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MATHEMATICAL VIEWS OF PUPILS AND FIRST-YEAR-STUDENTS

Abstract:

Beliefs concerning mathematics education are important for better understanding the process of learning of mathematics. A questionnaire constructed ten years ago by E. Pehkonen and B. Zimmermann was used among other things with seventh grade pupils' in Finland and Germany. A comparison of pupils' beliefs in Helsinki and Bielefeld gives a first impression of different beliefs as an indicator for different cicumstances in teaching mathematics. Between different cities in Germany (Aschaffenburg in Bavaria and Bielefeld in Northern Germany) there are also interesting differences to be seen. Finally the questionnaire was developed further for first-year-students. Especially the differentiation between the experienced "is-state" and the wanted "shall be-state" gives noticeable conclusions.

About ten years ago Erkki Pehkonen and Bernd Zimmermann constructed a questionnaire about beliefs concerning mathematics education of 12 to 14 years-old children. It was used proviously by Erkki Pehkonen and other persons in several countries (see e.g. PEHKONEN 1993, PEHKONEN & TOMPA 1994 and PEHKONEN 1995). I first used this questionaire for comparing the beliefs of Finish and German pupils in 1991 and later on in 1997 for looking at the differences of pupils beliefs in different parts of Germany. On the occasion of TIMSS all this research about beliefs will get a new importance.

Motivated by the workshops of the Finish-German researcher group called MAVI (Mathematical Views) this test was used again with first-year-students who want to become primary school teachers in April 1996 and January 1997 in Bielefeld. However this time the test was changed a little bit by differentiating between the statement about the experienced is-state and the wanted shall be-state. Also some items about teacher education in mathematics were added.

The pupils' test consisted of 32 items which can be arranged in four groups namely 1. mathematical content at school, 2. characteristics of mathematical learning, 3. method of mathematics instruction, 4. Teachers' conduct in mathematics instruction.
Each item had a scale from 1 (total agreement) to 5 (total disagreement). The test ended with three open questions about good and bad experiences in mathematics education and wishes for mathematical instruction.
The 32 items (their number and short reading) arranged in the above mentioned groups are:

(1) mental arithmetic, (3) written computation, (9) word problems, (19) math. for practical benefits, (22) calculation of area/volume, (6) drawing figures, (14) use of calculators, -
(17) topics are taught seperately, (28) get findings with concrete objects, (5) express exactly, (2) right answer is important, (7) get the right answer quickly, (16) give exact reasons, (30) it's important to understand everything, (21) not always fun, (29) requires a lot of exercises, (23) a lot of effort, (20) math. talent necessary, -

(10) there is always a scheme for results, (12) memorizing rules, (18) a lot of repetition, (8) strict discipline, (4) make guesses sometimes, (24) not only one way, (25) mathematical games, (13) work by oneself, (27) solve by oneself, (31) working in groups, -
(15) teacher helps at once, (26) teacher explains exactly, (32) teacher tells accurately what to do, (11) explanations must be understandable for everyone.

1. Beliefs of Seventh Grade Pupils in Finland and Germany in 1991

First results about the comparison of Finland and Germany were presented at the German conference on didactics of mathematics in 1993 (see GRAUMANN & PEHKONEN 1993). The most important points are the following:

The questionnaire was administered in different schools in Bielefeld and in Helsinki with approximately 250 pupils in each city. The means of all pupils in each city differentiated significantly (on the level of 0.1%) in the following 14 items (D:Germany):

