BERND WOLLRING,

EXAMPLES OF SPATIAL GEOMETRIC *EIGENPRODUCTION*[1] IN PRIMARY CHILDREN'S DRAWINGS – REFLECTIONS ON THE DIDACTICS OF MATHEMATICS FOR PRIMARY SCHOOLS

1. Introduction: The importance of *Eigenproductions*

In primary school many students, parents and teachers frequently associate mathematics and mathematics education with normative ideas. The mathematics they experience in primary school often contributes to this picture. Often, primary school teachers, who do not themselves have a positive attitude towards mathematics, concentrate - by necessity and lack of knowledge and skills - simply on drilling syntactical structures. This results in an unfortunate handing down of negative conceptions of mathematics from the teachers to the learners. Current efforts in mathematics education seek to break this cycle of didactic tradition.

In this respect, the integration of reflections on *Eigenproductions* it would be useful, in a fortnote, to explain “Eigenproduction” of children in teacher training and in classroom practice in order to supplement the didactics of mathematics with an essential empirical component seems to be helpful. This idea is not new, but has already been advocated by OEHL (1935) as well as KERSCHENSTEINER (1905) and certainly also by other authors before. In the past, empirical efforts in mathematics education were concentrated rather on measuring how far children's performances would deviate from the normative ideas of the teacher. In the meantime, there has been an increasing awareness of the importance to develop further and expand children's argumentative approaches and informal strategies in the primary mathematics classroom.

The analysis of counting and number sense of school beginners by SCHMIDT and WEISER (1982) is, in the opinion of the author, a guiding research study in this direction. At a time when qualitative empirical studies based on interviews were not yet as popular as they are today, SCHMIDT and WEISER, by analysing children's verbal statements and actions discovered that, in contrast to common professional knowledge, basic arithmetic competencies develop on the basis of ordinal rather than cardinal strategies. A more recent directive study is Selter's (1993) analysis of arithmetic *Eigenproductions* in the primary mathematics classroom.

But the perspective on *Eigenproductions* can also raise a variety of problems for primary teachers: In order to be able to interpret and understand children's argumentative approaches and to organise a mixed-ability mathematics classroom, primary teachers have to be familiar with these *Eigenproductions* not only because of frequently poorly systematically reflected personal classroom experiences but also based on systematic scientific processing during preservice teacher training. The inefficiency of mathematics teaching often lies in the fact that the argumentative approaches and strategies that children contribute are either not recognised or appreciated by the teacher or do

[1] „Eigenproductions“ are oral, written or material products out, which are produced by children or pupils.

not match with the standardised procedures introduced in the classroom. The core of preservice teacher education programs should not be restricted to the systematic training of classroom rituals but rather should concentrate on the analysis of children's *Eigenproductions*. Selter (1993) designates this theoretical analysis as "awareness" and recent German publications in mathematics education such as the monograph "Mit Kindern rechnen" ("Arithmetic with children") edited by Müller & Wittmann (1996) refer to this principle. The different texts suggest different working environments in which mathematics can be learned actively and in a discovery-oriented way (Wittmann, 1997).

2. Geometry in primary school mathematics: Superfluous or essential?

The teaching and learning of geometry is neglected in many primary school mathematics classrooms for several reasons: Geometry lessons require in general more preparation and planning than arithmetic lessons. Furthermore, many teachers find it more difficult to assess geometric achievements than arithmetic achievements. Frequently, geometry lessons are placed in small sections in between units focussing on arithmetic and word problems. This also has the effect of causing deficit in these tuo areas.

One of three reasons that Bauersfeld (1992) puts forward for a better integration of geometry into primary school classrooms is that "arithmetic conceptions cannot develop without geometric underpinning". Geometric illustrations that are introduced ad hoc into the mathematics classroom are by no means self-explanatory, they rather need to be learned. A second reason that Bauersfeld states is related to "changes in children's living environment". The world in which children of today live, is currently changing in such a way that subjective geometric experiences can only be assimilated and developed in the classroom on a much smaller scale than in the past. It is the responsibility of the primary school to create and offer those opportunities for experience that were previously available in children's leisure time. The third reason mentioned by Bauersfeld has already been introduced and deals with *preservice teacher training*. Teacher training programs deliver far too fewer fundamental experiences for the organisation of a geometry classroom based on *Eigenproductions*.

