

HANS-WOLFGANG HENN, DORTMUND

## THE CAS PROJECT *MOBILE CLASSROOM* IN THE GERMAN STATE BADEN-WUERTTEMBERG

### Abstract:

In the German federal state of Baden-Wuerttemberg, extensive tests on the use of computer algebra systems have been going on since 1993. We started a pilot project at the beginning of the school year 1996/97 that is at present unique in Germany. All students of five 11<sup>th</sup>-grade classes (age 16 - 17) from different towns in Baden-Württemberg have been equipped with a modern notebook and will work with MAPLE for three years until their final exam. One of our major goals is to bring more applications and modelling into the everyday teaching experience.

### 1 Introduction

The Federal Republic of Germany consists of 16 federal states which are all independent in matters of education and culture. I will talk about my native state Baden-Wuerttemberg, where I am responsible for the training of future high school teachers. Deficiencies in our teaching that were already known before TIMSS make it necessary to reflect on the contents, methods, and didactics of teaching in its present form [see Blum 1995]. After TIMSS brought mathematics teaching into public view we hope and believe, however, to be able to change something over the next years [see Henn et al. 1998b].

### 2 Computer Algebra Systems (CAS) in the Classroom in Baden-Wuerttemberg

We regard the sensible use of computers as a new didactical and methodological dimension. In particular computer algebra systems alleviate the numerical and algebraic “burden” and thus make it possible to concentrate on modelling activities [see Henn 1998a]. In Baden-Wuerttemberg, extensive tests on the use of computer algebra systems have been going on since 1993 [Henn 1996]. We will examine the influence of such systems on school mathematics and their educational goals and the ensuing consequences for a mathematics education fit for the 21st century. We will investigate questions such as the following: What exactly are the real potentials for the use of CAS in supporting mathematics teaching? Which conditions favour its use? Where do obstacles lie, are they located in the hardware or the software, at the cognitive level, in the didactic practice, or in the daily use of CAS? Luckily, we have in principle a free hand as far as the ministry is concerned. The excuse “I would like to do it but the guidelines won’t allow it” is not valid any more. At the outset, we started to offer courses for *Mathematics with CAS* at the higher secondary level in more than 40 schools. We work with the software DERIVE and MAPLE and, normally, the students work in the schools’ computer labs.

### 3 Framework of the Pilot Project

With the beginning of the school year 1996/97 we started our most ambitious pilot project, *Mobile Classroom*, which is unique in Germany. In order to optimize assessment of the long-term consequences of CAS, all students of four 11<sup>th</sup>-grade classes (age 16 - 17) in different towns in Baden-Wuerttemberg have been equipped with a modern notebook, and now a fifth school has joined this school year. Twelfth-grade students can choose between advanced-level and fundamental-level courses, then work with their notebook until their final examination, where they will receive special examination problems. The CAS MAPLE is used. "Regular" teachers instruct these students and it is exciting to watch this teaching and to discuss it with all participants! The teaching project receives a high level of public attention - a fact that is also appreciated by the respective members of the local state parliament. Often newspapers and local TV stations report on such project classes. The project received no recommendations regarding contents from the ministry (apart from allowing students to transfer into a "normal" Year 12 class). Project teachers report regularly and in detail on planning, teaching, results, and experiences with their classes.

We mathematicians of the Karlsruhe teacher training programme provide pedagogical advice for the project, especially with regard to educational concepts, proposals for teaching, exchange of experiences, design of syllabi, joint meetings with project teachers in Karlsruhe or other meeting locations, and informative classroom observations. The Internet is used as an electronic filing system for all reports, material, exchange of information, and e-mail contacts. Participating students and teachers are interviewed, and a detailed annual report is provided for our ministry.

### 4 First Experiences from the Instruction Experiments

In general, students are much more motivated, and there is a lively working atmosphere that differs pleasantly from the usual bored, passive behaviour in Year 11. Mathematics teaching, however, becomes more challenging.

Demands on teachers are high. Learning to use the CAS and preparing subject-related materials are very time consuming. The teacher's role is completely different, as the focus has shifted away from them being the central mediator of mathematical content. Now planning of courses and worksheets and giving advice to classroom workgroups has become our major activity and hence a high measure of flexibility is required.

During the first year the chronology of teaching in all four classes was motivated by the regular grade 11 curriculum, apart from the binomial distribution topic and tests which had to be omitted because of lack of time. For quite some time the handling of the CAS dominated teaching. The desired open approach was realized more and more towards the end of the school year. Typical problems then included, for example, the reinforcement of traditional teaching content by the transition to two-dimensional functions, the investigation of curvature as an important curve characteristic, and the development of problem-solving competence when using real-life problems. These insights into methods of mathematical modelling are neglected in traditional teaching and therefore pose special difficulties for students.

