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MY COLLECTION OF GEOMETRICAL IDEAS – EXPERIENCES AND SUGGESTIONS FOR A CLASSROOM PROJECT ON GEOMETRICAL EIGENPRODUCTIONS IN PRIMARY SCHOOL

Abstract:

This article reports about a project on geometrical eigenproductions in primary school. Initially it deals with the idea of eigenproductions in the mathematics classroom. It then describes the beginning, the progress and the organisation of a classroom activity, concerning pupils designing a 'book of ideas'. Various investigations into pupils' drawing ability and corresponding phase models are reported. Finally the main insights and practical experiences of the project are summed up. The text is enriched by typical examples of pupils' geometrical eigenproductions.

1. The idea of "geometrical eigenproductions"

Consider a familiar situation: people show a couple of holiday videos or lots of photographs to visiting friends; it lasts for hours. The hosts continue eagerly presenting their visual material which they do with growing attention and fervour, while the visitors soon appear exhausted and disinterested.

This situation can make us reflect on education in geometry. Figures in textbooks or in other learning materials are – how ever well they may be produced – in the first instance not particularly interesting and impressive for pupils. Nowadays children are more and more overwhelmed by visual stimuli. Working with children's visual eigenproductions in the classroom, however, makes it possible to arrive at quite a different relationship to, and attention for, pictures. High levels of engagement and attention seem attainable, since a totally different personal approach to the contents of their learning can be built up. More and more teachers try to give their pupils scope for their own ideas and discoveries in mathematics. In this way the mathematics classroom becomes more open, individual and child oriented. "Active discovery learning" or "Constructive discovery classroom" are labels often used for the description of this development. Geometrical eigenproductions can be one promising way of realising such kinds of classroom organisation.

In the case of language education a good deal of experience with, and practical examples for, pupils' own text writing and individual production of books still exists. There are also some practical ideas for the primary mathematics classroom, mainly in the field of arithmetic. Interesting ideas can be found in:

- GALLIN & RUF (1991), who let the pupils produce "travel diaries", i.e. documents describing ways of learning by solving arithmetical problems;
- SELTER & SPIEGEL (1997), who collected mathematical eigenproductions about pupils' notes on their own ways of calculating;
- GEERING (1994), who e.g. used 'number albums' as an individual approach to different number aspects at the beginning of primary education;
- SCHÜTZ (1994), who let the pupils design 'researcher's note-books' which are reviewed in 'mathematical meetings';
- CLAUSSEN (1995), who asked pupils to write self-formulated text problems, and found the pupils making use of self-collected, original materials;

– HERZOG (1994), who let the pupils produce 'discoverer copy-books' in so-called 'discovery classrooms', where the pupils reflected about mathematical self-portraits.

So far there are relatively few reports on experiences with, and research on, geometrical eigenproductions. The projects on geometry in primary education, described subsequently, report about pupils' spontaneous production of geometrical figures in different classes.

2. Production of geometrical book of ideas in the classroom

In principal it is possible to work with a 'copy book of ideas' or a 'book of ideas' in each year of primary school. Carrying out such work during the whole time in primary school, however, can give an almost complete impression of the pupils' development in geometry. Learning processes and different ways of thinking become 'visible'. Within one school year up to 10 classes should be devoted to this work, e.g. in the form of a project. Each child develops single sheets within the lesson time and collects them in a book of ideas of between 20 and 30 pages. In order to avoid unwanted and direct influence from home, the work should not continue at home. During lessons blank pages having only a headline about different topics are given out in the classroom, one for each pupil. The children are allowed to come to the front in a certain order and to choose one of these topic sheets.

The pupils draw without any specific instruction or help. They are given nothing but the headline. They also do not get any instruction concerning the ways and media of designing. They themselves decide the colours and the techniques they want to apply. They can draw freehand or with the help of a ruler or stencils. Magazines can be available in a corner of the classroom which enable the pupils to cut out and stick in pictures. This kind of designing, not linked to a specific purpose and a particular instruction, is to allow a variability of ideas as wide as possible, and not to impose restrictions from the outset. The children are given about 10 or 15 minutes for working at one drawing; afterwards they are allowed to choose further topic sheets, this time in reverse order, and to work on them. In subsequent classes the pupils are not expected to work on new topics. They can continue their work on previous topics; they have the chance to complete sheets, adding further ideas. At the end of a class each pupil has produced three or four sheets. They are given a folder for collecting the sheets (eventually put into transparent folios), and a brief instruction text. A cover page and a table of contents are individually designed by each child, but this should not be done before most of the topics are known. For the front page the pupils can return to, in their opinion, their best ideas (see Susi's work in attachment 1).

