**GÜNTER TÖRNER**

# VIEWS OF GERMAN MATHEMATICS TEACHERS ON MATHEMATICS

**Abstract:**
In a questionnaire with 77 items more than 300 mathematics teachers of secondary schools were asked for details into their view on mathematics. Thus we were questioning their mathematical beliefs, by which we understand the teachers' attitudes towards mathematics. In particular, we were interested in whether the mathematical world view can be recognised as a structure.
Attitudes towards mathematics are very complex and multiple; mathematics is recognised by teachers very refinedly and structured. The aspects "formalism-", "scheme-", "process-" and "application-aspect" which are known from former research are central dimensions of attitudes in the teachers' answering. These dimensions are relevant categories, by which teachers structure their recognition and cognitive representation of mathematics; within these, thinking, valuing and feeling of mathematics take places on a global level, and plans and intentions of acting are designed in these dimensions. By a factor analysis it could be proved that these aspects serve as dimensions, that is, as the essential global and constituent elements of a mathematical world view in the sense that they largely determine the main orientation and characteristics.
In the mathematical world views of teachers these four global dimensions form a global partial structure which we derive as a graph through the significant partial correlations. This structure corresponds with the theoretical preassumption of antagonistic leading conceptions of mathematics as a system as well as a process. The aspects of formalism and scheme correlate positively and represent both aspects of the static view of mathematics as a system. They are opposite to the dynamical view of mathematics as a process. However, the aspect of application of mathematics is only linked with the aspect of process.

## 1. The Starting Point

As early as 1973 the mathematician, René THOM, pointed out that all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics (p. 204). HERSH (79, p. 13) underlines this position emphatically: "*One's conceptions of what mathematics is affects one's conception of how it should be presented. One's manner of presenting, it is an indication of what one believe to be most essential in it ... The issue, then is not, What is the best way to teach? but, What is mathematics really about? ... Controversies about ... teaching cannot be resolved without confronting problems about the nature of mathematics.*" Independent from this earlier philosophical approach, many papers have come out since the 70s in the field of mathematics which concern views and their effects on the didactics of mathematics (see THOMPSON 92), mostly speaking of mathematical beliefs. However, the translation into and the terminology in the German language is not always uniform; one speaks of ideas, views, attitudes, conceptions whereby the objects are elements of mathematics or mathematics as the whole. It is worth mentioning that it consists of ideas, opinions, interpretations, viewpoints, attitudes etc. whereby the objects of the time are seen as elements of mathematics or as mathematics as a whole. At the same time, also processes and relationships concerning the teaching and learning of mathematics are addressed since they are involved quite naturally. Corresponding views or attitudes are characterized as beliefs and as belief systems in English literature. In America there are numerous research papers concerning these topics, contrasting to the few released in the German-speaking world see, for example, BAUER 88 resp. GRIGUTSCH; TÖRNER 94 resp. the literature in PEHKONEN; TÖRNER 96).

## 2. Mathematical World Views

### 2.1. Definition and Some Remarks

Mathematics as a world of experience and action can be assumed to be an extremely complex field. One may also assume that the corresponding attitudes towards mathematics are complex as well as diverse in nature. For certain there are not only the positive and negative attitudes, but also many differentiations arising from them. On the cognitive level we can assume that the subjective knowledge of mathematics and teaching mathematics comprises ideas in several different categories:

(1) Beliefs about mathematics
(2) Beliefs about learning mathematics
(3) Beliefs about teaching mathematics
(4) Beliefs about ourselves as practitioners of mathematics (self-concept as a mathematics practitioner: a self-evaluation of one´s abilities and causal attribution to individual success and failure)

At the same time, the category, "beliefs about mathematics," comprises a wide spectrum of beliefs which, at least, includes the following components: (1) beliefs about the nature of mathematics as such, (2) the subject of mathematics (as taught in school or at the university), (3) beliefs about the nature of mathematical tasks and problems, (4) beliefs about the origin of mathematical knowledge and (5) beliefs about the relationship between mathematics and empiricism (in particular about the applicability and utility of mathematics).

The cognitive component of beliefs can comprise a wide spectrum of single, integral parts, emotions and evaluations, which are connected with beliefs as well as behavioural dispositions and intentions induced by them and may be very complex. There are easily understood affections associated with each component (1) through (4).

Therefore, opposite to mathematics regarded as a world of gaining experience and acting there is a „world" of attitudes which we will characterise as a "mathematical world view". A "mathematical world view," as defined above, is a system of attitudes towards (integral parts of) mathematics. It is a hypothetical construction which, concerning attitudes towards mathematics, is yet to be proven and, therefore, of no empirical, but rather of heuristic value.

Information gained from two levels is significantly important to express a "mathematical world view“ concretely : (a) expressions of single attitudes and (b) relationships between different attitudes within the "world view". The relationships between single attitudes form a structure which is probably more important to the expression of a "mathematical world view" and its relevance to action than all the attitudes it contains. Conclusively, a "mathematical world view" is an *attitude structure*.