Obviously, the new tool was to be used as often as possible, which also conformed with students' expectations. Thus teaching topics such as equations of straight lines, simple manipulations of expressions, or schematic curve discussion were treated in too much detail with the CAS - topics where paper-and-pencil methods make much more sense. In some cases, learning MAPLE syntax itself became the goal and in a few cases students got lost fooling around in its computer programming aspects.

The CAS provided a special advantage for mathematical concept formation through its features for visualization, animation, and the automation of calculation processes. Problems resulted from the high technical and syntactical demands it imposed which left little time for explanation phases. All colleagues agreed clearly on this point. In the future this will have to be remedied by changes in the selection and weighing of teaching contents. Also, one must pay careful attention to the use of correct mathematical language, since students tend to talk in MAPLE commands.

The chosen schoolbook was rarely used. Teaching methods were characterized by the use of worksheets, an instructional mode which furthers independent work. However, a danger lies in too thoroughly "pre-thought" teacher solutions that can be followed by students without much effort ("Didactics of the return key"). On the other hand, a variety of different possible solutions arise within classes and these can be very valuable from a didactical point of view. Unfortunately time was often lacking for discussion and teacher-prepared solution sheets took over the learning control role. This is counterproductive and leads to a mere acceptance of the solutions.

As a rule tests were written in two parts, one part with and the other without the computer. Assessment was further influenced by class participation, project papers and coursework. Problems for the final examination in Baden-Wuerttemberg are set centrally. As some of these problems turn out to be trivial when using a CAS, we plan as follows: Participating colleagues pose three complete problems (consisting of various small problems from all topics covered). One of the problems is given to all students of this teacher and hence topics can be tested that have not been covered by all classes. An additional problem is worked out by us and given to all students.

An anonymous student questionnaire at the end of the school year showed that all the students would again decide to take part in the CAS project. They found the computational facilities of the CAS a great help with tedious calculations, while the visualization possibilities proved to be a great aid to understanding. Emphasis on independent and co-operative learning modes were seen as very positive. However, no one wanted to do completely without traditional teacher-centred instruction.

## **5 Computer Algebra Systems - Challenge and Chance**

In summary: The CAS computer should be used as a methodological supplement to classical media such as blackboard, exercise book, etc, but it should not be a substitute for them. As a rule the design of a problem solution has to be done using paper and pencil. The MAPLE slogan "The end of the pencil" is nonsense! This is also emphasized by students' comments such as: "One cannot think in front of a computer monitor" and our aim is certainly not to perfect the computer science aspects. A main point is the didactical reduction to central concepts of the subject, not the accumulation of new contents even if they may be easily accessible from

the CAS perspective. Thus, compared to the first course, time can be gained, time that is urgently needed to strengthen the explanation aspect. Traditional mathematics teaching is accused - and not without good reason - of placing too much emphasis on syntactical skills. It would be a catastrophe if this situation were to be strengthened by using computers without critically evaluating their contribution. The aim cannot be to automate classical curve discussion methods by using computers, even if these methods are part of each Abitur examination, but to help understand criteria and concepts and to apply them in appropriate situations. The new CAS tool enables us to do this, because it allows us to tackle open-ended problems that lead to individual new solution strategies thus motivating creativity. This conscious problem-solving process usually requires mathematical modelling, i.e. a translation of the situation under discussion into mathematical language [see Henn 1997].

## 6 A Look in the Classroom

### 6.1 Competence with Functions

The idea of looking at different aspects of our world in isolation and describing the connections between them using mathematical methods marks the beginning of the exact sciences. To understand the idea of functional connection, students have to be competent in the handling of functions. In principle, all types of functions are known from the lower secondary level: linear, quadratic and power functions, exponential and logarithmic functions and trigonometric functions.

The use of CAS makes it very easy to represent graphically all those types of functions and, for example, to investigate personally what produces the transposition of the term  $f(x)$  to the term  $a \cdot f(bx+c)+d$ . For a given graph, a function term can be determined and the correctness of the idea can be checked quickly. From the start, functions of several variables can be studied in a natural way, thus leading among other things to a deeper understanding of three-dimensional space. The following example is studied at the beginning of grade 11:

- Examine the family of lines  $f(x,t) = tx + t^2$  with parameter  $t$ .

At first students sketch some of the lines on a sheet of paper (Figure 1); this does not give much information. With the help of MAPLE, they can reduce the width of the steps of the parameter as they wish. This leads to such figures as in Figure 2. A usually “weak” student spontaneously exclaimed “that’s a parabola”. The graph leads to the assumption  $g(x) = -\frac{1}{4}x^2$  for the equation of the parabola, which can be checked immediately i.e. for every  $t$  the equation  $g(x) = f(x,t)$  leads to exactly one intersection point. Therefore, the respective line is always a tangent (still in a naïve sense).

