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CHANGES OF TEACHING MATHEMATICS BY COMPUTER ALGEBRA SYSTEMS (CAS)

Abstract:

Computers and especially computer algebra systems are able to do almost everything that in an operative way is normally asked for in mathematics teaching and that in most cases, is also extensively practiced. Thus, using CAS in mathematics teaching can and will change essential parts of the traditional, computerless mathematics teaching in an effective way. With these changes not only innovative progress, but also undesirable developments can be expected. These changes offer chances as well as dangers; great hopes are linked to them but considerable fear also arises.

In this paper I will outline some problems and questions that arise as soon as we consider changing mathematics teaching by using computers or CAS with regard to its *goals*, to its *content*, to its *methods* and to the *social behaviour and teaching manners* (cf. PESCHEK 1997).

1. Changes of Mathematics Teaching With Regard to Its Goals

Using CAS in mathematics teaching suggests, and I think that there is a high degree of agreement in this, an outsourcing of algorithmical, numerical and also symbolical operations to the machine. Creating tables, transforming terms, solving equations and inequations and systems of them, operating with matrices, differentiating and integrating numerically and symbolically, drawing graphs of functions (even 3-dimensional ones) can already be done far quicker and better by inexpensive software products than could ever be expected of an advanced pupil working by hand.

Some people (among those many teachers) state that mathematics teaching will be badly shaken in its foundations and basic values and they vehemently refuse to accept what they assume to result in the decline of Occidental Culture. After all, so they say, skillful calculations done by hand (even if they are no longer done in one's head) are coupled with many qualities which are still important in our society: persistence, patience, precision, concentration, consequent thinking in a stringent way and maybe even certain forms of logical thinking.

Arguments like these are not new and they have not just come into being with the availability of computers; they had been heard again and again in a similar way in connection with the introduction of hand-calculators and their use in schools (cf. for example KIRSCH 1985).

Others (among them a lot of didacticians) see computer technology as a chance to finally relieve the teaching of mathematics of uncreative routine operating and to gain room for didactically more meaningful and intellectually more ambitious goals. Often named in this context are:

- orientation of applications, modelling, authenticity and problem solving
- emphasis on aspects of representation and on interpretation within mathematics
- concentration on an adequate formation of concepts
- discussion of the possibilities and limits of mathematical procedures
- orientation of fundamental ideas of mathematics and their explicit treatment
- interdisciplinarity
- consideration and inclusion of historical or socio-philosophical aspects
- various affective and social goals of teaching mathematics

In the first place the question arises whether indeed all the qualities which are related to doing operations by hand get lost in a computer-supported teaching environment (presuming that this relation is at all justified). Further the question arises as to whether we could dispense with these qualities if necessary or if the promised alternatives offer an equivalent replacement. Finally, of course, the question arises as to what mathematics teaching could look like for example when it is consequently orientated on fundamental ideas, on the aspect of representation or on interdisciplinarity, and which concretely developed concepts of this kind (especially textbooks) already exist.

With regard to the first question, algorithmic-operative activities, as well as routine activities and the subsequent requirements on the pupils' precision, concentration and patience will not completely be put aside but much rather shifted to another level. From my experiences and class observations up to now I can rather say that the availability of a complex system of machines brings about an almost irresistible temptation (for teachers and pupils) to replace routine operations done by hand with operations done by machine which are at least as complex (thereby exhausting the possibilities of the machine). And it is common knowledge that the machine´s inexorable requirements on precision surpass by far the requirements of every teacher how ever rigorons he/she may be.

Concerning the question of the social meaning of such qualifications quite convincing arguments can be found in the literature, showing that their relevance still exists but in comparison to the social requirements of former times, it is obviously declining (cf. for example POSCH 1987 and 1991).
The alternatives offered by those who support these arguments refer to - which I cannot explain here in detail - "dynamical qualifications" such as creativity, independence, personal responsibility, initiative which - again according to P. POSCH (1987 and 1991) - gain increasing importance in our society today.

