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PRIMARY STUDENTS' *EIGEN-PRODUCTIONS* IN SPATIAL GEOMETRY - POSITIONS TOWARDS MATHEMATICS EDUCATION IN PRIMARY SCHOOLS

Abstract

In primary school mathematics classrooms it is generally a life-long impression which is formed not only of mathematics but also of mathematics learning. Often an image of mathematics determined by the syntactical learning of formal elements dominates, along with a general image of learning mathematics through individual activity.

An attempt to encounter these images effectively is made, for example, through *real-world problem solving* (German: Sachrechnen) *as a way of accessing one's environment*, *active-discovery learning* and *productive practising* (German: produktives Üben, see Wittmann 1997). These didactical concepts and approaches generally aim to present the formal conventions established in mathematics and their resulting contextual associations in an understandable and useful manner. Performing mathematical tasks mainly consists of sharing already established conventions which, through time, have become accepted by mathematicians. On the other hand, these elements too have, at some time or another, first been invented and have had to undergo a clarification process before finally becoming accepted components of today's mathematical knowledge.

It is beyond doubt that encounters with such elements and being able to utilise them, represent a fundamental part of mathematics education. While this element dominates in secondary mathematics it is also an essential element of primary mathematics. Especially in primary school, however, mathematics classes should not only focus on encountering mathematical conventions. In order to gain acceptance for the sense and purpose of mathematical concepts the teacher should create classroom situations in which the children must first be able to *invent* and then *collaboratively extend* ways to describe and quantify a working situation in a formal manner. If there is indeed a characteristic in mathematics education specific to primary mathematics as compared with other levels of schooling, in the author's opinion it is the desirable *balance between invention and convention*. The author argues that a child who has experienced the effectiveness of a self-invented, formal and quantifiable problem description, or problem solution in a working situation together with other children, is more likely to have an open mind for confronting existing mathematical procedures. They are also more likely to strive for semantic access to these procedures than a child not faced with such problem-solving and social demands who must overcome the mathematics presented to them.

With respect to primary mathematics teacher education, it is sensible therefore to qualify the teachers to teach mathematical facts and to establish basic mathematical working environments. Furthermore, they should be in a position to recognise the children's "little mathematical inventions", to observe their initial mathematical approaches to forming definitions and arguments and hence develop these in appropriate working environments. In their mathematics classrooms the primary teachers have to instruct and organise as well as to moderate and diagnose. For this they need not only *mathematics content training* but also *subject-related diagnostic training*. The mathematics content training is, in the first instance, not merely intended to supply subject knowledge as a basis for the selection and arrangement of the curriculum material. It is much more an essential prerequisite for a subject-specific diagnostic of the children's *eigen-productions* (e.g. Wollring 1998).

Analyses of *eigen-productions* and/or their documents have in recent times increasingly become the subject of didactical discussion; they can however already be found in Oehl (1935) and Kerschensteiner (1905) and certainly also in earlier work. A classical norm-orientated analysis of *eigen-productions* forms Gerster's (1982) analysis of the error patterns with respect to written calculation procedures. The author considers the analysis of Schmidt and Weiser (1982) relating to counting and the understanding of figures and the analysis of Selter (1994) concerning arithmetical *eigen-productions* and the book "Wie Kinder rechnen" (English: How children compute) by Selter and Spiegel (1997) as definitive. Despite their emphasis on arithmetic, the range of these works already demonstrates the difficulty of defining *eigen-production*. Those documents which children create whilst mastering (in the full sense of the word) a mathematical problem are called *eigen-productions* by the author. In them the children establish their initial statements for the solutions, their strategies and/or results.

According to the author, the study of eigen-productions can and will, along with the study of objective concepts, enrich empirical mathematics education considerably.

Analyses of eigen-productions are sensible not just within the frameworks of quantitative empirical analyses (these aim mostly at representative phenomena), but are especially important within the frameworks of qualitative empirical investigations, particularly in case studies.

It appears sensible to differentiate between *time-based eigen-productions* and others. Time-based eigen-productions allow recognition of their history of origin; such documents are transcripts or representations which have the intention of presenting a chronological sequence of their elements. However eigen-productions in which the time sequence of their composition is not intended, can occasionally still be *"sequenced"*, i.e. analysed with regard to their history of origin. In the opinion of the author, the study of time-based documents therefore seems to be a branch of empirical didactical research, which can, to a considerable degree, shed light on the processes of learning mathematics – both individually and during the interaction of several individuals.