In the same way, we can look at  $t$  as an independent variable and get the family of parabolas

$f(x,t) = t x + t^2 = \left(t + \frac{x}{2}\right)^2 - \frac{x^2}{4}$  with parameter  $x$  (Figure 3). The standard parabola has been moved along a parabola turned upside down.

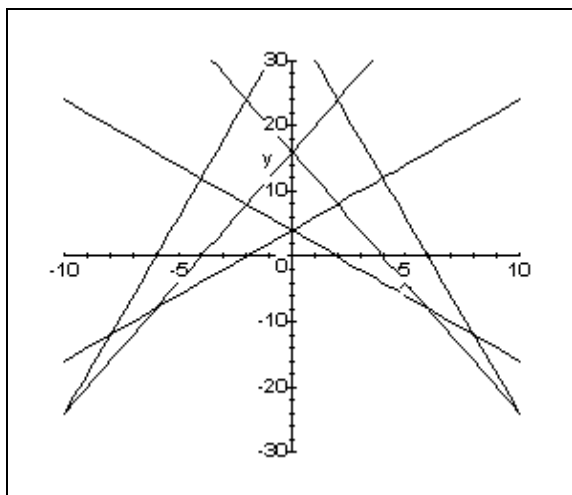


Figure 1

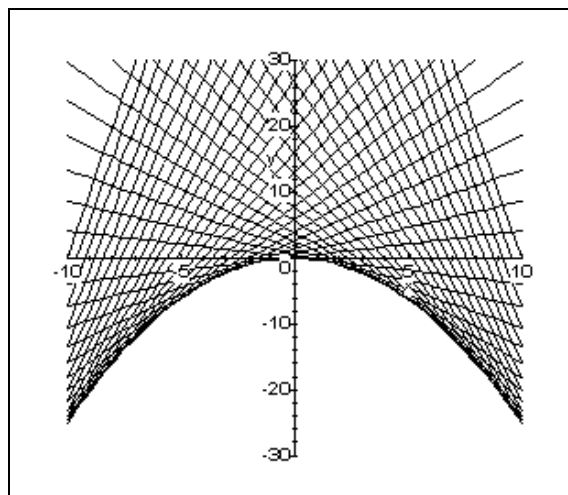


Figure 2

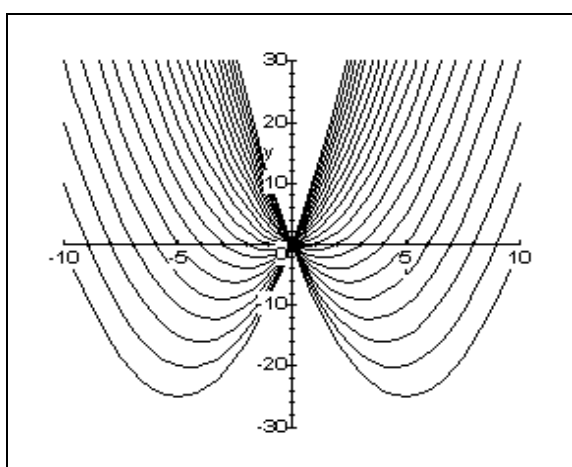


Figure 3

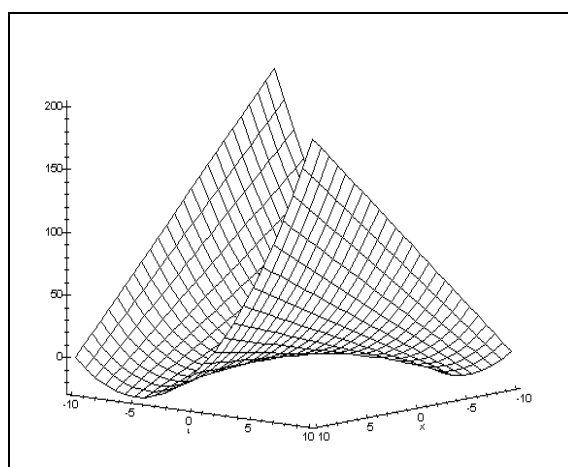


Figure 4

The three-dimensional embedding of those families of parametric curves, something which is certainly worth striving for, leads to a saddle surface (Figure 4), which can be drawn very easily with the help of MAPLE and can then be inspected from all sides. Later, you can re-examine such surfaces again with the help of analytic geometry. Many surfaces you can see in reality are saddle surfaces, as the two following examples show. The first (Figure 5) is a work of art near the harbour of Lausanne/Switzerland, the second (Figure 6) shows a roof put together with such surfaces in Xochimilco/Mexico.