The main topics should, on the one hand, be based on content covered by the curriculum; on the other hand, curriculum contents should not restrict the areas of their ideas. For grades one and two, topics, such as those which follow, seem to be particularly adequate: "basic geometrical shapes and plane figures"; "pictures formed by well known shapes and lines"; "geometrical ornaments"; "simple (three-dimensional) solids". For grades three and four, the already mentioned topics may be continued while other possible titles include: "symmetrical shapes"; "particular (plane) shapes"; "particular solids"; "enlargement and diminution". As a starting point in grade one the topic "geometrical figures in my favourite colour" seems appropriate. In this case all pupils can, exceptionally, start with the same topic. Apart from that, only the pages "my ideas for a picture" and "my discoveries in a picture-puzzle" provide a headline together with a few geometrical figures. In the case of grade three and four this is true for the pages "my discoveries in reflected image" (with errors in it) and "my ideas about funny drawings (so called Drudels)". In this way the pupils can be motivated to design, after gaining some experience with different topics, a topic of their own choice.

In our project we collected 60 topic areas for all primary levels. A complete overview on all the topic fields for grades one to four successfully tried out by us can be found in the booklet "Mein Ideenbuch für Geometrie" (SENFTLEBEN 1998).
The pupils' designing goes on with great eagerness, in spite of, or even because of, the lack of any direct help. Of course, the teacher can offer some suggestions and stimuli from time to time, particularly when the pupils are not yet quite familiar with this kind of work. A statement by MONTESSORI (1992) may characterise the way to help: "help me to do it by myself".
There is emphasis in the classroom not only on designing the sheets, but also in talking about them. The aim can be to discuss the geometrical eigenproductions from a technical perspective. We call that our "geometrical meeting". This can happen with the children sitting in one circle while, later on, larger classes can be formed into two circles. It begins by putting several sheets (up to 5) about the same topic on the floor and looking at them. All pupils can make their comments spontaneously. Stimuli for the talk may be: "What do the pictures represent?" - "What do you like particularly?" - "What is it you don't like?" - "Do the drawings fit the headline?" - "What is the main feature of the picture?" The pupils should also ask the author as they are able to give good explanations. Generally the pupils are very proud of their drawings and like to see their products commented upon very much.
The drawings stimulate conversation, which quite often leads to geometrical questions or forces the pupils to use geometrical terms and to reflect critically on their designs. The geometrical eigenproductions, therefore, do not have to be perfect drawings. Simply designed sheets often stimulate more discussion than artistic paintings. The teacher should try to engage the pupils in an intensive exchange of ideas, to remain in the background as long as possible and not reveal errors at once.
These geometrical meetings are highly demanding for all participants. The pupils have difficulties in expressing themselves clearly. Often they lack adequate technical terms but sometimes they invent their own words like "Rundeck" (round polygon) "Dreiecksäule" (triangle column) which often describe the situation appropriately. On the other hand, a certain culture regarding talking seems necessary. The children must learn not to speak at the same time, to respect different opinions, and to criticise reasonably. Initially so-called 'quick circles' have proved useful. In these all pupils in the circle are allowed, one by one, to say just one sentence. We have also had good results with the use of a stop sign which is shown when pupils speak at the same time, and indicates an order for silence. If somebody does not follow the rules of discussion they will be shown a yellow or a red ticket. Having such kinds of meeting regularly often results in a remarkable conversation culture where it is no longer necessary to exclude someone by showing them a red ticket.
In each school year the project can end up with a small public exhibition, where the children can take the role of a jury. For every topic the best pictures are picked out and awarded a prize. The exhibition should be open to all pupils, teachers and parents.
At this point I want to return to the story about holiday pictures mentioned earlier. The situation described there changes completely if pictures are shown to which the spectators can relate personally, e. g. if they too had had holidays at the same location. Geometrical eigenproductions can provide an adequate basis for looking at further figures in a text book or on a work sheet with much more interest and attention, and for solving problems linked to them with a higher level of pupil engagement. The book of ideas, therefore, is not intended to replace the textbook. On the contrary, the pupil's mind can be opened by their previous work with 'their own book'.

