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**TEACHING GEOMETRY IN ELEMENTARY SCHOOLS
RESULTS OF THE EVALUATION OF AN INQUIRY ON TEACHERS AND
TEACHING POST CANDIDATES**

Abstract

To obtain a snapshot of the actual educational classroom practices in geometry teaching was the goal of the evaluation of an inquiry. The inquiry was distributed to teachers and teaching post candidates of elementary schools in North-Rhine-Westfalia in 1995/96.

The lecture reports on selected results of the evaluation. For example: Which items of geometry are the most important for the teachers? Which items are realised in practice? How do the teachers rate the school book used in the classroom? From the teacher's point of view, will teaching of geometry be neglected? Which geometrical activities are articulated in other teaching disciplines?

Looking through the published articles addressed to mathematics teachers, an always repeated appeal for geometry teaching in elementary schools could be noted (e.g. Radatz 1989 et al.): Do not disregard teaching geometry!

Geometry teaching in elementary schools is thought to be badly accepted by most people. It was described in terms like: "just managing to keep alive", "even neglected" in many cases, "Cinderella" or "steps motherly life": also it was specified as the "poor relation" of the elementary school.

The authors found many weighty reasons for the actual state of teaching geometry. However the significance of geometry items and classroom activities in geometry depend on the attitude of the individual teachers. This is a very strong reproach which has to be submitted to many in the teaching profession.

The problem (neglecting geometry) is assumed to be founded in the teachers' personality itself, in a supposed lack of time, in the limits and the blindness of the commonly used priorities and also in the lack of courage for new enterprises. There is no courage to take the risk of fresh and novel experiences with oneself and with the children (Bauersfeld 1993, p. 11).

Neither the justification of the teachers nor the assignment of guilt was the aim of the applied questionnaire. The questionnaire was distributed from November 1995 to February 1996 in elementary schools in North-Rhine-Westfalia (NRW) randomly selected - mainly in the administrative districts of Detmold, Münster and Arnsberg. The addressees were teachers and candidates for teaching posts.

The main goal of the inquiry was to obtain an instantaneous picture of the actual educational classroom practices in geometry teaching:

- Which items of geometry are the most important for the teachers, which items are realised in practice?
- How do the teachers rate the school book used in the classroom with regard to the organization and planning of lessons?

- Which kind of materials are the teachers familiar with and how do they apply these materials in classes?
- How are the selected objectives “Support of spatial imaginative faculty” and “Contribution to environmental discovery and experience” ranked?
- From the teachers’ point of view, is geometry a neglected subject matter? If yes, which reasons are responsible for this situation?
- Which geometrical activities are articulated in other teaching disciplines or instructions?

As a result of these guiding questions, the questionnaire was divided into six thematic complexes. In order to get the most information possible about contents, different opinions, direct and personal experience, the questionnaire method was chosen for this investigation.

Although the questionnaire method is mainly an instrument applied in quantitative social research, an inquiry of transferable and generalizable data was not carried out and therefore the representative random sampling was not selected.

It was more important to motivate the teachers to “talk” about the subjects, in order to find out about their ideas and experiences close to their everyday life at school and their personal opinions (see Lenzen, 1989, p. 626) and to obtain knowledge about the “real state” in geometry teaching.

As the interviewed persons are academically educated and verbally competent, I took the liberty to ask a great number of open questions. In this respect the response to the questionnaire yielded very positive results. The questionnaires present evidence for a great variety of practical instructional ideas in teaching and for spontaneous expositions. But they also show a critical reflection about the teachers’ individual instruction method, their individual roles as teachers and a very differentiated perception of the children. I think that a great number of the interviewed persons are in an intensive conflict when teaching geometry. Therefore the image outlined by this questionnaire on geometry teaching is more positive than it can be assumed from the teaching practice actually applied.