Just as in the field of arithmetic, in the geometry classroom there is the danger of losing argumentative approaches and strategies introduced by the children, while solving geometric problems in class, through a premature standardisation of procedures. At the same time, the potential for *Eigenproductions* of primary children in this field is particularly rich. Wide areas of the science classroom are based on this potential while mathematics education seems to miss this chance. As a mathematics educator, it is the author’s experience that a good understanding of the mathematical potential of these working environments is in general only possible on the basis of substantial content knowledge, in this case geometric knowledge. The question arises as to why mathematics education during subject specific teacher training neglects the *material Euclidean* *geometry* of the pre-service teachers' living environment in favour of an axiomatic geometry which is unlikely to influence their primary teaching (Bauersfeld, 1992).

In particular, spatial geometry is a neglected discipline in primary classrooms, at least in mathematics classrooms. In the following, some exemplary spatial geometric *Eigenproductions* will be introduced with the aim to reveal their meaning. They will be connected with a supporting conceptual vocabulary that has developed in working environments which can be further expanded into effective learning environments for primary schools.

Frequently, geometric *Eigenproductions* are either drawn or constructed and their further development is often based on drawing or building. In general, not only in the mathematics classroom *action competency develops prior to language competency*. Usually, primary children are unable to describe their action-based geometric experiences with a pre-existing language, the language is rather developed in conjunction with or subsequent to these experiences. Just as the vocabulary required in language classes, the language specific to mathematics has to be developed in school. Research studies in mathematics education have highlighted a lead of up to three years of action competency over language competency for almost the entire school-time in some areas. In the geometry classroom children should be offered a rich array of *possibilities to articulate themselves* in both material and social working environments which form the fertile soil for an eloquent geometric language.

3. Spatial perception: Determined by availability of mental actions

Spatial perception capability is characterised by availabilities of mental action as well as articulation based on these mental actions. Spatial perception capability does not only involve objects existing in reality, every artist and engineer is dependent on his/her ability to imagine and articulate non-existent objects.

In the following, hand drawn products will be the focus of examination. We report results of analyses and from own repetitions of them and new analyses within our own work with teachers. The first main goal of these pilot studies is to check if the results are still valid or culture specific. The second main goal beyond this text is to check how the results are influenced in working environments with interaction.

With respect to the representation of spatial objects and situations in children's drawings, one can distinguish two main strategies, *morphological strategies* and *occlusion strategies* which can appear simultaneously in the drawing of a single child.

Morphological strategies are applied by children when drawing single massive solids. They are subject to developmental stages that are dependent on the shape of the solid (Mitchelmore, 1978;

Chen, 1985; Woodrow, 1991; Wiese & Wollring, 1995). Spatial perception is generally characterised by the ability to visualise objects and mentally move them around. Expanding this we define

spatial perception
as the ability to visualise configurations of spatial objects and the viewer
as well as the ability to change this configuration by mentally changing
the position of the viewer relative to the objects.

When drawing solid objects, one can observe a gradual transition from a mental viewer revolving relative to the object to a mental viewer who is stationary relative to the object. The morphological strategies when drawing solids could be correspondingly interpreted as *gradual transition from sequential to simultaneous coding of the spatial depth* in the drawings (Wollring, 1995 a and b).

When drawing several objects which form a spatial configuration, *occlusion strategies* are additionally used for the coding of foreground and background. Children tend to prefer these strategies when the objects are spatially difficult to represent or have no spatial depth (Freeman, 1980; Cox, 1985; Schuster, 1990). In order to expand further this concept we define

spatial perception
additionally as the ability to visualise an object in space
and to mentally reversibly disassemble, enlarge, or shrink or perform
any other reversible changes for the purpose of its visualisation.

This and the mental moving of objects and viewer has the effect that when drawing several objects or configurations in space, firstly objects which are modified in this sense or *"space slides"* are portrayed. We define space slides as separate pictures of planes, which have to be spatially perceived as one lying behind the other. In the course of development, lines that cannot be seen from the perspective of the observer are increasingly eliminated in the drawing. This leads to a gradual *transition of side by side separated depictions to representations with partial occlusions of objects standing one behind the other in space* (Wollring, 1995 a and b).

4. Working environments for spatial drawings

In the following, first experiences with some working environments in which children were asked to draw spatial objects without further guidance ("Draw what you see") are introduced. The non-guided drawing of spatial objects seems to be influenced by a variety of situative factors. Standardisations result in the loss of various original interpretations of children's drawings. It seems sensible not only to take into consideration rather classical studies which derive "standardised patterns of pictures" to characterise frequent types of drawings or particular stages of development.