3. Research into pupils' drawing ability

Since most pupils' geometrical eigenproductions are drawings, it seems sensible to look for research about drawing beyond the subject borders, mainly in the fields of art and art education. A quite interesting source of knowledge are the publications of KERSCHENSTEINER (1905). About 100 years ago he started to analyse children's drawings systematically. Over a period of seven years he reviewed about 500,000 drawings of about 58,000 pupils in Munich, and classified 300,000 of them. The results have, so far, mainly been noted by people who intended to reform art education. Drawing demands specific artistic abilities, but also abilities of spatial perception of the environment, abilities of mentally manipulating real objects (which are also abilities of mental geometry) and abilities for representing the space – i. e. abilities of spatial thinking which, as mentioned before, are essential objectives of geometry in education. KERSCHENSTEINER let pupils draw objects, plants, animals etc. from the heart, since he was convinced that the way to a clear imagination of shapes can only be achieved by operating one's personal imagination of pictures, by the "Gesichtssinn" (sense of vision), as he called it. He arrives at the conclusion that children in their first attempt – concentrating only on particular properties of the objects – produce what he calls a "Niederschrift der begrifflichen Merkmale" (a note of conceptual properties). Later on, their feeling for lines and shapes develops, leading to the ability to produce a realistic representation. KERSCHENSTEINER distinguished four different steps in developing this ability:

1. Stufe der Raumlosigkeit (step of spacelessness): Children wish to represent three-dimensionality in their pictures so they line up the objects in a row. They "narrate" singular events. Relationships between special positions such as 'on top', 'on the bottom', 'left' and 'right' are often made clear quite well, but the dimension of depth is not represented. Indications of that sequential way of drawing can be found in ancient Egyptian wall paintings.
2. Stufe der mißlungenen Raumdarstellung (step of failing representation of space): The children try to find ways of visualising three-dimensionality but their efforts frequently lack perspective. The drawings often have the character of maps or ground plans and front elevations are composed of different folds. The figures therefore lack the necessary perspective dimensions. Antique drawings often exhibit these traits.
3. Stufe der richtigen bildlichen Raumdarstellung (step of correct representation of space): Here the children draw perspective homogenously, but space is not yet represented as a comprehensive picture. There are failings in handling the problem of ground-horizon representation. The relief style of medieval art at the end of the Roman period contains examples of this evolutionary style.
4. Stufe der vollendeten Raumdarstellung (step of perfect representation of space): Here children succeed in representing all line, shape and colour perspectives as well as crossings and foreshortening. Light and shadow are included correctly. The scale of depth and surface outlines is properly incorporated.

PIAGET (1993) has also worked on the development of spatial thinking and children's ability to represent, pointing out three steps of development:

Step I: Inability of synthesis: The children do not succeed in proportions, distances and perspectives. They have problems with details. However, they achieve success in topological relationships e.g. neighbourhood, separation, inclusion or openness.

Step II: Intellectual realism: Children do not draw what they see but rather what they know about the object; e.g. they draw a meal in a stomach or an egg in a chicken. Imagination about space exists in an incoherent form. They mix up different incompatible perspectives. What is missing is co-ordination of different perspective views.

Step III: Visual realism: Perspectives, proportions, measures and distances are appreciated by the children. Even perspective foreshortening turns out quite well.

MÜHLE (1955) has critically reviewed the research of different experts on drawing, working extensively through the literature. Many of his indications proved helpful in the interpretation of our books of ideas. He refers to the fact that a geometrical-technical perception is not based on an intuitive qualitative impression but rather on realistic, emotionally indifferent order qualities like symmetry, parallelism and perpendicularity. In drawing, children demonstrate a tendency towards ordering and structuring as a baseline of their 'Gestalt'-experience. In particular the right angle is used by young children as a means of distinguishing directions. Three-dimensionality is represented by setting different planes upright into the drawing plane (tendency of setting upright). MÜHLE classifies this into "Geometrisierung" (geometrisation), i.e. formalisation into regular geometrical figures, and "Empisierung" i.e. normalisation into given relational norms. In his opinion, in the development of children's drawing this kind of "Empisierung" comes to the fore. When formally geometrised designs appear later on, they are (as in cubism) not to be understood on the basis of an original approach to perception and designing, but as procedures which "by the sophistication of reflecting and analysing observation, go back to previous approaches of handling the object arbitrarily, not having much in common with the originality of the intention which guides the drawing".

