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"BUT FIGURES AREN'T SUPPOSED TO HAVE ONE SYSTEM ONLY" - ABOUT THE DEVELOPMENT OF SUBJECTIVE NUMBER CONCEPTIONS OF ELEMENTARY SCHOOL CHILDREN

Abstract

In this paper results are presented of an ongoing four-years longitudinal study of subjective number conceptions of elementary school children. The main interest of this study lies in finding what internal number representation children possess, how they develop during elementary school years and how such subjective number conceptions influence the learning of the pupils in math classes.
To this, all pupils of one class during their 1., 2., 3., and 4. school years were repeatedly interviewed and case studies were carried out about individual learning processes in math classes. First conclusions may be deduced from the investigation results. It seems necessary to consider the respective subjective number conceptions of children when organising learning situations in elementary math classes.

1. Introduction: The starting point for the study

Since 1993 I habe been conducting a project of out-of-school advancement of mathematically interested and gifted (talented) third and forth graders. Week after week I could reckoning how the children improve their insight into demanding problems. To my surprise, I soon recognized that most children were fascinated of numbers in a special way and that they seemd to play with numbers in their own peculiar way. Some children had in fact their favorite numbers. Other children always used individually preferred numbers to solve problems. And there were children who found mystic conceptions hidden behind numbers. This fantasy world of numbers in the children's imagination enhanced my curiosity and I wanted to learn more about it.

2. The relevance of the theme within current mathematical didactics

Numbers are traditionally a central content matter of the first school years - and furthermore of the whole mathematical education. The goals and contents for the teaching of numbers in elementary school habe remained essentially the same for many years. Pupils are supposed to understand the natural numbers as cardinal, as counting and as ordinal numbers. They are expected to grasp also the function of numbers as measuring means and when acting as calculating units and operators. in connection with the four basic arithmetic operations. The use of coded numbers in every day context will also be part of their calculating ability.

During the last years several studies in mathematics education in different European countries have been published which tried to analyze the preliminary knowledge of school beginners and elementary school children. (e.g. Heuvel-Panhuizen 1993, Grassmann 1995 or Hasemann 1998). Those studies were generally set up in such a way that the preschool knowledge of the children have been investigated with respect to different number aspects before they started school and actual teaching of this subject matter. The results show that math educators and teachers so far often wrongly estimated the beginners' level of understanding and that they often underestimated the preliminary knowledge of the children with respect to their number conceptions. These results released a lively discussion about the consequences of the teaching of numbers in math class and an appropriate appreciation of the children's interest.

In what way, however, does the identification of children's preliminary knowledge suffice for the organisation of learning of the number aspects later on in math classes? Don't children have - as was told in the beginning - complex entries to and conceptions of numbers which also could be very different with each individual? And how do the different subjective number conceptions influence the learning of the children in elementary math classes - on the background of an understanding of learning as an individual be formed and actively influenced developing process? (e.g. Glasersfeld 1995, Cobb 1996, Bauer 1988, Bauersfeld 1983)?

For the authorization of the posed questions one finds in recent research literature here are some essential clues: In psychological experiments of various designs it was tried to develop models for the internal representations of numbers and the relationships of numbers (e.g. Mc Closkey 1992, Dehaene, Bossinie et all. 1993, Gray&Tall 1994).

Dehaene (1992), for example, assumes the principle that people conceive numbers mainly as cardinal numbers. Thus they have three representations of numbers:

- the verbal presentation of a number e.g. the numberword (e.g. "three"),
- the digital representation of a number ("3"),
- an analogous representation of its size (e.g. 3 m).

Dehaene and Cohen (1995) also presume that the number line underlies the the semantic coding of numbers. This means that both scientists assume that children conceive a number by hearing a number word, and then they think of the order in which numbers are represented on the number line (e.g. also Restle 1970).

Concerning these studies I must state two aspects of criticism: Firstly, these models of Dehaene concentrate on experiments of number conceptions with adults. In view of the fact that the fantasy of children using numbers are often still open and their experiences with numbers depend on sensual impressions one can assume that children develop very different and not only mathematical criteria of number conceptions. As far as that goes it is doubtful whether the models of internal representation of numbers which refer to the written form and to the linear ordering as in the number line, reflect the multiple associations of children to numbers.