- mechanical calculation (item No 3)	Fin: 2.00 / D: 1.53
- to always get the right answer quickly (7)	Fin: 3.74 / D: 3.30
- there is always a procedure to get the result (10)	Fin: 3.07 / D: 2.57
- math. teaching should be understandable for everyone (11)	Fin: 1.85 / D: 1.47
- different topics are taught and learned separately (17)	Fin: 2.77 / D: 2.02
- the teacher explains every stage exactly (26)	Fin: 2.08 / D: 1.51
- there will be as much practice as possible (29)	Fin: 2.34 / D: 1.99
- it is necessary to understand as much as possible (30)	Fin: 2.01 / D: 1.42
- you sometimes make guesses and use trial and error (4)	Fin: 1.98 / D: 2.63
- strict discipline (8)	Fin: 1.51 / D: 2.40
- much has to be learned by memorizing (12)	Fin: 3.30 / D: 3.71
- learning mathematics requires a lot of effort (23)	Fin: 2.87 / D: 3.18
- use of calculator (14)	Fin: 2.04 / D: 2.98
- studying mathematics with practical benefits (19)	Fin: 1.52 / D: 2.02

If we try to interpret these differences it could be true that in Finland there is more emphasis on tasks with practical use whereas mathematics teaching in Germany is more traditional with emphasis on learning content and calculation. This could be because much more differentiation in method is necessary in Finland where there is no outer differentiation with different school forms until grade 8. But it also underlines what came out of the TIMSS-videos from Germany.

2. Beliefs of Seventh Grade Pupils in Aschaffenburg (Bavaria) and Bielefeld (NRW)

Now I will refer to the comparison of the pupils‘ beliefs in two different cities in Germany. The questioning in Bielefeld is the same already used. The questioning in Aschaffenburg was done by one of my students in 1997.

137 pupils were questioned in **Aschaffenburg (BAY:Bavaria)**: "Main" school (HS) 34 , middle school (RS) 59 and grammar school (Gy) 44 . The number of questioned pupils in Bielefeld was originally about 250 but for comparison with Aschaffenburg we dropped out the

pupils from the comprehensive school (Gesamtschule) because such type of school does not exist in Bavaria. So the **number in Bielefeld (NRW:North-Rhine-Westfalia)** was **151**: "Main"-school (HS) 51 , middle-school (RS) 58 and grammar school (Gy) 42 .

In addition to these figures I should mention a special feature of the Bavarian school system:
1. At the end of the primary school the pupils can choose between "main" school and grammar school depending on their achievement. After grade 6 the middle school starts for good pupils at the „main" school and for poor pupils at the grammar school. The first year of the middle school (the seventh grade) is valid as test year.
2. The final exams especially of the grammar school are set with centrally chosen tasks which are the same for all students of Bavaria.

Let us look at the items with **differences between the means of NRW and BAY** larger than 0.3 whereas a positive difference NRW-BAY indicates more agreement in BAY than in NRW and vice versa a negative difference NRW-BAY indicates more agreement in NRW than in BAY.

No 28 (concrete objects)	[NRW-BAY = +0.33, neither-nor both]
No 5 (express exactly)	[NRW-BAY = +0.34, neither-nor both]
No 6 (drawing figures)	[NRW-BAY = +0.35, agreement BAY]
No 19 (practical benefits)	[NRW-BAY = +0.35, agreement both]
No 17 (topics sep. taught)	[NRW-BAY = +0.39, agreement both]
No 14 (use of calculators)	[NRW-BAY = +0.43, neither-nor NRW]
No 18 (a lot repetition)	[NRW-BAY = +0.46, agreement both]
No 8 (strict discipline)	[NRW-BAY = –0.45, neither-nor BAY]
No 9 (word problems)	[NRW-BAY = –0.60, agreement NRW]

These results must be differentiated a bit within one city because there are sometimes exceptions if we look at particular means. Therefore we will have a look at the **differences NRW-BAY according to school-types**.