In the recent literature SCHUSTER (1993) describes further interesting aspects of the development of children's ability in representing and drawing. From the viewpoint of mathematics education WOLLRING (1996) actually works intensively in the field of elementary school children's ability to represent spatial structures in unguided drawings.

All the step models mentioned above are certainly interesting and justified under their particular view on classification. It also seems important to know the different steps of development. Having said that however, we should avoid thinking too rigidly in terms of steps, and should not try to attribute our children's sheets of ideas to exactly one step in the developmental process. In particular the idea that children go through these steps in a linear way seems doubtful. It is essential to be conscious of the complexity of, and the multiple development required for, geometrical thinking and the ability of representation. Steps of development can assume a kind of 'hermeneutic function' in the interpretation of children's drawings.

4. Practical experience and insights

The way of building up geometrical concepts by means of eigenproductions turned out to improve motivation much more than the geometry education products created by others. The fact that their pictures were given a similar significance to figures in a textbook strengthened their self-concept, which again raised or even created new interest in mathematics. Only instructed by a topic (headline) and working unguided and free of a specific purpose enabled the children to express themselves freely and to expose their own ideas in the form of a drawing or a collage. In designing their book of ideas the children showed much responsibility and, since efficiency was not a primary concern, their work could then be made readily available. Many children appreciated that unsuccessful representations or mistakes were not immediately pointed out by the teacher, and that they were allowed to complete, correct and change their products later on. A general difference between girls and boys could not be found, however girls were often more anxious when they worked.

Although the production of sheets of ideas was organised in such a way that no two children worked on the same topic at the same time, the pupils exchanged their ideas in the break-time and passed ideas and techniques of representation to others.

The classroom organisation and the way of working with the sheets of ideas in “geometrical meetings” improved social learning and critical behaviour towards each other. The “talking about” i.e. the discussion about the representations became a valuable part of the class. It provided stimuli for the talks which helped guide the technical discussion. These phases of classroom activity proved quite essential for building up, and understanding, fundamental geometrical concepts. (In Christine’s picture “My triangles” her classmates at first admired a little Chinese then later on discovered triangles of equal size and reflected about the conditions under which two triangles do form a square; see attachment 2). Children’s drawings were evidently influenced by specific personal adventures. They used topics in their book of ideas to report about their own environmental experience, their interests and dreams.

In the case of a lot of topics the work with some of the sheets of ideas made it possible to realise objectives crossing subject borders. There were several links to art, handy-craft and textile designing (e.g. in creating patterns).

Spatial and perspective representation caused most difficulties for the children, whereas geometrical exactness and precision sometimes appeared to them of minor importance. However the creativity and imagination of the children were unsurpassed.. They liked pictures with “geometrical animals” and they knew of similar creations from puzzles such as TANGRAM or TRIDOMUS. (These kinds of pictures proved good motivation in naming geometrical shapes, in comparing them, and deciding on their number; see attachments 3 and 4).

In some cases the children represented objects which cannot be seen from outside. They intended to inform the reader about important observations and events and help them visualise these (Andreas drew an invisible staircase within a high tower – he wanted to explain to the reader the way in which a worker was able to get onto the top of the tower; see attachment 5).

Since the pupils always designed their ideas without guidance the results did not always mirror exactly the geometrical topic behind them. In most cases however, ‘mistake’ would not be an adequate category to be attributed to the representations. Different levels of development in the drawing ability and the construction of geometrical concepts, coupled with the individual’s interest in, or importance assigned to, the topic can be seen as reasons for the qualitative differences in the representations.

The task of working on a geometrical topic motivated most children quite intensively and gave good insight into pupils’ different levels of development. As an example the topic “My cube” was, for all pupils, a real challenge. All pupils in grade one and two thought of a die. Almost no children were content with drawing only one square showing a certain number of dots. Markus patiently worked separate visions into the cube, in order to make quite clear that six different dot numbers exist. He put less emphasis on the squared shapes (see attachment 6). Many of the sheets of ideas were similar to a net in the form of a cube, although up to then it had not been a classroom topic. Manuela at first drew a shape, then a net of five and later of six shapes, in order to be “complete” (see attachment 7). The interpretation of Miriam’s cube pictures is rather difficult. At first she drew a square (one side of a cube) with many more than six dots in it – evidently an ideal die for the game. Afterwards she tried a perspective representation in which she mentally examined the cube from the front-side, from the left, from the right and also from above, one after another. Finally she drew dots in a square, in a circle, and in a triangle. Square means for her that the cube is at rest; the circle means that the die is thrown, rolled or moved; and the triangle means that the die is put on an edge (no shape is on top so the die has to be thrown once more). It is interesting also to note the colour of the dots (see attachment 8).