During their preservice training, teachers should rather implement case studies with a constructivist perspective both with individual students and with small groups of children. In addition, particular interpretations of children can be exploited during interviews. A decisive factor in this respect is the understanding that children's drawings which at first sight appear to be "deficient", rarely indicate a deficiency in spatial perception. These drawings should not be interpreted in the sense of *visual realism* but rather in the sense of *structural realism*, that is, in the sense of an explanation of the spatial scene beyond the visible for a fictitious viewer.

4.1 Children's spatial drawings with occlusion strategies

In the following, we briefly introduce the *"hat-pin experiments"*, *"apple experiments"* and *"card experiments"* as examples for working environments in which children articulate their spatial perception abilities predominantly through occlusion strategies.

Hat-pin experiments: In the original version of this experiment (Clark, 1897) the task was to draw an apple which had been pierced by a hat-pin. In a later variation of this experiment (Freeman, 1980) the task was to draw an apple behind which a hat-pin protruded from a table. A new edition of these experiments (Wollring, 1995 a and b) confirms the results by Clark: Structural realists among the primary children draw the whole hat-pin going through the apple, whereas visual realists draw only the visible ends of the hat-pin protruding from the apple. In Freeman's variation, apart from visual realistic drawings we also find pictures in which the hat-pin has been drawn above or next to the apple, which we interpret as intended explanation of the scene.

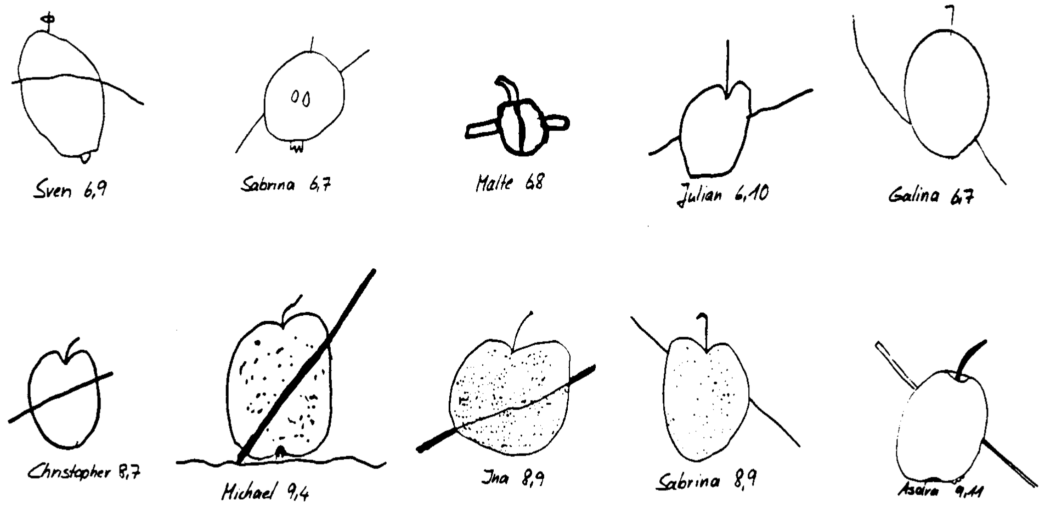

Figure 1: *"Hat-pin through apple": Drawings from new editions (1995) of the "hat-pin experiments" based on Clark. Top row: 1st grade students; bottom row: 3rd grade students*