No 28 (concrete objects)	HS = **+0.87**	RS = +0.09	Gy = +0.08
No 5 (express exactly)	HS = **+0.62**	RS = **–**0.37	Gy = **+0.95**
No 6 (drawing figures)	HS = **+**0.28	RS = +0.29	Gy = **+0.56**
No 19 (practical benefits)	HS = **+0.74**	RS = +0.24	Gy = +0.06
No 17 (topics sep.taught)	HS = +0.25	RS = **+0.42**	Gy = **+0.50**
No 14 (use of calculators)	HS = –0.08	RS = **+1.01**	Gy = +0.07
No 18 (a lot repetition)	HS = **+0.67**	RS = +0.16	Gy = **–0.67**
No 8 (strict discipline)	HS = **–0.66**	RS = **–0.37**	Gy = –0.04
No 9 (word problems)	HS = +0.05	RS = **–0.40**	Gy = **–1.35**
No 25 (math. games)	HS = **–0.81**	RS = +0.12	Gy = **–0.30**
No 32 (t. tells what to do)	HS = +0.28	RS = –0.27	Gy = **–0.69**
No 10 (scheme for results)	HS = **+0.99**	RS = **–0.59**	Gy = +0.09
No 16 (reasoned exactly)	HS = **+0.80**	RS = –0.21	Gy = +0.05
No 29 (lot of exercises)	HS = +0.11	RS = –0.15	Gy = **+0.60**

We can complete this picture by including the **results of the open questions** of the pupils in NRW and BAY.

Good experiences with mathematics instruction

mentioned keywords	**NRW**	**BAY**
Fun, amusing, relaxing, sometimes joking	21 %[1]	11 %
Understood a lot, learnt a lot, was interesting	17 %[1]	18 %
Teacher explained fine, good teacher	15 %[2]	17 %[2]
Nice, good-tempered and sympathetic teacher, teacher helped	9 %	7 %
Playful, vivid	8 %	3 %
Geometry	5 %	4 %
Good marks	0 %	16 %[3]

[1] nearly half of this HS [2] "expl. fine" mostly HS and RS, "good-temp." mostly BAY/Gy [3] 70% RS

Bad experiences with mathematics instruction

mentioned keywords	**NRW**	**BAY**
No fun, too serious, too much dry theory	6 %	12 %
Unnecessary/useless, illogical and meaningless things	0 %	10 %[4]
Too difficult, too fast, too many topics, so little time	13 %[5]	40 %[5]
Bad or complic. explanations, too many terms, bad teacher	7 %	25 %[6]
Rigorous/humourless teacher, no sympathy for poor pupils	18 %	9 %
Too much homework	12 %	5 %
word problems and percentage	6 %	7 %
Bad marks	5 %	10 %

[4] mostly Gy (no HS) [5] mostly RS and Gy [6] mostly RS (esp. pupils that changed from Gy)

Wishes for mathematics instruction

mentioned keywords	**NRW**	**BAY**
more fun/more change, relaxed and interest.instruction	13 %	25 %[7]
Useful and meaningful things for practical benefits	2 %	9 %[8]
Good and more explan.,everybody should understand	18 %	27 %[9]
Easier tasks, more exercise and repetition, better marks	10 %[10]	8 %[10]
Slower/more time, not so fast changes, less topics	2 %	9 %
Nice and good-tempered teacher, helpful t., symp. for poor	9 %	15 %
Not so much homework	11 %	3 %
math. games, group work, mental arithmetics	30 %	10 %
more geometry, more drawings	8 %	1.5 %

[7] mostly RS and Gy [8] mostly Gy [9] very often RS [10] mostly Gy

From the results of these **noticeable Differences between NRW and BAY** in the classes concerned (*and only for them*) we can get the following impressions:
Fun and relaxed instruction appear more in NRW than in BAY; useful and meaningful things appear more in NRW than in BAY; better explanations are wanted in BAY (RS and Gy); good marks have great importance in BAY (RS) [see the peculiarities of the Bavarian school system];

less homework is wanted in NRW- more geometry is wanted in NRW; playful mathematics lessons appear more in NRW but are also wanted more in NRW.

3. Beliefs of First-Year-Students

In the summer semester of 1996 and the winter semester of 1996/97 I had to teach first-year-students studying to become teachers in primary school (1. to 4. grade). In the state of North-Rhine-Westfalia (NRW) all of them have to study a little bit of mathematics and didactics of mathematics (about 20 hours a week within 6 semesters). In their first mathematics lesson at university I gave them a questionnaire about their mathematical view consisting of the 32 questions already mentioned and some questions about their forthcoming studies in mathematics. In contrast to the earlier questionnaire *the possibility of answering was changed i.e. once in respect to mathematics education they know from school ("is-state") and secondly in respect to their wishes ("shall be-state").* Already in 1991 it was my impression that we have to differentiate between both of these views. But at that time I wanted to use the same questionaire as used in Finland and other countries for better comparison.