The study of eigen-productions is proving to be an increasingly effective component of primary mathematics teacher education. It allows, on the one hand, very practice-oriented work, for example in the form of clinical interviews. On the other hand it permits a repeated analysis of existing empirical didactical research results and the development of new and actualised findings, for example with respect to interaction analyses (see Jungwirth et al. 1994 and 1998). It also facilitates ways of working which are closely practice-related. In this way research and teacher education interests have a shared basis and new synergy effects are created. Furthermore, optimising the working environments in which the eigen-productions are analysed can ultimately lead to the conception of suitable working environments within the classroom.

The systematic analysis of eigen-productions therefore appears to be suitable for combining components of objective concepts with diagnostic ones in such a way that not only useful concepts and findings are developed, but also that the demands with respect to mathematics education as a *design-science* (Wittmann 1995) gain increasingly more scope for fulfilment.

According to the author, the study of the primary school children's mathematical eigen-productions should

- become a "meaningful part" of the mathematics education courses during teacher training,
- form the object of early practice-oriented studies and experiences, for example as qualitative empirical investigations or clinical interviews prior to formal teaching practice,
- link up with the respective mathematics content courses,
- be the subject of advanced practice-oriented studies in connection with the design of concrete working environments for the classroom (following practical studies or school-based semesters), and
- be present as an integral part of the teacher exam in mathematics education.

Analysing eigen-productions during mathematics teacher training can foster personal practical-oriented learning without slipping off into casual discussions which are lacking in theory.

Many of the primary school children's eigen-productions found in didactical analyses have been specifically created for these analyses or for their authors as addressees. Hence all of the eigen-productions relating to the same task often offer a varied and confusing picture, making systemization more difficult. Two different forms of question can assist in the analysis of the eigen-productions:

- in the experience of the author, eigen-productions are in each case part of a *learning history* and are possibly decisively influenced by this development. If the guiding conditions of this learning history can be understood, then the eigen-productions can be interpreted more accurately;

- in many studies the underlying eigen-productions have been created in response to a very open set task and are not intended for a specific purpose or for a specific addressee. If insight could be gained into *the purpose which the child has assumed* or the *addressee which the child has imagined*, this could assist in interpreting the eigen-productions.

Eigen-productions are messages. They show which learning history and/or which level of prior knowledge the child assumes of the addressee and how they model the latter's need for information. Children who are given no firm guidelines often create eigen-productions for a fictitious partner, whom they model as a kind of archetypal copy of themselves.

The author therefore differentiates between *purpose-specific eigen-productions* and others. In the former case, the child knows which factual purpose the eigen-production is to serve, for example, as a contribution to a classroom discussion on children's own ways of doing mathematics or as a construction guide for a building-block structure. The author further differentiates between *addressee-specific eigen-productions* and others. In the former case the child knows for whom the eigen-production is intended and can, where necessary, make use of already existing understandings and agreements.

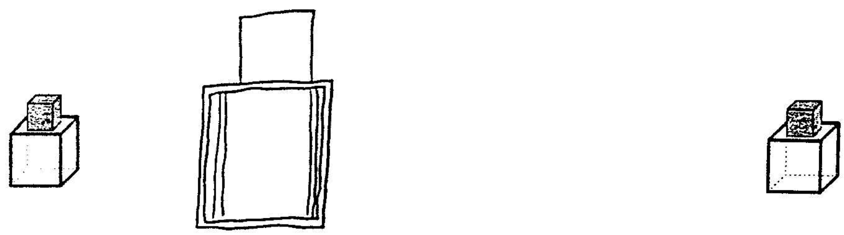
In addition, the author differentiates mathematical eigen-productions of primary school children in didactical research and primary teacher education into three contexts:-
Explorations of eigen-productions in relation to specific tasks serve to clarify certain patterns or structures. They help to clarify whether a given specific purpose in a certain working context influences the eigen-production.
Recognition tests infer how primary school children interpret eigen-productions produced by their peers which have no specifications regarding the purpose or the addressee.
Where the working environment is additionally provided with a *feed-back option* which can influence the eigen-production, *reconstruction episodes* are created. One partner, the "donor" produces an eigen-production as a purpose- and addressee-specific "document" for their partner, the "acceptor". The acceptor then attempts to fulfil the task. Finally their work is compared with the "correct solution" and in the case of deviations the "fitting" of the self-composed documents is discussed. Several reconstruction episodes during which donor and acceptor change roles alternately form a *reconstruction test*. Here the series of consecutive purpose- and addressee-specific eigen-productions can be analysed, which usually highlights the learning process. The organisation of working environments as reconstruction tests is a fundamental and effective model for both the empirical investigation of eigen-productions as well as for the organisation of classroom working environments, if the intention is to incorporate eigen-productions. This holds true for arithmetic, for real world problem solving and in particular for geometry.