Previously geometry was not always accepted as a teaching subject. Asking headmasters and headmistresses to allow the distribution of the questionnaire to the interested staff evoked scepticism and a guilty conscience: “Geometry, we don’t know if we can help you. Honestly, we only teach so little of the subject!” But it also evoked interest: “Geometry?! Yes, it is necessary to push forward the teaching of geometry in the future.” The feedback of the questionnaire shows that the teachers and the candidates for teaching posts were very positively engaged in the discussion about these questionnaire items, in spite of the costs and high expenditure of time and extra work. 128 (about 27 %) of a total of 480 questionnaires were returned (108 from teachers, 20 from teaching post candidates). I thank the Ministry of School and Further Education NRW for the permission to distribute the questionnaire.

For some selected results see the following text.

1. Associations and pictures:

2. “In my mind geometry teaching in elementary schools is ...”

Please complete the following sentence!

“In my mind geometry teaching in elementary schools is.....”

The first task was to complete the above sentence. This led to the expected broad range in the response spectrum. In my opinion, it is therefore I think not reasonable to assign the great entirety of answers quantitatively to definite criteria retrospectively. Furthermore, many individual answers include different aspects, making it very difficult to extract a general statement. Therefore I shall use some typical quotations instead.

In my mind geometry teaching in elementary schools is ...

„... of secondary importance, because I normally need all the four lessons of mathematics weekly to cover the remaining items.“

According to this teacher’s answer, geometry instruction is demoted to the second rank by the other mathematical contents. This is due to the lack of instruction time. Although the curriculum in NRW does not set priorities to arithmetic, geometry or measurement (money, length, time, weight, volume, area), geometry is degraded with respect to the other topics in the hidden curriculum of teachers. This was described by another teacher. Here there seems to be a need for justification regarding the fundamental ideas in teaching geometry, showing it to be worthwhile to be taught despite time deficiency and the overburden of items.

In my mind geometry teaching in elementary schools is ...

„... an appropriate means to liven up the occasionally monotonous numerical calculations.“

This statement shows clearly the delimitation between geometry and teaching numerical calculations. Calculating, often experienced as rigid and stupid, must undergo changes in order to live up with regard to the children and the teachers as well.

Nevertheless the idea to liven up leads to the conclusion that geometry lessons are seen as a kind of break in the normal teaching of the subject matter. This implies that teaching numerical calculation is consuming most of the time in the lessons of that teacher.

The delimitation between geometry and numerical calculation shows that both are regarded separately with only a few points of contact (in its contents, and also referring to the methodical aspects).

In my mind geometry teaching in elementary schools is ...

„... often of minor importance, because in my childhood I didn’t like geometry very much.“

In this case a very personal argument for the neglect of geometry is given, i.e. this teacher's own negative classroom experience in geometry. It is a deeply-rooted experience which was apparently not assessed and reflected until now. There still remains the risk of infection while the own emotional rejection of geometry is transmitted from the teacher to the children. Since geometry is considered to be „of second rank“ it rarely occurs in the teaching practice of this person.

In my mind geometry teaching in elementary schools is ...
 „... an exciting matter. But it is not always easy to imagine what happens in the children's minds.“

“Exciting”, that includes positive and negative aspects as well. For example, it is exciting - especially for new teachers - whether the course of a lesson runs as it was organised and planned. It is exciting to observe in which manner the children handle geometrical tasks, which kind of ideas they have, which ways of problem solving they find and whether they can be motivated by geometrical tasks. It may be that geometry lessons will become exciting, because many ways of solution - and also different results - are often possible.

But each time this excitement also involves an uncertainty. Geometry cannot be organised and planned exactly and the reactions of the children cannot be foreseen accurately - perhaps because geometry lessons permit a wide range of open teaching. Therefore the teacher is not always able to be one step in advance of the children to understand their arguments and to internalise what their thoughts are. To stand this excitement is probably a greater performance than the exact planning of each lesson - but it also leads to uncertainty and doubts with respect to one's own competence of action.