Our research focuses on "mathematical world views" and not on single attitudes because:

(1) in contrast to mathematics, there is a spectrum of beliefs and attitudes which possibly influence each other (see above) and (2) in this case, the overall structure of attitudes is possibly of greater importance than individual attitudes, namely

– regarding *expression:* it is possible that the expression of a "mathematical world view" or even of single attitudes is determined considerably by the relationship between attitudes.
– regarding *their relevance to action:* attitude structures may offer better explanations and predictions of certain ways of acting than single attitudes.

– regarding *change:* it is possible that an attitude's susceptibility to influence depends considerably on the number and strength of connections through which it is woven into a net of attitudes.

**2.2. The Functional Meaning of Mathematical World Views**

Due to the fact that the term attitude, which we want to use in regard to mathematical world views, is not clearly defined in the literature, a functional analysis can lead to a more precise definition.

(1) *Organisational function*: Attitudes function as a selection, emphasis, fixation, and organised configuration of objects, i.e. they structure and simplify the complex variety of stimuli in our environment. As a result, attitudes help the individual to understand his or her environment. KATZ even mentions the need of an individual to structure and understand his/her world, thus, attributing a cognitive function to attitudes.

(2) *Adaptational function*: Attitudes activate an established repertoire of reactions if a known category of objects relevant to attitudes has been identified. They define a schematic reaction of a subject to a familiar situation / stimulus. A single situation does not require a completely new concept of clarification nor a program of action. Schemes of action are already adopted because one's past experiences are made active.

Whether the activated scheme of action is optimal or not depends on whether the object to which an attitude refers has been classified correctly, whether this object is similar to former objects and whether one's past experiences have been successfully translated into a scheme of action.

In summary, we can conclude that attitudes fulfil an *orientational function.* In an "otherwise chaotic environment" (WILLIAM JAMES) attitudes practice a selective and leading function while the individual perceives and values objects. This results in a selection of action patterns. The existence of ideas and emotions enable a person to find orientation in his environment. Only lasting orientation patterns are applicable to more than one situation, which determine the relationship between the individual and the object, provide stability and continuity. (see MEINEFELD 83, p. 92)

This can be also be taken as an assumption: the assumption that man is in need of such an orientation described above is essential to the theorem of consistency and is, therefore, essential to the three-components-approach (cp. MEINEFELD 83, p. 92).

Apart from these rather general functions, attitudes fulfil some specialised functions as well.

(3) *Self-assertion function*: If the object to which an attitude refers is the individual him/herself, attitudes often function as a mean of self-assertion: they protect the feeling of self-esteem by rejecting or ignoring unpleasant truths.

(4) *Self-portrayal function*: Attitudes serve as an avenue in which one can express his/her own convictions. The outward expression of one's individual attitudes is very unpleasant because they represent those basic values which the individual holds to be extremely important. (cp. TRIANDIS 75, pp. 6-)

The functional character of the above considerations regarding mathematical world views can be summarised by the following teaching and learning processes:

(a) Mathematical world views function as a regulating system.
(b) Mathematical world views function as an indicator.

(c) Mathematical world views contain a degree of inertia.
(d) Mathematical world views are worth prognosis.

It is obvious that the substantial organizers of learning processes, i.e. the teachers, play a most important role regarding their mathematical view of the world. To ascertain this fact was the goal set forth by this inquiry.

# 3. The Inquiry

## 3.1. The Beginning of the Inquiry

During the Annual German Congress on Mathematical Didactics in Duisburg in 1994, a survey of 310 secondary school mathematics teachers was conducted by Dr. GRIGUTSCH and the author to extract a mathematical world view and its structure. We regard the various attitudes compared with mathematics as being extremely essential. Within the attitudes of mathematics we restricted ourselves to the essence of mathematics (mathematics as a field and not as a subject taught in schools) and the main ideas of "process" and "system" as well as the estimation of the use of mathematics. Our inquiry, therefore, related to four aspects of a mathematical world view: "scheme," "formalism," "process" and "application." Each one of these four aspects is operationalised through ten items, demanding a methodological and statistical approach. With this in mind the questionnaire is long enough so that no further aspects can be raised. A factor analysis justifies these aspects as permanent dimensions. They are dimensions in which the teachers structure their acceptance and cognitive representation of mathematics.

These four dimensions build a global part-structure in the mathematical world view. This structure corresponds to the theoretical presumption that the antagonistic, underlying ideas of mathematics are represented both as a system and a process. The formalism and schema aspects are known in a positive frame of reference, and both represent the static view of mathematics as a system. They are in contrast with the dynamic view of mathematics as a process. The aspect of the applicability of mathematics is only connected with the process aspect.

## 3.2. Carrying Out the Inquiry

Approximately 400 questionnaires were given out, of which 310 were filled out and returned. We were satisfied with the number of responses. This spot check can not, however, be classified as fundamentally representative of mathematics teachers at the secondary level. It represents teachers who did not put up with the slight expense of taking a continuing teacher training course. To some extent this population, under which a known multiplication function can be assumed, has especially earned our attention.