But in that case why do teachers not enthusiastically change to an appropriate computer-supported teaching of mathematics? Why do even in schoolyear 1994/95 more than 90% of secondary school pupils in Vienna have maths lessons in which computers are rarely or not at all used and why do only 2% of the secondary school pupils have seriously computer-supported lessons (cf. HUMENBERGER 1996)? In Germany too, the situation seemed to be similar at least until a short time ago (cf. MELHASE 1992).
I think it is because the fear lies deep down and is very legitimate: fear of asking too much of weak pupils in lessons which are too difficult (cf. SCHÜLLER 1994), fear of producing not enough work visible to others and even less to assess (a page of a schoolbook calculated by hand is still more impressing - you can quite clearly see the efforts - and it is more familiar and easier to assess than any more or less desired output on the computer screen), also the fear of not having the expected success with even more ambitious goals and thus of having been poorly instructed.
It has been really quite interesting for me to learn that even computer enthusiastic teachers for example do not allow linear systems of equations to be solved by the „solve“ - command but step by step with computer-supported equivalence-transformations or also that in the well known Austrian Derive Project the so-called White Box - Black Box - principle is used in a dominant way; a principle that says, formulated a little polemically, that first the pupils have

to master the mathematical procedures in a traditional way, before they find out that they do not need what they have learned because in the future they are allowed, more or less as a reward for their efforts, to hand over the work to the computer.
I think, the essential problem lies in the fact that although there are a lot of very good didactical papers and investigations on the problem of more ambitious goals (as for example the aforementioned), these papers generally deal with the problem of changed goals only in a very local-exemplary way or in a very fundamental way and without explicit practical concretisations. There appears to be a lack of appropriate textbooks and teaching aids with explicit concretisations for the lessons and for the use throughout the schoolyear (instead of those concentrating only on specific parts of the mathematics curriculum) and a lack of didactically reasoned conceptions for teaching that would allow teachers to engage in the experiment of computer-supported mathematics teaching with acceptable expenditure and with a calculable risk (for themselves and for the pupils).

2. Changes of Mathematics Teaching With Regard to Its Content

You almost cannot fail to notice that changes concerning the mathematical contents (procedures, concepts) have become possible by the permanent availability of computers and CAS:
For one thing *new content* gain school practical relevance with the availability of computers. The examples mostly given for this are dynamical systems, chaos theory. Therefore, a lot of developed examples for the lessons and a lot of practical teaching experiences already exist (cf. for example OSSIMITZ 1990). Nonetheless, reworking lessonplans cannot include such comprehensive didactical treatment, penetration and reflection as we (and the teachers too) are used to find in traditional lessonplans. On the contrary, these lessonplans must even be prepared for quite fundamental questions such as: What makes chaos theory and dynamical systems so interesting from a didactical perspective that they should be included in the maths lessons already overloaded with topics? Which fundamental ideas, which archetypes of the human thinking should be shown by them? Which (general) capacities should be developed by them? Which fields of application are typical? Which methods of teaching are appropriate? Which examples are possible under such school conditions? Which are useful? Didactical answers to such and similar questions (also in the way of detailed and reasoned lessons) would be necessary to convince more teachers to use such new content and to enable them to teach appropriately.

Computer-supported mathematics teaching makes possible new content, and also *questions traditional lessonplans*. This is valid not only for many algorithmic-operative procedures (for example numerical procedures for solving equations and systems of equations or procedures of linear optimization, various rules of differentiation and also integration), but also for various rules of combinatorics, for normal distribution in a statistics course where the normal distribution is only used for approximation of the binomial distribution, and perhaps it is also valid for the binomial distribution if this distribution is used for nothing but the approximation of a hypergeometrical distribution.

However, changes in the content can also be a consequence of *new approaches and new emphasis within traditional content,* approaches and emphasis that until now had not been included in the lessons because of the great efforts needed for hand-calculations or also because of difficulties in calculating: recursion is one example often given in this connection, discretisation and elementarization are other examples.

Recursion: Using CAS a recursive definition of exponential functions (exp: $f(n+1) = f(n) \bullet (1+a)$) is not only possible but the expenditure of calculation is also manageable. A recursive definition explicates the fundamental idea of exponential growth or exponential decrease far better than this could be done by other representations such as the graph of a function or the algebraic equation of a function. Thus, the theoretical formation of concepts (in the sense of mental construction of the constitutive and action-oriented property of exponential functions) could be supported. But at the same time other important possibilities and properties also recede into the background by a recursive representation of exponential function, for example the possibility of calculating the values of functions in a direct way or of „reading" them off the graph of the function.

Elementarisation: Of course a stronger emphasis of elementary „open" (cf. FISCHER 1995) methods is a difficult point. A lot of mathematical methods have been developed just for creating calculation advantages and for limiting the efforts of operating to a tolerable extent (combinatorics, calculuses of differentiation instead of calculation of limits, formulas of compound interests and of annuities instead of calculation of savings account books and redemption plans, etc.). Using CAS the calculations could be outsourced to the computer so that in many cases the troublesome („elementary", „primitive") variant becomes not only manageable but also more meaningful.