One research focus of the author places particular emphasis on questions concerning the didactics of *spatial geometry in primary schools* (Wollring 1995 and 1998). Bauersfeld (1992) and Wollring (1998) highlight the necessity of didactical research and developmental work especially with respect to geometry. As an example of the method of work described above an experiment which focuses on spatial-geometric eigen-productions of primary school children is presented pictorially. The eigen-productions at the centre of the research interest are *children's drawings as documents of their notion of space*. More precisely the goal was to establish to what degree primary school children are able to communicate with each other about certain spatial objects consisting of two dice primarily on the basis of their own respective drawings.

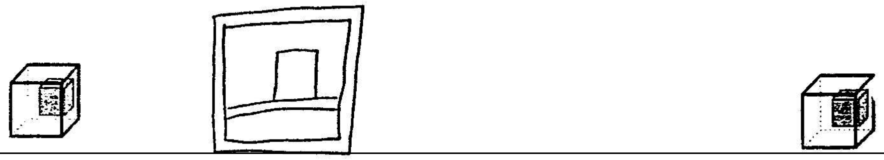
The experimental research design was inspired by a field study by Ingram and Butterworth (1989) which investigated how children between four and twelve years draw configurations of two cubes, one with approximately 7cm edge-length and a smaller one of approximately 5cm edge-length, which can be placed either in front of, within, behind, on, or next to the large one. Using sample pictures in age-dependent steps, Ingram and Butterworth describe the morphology of respective eigen-productions which however were not created for a specific purpose or addressee.

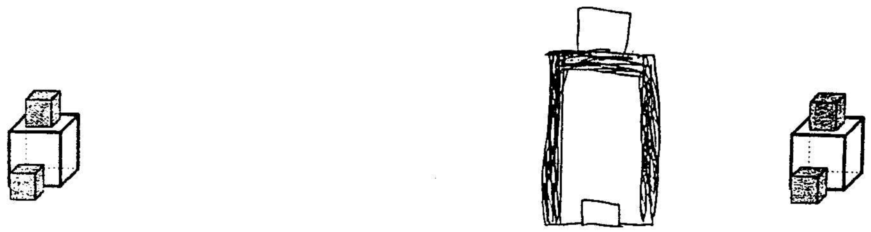
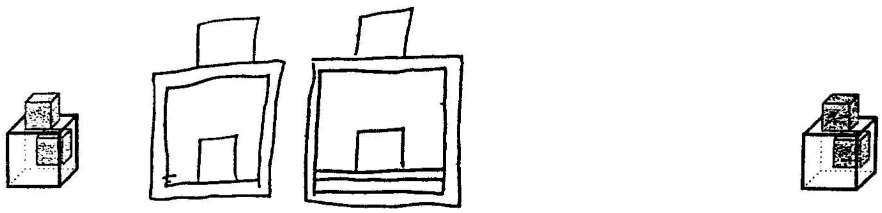
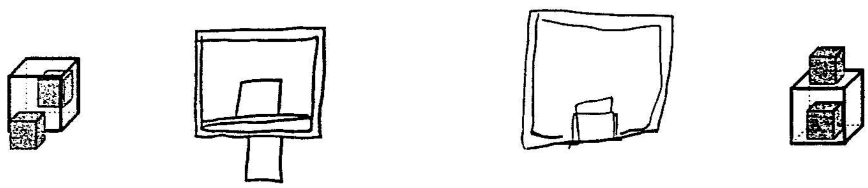
The amended experiment, designed and directed by the author, was conducted by student teachers working with 16 pairs of third-graders (children aged 8 to 9 years) as a case study in the form of *spatial reconstruction tests*, each consisting of eight episodes. In the individual episodes, the "donor" had to draw the dice configuration, knowing that his partner the "acceptor" would have to “build” the given dice configuration on the basis of the drawing. After completion of the drawing the original structure was covered and the acceptor produced a suggestion for a reconstruction which was then compared with the original. When comparing the original structure and its reconstruction, the deficits of the drawing were discussed and, where necessary, suggestions made for improvements. After exchanging roles the next structure was reconstructed and discussed.