In my mind geometry teaching in elementary schools ...
 „...has not been of importance, in its real meaning as geometry teaching. More to the point, it appears in all sorts of disciplines in elementary schools.“

This teacher is sure that geometric tasks seem to be distributed to other subjects in the classroom practice. In this case, geometry is seen more as a teaching principle than a discipline of mathematics.

I think nothing would be lost if the basic geometrical ideas could be experienced in different subjects on the basis of factual depth and content - by comparison it would be fatal if the pushing of geometry to other educational subjects only had a cover-up role to conceal its neglect in the elementary schools.

Which image of teaching geometry do the questionnaire participants and teaching post candidates have?

Different accents have already been given by the small number of these citations.

- methodically orientated accents, focussing on the way of learning geometry;
- motivationally oriented accents, describing the pleasure in learning and the interest of the children and characterising geometry as a means to liven up mathematics lessons;

- teaching-matter-related and content-related accents listing general mathematical and specific geometrical teaching goals;
- institution specific accents, where geometry is seen in the context of teaching in general and curriculum;
- accents which regard geometry to be neglected etc.

2. Taught items and activities

What are the geometric content and activities which are really taught in classroom? On the basis of the NRW curriculum, a list of geometric items was presented to the questionnaire participants. As a first step, questionnaire participants were asked to mark all items and content taught in their classrooms. The following items were at the top of list:

The most frequently taught items, contents and activities of the inquired persons		
Item/activity	Number of answers	Class according to the NRW curriculum
• Positional relationships: spatial orientation	100	1.
• Figures: laying of patterns	98	1., 2., 3., 4.
• Positional relationships: orientation by city maps, quarters	94	3.
• Figures: discerning in the real world	94	2.
• Figures/symmetry: mirroring	94	3.
• Figures: drawing	93	1., 2., 3., 4.
• Bodies: discerning in the real world	93	2., 4.
• Measures: drawing and measuring of distances	93	2.
• Figures: finding of properties	92	1., 2.

Table 1

Due to their lack of practical classroom experience, student teachers' answers were not taken into account.

Table 1 shows that content and activities in the field of figures and spatial relationships are predominantly taught in geometry by the questionnaire participants. It shows also that the given items can be assigned to teaching goals mainly in the first and second class according to the NRW curriculum. Table 2 shows in contrast those items of minor importance:

The most rarely taught items, contents and activities		
Item/activity	Number of answers	Class according to the NRW curriculum
• Measures: determination and comparison of volumes	27	3., 4.
• Symmetry: discussing purpose and function	38	3.
• Bodies: discussing purpose and function	47	4.
• Figures: enlarging and reducing	47	4.
• Bodies/symmetry: mirroring	50	3.
• Figures: discussing purpose and function	52	2., 4.
• Symmetry: discussing the experience with mirror images	52	3.
• Bodies: drawing	53	4.

Table 2

In this case, content belonging to the category “bodies”, is slightly predominant. The assignment of content and activities to the corresponding classes show that mostly third or fourth class items are neglected or less frequently discussed in many cases.

Different reasons are suspected to be responsible for the fact that first and second class items are especially popular while third and fourth class items are partially neglected.

- Perhaps, the lack of time growing from school year to school year may be the cause for the disregard of geometric items. This thesis contradicts the number of geometry hours taught in school, which is at its maximum in the third and fourth class (see Table 2): weekly geometry teaching 14-15 min in the 3rd and 4th class while it takes 11-12 min in the 1st and 2nd class.
- Additionally, those seemingly neglected items are of a greater complexity, but on the other hand they do not belong to the traditional items of geometry (e.g. discussing the purpose and function of bodies, figures and symmetry). Consequently are these teachers only capable of teaching the geometry content of the first and second class, due to their own knowledge level?
- It has also to be investigated to what extent school books contain material covering these items, material which the teachers believe they can use in their classes.