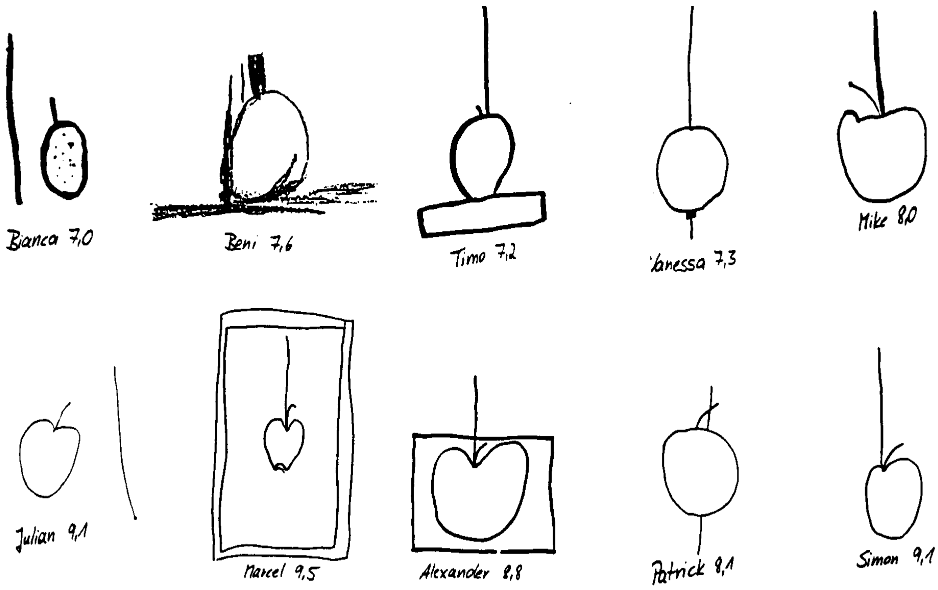

Figure 2: *"Hat-pin behind apple": Drawings from new editions (1995) of the "hat-pin experiments" based on Freeman. Top row: 1st grade students; bottom row: 3rd grade students*

Apple experiments: In these drawings the spatial perception ability in the sense of the previously described perception in the children's drawings is not only articulated in such a way that the position is varied relative to the scene but also that the objects to be drawn are represented in a modified way. That is not in order to suggest real changes, but rather in order to explain the relative spatial position of the objects in the drawing. This phenomenon reveals itself in our apple experiments in which children were asked to draw either two apples one behind the other or three apples in a row touching each other. Structural realists draw the apples sometimes separated either one above the other or next to each other, sometimes one including the other, sometimes enlarged and sometimes from a different mental viewpoint, which they were de facto not allowed to take.

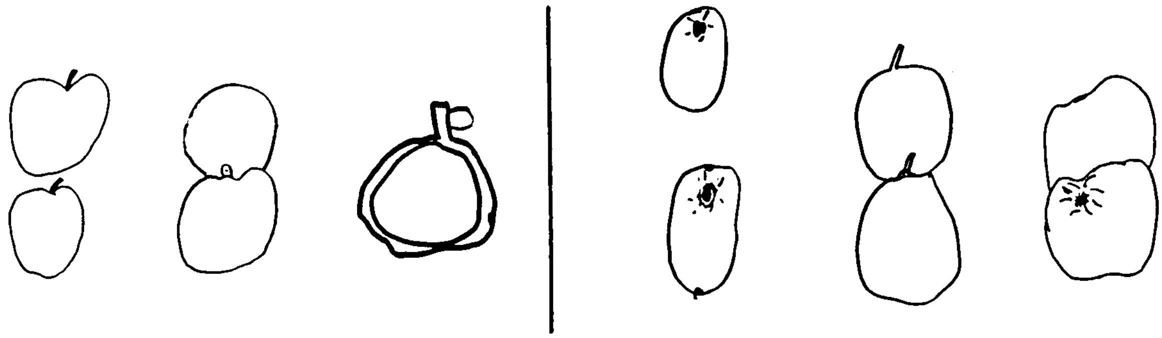

Figure 3: *Children's drawings from the "Two apple experiment". Left: 2nd graders; right: 4th graders*