Figure 1: Richard and Joachim (3rd grade)
reconstruct dice configurations according to their eigen-produced drawings.
The large die is transparent, the small ones are not.
Left: structure from the donor’s perspective, middle: donor’s drawing, right: acceptor’s reconstruction.

Episode 1 2.34 – 5.41 *Donor:* Joachim *Acceptor:* Richard Successful reconstruction	

Episode 2 6.06 – 8.16 *Donor:* Richard *Acceptor:* Joachim Successful reconstruction	



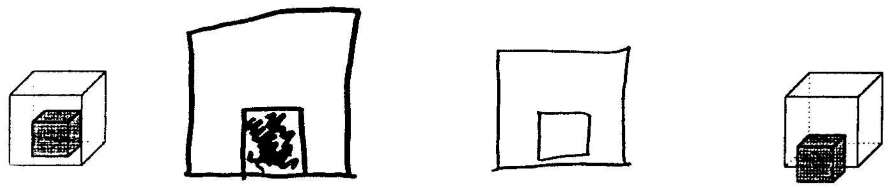
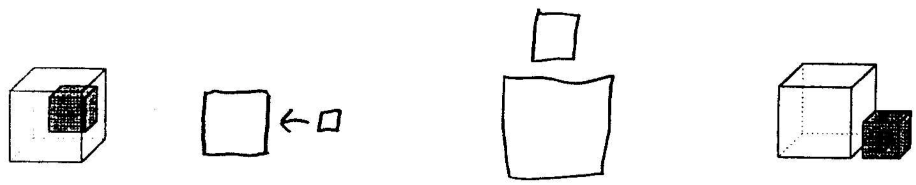
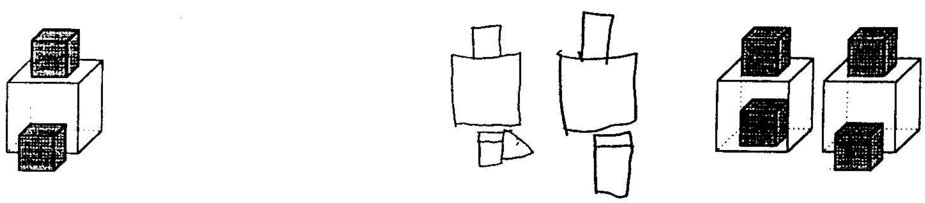
Episode 3 8.41 – 12.17 *Donor:* Joachim *Acceptor:* Richard Successful Reconstruction	
Episode 4 12.43 – 14.53 *Donor:* Richard *Acceptor:* Joachim Successful Reconstruction	
Episode 5 15.18 – 18.50 *Donor:* Joachim *Acceptor:* Richard Successful Reconstruction	
Episode 6 19.20 – 21.15 *Donor:* Richard *Acceptor:* Joachim Successful Reconstruction	
Episode 7 21.40 – 27.00 *Donor:* Joachim *Acceptor:* Richard Unsuccessful Reconstruction	

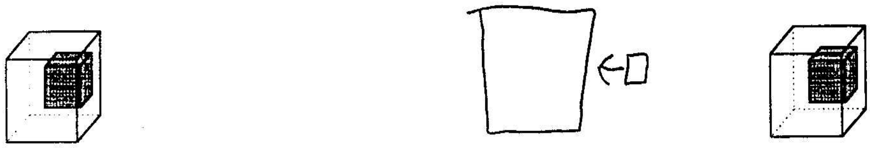
Episode 8 27.25 – 29.27 *Donor:* Richard *Acceptor:* Joachim Successful Reconstruction	
Episode 9 29.48 – 32.34 *Donor:* Joachim *Acceptor:* Richard Successful Reconstruction	

Due to space restrictions, only a cursory presentation of some of the typical main results can be provided, without elaborating on the interaction analyses or the theory-guided interpretations on the basis of various concepts of spatial creativity. The episode plans of the video-documented reconstruction attempts of the two pairs *Richard and Joachim* and *Sirkka and Jan-Martin* are shown in Figures 1 and 2 respectively. In each case on the left is the construction presented to the donor, in the centre is the drawing as a purpose and addressee-specific eigen-production of the donor, and on the right is the reconstruction of the acceptor based on the drawing.