Concerning the most frequently mentioned items, we find mainly non-verbal activities like “orientation”, “laying”, “discerning”, “mirroring”, “drawing”. Contradictorily, in the less frequently elaborated items it is surprising that the word “discussing” is mentioned four times. With regard to the questionnaire participants, it seems that ‘doing’, i.e. the handling of concrete objects, is of great importance. This level is less frequently achieved when the acquired experiences must be described verbally to given feedback on the activity.

Moreover the questionnaire participants were asked to mark three items/activities of special importance and significance. They were also asked to give the reasons for this assessment (see Table 3).

The emphasis items and activities displaying a direct relationship to reality or environment is very evident showing a special accent on stereometry. Reasons like “closeness to reality” and “help in one’s life” are the main decisions when referring to the relevance of geometric content.

Under these aspects the assessment of the questionnaire participants seems to be less critical, i.e. it should be differentiated between “reference to the environment” and “closeness to life” as an activity. For example “orientation by city maps and quarters” is of secondary importance for the childrens’ actual situation in life.

For the choice of a particular geometric item, it can be concluded that further considerations play a larger role than purely internal mathematical importance.

Items and activities, most important for the questionnaire participants		
Item/activity	Number of answers	Class according to the NRW curriculum
• Spatial relationships: spatial orientation	65	1.
• Spatial relationships: orientation by city maps, quarters	62	3.
• Measures: measuring and drawing distances	41	2.
...		
• Figures: discussing purpose and function	7	2., 4.
• Bodies: mirroring	7	3.
• Figures: laying of patterns	6	2., 4.
• Bodies: specifying by name	5	2.
• Symmetry: discussing the experience with mirror images	3	3.

Table 3

The comparison of the items appearing most frequently to those of less importance in the questionnaire participants’ point of view makes the following differences become evident. “Specifying of bodies by name” is seen as less important (only 5 citations) but many of the teachers questioned selected this item for teaching purposes (78 citations). A similar example: only six teachers think that “laying of pattern” is an important activity but 98 persons used this skill in their classroom teaching. The question remains: which geometric items/content are really important (see Bauersfeld 1993, p.8)?

The participants’ opinions on this point vary greatly. Although items were assessed as important by the teachers, they were not given a high ranking in their educational practice. At the same time instruction time is wasted by less important content and activities.

A consensus which helps to separate essentials and fundamentals from trivialities is lacking - a chain of reasoning which shows what it means to withhold the discussion of certain items in the classroom from the children.

3. Demands on school books

The questionnaire participants were requested to assess the supporting role of school books and teachers' manuals used for the preparation or realisation of geometry lessons.

The books named by teachers and teaching post candidates are the school books most commonly used in NRW.

The valuation of individual aspects is very different. Some teachers regard the content as sufficient while others describe the same school book as "poor" and less profitable for the preparation of lessons and for teaching practice.

Remarkable: not a single book was really praised and well accepted. The reactions and inputs are characterised as "satisfactory", "average quality" and "sufficient" in most of the answers. It seems as if the expectations of teachers are not very high even before they start reading school books. Typical quotation: "The teaching aids or proposals are sufficient for a teachers' manual". The following comments should not be regarded as an attempt to recommend selected school books or as a valuation of their usefulness, but as a collection of teachers' requirements. For this reason the most frequently cited points of criticism were not assigned to particular books or teachers' manuals. Table 4 represents a collection of teachers' wishes or demands on school books summarised from the questionnaire participants' inputs.