## 3.3. Evaluation of the Inquiry

First of all, we did a factor analysis to form groups of statements which were part of the questionnaire, mainly, for three reasons each of which necessarily requires such a grouping

### 3.3.1. The Conceptual Argument

The concept "attitude" is characterised by a consistancy among reactions. Only in this case can the existence of such a characteristic within mathematics, which is the object of an attitude, be estimated (necessary, not sufficient condition).

### 3.3.2. The Argument of Validity

A special problem concerning the validity of questionnaires arises from verbal communication, i.e. whether there is a mutual agreement and understanding using the written word. In most cases this problem is reduced to demanding that items of the questionnaire be understandable. In this inquiry we are not concerned with pairs, but rather *groups* of statements which are semantically connected according to the author. The question is, now, whether or not these groups are being reproduced from the examinees. The statements are being grouped in the factor analysis by assumptions made from observed correlations among the statements. We demand that these groups consist of statements which are homogeneous in content. Whether or not the statements made within a group are formally homogeneous or not, we will judge using Cronbach´s Alpha.

### 3.3.3. The Measure-theoretical Argument

In many cases attitudes contain varying dimensions. It is an essential task of measuring to find out the dimensions to which extent the experiences of the groups of examinees are to be categorised. The most well-known technique for solving this problem is the factor analysis. The question concerning the number and quality of attitudes has often been a central topic of various mathematical investigations concerned with didactics (see, for example, JUNGWIRTH 94; PEHKONEN 92; PEHKONEN; LEPMANN 94).
For further emphasis in connection with the measuring of attitudes, we refer to GRIGUTSCH (94).

## 3.4. Range of Variables and Sample Data

The items were scaled as follows: 5 = totally agree, 4 = agree for the most part, 3 = undecided, 2 = partly agree, 1 = do not agree. Furthermore, we used 'listwise deletion' provided a person did not have a positive value in one of the items.

Each factor analysis consisted of 75 items. Items 1 to 77 were included except for items 22 and 23. As for items 22 and 23, the large number of refusals to answer them resulted in too many observations (subjects) being excluded from the factor analysis; furthermore, it was questionable in principle whether items to which many subjects refused to respond could be taken into consideration during the evaluation of the questionnaire. Those items concerning causal attribution for success or failure (items 78 to 105) were not included in the factor analysis due to numerous refusals to respond to these items and their content, which differed from the first 77 items: while items 1 to 77 aim at the view of mathematics, the causal attributions were concerned with an aspect of one’s self-concept (in this case: of the conception of pupils) when doing mathematics.

The range of the sample amounted to 310. During the factor analysis only 207 persons, however, were taken into consideration because each person who did not have a value in all of the 75 variables was excluded from statistical procedures.

## 3.5. Scree-Test

First of all, an analysis of the principal components was calculated in order to determine the eigenvalues and to carry out the Scree-test. There are 25 self-values which exceed 1. When taking data from the scree-plot, an analysis based on four factors seems to be recommended.

# 4. Results

## 4.1. The Four-Factor Solution

In performing principal components analysis, 4 factors were assumed and then the varimax-rotation as a transformation method was applied (using StatView Software). As to the orthogonal solution, each factor was determined by items whose loadings exceeded .39.

At first, we were interested in finding out whether the factors were homogeneous in content and whether they could be meaningfully interpreted.

| Item | **Factor F = Formalism Aspect** | loading |
|---|---|---|
| 30 | Logical strictness and precision, i.e. "objective" thinking, are essential aspects of mathematics. | .699 |
| 28 | Math is characterised by strictness, namely a definitory and formal strictness of mathematical argumentation. | .678 |
| 50 | Clarity, exactness, and unambiguity are characteristics of mathematics. | .650 |
| 32 | Conceptual strictness, i.e. an exact and precise mathematical terminology, is indispensable to mathematics | .614 |
| 38 | Mathematical thinking is determined by abstraction and logic. | .597 |
| 26 | Mathematics is a logical, indisputable thought process with clear, precisely defined ideas and unequivocal, provable statements. | .583 |
| 48 | Crucial fundamental elements of mathematics are its axiomatics and the strict, deductive method. | .543 |
| 36 | Mathematics particularly requires formal, logical derivation and one's capacity to abstract and formalise. | .530 |
| 40 | Central aspects of mathematics are flawless formalism and formal logic. | .527 |
| 3 | In the mathematics classroom the students must think extremely logically and precise. | .486 |
| 17 | In the mathematics classroom the students must employ technical terms correctly. | .475 |
| 45 | Mathematics originates from setting axioms or definitions, then, by deducing theorems according to formal logic | .469 |