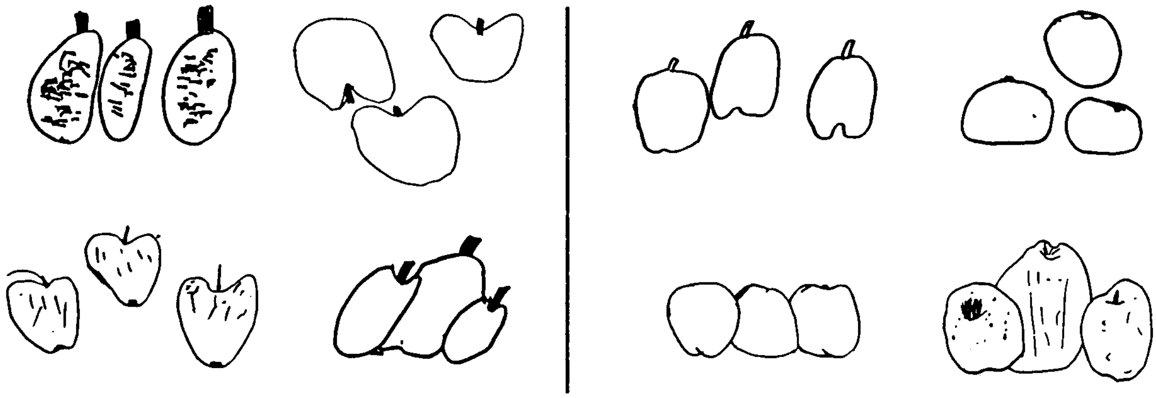

Figure 4: *Children's drawings from the "Three apple experiment" (middle apple behind)*
Left: 2nd graders; right: 4th graders

Card experiments: These strategies are articulated even more clearly in our card experiments. The task was to draw three cardboard squares standing up on a table in a row, either one standing behind the other or diagonally staggered. The majority of the primary children's drawings were structural realistic representations, visual realistic pictures were rarely observed. The structural realists drew the row with one card standing behind the other sometimes completely separated above or next to each other, sometimes completely occluding each other, sometimes partly overlapping with or without a concept of base-line and mostly neglecting the square shape. The pictures of the diagonally staggered cards contain more visually realistic elements, but the previously mentioned characteristics of a structural realistic representation predominate.

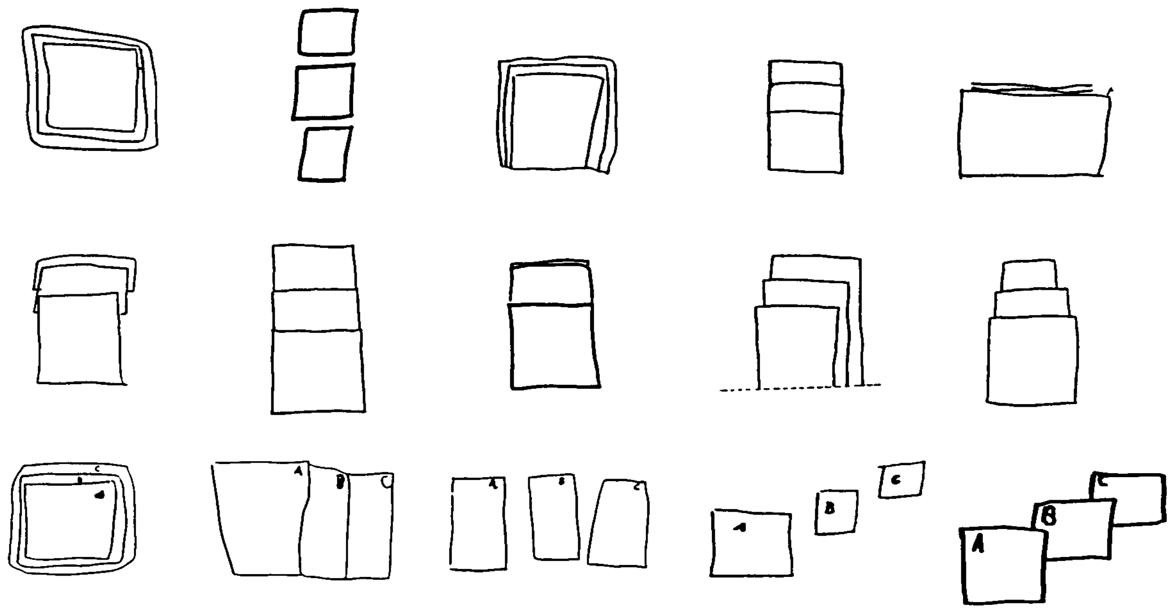

Figure 5: *Children's drawings from the "Three card experiment" (cards lying one behind the other)*
Top row: 2nd graders; middle row: 4th graders; bottom row (staggered diagonally): 2nd graders

4.2 Children's spatial drawings with morphological strategies

Valuable starting points for the design of individual working environments are the investigations by Lewis (1963) and Mitchelmore (1987). The experimental design of Lewis is particularly suitable for the repetition of the experiment with a small group and for modifications. Lewis took a cube with a door drawn on the front side face and a window on both left and right side faces, moved it back and forth in front of the children's eyes and then instructed them to "Draw what you see". She presented her results as a sequence of patterns of pictures which she called *pre-schematic*, *schematic*, *pre-realistic* and *realistic*.