Wishes with reference to the teachers' manual		
Aspects	Wishes	Number
Subject related background knowledge	<ul style="list-style-type: none"> • (More) subject related explanations • Comprehensibility of the subject related explanations also for non mathematicians • More literature references 	17
Advice for the teaching practice	<p>More ideas, proposals and for impulses:</p> <ul style="list-style-type: none"> • First steps in the subject • Differentiation • Learning by discovery and self-activity of children • Interlinking with arithmetic • More realisable formulation of problems • Offers for free (independent) classroom work • Projects and interdisciplinary topics <p>Didactical advice:</p> <ul style="list-style-type: none"> • More detail • Illustration of links between the individual contents 	44

Table 4

Compared to teachers, candidates for teaching posts in particular tend to criticise in a rather negative way. Probably they are looking for examples of demonstration lessons in the school books as

a help for their own teaching. Typical quotation: “The ideas are unsuitable for demonstration lessons.”

Furthermore the questionnaire participants criticise suggestions to “measure a kilometre” as impractical. They feel that such suggestions do not comply to classroom practice at all.

Particularly teaching post candidates and “self-taught mathematician” (36% of the questionnaire participants) articulate the desire for more subject-related background knowledge.

It has to be emphasised that school book tasks often only deal with partial aspects of a main topic. Another deficit is the lack of suggested further reading matter and literature references expanding upon books’ contents.

Do the authors of school books underrate the group of teachers who are not willing to work through a school book page after page but would be grateful to get more information and advice giving them more competence and self-confidence in their own teaching practice and help to meet the individual requirements of their learning groups in geometry?

Wishes with reference to the pupil’s book		
Aspects	Wishes	Number
Volume and distribution of the chapters	<ul style="list-style-type: none"> • Inclusion into a framework • Putting together the chapters to a course • References with other contents • More geometry • More exercises, also repetitions • Improvement of the succession 	35
Setting of tasks	<ul style="list-style-type: none"> • Less text, more figures • Simplification of the texts • Clear working instructions • More orientation to the childrens’ world • More motivating, children-related, actualised exercises and interesting tasks • No mental overstrain, but also no trivialities • Less theoretical abstractions • Broad range of skills • Pleasing and attractive games 	41

Table 5

The small volume of the geometry chapters in comparison to the volume on numerical calculation is very seriously criticised. Due to this fact, teachers have to prepare work sheets themselves.

Additionally it is pointed out several times that the exercises often do not correspond with the childrens’ intellectual capabilities. In most cases the exercises are described as trivial and poor, so that the children are not motivated.

To solve this problem, the development of interesting, constructive and upgradeable exercises, is necessary, which should supersede drawing patterns, counting edges and corners - so that the rare time is filled with reasonable tasks.

A remarkable proposal is to counteract the isolation of the geometry items by combining the different parts as a short separate course - comparable to the traffic instruction. This proposal describes the observation that the distribution of geometry chapters between the arithmetic pages does not permit an appropriate inter-linking of the themes in most cases. The question remains: Does the separation of geometry make it even more neglected, or does its separation draw attention to its importance? It seems that teachers find only a few suitably prepared items of interest which are also orientated to the life of a child.

4. Reasons for neglecting geometry

Teaching geometry often needs justification - in comparison to the remaining mathematical items taught (time is short): a justification for the child (they say: "This is not mathematics"), for colleagues (having taught more of the subject matter), for parents (they think this is only gambling). Nearly 80% of the teachers agree with the statement that geometry teaching is neglected.