All twelve items of the first factor show only loadings higher than .46 and possess no secondary loadings. They are homogeneous in content and can be meaningfully interpreted as an aspect of mathematics which puts particular emphasis on formalism: mathematics is characterised by strictness, exactness, and precision on the terminological and language levels concerning thinking ("logical", "objective", and "flawless thinking"), argumentation, giving reasons and proof of statements as well as theoretical systematology (axiomatics and the strict deductive method). We call this factor the *formalism-aspect* of mathematics.

| Item | **Factor A = Application Aspect** | loading |
|---|---|---|
| 68 | Knowledge of mathematics is very important for the students later in life. | .695 |
| 72 | Mathematics helps to solve daily tasks and problems. | .659 |
| 70 | Only a few things learned from mathematics can be employed later in life. | -.638 |
| 66 | Many parts of mathematics are either of practical use or are directly relevant to application. | .600 |

| | | |
|---|---|---|
| 67 | In the mathematics classroom one can become independent from that which is being taught and can very little of what is to be used in reality. | -.596 |
| 69 | Mathematics is of general, fundamental use to society. | .591 |
| 71 | Mathematics is of use to any profession. | .580 |
| 76 | Math lessons are concerned with tasks which are of practical use. | .512 |
| 75 | Mathematics is a game free of purpose. It is occupying oneself with objects without any solid relevance to reality. | -.438 |
| 74 | With regard to application and its capacity to solve problems mathematics is of considerable relevance to society. | .444 |

As for the second factor, ten items can also be found possessing a loading of more than .4 and eight with a loading of over .5. None of these items reveals a secondary loading. Every item is a uniform expression (note the negative loadings of three items!) of the immediate relevance to application or the practical use of mathematics. The pupils' knowledge of mathematics is important to their future life : mathematics either helps to solve everyday tasks and problems or it is useful to one's occupation. Apart from that, mathematics is of a general, fundamental use to society. This factor is homogeneous in content and can be meaningfully interpreted as an *application-aspect* of mathematics.

| Item | **Factor P = Aspect Process** | loading |
|---|---|---|
| 43 | With mathematics one can find and try out many things for him or herself. | .603 |
| 31 | Mathematics requires new and sudden ideas. | .570 |
| 52 | Mathematical tasks and problems can be solved in various ways.. | .502 |
| 54 | If one comes to grip with mathematical problems, he/she can often discover something new (connections, rules and terms, for example). | .468 |
| 61 | Any person can invent and re-invent mathematics.. | .454 |
| 58 | It is common to be able to solve tasks and problems in more than one way. | .450 |
| 41 | Above all, mathematics requires intuition as well as thinking and arguing, both relating to contents. | .419 |
| 55 | In order to solve a mathematical task, there is usually only a single method which one must find. | -.419 |
| 37 | Doing mathematics means: understanding facts, realising relationships and having ideas. | .414 |
| 35 | Mathematical activity is comprised of inventing or re-inventing (re-discovering) mathematics. | .413 |
| 25 | Mathematics is an activity which is comprised of thinking about problems and gaining knowledge. | .407 |
| 46 | Contents, ideas and cognitive processes are central aspects of mathematics. | .404 |
| 49 | Wanting to understand mathematics means wanting to create mathematics. | .393 |

There are thirteen items connected with the third factor with loadings over .4 without any secondary loadings. When describing mathematics from a constructivist point of view as a process, the items correspond to each other regarding contents. Item 49 with its loading of .39 also corresponds to this view of mathematics. Mathematics is characterised in this factor as a process and as an activity in thinking about problems and gaining knowledge. On the one hand, this cognitive process is about creating, inventing or re-inventing (re-discovering) mathematics. On the other hand, it also includes the comprehension of facts and understanding connections. This problem, the oriented cognitive process of understanding, decisively requires thinking and arguing in regard to content as well as sudden and new ideas, intuition and experimenting. The *process-aspect* expresses the dynamic view of mathematics.

| Item | **Factor S = Schematic Orientation (Schema-Aspect)** | loading |
|---|---|---|
| 44 | Mathematics consists of learning, recalling and applying. | .642 |
| 24 | Mathematics is a collection of methods and rules, which precisely determine the solution of a task. | .600 |
| 73 | It is certainly quite an achievement if maths lessons quickly impart that knowledge which is needed for application, occupation, or life; everything else is a waste of time. | .543 |
| 62 | Trying to solve a mathematical task, one needs to know the only method that is correct; otherwise, he/she will be lost. | .513 |
| 39 | Mathematics is the memorising and application of definitions, formulas, mathematical facts and methods. | .482 |
| 34 | Doing mathematics demands a lot of practice in following and applying calculation routines and schemes | .468 |
| 42 | Doing mathematics requires extensive practice in correctly following rules and laws. | .417 |
| 20 | In order to successful in the mathematics lesson, one must have a strong working knowledge of many rules, terms and methods. | .399 |
| 29 | Almost any mathematical problem can be solved through the direct application of familiar rules, formulas, and methods. | .376 |

In Factor S, seven items indicate loadings over .4 without any secondary loadings. They are homogeneous in content and operationally define a view of mathematics which is seen as a "tool-box and bundle of formulas" and an idea oriented with algorithm and schemes. Mathematics is characterised by a collection of methods and rules which precisely determine how to solve a task. The consequence with dealing with mathematics is: doing mathematics consists of remembering and applying definitions, rules, formulas, facts and methods. Mathematics consists of learning (and teaching!), practising and the remembering and applying of routines, schemata and applications. We call this the *schema-aspect* of mathematics. We have added two further items with loadings of .399 and .379.