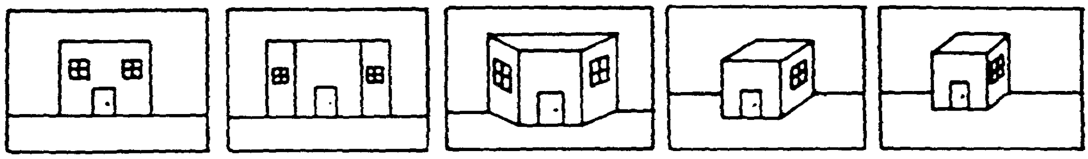

Figure 6: *Patterns of pictures from children's drawings of a painted cube following Lewis (1963)*

Mitchelmore's experimental design is further standardised and proved suitable for repetition with individual children. He asked children to draw five solids, which were placed in single closable boxes, from a certain fixed viewing position. He presented his results in the form of a picture matrix which documents the typical stages of development of the different solids. For cubes and cylinders it did not seem to make any difference whether the children could view them permanently or only for a short moment.

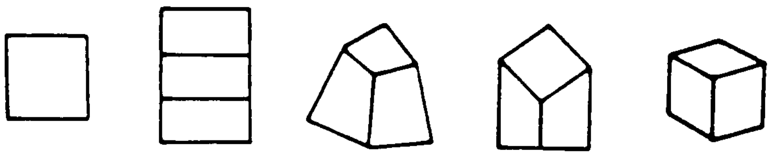

Figure 7: *Patterns of pictures from children's drawings of a cube following Mitchelmore (1978)*

Although Lewis and Mitchelmore support by test-statistical evidence that their results do not arise accidentally, the children's drawings appear to be strongly influenced by the respective context. It seems to be problematic to ascribe the documented stages of development to certain age groups. On the contrary, the qualitative courses of development outlined in the patterns of pictures appear to be more useful categories for the interpretation of one's own research results.

Woodrow (1991) applies this idea in a very simple but effective working environment, that seems to be appropriate for primary students: Children were asked to draw an asymmetric u-shaped construction made from six connectable cubic building blocks while they were sitting in a circle around it. A vast variety of children's drawings are documented. They are only coarsely classified by age and single characteristics and leave enough room for interpretations and in this way challenge analysis as well as one's own further experiments. The drawings of the four and five year old children indicate that children of that age are not primarily interested in the spatial structure of the construction but rather document in their drawings in which contexts the connectable cubic building blocks are meaningful to them. (These connectable cubic building blocks are didactic material rather than toys.) Some drawings of the nine up to fourteen year olds demonstrate their analysis of the spatial structure of the construction while others only show a plane view.

In our own investigations we have asked primary children (grades 1-4) and junior secondary students (grades 5 and 6) to draw a construction made of cubes, as it was used by Woodrow, under various conditions. Each child was given his/her own construction which for the primary children consisted of connectable cubic building blocks which are customary in primary school. The junior secondary students on the other hand received constructions made from smooth wooden cubes (3 cm). The only instruction was "Draw what you see!" and the drawing had to be made on blank paper. The following drawings show a selection of *Eigenproductions* of students from grades one, four and five.