The total number of tests which could be evaluated is **200 for the questioning in April 1996** and 41 for the questioning in January 1997. Because the primary school teachers are mostly female the rate of male students was only 16 %. The age of the students ranged from 19 to 37 but about 80 % were 19 to 22 years old. Most of them (87 %) came directly from school, i.e. had done their "A levels" (called "Abitur", i.e. school-leaving exam and university entrance qualification) at grammar school.

I cannot give an evaluation of all questions but will **pick out eight questions** which are interesting in some respects.

Item 3 (written computation / mechanical calculation)

Scale	**1**	**2**	**3**	**4**	**5**	***no answer***
Is	55	104	22	8	0	*11*
Shall be	64	91	27	5	2	*11*

The means of this sample are: 1.91 (Is) 1.89 (Shall be)

With this result we see no difference between is-state and shall be-state. Both show agreement. (All other questionings like those in 1991 and 1997 also showed agreement with this item.)

Item 8 (strict discipline / concentration)

Scale	**1**	**2**	**3**	**4**	**5**	***no answer***
Is	36	90	51	16	1	*6*
Shall be	42	105	33	8	0	*12*

Means: 2.26 (Is) 2.04 (Shall be)

With this question we already see a noticeable difference between the means of is-state and shall be-state. Remarkable is also the number of no-statements, especially the difference between is-state and shall be-state, might be interesting. I think we can interpret this as an expression of unsureness about the role of discipline in mathematics education or the meaning of the question.

Because of interest in teachers' conduct and the method of mathematics instruction in Germany (especially the TIMSS-discussion in mind) we now will have a look at the following four items.

Item 11 (explanations should be understandable for everyone)

Scale	**1**	**2**	**3**	**4**	**5**	***no answer***
Is	4	10	43	72	67	*4*
Shall be	125	36	7	11	17	*4*

Means: 3.96 (Is) 1.77 (Shall be)

First we notice the mean of nearly 4 (i.e. disagreement) for the is-state whereas the shall be-state indicates agreement. This gives us a hint that (at least in the higher secondary school in Bielefeld) the explanations in mathematics teaching often are not understandable for all students although it is a big concern of all. The big difference between is-state and shall be-state is striking.

To get more information about the students' answers it is good not only to look at the differences in the means but also at the **distribution between is-state and shall be-state with each test person**.

Shall be / Is	**5**	**4**	**3**	**2**	**1**	***no a.***	**Sum Is**
5	1	1	1	6	49	-	67
4	1	6	1	17	47	-	72
3	5	0	4	11	23	-	43
2	1	2	1	2	4	-	10
1	-	2	-	-	2	-	4
no a.	-	-	-	-	-	*4*	*4*
Sum Shall be	17	11	7	36	125	*4*	200

With this we can see that the number of test papers with $|\text{Is} - \text{Shall be}| \geq 2$ (which might be a good indicator of marked differences) is *152 (76 %)*.
From this we can see very clearly that in questionnaires about beliefs like this one **we have to differenciate between the is-state and the shall be-state**.

Item 26 (teacher explains each step exactly)

Is / Shall be	**5**	**4**	**3**	**2**	**1**	***no a.***	**Sum Is**
5	1	-	2	3	11	-	17
4	2	1	2	11	21	*2*	38
3	-	7	15	18	16	-	56
2	1	10	9	18	29	*1*	69
1	2	1	2	1	8	-	14
no a.	-	-	-	-	-	*6*	*6*
Sum Shall be	6	19	30	51	85	*9*	200

Means: 2.87 (Is) 2.01 (Shall be)

Here we also see differences between is- and shall be-state clearly. The number of test papers with | Is - Shall be | ≥ 2 is *79 (40 %)*. Also noticeable is the relatively high agreement with the shall be-state. More than 40% voted for strict agreement and another 25% join them with agreement. This gives us a hint to the importance of teacher explanations at least in Germany.