The variance figures show,

| Factor | variance share in the communitality summary (=18,26) | | variance share in the overall standard variance (= 75) | |
|---|---|---|---|---|
| | | accumulated | | accumulated |
| F | 31,7 % | 31,7 % | 7,7 % | 7,7 % |
| A | 22,9 % | 54,6 % | 5,6 % | 13,3 % |
| P | 22,0 % | 76,6 % | 5,4 % | 18,7 % |
| S | 23,3 % | 99,9 % | 5,7 % | 24,4 % |

that each of the four factors accounts for more than five percent of the overall standard variance, so it exceeds the general minimal value, therefore, bears sufficient formal significance. As the 4-factor-model accounts for just 25 % of the response variance, it is even more surprising. Again, it becomes apparent that the four factors describe and structure only a fraction of attitudinal thinking about mathematics; they are significant, however, and further significant factors could not be found during an even 9-factor-solution.

Another characteristic of these factors, which often represents a criterion for their selection, is the simple structure of the selected items. The items load in regard to only one factor respectively. Thus, the set of items chosen can be subdivided into four disjointed groups by means of these factors, i.e. there is no overlapping. Concerning the factors, this implies: each factor is operationally defined by items which bear a loading exclusively on said factor and which are entirely homogeneous in content. Consequently, each factor is operationally defined independently by a set of items of the remaining factors, and, therefore, each factor itself is independent of all the other factors.

Those items which operationally define a factor were grouped according to correlative connection. For the time being the importance of this grouping is left open. Being the most important reason, homogeneity of contents was tested. Now, the homogeniality within a particular grouping should be formally tested with Cronbach's Alpha.

Cronbach's alpha amounts for 0.83 (out of ten items) of the application scale, 0.85 of the formalism scale (out of twelve items), .76 (out of 9 items) of the schema scale and for 0.72 (out of 13 items) of the process scale. Due to the fact that all values are more than 0.7, the homogenality of an item group is given also after this formal criteria.

### 4.2. First Conclusions and Interpretations

We believe that the first four factors can be recognised as significant and interpretable in regard to their content. Factors F (= formalism aspect) and A (= application aspect) include ten homogeneous items respectively, so they are significant and interpretable. This also applies to Factors P (= process aspect) and S (= schematic orientation). These four factors are homogeneous and interpretable in relation to content, and they represent significant dimensions compared with the number of items. We assume that ,in this way, those attitude patterns, which are relevant and characterise the range of reactions to this questionnaire, are best understood. Conclusively, those teachers included in our sample have a very differentiated and structured view of mathematics.

### 4.3. Relations among the Dimensions

The four dimensions are independent attitude objects which, first of all, were to be diagnosed and evaluated. In addition, attitude theory is also concerned with those structures formed by individual attitudes or attitude dimensions. Within the semantic network of one's memory, cognitive structures exist on of both levels of ideas and associated affections as well as action schemes. One may substitute the hypotheses with the idea that cognitive structures are linked to attitude structures, as well. These structures are discussed in various respects, e.g. with regard to their quasi-logical structure, to their psychological significance or to their cluster property (see PEHKONEN 94). Probably, it is these very structures, in our estimation and less probably the markedness of individual attitudes, which determine the effects and suggestibility of attitudes to a considerable extent.

The four scales refer to the following partial correlations: (n = 253):

| | F | A | P | S |
|---|---|---|---|---|
| F | 1,000 | ,042 | -,127 * | ,364 *** |
| A | ,042 | 1,000 | ,127 * | ,087 |
| P | -,127 * | ,127 * | 1,000 | -,146 * |
| S | ,364 *** | ,087 | -,146 * | 1,000 |

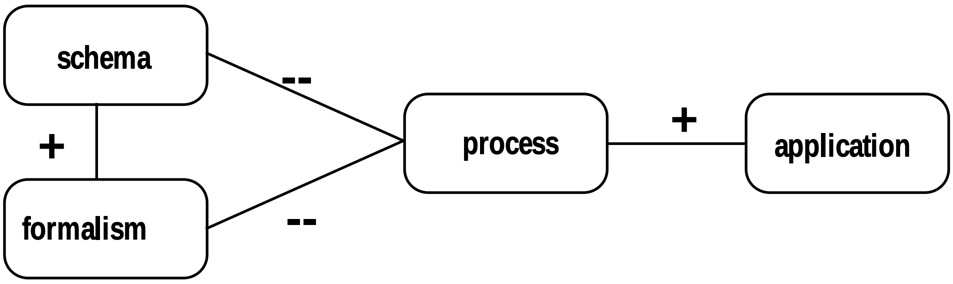


Figure 1: Intercorrelative relation between the scales

The correlation coefficients are certainly very low. This may be attributed to the fact that the connections between the dimensions are not very strong, i.e it is a matter of different dimensions. In our opinion, it is essential that one can still interpret the significant correlation coefficients. First of all, they are significantly different from zero, i.e. the represented connections exist in each case according to tendency (sign of the correlation coefficient). Secondly, the lower, significant correlation coefficients are of importance to the content if one considers that the correlation coefficients are lowered by the mistakes in measuring and that the coefficients express a connection among the various dimensions. There are different dimensions under consideration, however, which are still in connection with one another.