But does this mean that in teaching mathematics we should dispense with advanced mathematics? Under which conditions could or should we dispense with it? When would it be completely wrong to dispense with it? And why? But this last question leads back to the changes concerning the teaching goals.

Often a contrary tendency in computer-supported lessons can be observed: No new content, no changes in the content within traditional content rather *the treatment of more and more complex examples* with increasingly tricky and complex mathematical constructions (for example solution of even more complex equations, even more extensive systems of equations, differentiation of even more tricky terms, treatment of more complex data records,...). This is often argued with authenticity and legitimated by the simple mastering of calculation with the computer. I would venture to question if such developments really decisively further the teaching of mathematics in its fundamental intentions.

I think that even these brief comments show the need for more detailed didactical analyses of this kind, comprehensive concepts as well as well-documented practical teaching reports of experiences, if teachers are expected and required to carry out possible changes in content by using computers in a didactically appropriate way in their lessons.

3. Changes of Mathematics Teaching With Regard to Its Methods

If we see CAS as "calculation-slaves" which solely serve to take over those tasks that can be done by hand either with great effort or not at all, then not too many things will change on the methods of teaching mathematics.

However, we almost always expect changes in the methods of teaching mathematics when using computers, and these methodical changes are often even mentioned as a decisive argument for the use of computers in mathematics teaching. In most cases the expected and proclaimed changes are directed towards more intensive explorative, experimental and heuristic processes. For example Hillel (cf. HILLEL 1991) emphasizes the role of computers as 'investigating tool' which suggests experimentation by the pupils themselves and lets the pupils see what happens. The expectations and proclamations are primarily based on the good graphical and tabular possibilities of representation and visualization of CAS, on the easily accessible possibilities of iteration or recursion as well as on the interactivity. So the pupils are able to work out quite comfortably by themselves properties of functions and their parameters in an experimental way using the aid of computers. Hypotheses about possible properties can hopefully be developed by graphical, tabular and symbolical representations, they can be tested and revised on any other examples. Thereby, the essential point is that the pupils can repeat their investigations over and over again with almost any number of examples in order to test their hypotheses until they can no longer question the correctness of their hypotheses and then verify them as mathematical facts.

What does this mean for the learning of mathematics? On the one hand, such an explorative, experimental process requires more individual action by the pupils, dealing actively with problems posed, the development of strategies for recognizing patterns and relations by themselves etc. Thus it requires altogether creative capabilities which could well support the treatment of mathematics and the mathematical understanding (cf. for example WEIGAND 1995). On the other hand, an experimental process always involves a certain amount of danger, as in this case, that the pupils will not surpass a phenomenological understanding of concepts, they will not even see a necessity for a mental (theoretical) continuation of their empirical observations. An experimental-heuristic process concentrates strongly on the experimental and descriptive aspects; the phenomena (cf. GRAF et al. 1992) are in the foreground. The essential question is "What if?", the question "Why?" barely occurs to the pupils and doesn't occur at all if they are able to be successful without responding to it.
Without any doubt, experimental processes can be mastered well and in a didactically meaningful way with CAS - and various observations of lessons have shown again and again that pupils can enjoy doing this kind of mathematics. Positive reactions of students could be observed for example by Heid (cf. HEID 1988) within her project concerning a computer-supported calculus course. Nonetheless, the following questions have not been dealt with in a satisfactory manner until now. Is it not so that a very restricted and one-sided image of mathematics is created by a strongly emphasized experimental-heuristic process? Where and how should such experimental, computer-supported mathematics teaching be organized to deal effectively with such deficits?
An important and, in my point of view, until now hardly answered question concerning experimental processes is in what way and to what degree of intensity the pupils should be directed in their explorative-experimental acting? Too strong a guidance of the pupils through detailed tasks poses the danger of the experimentation as well as the individual action being strongly pushed into the background. Tasks which are too open could have pupils' loss of orientation as their consequence which could also influence the pupils' motivation in a

negative way. Further considerations of this problem can be found for example in WINTER (1991) and HILLEL (1991).

From numerous discussions with colleagues at schools I know that many teachers are also insecure when it comes to the methods of computer-supported mathematics teaching - in a similar way as they are with goals and content of such a kind of teaching - and that they feel left alone by the computer-freaks as well as by mathematics education.

4. Changes of Mathematics Teaching With Regard to Its Social Behaviour and Teaching Manners

The use of computers is often also accompanied by changes in social behaviour and teaching manners. This reorientation is taking place into the direction of more individual action and independence on the part of the pupils as well as in the direction of a greater extent of working with a partner or in a team (cf. for example WURNIG 1996, HEUGL et al. 1996, NOCKER 1994, SCHNEIDER 1997). I call this reorientation because almost all studies concerning the culture of teaching traditional mathematics, including more recent studies such as those by HUMENBERGER (1996), NOCKER (1994) or SCHNEIDER (1998a, 1998c), show strong teacher dominance and, at the same time low pupil individual activity and independence and they point out that cooperation between pupils plays only a subordinate role in everyday classrooms.