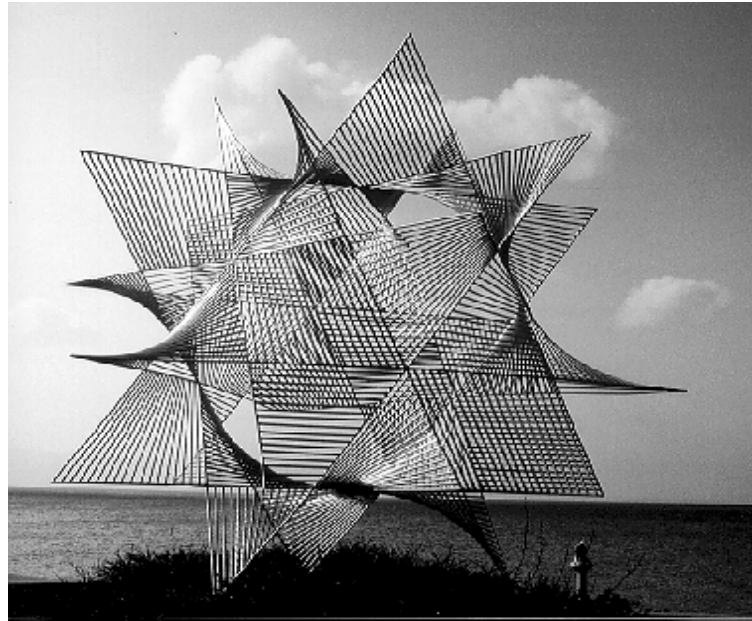


Figure 5

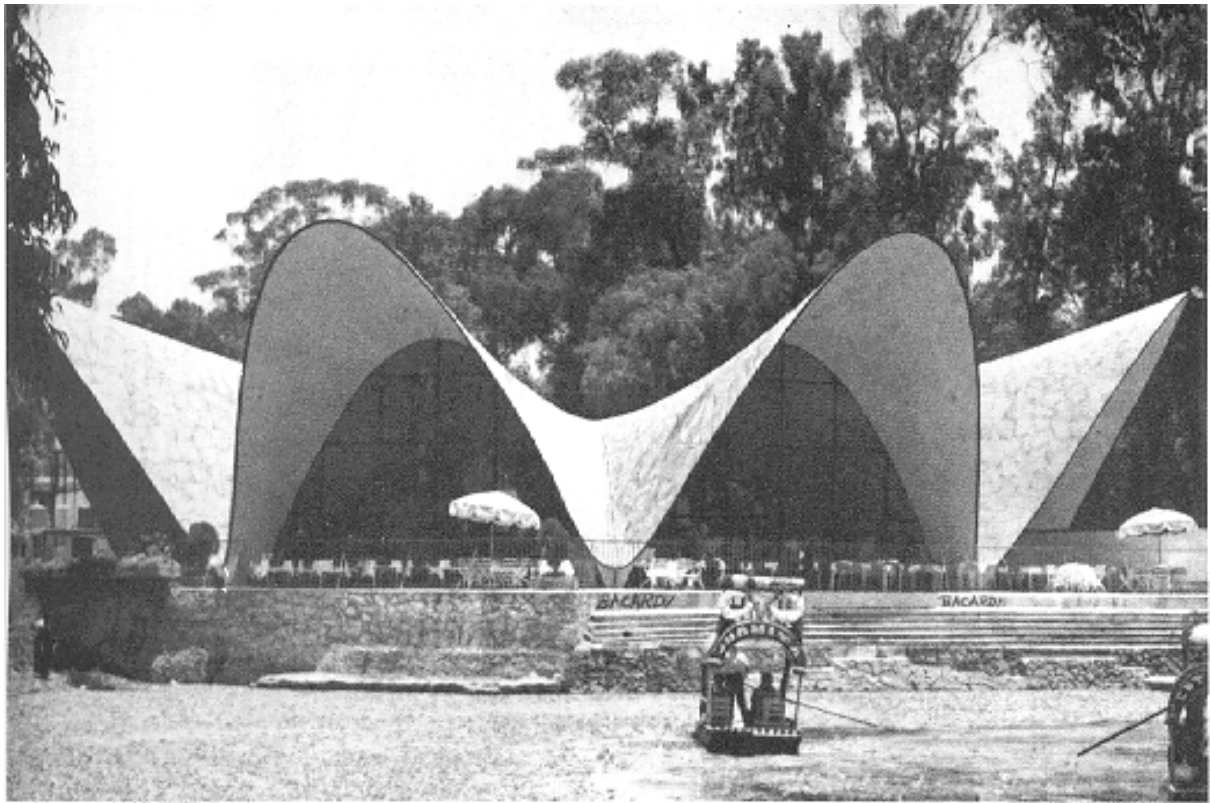


Figure 6

## 6.2 The CAS helps to solve problems ....

The following problem is typical of a relatively open-ended problem formulation that is still stated in a way to enable all students to work on it according to their individual abilities:

### Discover and prove!

- $f$  is a polynomial function of third degree with zeros  $-1$ ,  $2$ , and  $5$ . Draw the tangent to the graph of  $f$  for  $m = 3.5$ . What do you notice?
- Does your conjecture also hold true for a polynomial function of third degree with three other zeros?
- Can a corresponding conjecture be made if a polynomial function of third degree has only two zeros?
- Can a corresponding conjecture be made if a polynomial function of third degree has only one zero?
- Are there any generalizations to other functions?

Without much effort students formulate a first MAPLE worksheet. Only a little MAPLE syntax is needed. MAPLE draws the graph of  $f$  and the tangent  $t$ . Numerical values have been chosen in such a way that the desired theorem stating that, for three zeros  $n_1$ ,  $n_2$ , and  $n_3$  the tangent for  $m = (n_2 + n_3)/2$  passes through the third point  $(n_1, 0)$ , becomes obvious (Figure 7).

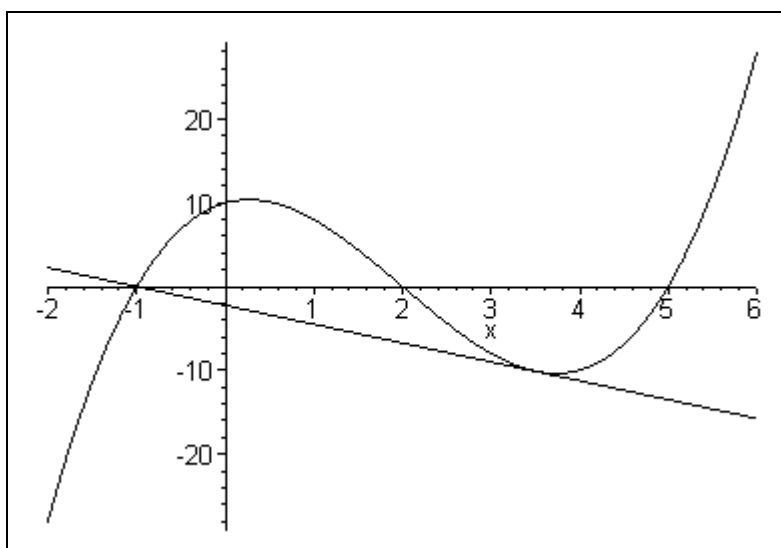


Figure 7

An abstractly thinking student is going to choose the approach

$$f(x) = a(x - n_1)(x - n_2)(x - n_3)$$

immediately and then check with MAPLE that the equation  $t(x) = 0$  has solution  $n_1$ . They have then already answered questions b. and c. by writing down the double zero twice. A more visually minded student will investigate further examples, which is easily done by varying their own worksheet. Question d. can be seen as trivial or, if a more difficult direction is taken, our theorem can, for example, be generalized by substituting the three zeros by three arguments with the same function value  $a$ . Finally, the last question e. is completely open-

ended, for example the question can be reversed by searching for functions  $f$  with three zeros  $n_1$ ,  $n_2$ , and  $n_3$  with a tangent to  $m = (n_2 + n_3)/2$  having zero  $n_1$ .

### 6.3 .... but the CAS can also be counterproductive!

The harmonic series provides a nice example of how a problem can be suitably solved by thinking after the computer has failed. Addition of the first  $n$  terms of the series by computer shows on the one hand that computation time increases dramatically with growing  $n$ , whereas the series ultimately seems to converge. This result should not be withheld from your students. By appropriate addition of the next  $2^m$  terms it is shown by hand that each time more than 0.5 is added to the series sum, the harmonic series diverges. The power of MAPLE can create false ideas among students if the system is used meaninglessly e.g. for small values of  $n$  MAPLE calculates at once the exact value of  $\sum_{i=1}^n \frac{1}{i}$ . Unfortunately, MAPLE provides a good approximation for any  $n$  almost immediately, because much mathematical knowledge has been incorporated into the system. MAPLE recognizes series such as the harmonic series and knows that its value can be expressed by means of the digamma function. When the sum is calculated for a variable upper limit  $n$  the MAPLE command

```
> s := sum(1/i, i=1..n);
```

is at once answered correctly by

$$s := \Psi(n) + \gamma.$$

Then, if one asks MAPLE

```
> limit(s, n = infinity);
```

the right answer

$$\infty$$

is given. But, who wins anything if students are confronted with this misunderstood “general abstract nonsense” without any comment?