The intercorrelations (see Figure 1) result in a partial structure of the "mathematical world view" which corresponds to our theoretic pre-assumption of antagonistic ideals. The formalism and scheme scale represent both aspects of the static view of mathematics as system and intercorrelate highly. Both parts of the static paradigm correlate with the process scale in a significantly negative way. This corresponds with our thesis that both views are opposed antagonistically (at least in a paradigmatic analysis). The application aspect of mathematics correlates significantly only with the process aspect of mathematics. This agrees with our pre-theoretic assumptions in that scheme and formalism express a static property which does not include that solving problems of reality is not a primary goal. From a formalist point of view, mathematics largely refers to itself, to a precise conceptualisation, to purely formal-logical verification of statements and to its logical, systematic structure. From a schematic point of view, mathematics is a collection of calculation techniques and algorithms which are considered more suitable for a math-related routine than for concrete applications and solutions to problems of reality (the „non-connection“ with the application scale must be interpreted that way). In contrast, the process aspect aims at developing knowledge through a problem-related, cognitive process emphasising the importance of seeing connections of ideas and intuition. This dynamic concept of mathematics may be more likely suitable for application, and this is expressed through the teachers' attitudes.

The mathematical world view, at least in these four dimensions, is relatively consistent and coherent in its structure. The statement serves only for the temporary understanding , i.e. for the static of a mathematical world view. Alternative hypotheses form the change  of attitudes, which we cannot go into at this time.

## 4.4. The Comparison of the Means

Concerning the following investigation, scale values referring to each of the four dimensions respectively were set for each person involved. Those dimensions were operationally defined by 8 to 13 items, respectively. The score concerning the statements of one dimension was added up for each teacher involved. A transformation and stretching of the scale results in each teacher having a scale value in each dimension ranging from 0 to 50; 0 to 10 represent utter rejection; 40 to 50 in full agreement.