Item 10 (there is always a scheme to get the results)

Is / Shall be	**5**	**4**	**3**	**2**	**1**	***no a.***	**Sum Is**
5	7	-	-	1	-	-	8
4	3	8	7	6	1	-	25
3	2	8	18	11	4	*1*	44
2	3	17	12	36	17	-	85
1	4	5	5	7	12	-	33
no a.	-	-	-	-	-	*5*	*5*
Sum Shall be	19	38	42	61	34	*6*	200

Means: 2.38 (Is) 2.65 (Shall be)

With this item the difference between is- and shall be-state is not so large. But what is noticeable is the distribution over nearly the whole field and the high amount of agreement in is- and shall be-state.

Item 4 (you sometimes make guesses and use trial and error)

Is / Shall be	**5**	**4**	**3**	**2**	**1**	***no a.***	**Sum Is**
5	-	-	1	1	2	*1*	5
4	2	6	3	14	6	-	31
3	1	7	15	23	11	*2*	59
2	1	6	9	42	22	*3*	83
1	1	2	1	7	9	*1*	21
no a.	-	-	-	-	-	*1*	*1*
Sum Shall be	5	21	29	87	50	*8*	200

Means: 2.58 (Is) 2.19 (Shall be)

This item also shows us differences between is-state and shall be-state but not as marked as before. The is-state is more neutral but there are also a lot of wishes for more self-activities. Probably the TIMSS-videos indicate a typical feature of German mathematics education concerning this point. Even in the wishes of the students we do not find as much interest in discovery learning. The two items discussed before underline the picture of German mathematical education pointed out by the TIMSS-videos.

Another interesting question in different contexts concerns the application of mathematics. Therefore we will have a look at one typical item for this point.

Item 19 (studying mathematics with practical benefits)

Is / Shall be	5	4	3	2	1	*no a.*	Sum Is
5	-	-	2	8	14	-	24
4	-	1	3	31	34	-	69
3	1	1	6	29	34	-	71
2	1	1	3	8	18	-	31
1	-	-	-	1	2	-	3
no a.	-	-	-	-	-	*2*	*2*
Sum Shall be	2	3	14	77	102	*2*	200

Means: 3.39 (Is) 1.62 (Shall be)

The evaluation of this item again underlines the probable differences between is- and shall be-state. The number of test papers with $|\text{Is} - \text{Shall be}| \geq 2$ is *122 (61 %).* The table also shows that nearly 87 % wish (59 % strictly wish) to study more mathematics for practical benefits, even most of those who have had studies of mathematics for practical benefits (18 %). Only 3% of them have the feeling of too much mathematics with practical use.

The main result of the questioning of the first-year-student is the often seen difference between the memory of the reality in mathematics instruction (Is-state) and the students' ideal about mathematics education (Shall be-state). If we balance this result on one hand we demand clarification about asked is-state or shall be-state with questionaires like this. On the other hand we should conduct more research into the background of this difference.

4. Beliefs about the study of mathematics for primary school teacher

As already mentioned the questionnaire for the first-year-students also consited of some questions about their coming study in mathematics. The question was: *What do you think should be part of a mathematics course for future primary school teacher ?* Ten items were put down (with a scale from 1 to 5 for is-state and shall be-state like before):

(41) natural numbers and the operations with them
(42) to master computation with fractions
(43) elementary geometric concepts and theorems as well as imagination of space
(44) to know more than only primary school mathematics
(45) able to offer proofs
(46) to reflect on mathematics
(47) to make exercises regular
(48) to tackle exemplarily with one problem field of elementary mathematics by oneself
(49) to deal with history of numbers and mathematics
(50) to deal only with didactical questions about mathematics education

On the basis of the questioning from Januar 1997 with **41 evaluated tests** we get the following picture:

No	IS-state							SHALL BE-state							Diff.
Scale	5	4	3	2	1	no	$\overline{x_1}$	5	4	3	2	1	no	$\overline{x_2}$	$\overline{x_2} - \overline{x_1}$
41	0	1	3	20	16	1	1.73	0	0	1	14	25	1	1.40	- 0.33
42	1	5	6	19	9	1	2.25	1	1	7	15	16	1	1.90	- 0.35
43	0	0	7	27	6	1	2.03	0	0	5	17	18	1	1.68	- 0.35
44	0	1	8	12	18	2	1.79	0	4	7	21	8	1	2.18	+0.39
45	2	6	12	11	9	1	2.53	7	7	12	11	3	1	3.10	+0.57
46	4	4	14	14	2	3	2.84	1	1	5	19	13	2	1.92	- 0.92
47	0	1	6	19	14	1	1.85	0	0	7	15	18	1	1.72	- 0.13
48	2	8	16	11	2	2	2.92	1	2	13	16	7	2	2.33	- 0.59
49	5	9	11	13	2	1	3.05	1	8	9	15	7	1	2.53	- 0.52
50	7	14	11	5	2	2	3.49	7	8	10	7	6	3	3.08	- 0.41

The **interpretation** of these results could be the following:

1. We see that the is-state and the shall be-state often differ.
2. Natural numbers and elementary geometry are taught already but students are asking for more. Also more mastering of fractions is asked for.
3. Proofs are not taught so much but should be treated even less.
4. Primary school mathematics is important but should be treated less.
5. History of numbers and mathematics is done not so often and should be done more.
6. Reflection about mathematics is important and should be done more.
7. Exercises are very important.
8. To work on one problem on one's own is done not so often and should be done more.
9. Only a few students think that their study should focus only on didactics of mathematics.

General conclusions cannot be drawn from these results because of the small number of experimentees on the one hand and special situations at other universities on the other hand. But I think hints for discussions about the mathematical education of future primary school teachers' are given.

To complete the impression about the mathematical topics I also used an **additional questionnaire** during my lecture in summer semester 1996 and winter semester 1996/97. The idea for it I got at a conference (see MOSEL-GÖBEL & STEIN 1996). After finishing a special topic in my lecture I asked the students how they had been able to follow the lecture and if they thought the lecture was helpful for their later profession. The scale went from -3 (total disagreement) to +3 (total agreement) and the topics were as follows:

1. Types of numbers and relations between the operations
2. basic concepts of set formalism
3. basic concepts of relations and functions
4. Natural numbers as cardinal numbers and systems of notions for numbers
5. fundamental concepts and problems of elementary geometry

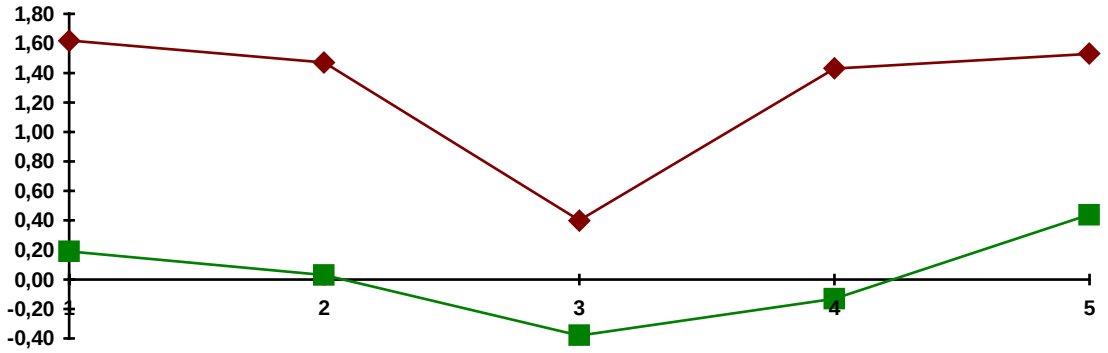
The result of this questionnaire are as follows (because of total numbers fluctuating from 100 to 200 I always give the percentage rate):