### 6.4 The CAS supports concept formation

Rates of change provide an appropriate approach to calculus and these ideas open the way to many applications. The rate of change is often known (directly or by modelling) and the function can be reconstructed from this. The correct understanding of the basic principle leads to all differentiation and integration rules and clarifies the ideas which are behind curvature and arc length. Just as we leave the calculation of sine values to the pocket calculator, we leave the practical calculation to CAS. In principle, it carries out algorithms which are understood and done by hand for the simplest examples.

The following example shows a graphical approach to the derivative as the local rate of change and to the integral as its reconstruction. Starting point is a current test report about the new Porsche 911 GT1 taken from an automobile journal. In the table of data and measured values (Table 1), among other things there are the times of acceleration from 0 to  $v$  for different velocities  $v$ .



**Table 1:** Table of Data and Measured Values

<b>Beschleunigung</b>		
0 - 50 km/h		2,1 s
0 - 100 km/h		3,9 s
0 - 130 km/h		5,4 s
0 - 160 km/h		7,1 s
0 - 180 km/h		8,8 s
0 - 200 km/h		10,5 s
0 - 250 km/h		17,4 s
400 m mit stehendem Star		11,6 s
1 km mit stehendem Start		20,7 s
<b>Elastizität</b>		
60 - 100 km/h (IV.G.)		6,8 s
60 - 100 km/h (V.G.)		9,1 s
60 - 100 km/h (VI.G.)		11,9 s
80 - 120 km/h (IV.G.)		5,3 s
80 - 120 km/h (V.G.)		8,5 s
80 - 120 km/h (VI.G.)		11,1 s
Höchstgeschwindigkeit		308 km/h
<b>Verbrauch</b>		
Super Plus	16 bis 29 Liter/100 km	
<b>Bremsweg/Verzögerung</b>		
aus 100 km/h	36,0 m/10,7 m/s <sup>2</sup>	
aus 200 km/h	130,8 m/11,8 m/s <sup>2</sup>	
<b>Preis</b>		
Porsche 911 GT1	DM 1.500.000,-	

You can immediately make qualitative statements like “the car accelerates quicker at the beginning than later”. The formulation of this sentence, in the sense of a quantitative statement, is a typical problem in science! For example, you can say: For  $\Delta v = 30$  km/h (from 100 to 130 km/h) you need  $\Delta t = 1.5$  sec. The given intervals of velocities as well as the intervals of times are different. For comparison, you should refer to the same  $\Delta t$  or the same  $\Delta v$ . Both possibilities are realistic and meaningful as the different statements on the consumption of fuel show: litres per 100 km in Europe, miles per gallon in the United States. For the acceleration, we reach agreement about equal times, for example always set  $\Delta t = 1$  sec. Now, we are able to calculate averages from the given data. In the example above, the average acceleration between 100 and 130 km/h is

$$\frac{30 \text{ km/h}}{1.5 \text{ sec}} = \frac{20 \text{ km/h}}{1 \text{ sec}} = 5.56 \frac{\text{m}}{\text{s}^2},$$

which you have to convert into the unit  $\text{m/s}^2$ . The measurements in the table and the calculated average accelerations can easily be represented graphically (Figures 8 and 9).