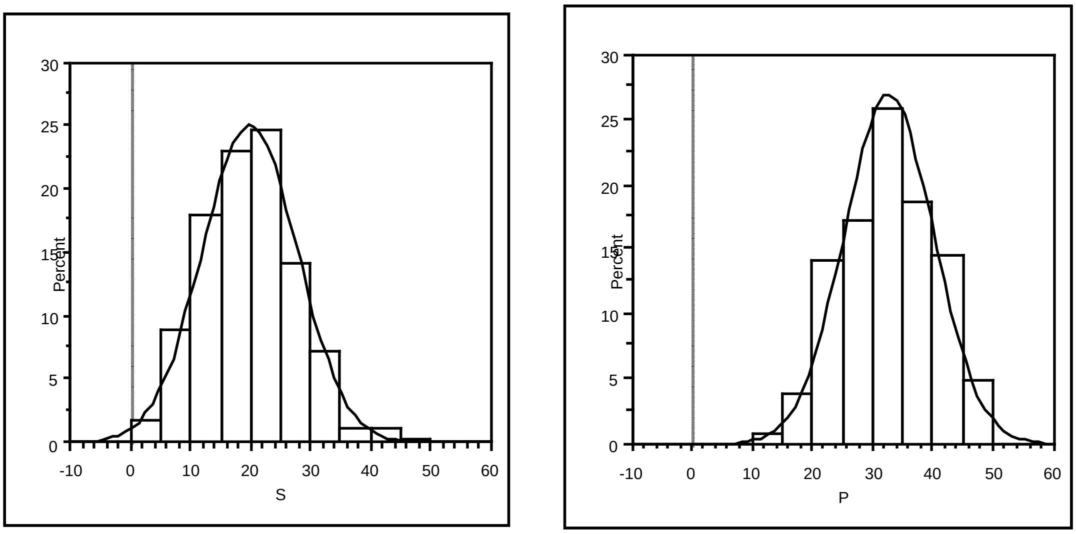


Figure 2: ‘System’ und ‘Process’ Aspects

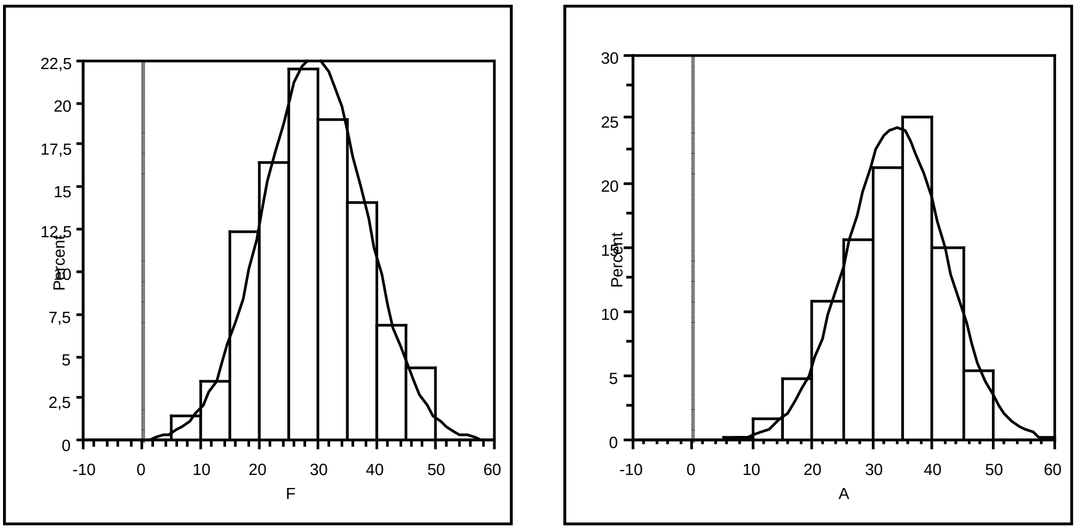


Figure 3: ‘Formalism’ und ‘Application’ Aspects

Concerning the mathematics teachers' views, the scheme-aspect is considered rather unimportant and negative whereas formalism is of moderately high importance. As to this image, the application and process aspects, which must be considered as indistinguishable, are regarded as significant.

### 4.5. Evaluation of some Results

Finally, we would like to point out three results which were really surprising. We will introduce our evaluation and want to leave it open to discussion.

#### 4.5.1. A Positive Image of Attitudes

First, we must assess the mathematical view of teachers, obviously against the background of our own view, very positively. In reference to the average mathematical view of teachers, the process and application aspects of mathematics are rated relatively high, while the scheme aspect is rather rejected. Thus, mathematics is understood primarily as a process in which realisation and understanding are stressed. The static view, on the other hand, is rejected in that mathematics grows stiff through learning, practising, applying arithmetical schemes and routines and through a mechanical drill of procedures. Furthermore, the process aspect, the development of knowledge in a problem related process of discovery and understanding, is linked to the applicability and profit of mathematics of which most teachers are convinced .

#### 4.5.2. Similar View in Different School Types

Secondly, with respect to the global view of mathematics of teachers in different school types the aspects of formalism, process and application must be rated similar, as our results tell us. Certainly, there are differences in the global goals of teaching, in the curriculum and in the contents of classroom discussion which also consider different dispositions and interests of students. Then, on the one hand, it is justified and desirable that differences in the curriculum correspond with a different mathematical world view of the teacher. On the other hand, we are surprised and we think it is good that teachers of the various schools types seem to have a similarly sophisticated and multi-layered view of mathematics. We think it is good that teachers in a *Hauptschule* or *Realschule* have a similar global view of mathematics and know how to convey it to their students just like teachers in a *Gymnasium* despite the many differences in curriculum. This is a view which shows how multi-layered the essence of mathematics is and how it is not limited to isolated aspects. This would be possible because, in our opinion, all aspects of the mathematical view can be conveyed independently of age, of the disposition of the students, of the type of school and, first and foremost, independently of the concrete topic in class.

#### 4.5.3. Mathematics as a Favourite Subject to Teach

The teachers' description of their motivation to teach mathematics is another positive result. Nearly all teachers (97 %) testified that they liked teaching mathematics. Two thirds of them admitted this even without restrictions. The circumstances could also possibly be reflected in these positive figures in that the participants of this inquiry must be evaluated as positively motivated due their preparedness and voluntary participation at the Inservice-Teacher Training as mentioned above.

### 4.6. A Critical Review

Because these results did not fully agree with our ideas, we have critically looked for explanations without intentionally wanting to bring the results into question. For a contrary estimation there are four statements of clarification.

(a) As previously mentioned, the spot-check for teachers is not representative of the „ones interested“ in the continuing teacher training.

(b) One can also doubt the validity of a written questionnaire. The teachers answered the questionnaire at a university on the occasion of the National Conference of Didactics in Mathematics (Bundestagung für Didaktik der Mathematik) in a scientific atmosphere. They might have anticipated the supposed expectations of a field of science, or they might have felt like being back in school.

(c ) The view of mathematics was thought of as a special field. However, there are other ideas such as: attitudes which influence the every day life in class; attitudes towards teaching and learning mathematics; attitudes towards the students (learning ability, eagerness to learn) attitudes toward school as an institution and the optimal behaviour in this institution; attitudes towards the economical behaviour in one's job and also one's own motivation; interests and needs as a math teacher and as a human being, wife or husband, mother or father, etc.