Secondly, these models are to be assessed as too narrow from a mathemacial point of view, since they do not reflect the computing relationships between numbers nor do they consider the special significance of numbers in mathematical structures.
Seron a. A. (1992) describe interesting results to the internal representation of numbers which also were found with adults. They refer in their investigation and experiment with 194 psychology students to actual cases of a visual representation, of numbers which possess to some extent a synesthetical character. Moreover the questioned adults with individual differences assigned distinct colours to numbers (e.g. also Lietzmann 1919 , p.143)

With these results Seron a. A. show a comparitively more differentiated view on possible subjective number conceptions. It is still not solved which specific internal number represenations children possess or develop and in which manner such number conceptions consolidate during elemenatry school years. (The last question is relevant because Seron a. A. found out with the results in their last study that subjectice number conceptions consolidate at the age of 5 to 8 years).

Gray&Tall (1994) eventually performed special studies on a selected aspect of individual number conceptions with children, namely the internal representation of computational relationships of numbers. Their research resulted in a distinct difference with respect of the quality of perception and grasping of content significance of concrete computational relationships like for example, the formal expression "3+2" with elemetary school children. They formulated a hypothesis in which they maintained that successful mathematical thinkers perceive more in such number combinations as other children. Already with the first perception of such number combinations successful thinkers combine consciously or unconsciously several different number relationships which bring them substantial advantages in their further occupation with number combinations (Gray&Tall 1994, p.116).

These investigations show that subjective number conceptions also influence also the quality of mathematical abilities. In which way different internal representations of numbers influence the learning of children in elementary math classes is still not clarified. Lorenz only found in single case research of slow-learning children that for them subjective imaginations of numbers and other mathematical contents, which differ with officially correct interpretation, additionally hamper their learning (e.g. Lorenz 1992).

The opinions and results of the listed studies support my observations made at the beginning: Children are equipped with different approaches and number conceptions which are individually stamped and which influence the way and the quality of their mathematical learning. We still know very little about how this occurs and how such subjective number conceptions develop. These apects are also not articulated in mathematics curricula and - as far as I know - play a minor role in the thinking of theachers.

With a view to this theoretical and practical inadequacy I accounted for my intentions to find out more about the subjective number conceptions of children, about the individual development of such conceptions during elementary school years and eventually their influence on the learning of mathematics.

3. Goals and design of the study

The goals of this study, which has been performed since 1995, consisted in the following points

- to enquire in open problem settings the internal number representations of children during their first to fourth school years,
- to inverstigate their internal stability over the time of 4 school years,
- to examine if the results of Seron a. A. found out with adults were also valid with the children of my research,
- to investigate through description of portotypes if and how such subjective number conceptions influence the children's learning in elemetary math classes.

Design of the study

In order to answer the great variety of internal number representations of children I have chosen an open problem setting. All the investigation aspects of Dehaene&Mehler (1992), of Dehaene &Cohen(1995) and of Seron, Pesenti a. others (1992) and also of Gray&Tall (1994) had been induced and were taken into consideration..

The individual interviews with the children revealed their subjective number conceptions with repsect to

- the ordering and the areal representaions of numbers
- the personification of numbers (existence of favorite and bad-luck numbers), the subjective assignments between numbers and persons or objects
- to relate numbers to colours.

In my first inquiry 27 school beginners (a whole class: 10 girls, 17 boys) were interviewed.
I repeated the questioning twice with the same children within the first school year and once during each of the following 2., 3., and 4. school year. In order to make a comparison of the results possible I designed the individual conversations on principle after the same content and organisational aspects.

By the way of introduction I told every child that our conversation will be dealing with their personal number conception. Then each child was assigned the first task:

> *"Try to imagine numbers, like for instance the numbers 1 to 10! - Can you place the numbers? Where to you thin,k for example, the number 3 must be, where must be the number 1, were must be the 6 or the 10? - Here is an empty sheet of paper. When you know how to place the numbers, and when you think the numbers should be written in a certain order, then you can write this order on the sheet of paper! And don't be shy to write the numbers where you think they ought to have their position in your opinion!"*

After each child drew her personal "numberpicture", I interviewed them for their favorite, their bad-luck-number, e.g. "evil numbers" and their subjective coordination of a number and a person, an object, an event or a colour. In this comparatively open conversation I questioned also reasons for the selection and the assignment of subjective number conceptions. The answers of the children were either documented by a protocol or recorded on tape and transcribed afterwoods.