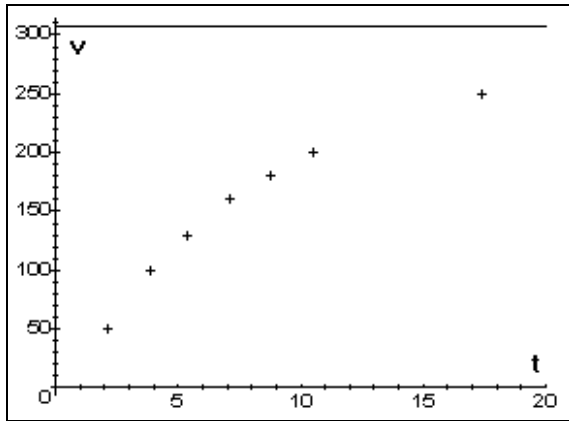


Figure 8: measurements of the velocity

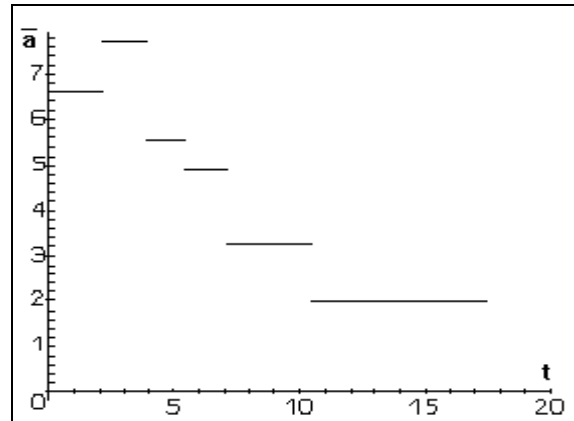


Figure 9: average acceleration

To improve these unsatisfactory results, we should have more information about the connection between time  $t$  and velocity  $v$ . We can obtain this information indirectly by drawing a curve through the measured points. The type  $v(t) = 1 - 2^{-t}$  is the most adequate of the function types known from lower secondary level. A corresponding function has first of all to be adapted by starting with  $v(t) = a(1 - 2^{-bt})$ . The obvious value for  $a$  is the maximum velocity. As usual, you get a value for  $b$  by substituting a point. MAPLE performs the calculation and the graphical representation (Figure 10):

$$v(t) = 308 (1 - 2^{-0.1384371003t}), \quad v \text{ in km/h, } t \text{ in sec.}$$

With it, we are able to compare the acceleration values in a more meaningful way advising MAPLE to calculate for a fixed  $\Delta t$  the average rates of change  $\bar{a}(t) = \frac{v(t + \Delta t) - v(t)}{\Delta t}$  and to

represent them graphically. This is done in Figure 11 for  $\Delta t = 1$  sec. Now the acceleration behaviour of the Porsche can be seen much more clearly.

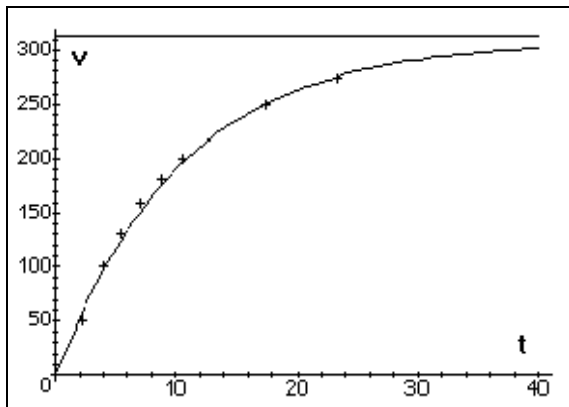


Figure 10: velocity  $v$

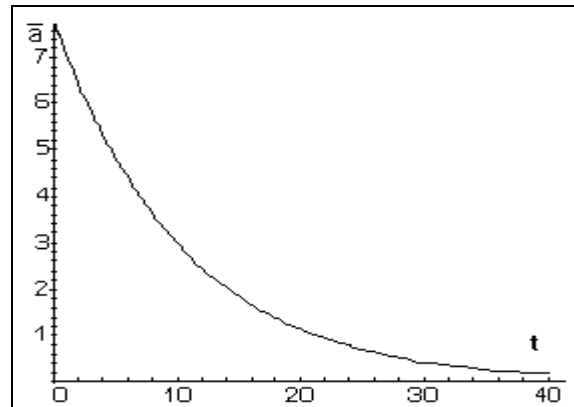


Figure 11: acceleration

Particularly clever students propose to get a more exact insight by reducing the interval  $\Delta t$  and the CAS allows them to check any such proposal immediately! However, the graphical representation, for example for  $\Delta t = 0.1$  sec, does not provide any more visible information.

For a more precise analysis, MAPLE has to calculate the average accelerations for a fixed value of  $t$ , for example  $t = 5$  sec, and for the time interval from  $t$  to  $t + \Delta t$ . Doing this for  $\Delta t = 1$  sec, 0.1 sec, 0.01 sec, and so on, the stabilization of the values becomes clear quickly!

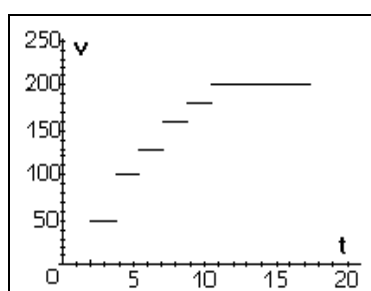
$\Delta t$	$\bar{a}(5) = \frac{v(5 + \Delta t) - v(5)}{\Delta t}$
1	4.844927917
0,1	5.056797223
0,01	5.078658334
0,001	5.080833334
0,0001	5.080833334

Moving from the average to the local rates of change and relating this with the graphical background of the transition from the secant to the tangent is done informally.

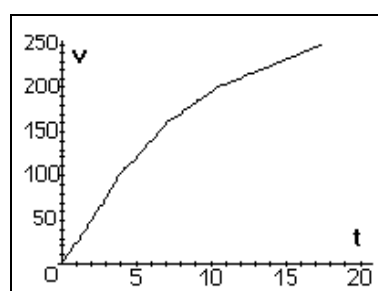
A second question is added to our velocity data. How far has the Porsche travelled by accelerating from 0 to 250 km/h? The basic idea is to reconstruct a function through its rates of change. Here we get a first estimate, assuming a constant velocity between the two points of measurement. That means 0 km/h as the lower and 50 km/h as the upper estimate between 0 and 2.1 sec respectively. Usually the students suggest the starting point with a velocity growing linearly in every time interval. The graphical representation of the three estimates in Figures 12 - 14 leads to the integral and to the trapezoid rule.