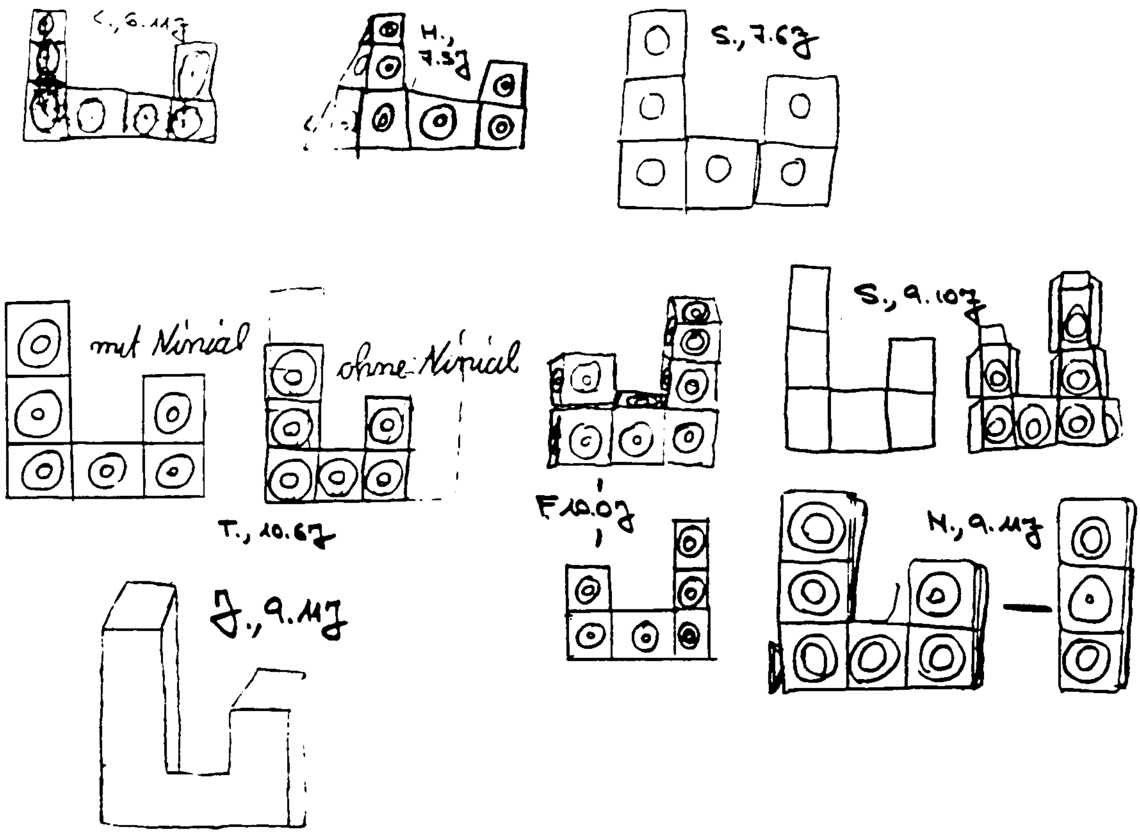

Figure 8: *Children's drawings of a "Woodrow-construction" made of connectable cubic building blocks (Wiese & Wollring, 1995). Above: 1st graders; below: 4th graders*

When entire school classes solve this task together, one cannot expect the same variety of drawings as observed in individual experiments. The children communicate verbally as well as non-verbally and informal conventions develop regarding the proposed solutions, so particular patterns occur in some classes but not at all in others. Within individual classes one can often distinguish between stages of development comparable with patterns of pictures documented by Lewis and Mitchelmore. In addition, there are other cases in which spatial perception is articulated in such a way that the drawing represents the construction in a certain modified form, for example divided or transparent regarding certain parts or rotated or shifted in certain elements.