Within the same class we could observe at the same time different levels of development or quality in pupils' ability to represent. The same differences could, of course, also exist between different grades of primary school but even these were less than expected. This may be due to the fact that, e.g., children in fourth grade had no experience with the geometrical sheets of ideas from previous school years.

Quite often we could observe that the same pupil showed different qualities in the representation of geometrical shapes in reference to different topics. As an example, some children succeeded in perspective representation quite well in the case of a particular topic. In representations for other topics, however, spatiality was completely missing. We could not prove models of children's development by our research. In some extreme cases we could even see different qualities of representation on one and the same picture e.g. Thomas, 8 years of age, succeeded in producing notable perspective representations of his bicycle. However, he drew the two people in the pictures as line figures as it is typical for a pre-school child to do (see attachment 9).

References

CLAUSSEN, C.: Öffnung des Sachrechnens - sich zusammen mit Kindern auf den Weg machen.-In: Grundschulunterricht, Heft 11, 1995

GALLIN, P. / Ruf, U.: Sprache und Mathematik in der Schule, LCH-Verlag, Zürich 1991

GEERING, P.: Das erste Zahlenalbum, ein Mathematikjournal für Anfänger.-In: Beiträge zum Mathematikunterricht 1994, Franzbecker-Verlag, Hildesheim 1994

GEERING, P.: Erste Mathematik mit dem Zahlenalbum.-In: Sachunterricht und Mathematik in der Primarstufe, Heft 11, 1994

Heckt, D.H.: Veränderung braucht Methode(n), Moderationstechniken im Schulalltag.-In: Grundschule, Heft 4, 1997

HERZOG, S.: Mathematische Erfindungen und Selbstporträts im Anfangsunterricht.-In: Die Grundschulzeitschrift, Heft 74, 1994

KERSCHENSTEINER, G.: Die Entwicklung der zeichnerischen Begabung, Verlag Carl Weber, München 1905

KÖNIG-WIENAND, A.: Moderations- und Präsentationsformen in Reflexionsphasen des handlungsorientierten Mathematikunterrichts in der Primarstufe.-In: Beiträge zum Mathematikunterricht 1998, Franzbecker-Verlag, Hildesheim 1998

MONTESSORI, M.: Das kreative Kind. Der absorbierende Geist. Herder-Verlag, Freiburg 1997

MONTESSORI, M.: Kinder sind anders. Deutscher Taschenbuch-Verlag München 1992

PIAGET J. / INHELDER, B.: Die Entwicklung d. räumlichen Denkens beim Kinde, Klett-Verlag, Stuttgart 1993

MÜHLE, G.: Entwicklungspsychologie des zeichnerischen Gestaltens, Verlag J. Ambrosius, München 1955

ROTHERT, M. / SENFTLEBEN, H.-G.: Tridomus. Geometrische Erfahrungen mit einem Legespiel (Lehrerband), Klett-Verlag, Leipzig, 1998

SCHUSTER, M.: Die Psychologie in der Kinderzeichnung, Springer-Verlag, Berlin 1993

SCHÜTZ, P.: Forscherhefte u. mathematische Konferenzen.-In: Die Grundschulzeitschrift, Heft 74, 1994

SELTER, C./ Spiegel, H.: Wie Kinder rechnen, Klett-Verlag, Leipzig 1997

SELTER, C.: Eigenproduktionen statt Fertigprodukt Mathematik.-In: Die Grundschulzeitschrift, Heft 110, 1997

SENFTLEBEN, H.-G.: Mein Ideenbuch für Geometrie, Wolf-Verlag, Regensburg 1998

SENFTLEBEN, H.-G.: Mein Ideenheft für Geometrie.-In: Grundschulunterricht, Heft 9, 1998

SENFTLEBEN, H.-G.: Grundschulkinder lösen kopfgeometrische Aufgaben.- In: Grundschulunterricht, Heft 1, 1996

WIESE, I.: Ein Bruchalbum für Kinder.-In: Beiträge zum Mathematikunterricht 1996, Franzbecker-Verlag, Hildesheim 1996

WOLLRING, B.: Räumliche Strukturen in unangeleiteten Zeichnungen von Grundschülern.-In: Beiträge zum Mathematikunterricht 1996, Franzbecker-Verlag, Hildesheim 1996

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