to different experiences in the children's everyday life. Moreover, almost all children interviewed have got emotional relations to certain numbers. A pattern only referring to an abstract symbolic representation of numbers doesn't meet the needs of child-oriented conceptions. Another result of this study is that subjective conceptions have a strong influence on motivation and ways of dealing with mathematical aspects. Furthermore, subjective conceptions can be of great relevance to comprehension processes. On the one hand a subjective conception varying from the officially correct conception, creates an extra hurdle for learning mathematics for less mathematically gifted children. On the other hand strong math-talented children are able to deal flexibly with their subjective conceptions and they develop original solutions by means of their diverse subjective conceptions. The question emerges whether, in the process of gaining experience in school as well as achieving the ability to abstract, the subjective conception is lost and whether this may lead to less interest in mathematics.

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