All three methods can be improved immediately by reducing the time interval  $\Delta t$  and making use of our function  $v$ . The total time  $t = 17.4$  sec has been divided into  $n$  equal intervals of length  $\Delta t = \frac{17.4}{n}$  sec. MAPLE then calculates improved lower and upper estimates for the total distance:

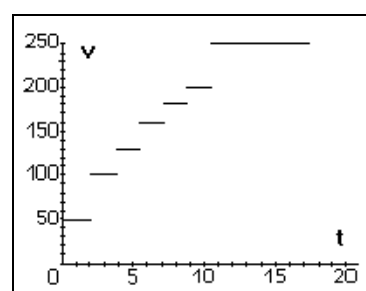
$$s_{\text{lower}} = \sum_{i=0}^{n-1} v(i \cdot \Delta t) \Delta t, \quad s_{\text{upper}} = \sum_{i=1}^n v(i \cdot \Delta t) \Delta t.$$



**Figure 12:** lower estimation of  $v$ , distance = 672 m



**Figure 13:** piecewise linear growth of  $v$ , distance = 770 m



**Figure 14:** upper estimation of  $v$ , distance = 962 m

Again, some experiments show that the values stabilize with growing  $n$  and hence lead to the definite integral! The following table contains some values:

$n$	$s_{\text{lower}}$ in m	$s_{\text{upper}}$ in m
10	702.86	823.70
100	758.90	770.98
1,000	764.36	765.56
10,000	764.90	765.02

Eventually we are able to check two further statements from the automobile journal. Among the acceleration data, two times are given for driving a distance of 400 m and of 1 km, beginning at zero velocity and with maximum acceleration. We can control these times with respect

to our velocity curve. We substitute  $n = \left\lfloor \frac{t}{\Delta t} \right\rfloor$  ([..] means the floor function) in the sums for

the time interval  $\Delta t = 0.1$  sec and obtain two values to estimate the distance  $s(t)$  from 0 to  $t$ , accelerating from a stationary start. MAPLE calculates for the 400 m and the 1 km distance:

1 km from stationary start		
Measurement	Model	
11.6 sec	$s\_lower(11.6 \text{ sec}) = 391 \text{ m}$	$s\_upper(11.6 \text{ sec}) = 397 \text{ m}$

400 m from stationary start		
Measurement	Model	
20.7 sec	$s\_lower(20.7 \text{ sec}) = 998 \text{ m}$	$s\_upper(20.7 \text{ sec}) = 1005 \text{ m}$

Such an approach to the basic concepts of calculus avoids assuming that derivatives are only tangents and integrals are only areas!

## 7 Conclusion

The use of computers has now become a cultural skill, indeed you cannot imagine the professional working day of the 21st century without multimedia applications. You must learn to use the new technologies responsibly and certainly education cannot shut itself off from them.

“Education is the only raw material, out of which successes may grow in the future for German exports”, said Prof. Schlafke of the Institute of German Economy. The TIMSS study shows that we have to take counter-measures in Germany to justify this claim again. It is one reason why the question about the goals of mathematics education has become the centre of attention. The crisis in mathematics education has become obvious through the advent of computers. “The computer forces us to think about questions which we would have had to address a long time ago did we not have computers”, Prof. Schupp pointed out.

The development of mathematical concepts is only possible with many examples and counter-examples experienced at first-hand, then by classifying and organizing. Allowing students autonomy in thinking and acting, as well as in visualizations and dynamic figures supports this process. In the States, those ideas are combined in the “Rule of Three”:

*Every topic should be presented geometrically, numerically and algebraically.*

Lessons which are classical and focused on calculations lose their importance and thereby opportunities arise for a reorientation of mathematics lessons. The second basic principle of the American curriculum reform calls this the “Way of Archimedes”:

*Formal definitions and procedures evolve from the investigation of practical problems.*

The computer opens up quite new dimensions for these new approaches to mathematics. Let us take the chance!

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Hans-Wolfgang Henn

*former adress:* Staatliches Seminar für Schulpaedagogik  
(Gymnasien), Karlsruhe, Germany

*now:* Universitaet Dortmund, Fachbereich Mathematik  
Institut für Erforschung und Entwicklung des Mathematikunterrichts  
Dortmund, Germany

e-mail: Wolfgang.Henn@math.uni-dortmund.de