Question A: "**I can follow the lecture**"

Topic No	-3	-2	-1	0	+1	+2	+3	Mean
1	2.8 %	1.9 %	1.9 %	5.7 %	28.0 %	31.3 %	28.4 %	+1.62
2	1.1 %	2.8 %	4.5 %	12.3 %	25.1 %	27.4 %	26.8%	+1.47
3	8.7 %	7.6 %	13.0 %	19.6 %	26.1%	6.5 %	18.8 %	+0.40
4	2.8 %	3.4 %	4.5 %	11.8 %	21.9 %	26.4 %	29.2 %	+1.43
5	2.3 %	2.3 %	0 %	0 %	37.2 %	46.5 %	11.7 %	+1.56

Question B: "**The lecture makes sense for my future profession**"

Topic No	-3	-2	-1	0	+1	+2	+3	Mean
1	6.0 %	10.3 %	12.0%	29.9 %	21.4 %	13.2 %	7.3 %	+0.19
2	4.5 %	12.4 %	13.5 %	33.7 %	19.1 %	15.1 %	1.7 %	+0.03
3	14.1 %	8.7 %	15.2 %	39.1 %	14.1 %	3.3 %	5.4 %	-0.38
4	12.3 %	6.1 %	19.0 %	23.5 %	26.8 %	8.9 %	3.4 %	-0.13
5	0 %	4.6 %	14.0 %	32.6 %	30.2 %	18.6 %	0 %	+0.44



The results of this questionnaire are similar to those Mosel-Göbel & Stein got: All topics but topic 3 (relations and functions) are understandable. The envisaged importance for the future profession of the means of all topics is nearly 1.5 less. The best topic especially in the last view is elementary geometry.

5. Summary of the open answers of all students

Finally I will give a summary of the open answers the students gave in the first mentionend questionaire. For this I evaluated all tests from April 1996 and January 1997. A lot of comments were very similar. So I divided them into eight categories. The total number of evaluated tests is 241, but to get the weight of one class I counted the number of comments (if someone made two comments these were both counted). If there was no comment I counted it as one comment - because I think "no comment" is also a comment. The total number of all comments by this counting was **295**. (This is the basis for the given percentage.)

The **wishes of students concerning their studies** that came out of the open questions are as follows:

1.	No comment	126	(42.7 %)
2.	Stressing references to the practice in school	41	(13.9 %)
3.	Stressing didactics strongly and giving hints for teaching	41	(13.9 %)
4.	Good Explanations, good structuring and own experience	25	(8.5 %)
5.	Not so many proofs and formalistic mathematics	20	(6.8 %)
6.	Information about learning strategies and problems of pupils	16	(5.4 %)
7.	Learning for games, fun and own experience in school	16	(5.4 %)
8.	Sound knowledge, logical thinking and demanding mathematics	7	(2.4 %)

An interpretation of the results concerning training in mathematics for primary school teachers is not so easy because the different situations in other cities have to be taken into account as well as goals which cannot be foreseen by first-year-students. But I think that these results can give some hints for discussion on that topic.

6. Conclusions for future research on beliefs

If we summarize all mentioned results and add results of some other items (I cannot show here) I would say that we should look especially at the following four aspects in future:

1. The aspect of "*application / environment concern*" is represented as a big wish of all students but too little attention is paid to it in school reality.

2. "*Clearness / good explanations / vividness of mathematics instructions*" is also one aspect represented as a big wish of all students while reality is somewhat lacking on this score.

3. "W*orking on problems / seeing different ways for solution / no strict scheme*" is another aspect represented as a wish of the students while classroom reality in this aspect is mostly questionmarked or felt to be lacking.

Special questionnaires for one or two of these points should be constructed and given to pupils of different ages. Parallel to this interviews with some of the tested pupils as well as their teacher are necessary.

Finally it should be mentioned that aspects of *typical items for mathematics teaching* which correspond in is-state as well as in shall be-state consist of "mental calculations", "written computations/mechanical calculations" and "discipline".

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