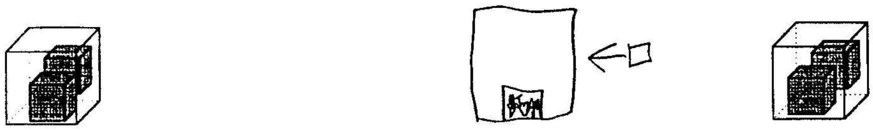
The documents prove that the drawings are for the most part *"reconstruction-effective"* i.e. they enable the respective acceptor to make a corresponding reconstruction of the original. Furthermore it can be noted that the *partners* obviously *learn through comparative feed-back*. The donors organise their drawings according to schematic points of view, which in most cases the acceptor can correctly interpret. The picture sequences during this process indicate two *forms of mutual understanding*, those concerning how the children retain and expand on their own drawing strategies but increasingly learn to understand that of their partner - as in the case of Richard and Joachim - and those where elements of the drawing strategies are adopted by the partner - as in the case of Sirkka and Jan-Martin.

Figure 2: Sirkka and Jan-Martin (3rd grade)
reconstruct dice configurations according to their eigen-produced drawings.
All dice are non-transparent.
Left: structure from the donor's perspective, middle: donor's drawing, right: acceptor's reconstruction.

Episode 1 2.04 – 3.32 *Donor:* Sirkka *Acceptor:* Jan-Martin Successful reconstruction	
Episode 2 3.53 – 5.57 *Donor:* Jan-Martin *Acceptor:* Sirkka Unsuccessful reconstruction	
Episode 3 6.16 – 8.57 *Donor:* Sirkka *Acceptor:* Jan-Martin Unsuccessful reconstruction	
Episode 4 9.10 – 12.12 *Donor:* Jan-Martin *Acceptor:* Sirkka Successful reconstruction	
Episode 5 12.29 – 13.50 *Donor:* Sirkka *Acceptor:* Jan-Martin Successful Reconstruction	

Episode 6 14.12 – 15.15 *Donor:* Jan-Martin *Acceptor:* Sirkka Successful reconstruction	

Episode 7 15.34 – 17.25 *Donor:* Sirkka *Acceptor:* Jan-Martin Unsuccessful Reconstruction	

Episode 8 17.42 – 18.59 *Donor:* Jan-Martin *Acceptor:* Sirkka Successful Reconstruction	

In the opinion of the author, however, the most remarkable finding is the fact that the series of the children’s drawings do not develop in the sense of an increasingly realistic visual presentation, for example in the form of slanting pictures. Rather the contrary is the case. The drawings show an *increasing tendency towards formal abstraction.* Similar to parts of a mathematical formula, the donor only represents those elements which are considered necessary for effective interpretation by the acceptor. The drawings hardly show any redundancies. Richard for example, from the second drawing onwards, only draws the ‘frame’ formed by the front edges and in addition only the rear bottom edge. This he uses in order to designate the position of the smaller die unmistakably. Joachim learns to interpret this way of drawing, although he himself draws differently. Sirkka "invents two signs" as she explains in the following interview - the dark shading to signify "inside" and the arrow for the mental displacement to designate "behind this". Jan-Martin learns, and he adopts these signs with increasing success in his own drawings.

The children’s drawings document *genuine self-organised mathematical learning processes* - the interactive development of an efficient means of communication in a geometric working situation. Such working environments are effective research instruments as well as models for working environments in the classroom. Even if, in many cases, the expressions and procedures conventionally used in mathematics are not found or invented, it can still be assumed that children who have worked on such tasks are made sensitive to respective problem posing and, influenced by the pressure of invention, they also approach the corresponding conventions differently.

Certainly the entire primary mathematics curriculum cannot be organised according to such patterns, but the main idea behind these experiments can act as one of its guiding principles. The teacher should attempt to build on the children’s eigen-productions and allow them to develop in a purpose- and addressee-specific manner in social learning so that the children’s productive original approaches and ideas are not overwhelmed by norms too early. In this context the analyses of eigen-productions carried out by the student teachers involved in this project can serve as effective and also well accepted elements of a teacher education programme as they

- provide *didactical foundations for decision-making instead of didactical patterns for decision-making*,
- give encouragement and self-confidence,
- reinforce design-competence and can in some cases lead to an even more positive re-evaluation of mathematics course contents (see Wittmann 1985).

The author firmly believes that the study of eigen-productions is suitable for forming key focus areas during primary mathematics teacher education programmes or as a supplementary course. Such a *"diagnostic-module"* aims at expanded knowledge for quantitative and qualitative empirical didactical research and eigen-productions. It also addresses learning processes which include the analysis of eigen-productions and the development of diagnostic tools and procedures in order to facilitate individual support and enhancement.

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