As things are, the availability of CAS alone is by no means a guarantee for changes regarding the social behaviour and teaching manners of maths lessons.

It is true that the computer is a medium that can be considered to be supportive of dialogues. An intensive dialogue directed by the pupils between human being and machine can take place by the interactivity of computer representations. Thus, I was able to collect a lot of thoroughly positive empirical data in the course of my doctoral thesis (cf. SCHNEIDER 1994).

But the computer is also a medium that can have a communication restraining effect in a social sense and can intensify the danger of social isolation. We all know the picture of a computer freak who can live almost without fellow-men but not without the internet and computer-games.

In order to have good computer-supported mathematics teaching, it will be necessary to find a way of avoiding the computer’s hindrance of communication (in social sense) without giving up the advantages of computers (ability of dialogue). This could be realized by selecting appropriate social forms which support interpersonal communication, the processes of interaction and cooperation and thusly the exchange of meanings. An obvious possibility would be working with a partner or in groups at the computer.

For example, in regard to social behaviour SHEETS/HEID (1990) have made interesting observations during a computer-supported calculus course (drafted in a new way) where the students should examine different (mathematical) content in an individual and independent way: The students were free to choose whether they wanted to work by themselves or in groups. Many students who had decided to work alone at the beginning of the course joined a group with continuation of the course. Their initial motivation for working together was simply to make efficient use of the limited quantity of computers, but with continuance of the cooperation these pupils also worked together when the common use of one and the same computer was not necessary any more. Further changes in the social behaviour of the students could be observed in connection with problems, unclearness: At the beginning of the course in such cases the students always asked the teacher (as they were used from teacher-centered lessons). With continuance of the course they more and more often discussed their questions with their collegues. Finally, the computer-supported calculus course drafted by SHEETS/HEID seems to result in a more open and wide-spread discussion about mathematical concepts and their applications.

Regarding social behaviour and teaching manners there still exists among teachers and even among experts a great insecurity about desirable and undesirable developments which can veritably be felt in the following quotation from the report about the Austrian DERIVE Project (HEUGL et al. 1996, p. 207): 'Frequently working with a partner is due to the hardware equipment. We have 14-15 computers in the laboratories, thus working with a partner was a necessity.'

5. Project "The Use of the TI-92 in Teaching Mathematics in Business Colleges"

Though they are only pointed out in a exemplary way in this paper, the didactical questions, problems and insecurities arising for each teacher in transforming to computer-supported teaching mathematics and hardly mastering it themselves have moved us - W. Peschek (head of the project) and me - to examine the problems in detail and especially on site (that means in the classrooms, in concrete lessons and together with teachers who until now have had no computer-experience). In the course of a research project lasting three years we attempt to find out possibilities of a didactical accompaniment and support for these teachers willing to change.

Therefore, we have chosen two teachers at two different schools both of whom were interested in the use of computers, in particular TI-92, in mathematics teaching and the possibilities connected with that, but had never had any experience using such technology. They were, however, not prepared to make such fundamental changes in their teaching without (external) support.

After organizational preparatory work in March 1997 the period of development has started with observations of lessons, collection and analysis of teaching materials, common development of teaching materials (complete, didactically reasoned textbooks for teachers and pupils), testing of these materials in the lessons.

The TI-92 Supported Mathematics Classroom

By using TI-92 and having the corresponding (high-performance and easy to use) CAS permanently available in the classroom clear adjustments in the teaching of mathematics of both teachers on the levels of goals, of contents, of methods and of social behaviour and manners of teaching could be seen. These adjustments could be detected in a first phase by comparing and analyzing both teachers' teaching aids (workbooks, test problems) from the previous year („pre-TI-92“) with those being used in the TI-92 supported classroom. The details of the results of this comparison cannot be discussed in this article (see Schneider 1998a, 1998c). In this paper I will confine myself to certain selected aspects.

Briefly described, previously when considering their goals, content and methods of teaching, the concentration in the (computerless) classrooms of both teachers essentially lay in the solving of inner mathematics problems practising certain specific procedures. The complexity of these problems was steadily increased. By implementing these problem sequences the content of the prescribed mathematics curriculum was introduced (presented by the teacher), practiced (mainly reproduced by the students) and tested. Solving the problems was carried out almost solely on the algebraic level.