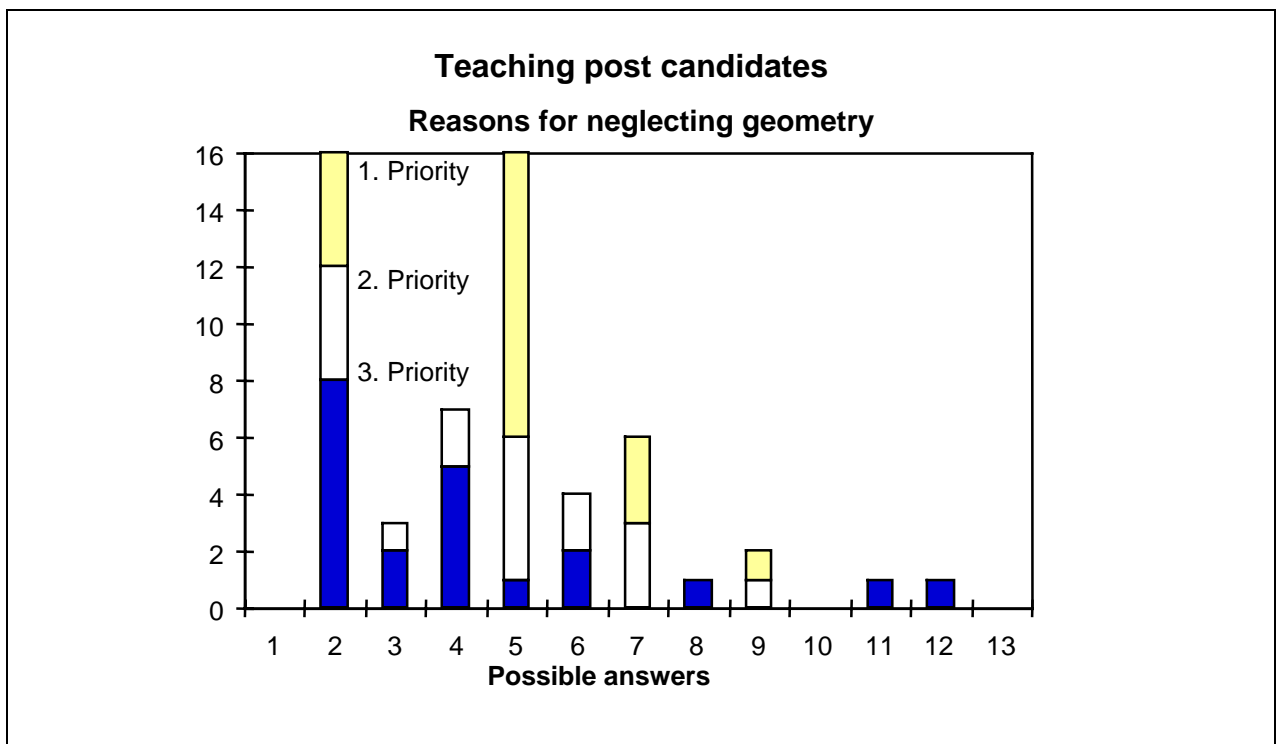
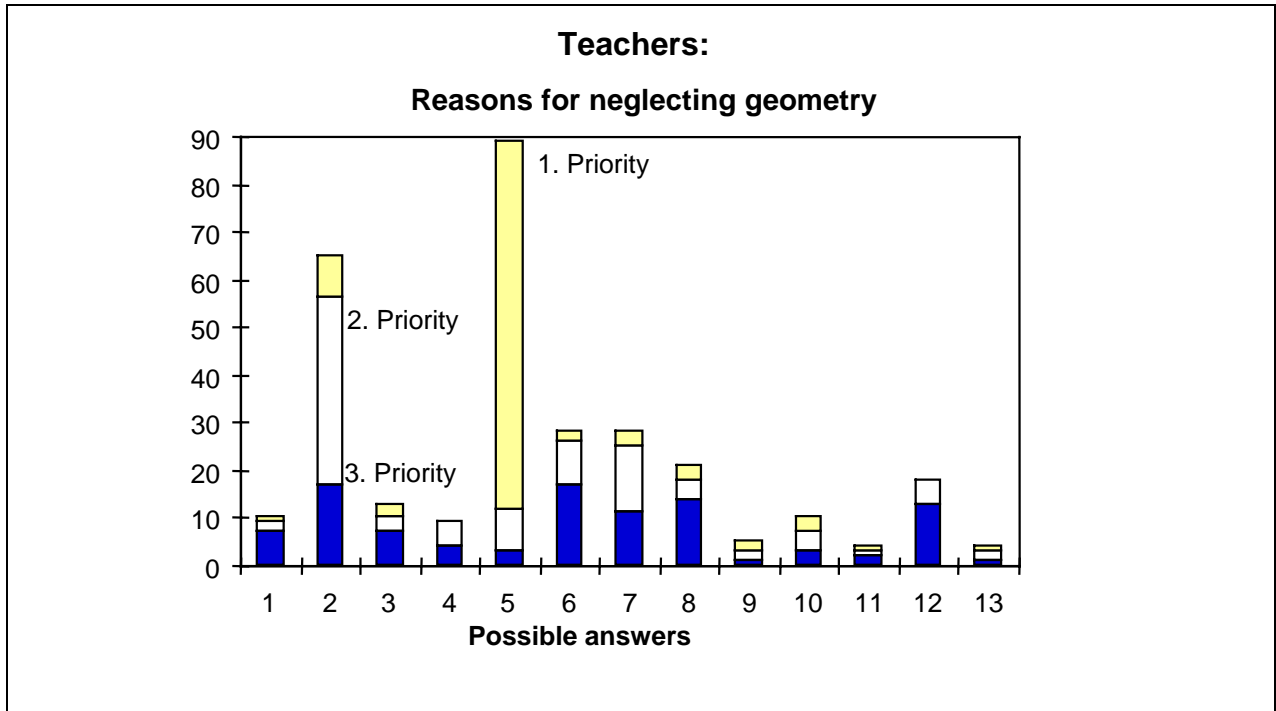
In order to search for reasons which contribute to this neglect, the following list was presented to the questionnaire participants. They were requested to tick off three main reasons.