(d) The statements of the questionnaire might not have been sufficient enough in their variety of content to raise a valid picture of the mathematical view of teachers, or important questions and aspects could have been left out which would have presented a more realistic picture. For example, the following statements could have been claimed, which aim more at the needs and potentials of the students, and furthermore could have included a motivation aspect of teaching math which stimulates the "social relation student-teacher": "a toolbox full of schemes and algorithms is important to give the less talented students a chance to succeed." "Often it is a great progress if the students know how to use the packet of formulas 'mathematics', e.g. the table of integrals" "I think Freudenthal's claim for a process-oriented teaching is far from realistic." If one had considered these statements he/she could have possibly noticed less process-orientation, and more scheme-orientation. The statements of the questionnaire have - as we believe - (i) possibly registered the essence of mathematics (ii) globally and probably validly. Questions (i), to certain aspects of math instruction and learning of mathematics, would possibly have at least shown relation to the positive view; the teachers who were questioned (ii) would possibly have been inclined to correct or even give up their global, positive view on mathematics if they been asked about marginal areas.

The question, whether or not these three, positive results of the mathematical view of teachers also influence the class behaviour ,cannot be answered. Hopefully, the positive view is also relevant for math instruction in that it determines the attitudes towards teaching and learning mathematics and that the ideas are realised in class and recipated by the students.We also hope that in all types of schools (despite the differences among students, curriculums, frame conditions and even if the math instruction and subject matters are different) the mathematical world view is equally multi-layered and positive.

References


BAUER, L.A. 1988. Mathematik und Subjekt. Wiesbaden: Deutscher Universitätsverlag.

GRIGUTSCH, S.; TÖRNER, G. 1994. Mathematische Weltbilder bei Studienanfängern - eine Erhebung. *Journal für Mathematikdidaktik* **15** (3/4), pp. 211 - 252.

GRIGUTSCH, S. 1994. Methodische Probleme bei der Erforschung von Schülerhaltungen gegenüber dem Mathematikunterricht. Beiträge zum Mathematikunterricht 1994,pp.119- 122.

GRIGUTSCH, S.; RAATZ, U.; TÖRNER, G. 1997. Einstellungen gegenüber Mathematik bei Mathematiklehrern. To appear: *Journal für Mathematikdidaktik.*

GRIGUTSCH, S. 1996. Mathematische Weltbilder von Schülern. Dissertation. Duisburg: Universität Duisburg.

HERSH, R. 1979. *Some Proposal for Reviving the Philosophy of Mathematics*. In: T. TYMOCZKO (Ed.) 1986. *New Directions in the Philosophy of Mathematics.* (pp. 9 - 28). Boston: Birkhäuser.

JUNGWIRTH, H. 1994. Erwachsene und Mathematik - eine reife Beziehung? *mathematica didactica* **17** (1), pp. 69 - 89.

MEINEFELD, W. 1983. *Einstellung.* In: R. ASANGER; G. WENNIGER (Eds.) 1983, *Handwörterbuch der Psychologie.* (pp. 92 - 99). Weinheim und Basel: Beltz, 3rd Ed.

PEHKONEN, E. 1994. On Teachers' Beliefs and Changing Mathematics Teaching. *Journal für Mathematikdidaktik* **15** (3/4), pp. 177 - 209.

PEHKONEN, E; LEPMANN, L. 1994. Vergleich der Lehrerauffassungen über den Mathematikunterricht in Estland und Finnland. Research Report No. 150. Helsinki: University of Helsinki. Department of Teacher Education.

THOM, R. 1973. Modern mathematics: does it exist? In: A.G. HOWSON (Ed.) 1973, *Developments on Mathematical Education.* Proceedings of the Second International Congress on Mathematical Education. (pp. 194 - 209). Cambridge: Cambridge University Press.

THOMPSON, A.G. 1992. *Teachers' beliefs and conceptions: a synthesis of the research.* In: D. A. GROUWS (Ed) 1992, *Handbook of research on mathematics teaching and learning.* (pp.127 - 146). New York: MacMillan Publishing Company.

TÖRNER, G. (Ed.) 1995. Current State of Research on Mathematical Beliefs. Proceedings of the MAVI Workshop October 1995, University of Duisburg. Schriftenreihe des Fachbereichs Mathematik. Duisburg: Gerhard-Mercator-Universität Duisburg. Heft 310.

TÖRNER, G.; PEHKONEN, E. 1996. Literature on Mathematical Beliefs. Schriftenreihe des Fachbereichs Mathematik. Duisburg: Gerhard-Mercator-Universität Duisburg. Nr.341.

TRIANDIS, H.C. 1975. *Einstellungen und Einstellungsänderungen.* Weinheim und Basel: Beltz.

Prof. Dr. Günter Törner
FB Mathematik, Universität Duisburg, D 47048 Duisburg, Germany
toerner@math.uni-duisburg.de