On the goal level, in their TI-92 supported classrooms both teachers now were *outsourcing the procedures* (cf. Peschek 1998b: „*Didactic principle of outsourcing*") and putting an increased emphasis on *representation* and *interpretation* (see Schneider 1998b). Furthermore, a more intense concentration of *application orientation, closeness to reality* and *problem solving* could be observed. For example, the students conducted a study of the development of world population to practice the field of exponential and logarithmic functions. They had been provided with real data and prognosis values from the U.S. Bureau of the Census and Long-term Prognoses from the United Nations. The development of world population was to be described on growth models which had been introduced in the lessons, and the data it contained and those made available in the official studies provided were to be compared and judged, criticizing each model.
Furthermore, in TI-92 supported classrooms learning processes tend to focus on *concept formation* rather than on mere learning of procedures. Great importance is attached to and much time is spent in the classroom working out the constitutive properties of exponential functions (constant percentile increase). Or consideration of the concepts of difference quotient and differential quotient is given a much higher placement than the symbolic or numeric calculation of derivatives (which have largely been outsourced to TI-92).

On the content level, for example, as a result of observing limits of exponential growth models more detailed growth models as the *model of limited growth* or the *model of logistic growth* are dealt with and analyzed in TI-92 supported classrooms. Dealing with such models was only inadequately possible in previous CAS-free classrooms (whereby certain aspects had to be neglected).
A further example for changes in content is the treatment of *recursion* as a form of describing (non-)mathematical behavior. The fundamental property of exponential functions, or of linear functions can much better be explained by recursive representations as is the case with only the aid of function graphs or function equations. Sensibly learning to deal with such recursive representations in the classroom has only been made possible by outsourcing the procedures to TI-92.

Differences on the method level have been moving increasingly in the direction of *experimental work*. For example, the properties of exponential and logarithmic functions or the effects of various parameters of logistic growth models on population behaviour or the formula for the derivative of power functions or addition and multiplication rules with a constant for derivative functions are being dealt with experimentally (increasingly independently by students).
On the method level conscious attention is increasingly being paid to *changing forms of representation* (graphically, algebraically or table). This is occurring in the introduction of

mathematical concepts as well as in the application of concepts which have already been learned.
Clear changes can also be observed in classrooms of both teachers on the level of social behaviour and manners of teaching as compared to their „pre-TI-92“. In this paper, these comparisons are shown in the observation data gathered for one of the teachers:

Study Groups (social behavior)

	without TI-92	*with TI-92*
1) plenum (teacher lecture, questioning-developing)	90.7%	*39.0%*
2) group work, partner work	0.0%	*32.8%*
3) individual work	7.9%	*2.8%*
4) combination of 2) and 3)	1.4%	*26.0%*

Approximately 90% of classroom time, which in a traditional classroom would have been conducted in plenum, was reduced to approx. 40%. In TI-92 supported classrooms; approx. 60% of classroom time is spent with the students working independently (alone, or in small groups).

Dominance of Communication

	without TI-92	*with TI-92*
Teacher dominated communication	**89.3%**	***45.6%***
teacher lecture	15.4%	*4.2%*
questioning-developing classroom	48.5%	*20.4%*
answering teacher questions	25.4%	*21.0%*
Student dominated communication	**2.3%**	***53.3%***
student discussion	1.5%	*21.6%*
answering student questions	0.8%	*31.7%*
no verbal communication (silent work)	**8.5%**	***0.0%***

The 90% of the classroom communication which had previously been determined and dominated by the teacher is nearly equally divided between teacher and students in a TI-92 supported classroom.

Active Participation of Students in the Lessons

	without TI-92	*with TI-92*
one student	0.7%	*0.6%*
a few students	67.6%	*23.9%*
many (the majority of) students	31.7%	*75.6%*

In a TI-92 supported classroom, the relationship of active participation of the students as compared to „pre-TI-92“ classroom is reversed.

6. Concluding Remarks

The permanent availability of technology such as TI-92 in the teaching of mathematics and the algebraic, graphic and numeric possibilities connected with it, puts teachers not only technologically but also didactically in a new and very challenging position calling for reorientation. Within the framework of our project we were able to observe that the teachers whom we were advising not only accepted the technological challenge but the didactic challenge as well. This lead to a clear reorientation in the organization and set up in the way they taught mathematics. This is a didactic reorientation which has been demanded by mathematics didactics for a long time; changes which also could easily be adapted to a computer free mathematics classroom. But it is also a didactic innovation (see Peschek 1998a), which has only been able to be realized in a didactically sensible manner using the possibilities of CAS.

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