In your opinion, which reasons are most responsible for the neglect of geometry teaching (GT)? Please tick off the three reasons and assign 1 to the most important reason, 2 to the second, and 3 to the third.

Possible reasons	
1	• GT requires a great amount of subject related preparation
2	• GT requires a high expenditure with regard to making and supporting materials
3	• GT requires a high methodical preparation
4	• It is difficult to test the achievements of the pupils
5	• The subject matter of arithmetic has to be studied thoroughly, there is no time left for geometry
6	• There are only a few supporting aids for the preparation of GT
7	• Teachers think that their own pre-knowledge in geometry is insufficient
8	• Teachers are feeling insecure in teaching geometry
9	• The goals of GT are not really important because...
10	• The development of geometric capabilities in children depends on an internal process of maturing which can neither be accelerated nor retarded
11	• Teaching of GT starts in secondary school
12	• Many proposals described in school books are trivial, not challenging enough
13	• ...

Table 6

The following diagrams show the frequencies of the reasons given by the teachers and teaching post candidates - subdivided into the first, second and third priorities.



As expected, teachers and also teaching post candidates think that the main reason is the dominance of arithmetic items and the accompanying time expenditure (89, 16 citations respectively) on the one hand, and the requirement of a large supply of materials on the other hand (65, 16 citations respectively). 55 teachers gave combinations of both reasons.

The curriculum, which is “too full” to be practical, is the reason for lack of time. “The arithmetic items have to be done” - this imperative statement was often emphasised by the questionnaire participants. Quotation: “To be done! - we are too quick in accepting this order”.

The reason why the geometric items in particular are pushed aside due to the lack of time is due to the tradition that arithmetical calculation skills are thought to be the most important ones (see Radatz/Rickmeyer 1991, p. 4 and Besuden 1988, p. 5).

Another reason behind this argument is as follows: “The aims of geometry teaching are not so important”, as one teacher comments, “because geometry starts in secondary school ‘from the beginning’ whereas the four mathematical operations are necessary basics which have to be elaborated upon until the child leaves elementary school”.

On the one hand the responsibility for teaching geometric skills seems to be chronologically shifted to advanced education, and on the other hand it is the public opinion, as described by a teacher, “to categorise basic geometric experiences as less important”. A candidate for a teaching post wrote: “The goals of geometry are not important for the performance-oriented society.”