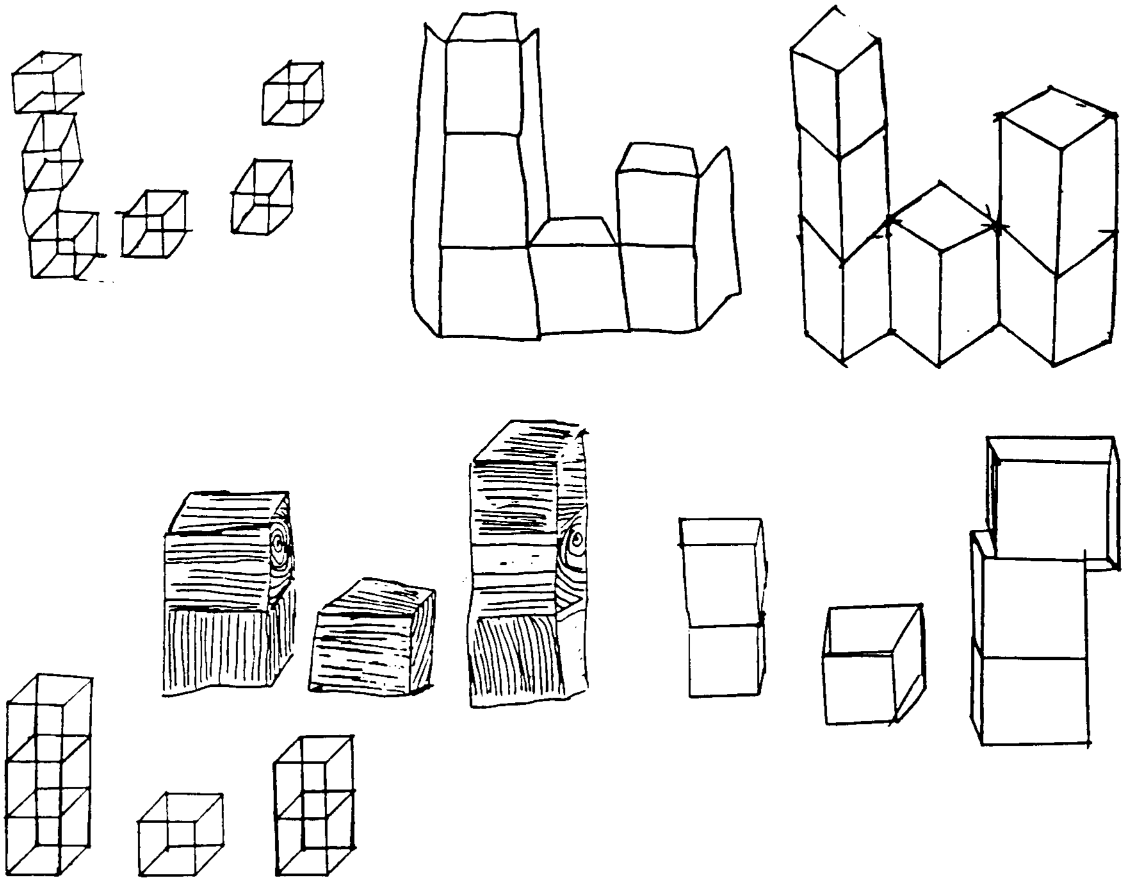

Figure 9:*Children's drawings of a "Woodrow construction" made of wooden cubes (Wiese & Wollring, 1995)*

Above: Remedial group for 5th grade low achievers

Below: Remedial group for 5th grade students with difficulties in reading and writing

Dominating are plane views of the u-shaped faces, *spatially intended "folding pictures"*, non-guided orthogonal projections from two directions and representations in "*Kavaliersperspective*", which German children obviously prefer even before they are given specific instruction. This preference may be the consequence of the customary use of squared paper. Our results also substantiate transitions from sequential to simultaneous coding of the spatial depth in the drawings. One child from a remedial group asked in response to our instruction to draw the cube based construction "Why?" in order to find out the purpose of the drawing. This indicates that the drawing might be *influenced by the intended purpose*.

The fact that Woodrow's cube based construction only consisted of one layer of cubes may be the reason why many students seem to regard a plane side view as sufficient in order to characterise the given construction unmistakably. Some of the primary students additionally draw the one layer thick side view.

5. Prospects: *Eigenproductions* - Possibilities for self-organising learning environments

The *Eigenproductions* of children previously introduced have been deliberately described only in an exploratory way. Further information becomes available once the children comment on their drawings in interviews. This leads the author to the following constructivist perspective on the demonstrated geometric *Eigenproductions*: *Children not only draw what they see but also what they know, and they not only draw what they know but also what they would like to communicate.* In order to analyse the selection referred to in the second differentiation we need to know *for whom and why* the children made their drawings. In the understanding of the author, the variety of the phenomena described above develops to a lesser extent as a result of the inability of children to draw the objects, but rather because the children had deliberately not been informed about the recipients and the purpose of the drawing.

One interpretative hypothesis in this situation is that the children anticipate themselves or an archetypical self-copy as recipients, that is that they draw what appears to be important from their personal point of view. A child would possibly substantially modify his/her drawing, if he or she were drawing for a friend or a familiar adult. *Geometric Eigenproductions are messages.* They reveal in how far the child models the need for information of the anticipated recipient.

Up to now we have not taken into consideration that geometric *Eigenproductions* of primary children are *part of a history of learning* and therefore are subject to change and modification. This will be part of our future work. The author is convinced that the central structure of the working environment which advances the history of learning and which represents the decisive motivation for achievement is the *appropriation of Eigenproductions*. This is the basis of self-organising and self-differentiating working environments as they have been advocated by Faust-Siehl (1996) as desired working environments for the primary school of the future. In order to allow this freedom of self-organisation the teacher needs a diagnostically trained overview and - resulting from that - the authority (or possibly autonomy) to give childlike argumentative approaches both shelter and freedom for their development.

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Prof. Dr. Bernd Wollring
Didaktik der Mathematik
Fachbereich 17 Mathematik-Informatik Universität GH Kassel
Heinrich-Plett-Straße 40
D-34132 Kassel
e-mail:wollring@uni-muenster.de