Once more: Until now no convincing concept has been found, which clearly shows the great advantages and opportunities of geometry with respect to the cognitive and emotional development of children.

An additional reason for this neglect is the high expenditure the preparation and supply of materials. This argument has the first priority for the teaching post candidates, perhaps in respect to their demonstration lessons.

Geometry teaching is of high expenditure: requiring time, effort, personal initiative and cost. A person is only prepared to pay this price if the results are worth it, i.e. if the children are highly motivated and ready to learn, and if both colleagues and parents recognise and respect this personal effort.

In particular I want to emphasise the following frequently cited reasons “Teachers have insufficient pre-knowledge” (28 teachers, 8 teaching post candidates). “Teachers do not feel competent teaching geometry.” (21 teachers, 1 teaching post candidate).

43% of the questionnaire participants who state that they have not been sufficiently educated in the material, indicated that they are “mathematically self-taught”. From those statements it can be concluded that these are really very person-related statements. Additionally 29% of questionnaire participants studied mathematics more than 20 years ago, whereas 21% had been members of staff for less than five years. This possibly also reveals a deficit in the current and advanced education of teachers. (Incidentally, only 20% indicated that they had acquired their geometry knowledge in an advanced training course.)

5. Geometry in other teaching subjects

The statement that geometry is neglected in elementary schools appears in a different light, should the following question be posed:

Geometric activities may arise in other teaching subjects. Which possibilities do you see within other teaching subjects - also in subjects which you are not teaching?

The questionnaire participants came up with an abundant (but partly overlapping) list of geometric activities.

Number of the cited geometric activities in other teaching subjects						
Subjects	Science/ Social Studies	Language	Religion	Arts/ Crafts	Music	Sports
Teachers	197	60	29	193	49	108
T.p.c.	29	16	5	37	6	19
Σ :	226	76	34	230	55	127

Table 7

The great number of citations in arts and crafts is not surprising. The preparation of patterns using different techniques, construction and design of geometric bodies, folding work and folding cuts - activities which mainly emphasise aesthetic aspects of the prepared shapes are predominant activities.

In science/social studies dominating activities are handling maps and plans, constructing bodies and objects, for example a sun clock, a castle, a kaleidoscope as well as talking about and testing the functionality of shapes and bodies, for example the structure of crystals and honeycombs - here the main emphasis is the physical and functional aspect of the shapes.

Which geometrical features can be regarded in religious education? As to be expected, religion is very far from geometry (34 citations). The aspect of "dimension" in a universally religious sense is cited repeatedly. "Dimension" is used in religion to show how to experience the multidimensionality of the reality in signs and symbols. Therefore symbols were cited like: labyrinth, middle, cross and god as the 4th dimension.

Teaching religion and teaching geometry can be regarded as "schools for learning to see", to lead the children to a more conscious understanding of the real environmental life. The conscious perception of the "empirical dimension" belongs to both - and it is also precondition to reach a meta-level where features of the everyday life, exceeding their empirical meaningfulness, get a symbolic meaning and become an important matter of faith.

As a matter of fact many experiences in several geometric fields can be made in other subjects. Yet the question remains whether geometric activities in other teaching subjects just remain in the status of doing or if they are also reflected, if the experiences are structured and arranged into a primary geometric context.

Regarding the great amount of geometric activities in other subjects the impression arises that the situation of geometry in primary schools may be not so bad, and the thesis of neglection may be doubted. Perhaps we need a new concept of geometry teaching- a concept which does not assign to mathematics but which integrates it into other subjects as an obligatory part of the curriculum. Following up these thoughts, the main tasks of mathematics would be to structure the activity-based experiences of the children, to scrutinise these activities closely, to reinforce and to consolidate them. A decisive assessment criterion for such a concept would be to examine whether by these changes geometry teaching is given an accidental or even trivial function or if the conscious experience of the complexity of geometric elements and relationships could be really scrutinised.

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