For the interpretation I used e.g. the group "Interpretative Unterrichtsforschung of the GDM-Tagung 1998 in München. With the analysis of all answers of the children I compared the results of one child within the different interviews and also the results of all childrenn in view of the different content aspects. In order to get results of "subjective conceptions" out of the manifold answers I tied them up in similar answers to the different aspects. Furthermore there were conversations with the chidren's teachers and temporary attendances at math classes to observe how subjectice number conceptions had influenced the learning in those forms.

The investigations could be carried out till 1999 according to plan. In my assessment the children were openhearted to the interviewers and they gave sincere and frank answers.
As 3 children were newcomers in class and four others left us in the meantime it was not possible to question all 27 children, but only 23 children from the first to the fourth grade.

4. Essential findings

The following summary contains the main points of the study in short statements.

1. Outcome:
The questioned children entered school with very different subjective number conceptions. These conceptions developed during the first four school years with individual differences. The number conceptions up to the end of elementary schooling were not restricted to number

aspects taught in math classes, but enclosed also "nonmathematical" number orderings and number contents experienced and internalised in every day contexts and sometimes were of "animistic" and "mystical" quality.
An example for an animistically affected conception:
"*Numbers always belong in groups together. But big numbers are strong, they can stand for themselves. Small numbers are week and need help.*" (Josephine, 1. class).
This and other examples show that Piaget's model of 5 different stages of children's animistic thinking should not be seen as fixed. After Piaget's model only children up to the age of 7 possess animistic number conceptions (e.g. Mähler 1995).

2. Outcome:
Relative to the subjective number conceptions a linear ordering of numbers prevailed among the questioned children in all classes according to the counting order. The total part of this ordering pattern was 58%. Moreover the children wrote also nonlinear number orderings along with the counting sequence (5 per cent), number orderings after a special mathematical structure (11 per cent), after a pattern from the every day context (10 per cent) and after a fantasy pattern (16 per cent).

Here are some examples of comments where children preferred a linear ordering according to the counting sequence:

Nino (6 years, 1.class)
"*This is the way I learned it, thus you can calculate easily. Every day I compute with my dad a little bit and this sequence is helpful.*"

Martin (7 years, 1. class)
"*I have already counted this way as a child. It would not look good otherwise.*"

Felix (7 years, 1. class)
"*Because it's proper and because I have already done it at home like this.*"

Karsten(6 years, 1. class)
"*The numbers belong onto a straight line and they should become bigger and bigger because their value also becomes bigger.*"

Daniel (8 years, 2. class)
"*The numbers belong into a series and not in all directions. But with me number one has to be written very big, because it is the beginning. Zero does not belong to it, it has no value.“*

Phillip (9 years, 4. class)
"*I like this order because it helps me to get along better among figures.*"

Benjamin (10 years, 4. class)
"*I would do it like this, because we have learned it like this. But eleven and twelve do not belong here. With thirteen it is clear. It would be otherwise, when it were called ‚one ten‘ and ‚two ten‘. Then they would belong to it.*"

Christian (10 years, 4. class)
"*I have written the numbers because we have learned it like this. So it is a law. I don't understand it properly, but it is like this. For example - when one puts zero behind it, it is called 10. Or 2 - when one puts zero behind it, it is called 20. I don't understand it. And the way it is spoken I really don't get.*"

The comments of the children confirm the great importance of counting and the counting sequence when children's learn the number concept (e.g. learning the meaning of more figures). This is in accordance with the studies of Hasemann (1998) according to the early number competence of preschool children and beginners. The comments of Benjamin and Christian also show that children critically question the conventions of the numbers. Or they don't realize which problems occur when learning the complex system of natural numbers.
That the childrens' ordering of the numbers according to the counting sequence could be different throughout the first four years is confirmed by the figure order of Johannes. At the beginning of the first school year the boy arranged the numbers like this:

His comment:

> *"In the news papers I see this often. I am used to it. This is right. Sometimes I have problems in school because I write it the opposite way. Sometimes I am confused, but I can pay attention. I don't like math, nor German. But you have to do it."*

The direction of writing and reading of Johannes is typical for lefthanders. Therefore I questioned, if Jonhannes was a lefthander and he answered: "*No, I am not a lefthander, but every now and then I write with my left hand"*. Further inquiries revealed that this boy was a retained righthander. His parents pressed him before school began to do all important actions mostly with his rigth hand. The boy strictly followed this instruction so that his lefthandedness did not show to the teacher. Only through questioning this problem became disclosed. Therefore a possible explanation could be given for the slowness of the boy's thinking who otherwise was an efficient learner.

During the following questionings Johannes again and again changed the arrangemnt of the numbers in their counting sequence from left to right and from top to bottom.
In his second school year he commented his number order "from left to right" with the following words:

> *" I have written like this, this is the way the numbers come. There is really my most important number because I have once been the third one after the winner. When calculating thisis important, then I think of my ordering."*

During the 3.rd and 4.th school years he gave the following comments:

> *"I have writtten the number one beneath the other, because to put them next to each other seemed boring. In supermarkets, on price tags one finds them often underneath each other."*
> *"The numbers are ordered better when one is beneath the other. We write side by side. So this is a change. And I find it really good. The order how you count them is important."*

The comments shows that Johannes has found a "neutral" solution for his problems: the vertical ordering of the numbers (the one that is recommended by A. a. Ellrott (1995) for lefthanders). Furthermore his words express a certain criticism using only one single number arrangement in class *("side by side each other is boring").* After he - like most children - estimated the "standardised" that is the conventional arrangement of the numbers on the number line and on the one-hundred field as sensible and helpful to learning, he considered to these

systems during the third and fourth class as more and more "rigid" and "boring" (cf. also the following comments). The example of Johannes shows that the analysis of a subjective number conception is important for diagnostic purposes. (You can find more examples on how subjective number conceptions influence the learning in math classes in: Käpnick 1998).

3. Outcome:
Half the children ordered the numbers identically after the same principle throughout the first four classes. Hereby the ordering according to the counting sequence dominated. But 50% of the children changed their arrangement several times. The comment of these children show that many elementary school children battle with themselves in an "inner" conflict between the official right number order prevaling in math class and a different subjectively favoured number arrangement (of which the teacher normally had no knowledge). Striking is that the children did not want to order the numbers only according to their mathematical structure. They wanted various arrangements.

Examples of such comments:

Phillip (8 years, 3. class)
"*I find the order 1,2,3,4,...good. This is normal.*"

Toni (9 years, 3. class)
"*1,2,3,4,...is good and bad. It is boring, but, sometimes makes fun.*"

Christina (10 years, 4. class)
"*When you write a sequence, that is 1,2,3,4 and so on, then this looks always uniform. I find it better, when the numbers have different positions.*"

Luisa (10 years, 4. class)
"*I would not write the numbers like this, as the teacher does it, because this looks like an order: Always 1,2,3,4 and 5- this is boring. I would like the numbers jumbled up. When they stand behind each other they look like soldiers.*"

Andreas (10 years, 4. class)
"*I find it simply better like this.* (Andreas points to his fancy arrangement of numbers). *The numbers should not have a system at all. when you claculate there are systems. I like to compute and there I appreciate systems. But figures aren't suppose to have one system only.*"

Michael (9 years, 3. class)
"*I find when numbers stand disorderly then they look more varied. It is a fancy arrangement. I believe that each child has a different fancy arrangement. And I find this good. That we have only one order in school I find boring. Math belongs to my favorite lessons. When I add figures together I do it after my own fancy and like best to add. My teacher does not know this. I prefer to calculate in all directions and solve puzzle problems.*"

4. Outcome:
The varying subjectice number conceptions of elementary school children are apparently based on different everyday experiences and upon the richness of the children's imagination. This becomes apparent with the various criteria for subjective number conceptions of the children.

With nonlinear number arrangements I found for example circle forms, zickzack forms and arrangements like waves which apparently reflect the children's specific feeling of rhythm when counting.

Examples of the children's number arrangements

The number orders of the children which originated in everyday experiences are the following
- Number orders on telephones and automatic machines,
- Number patterns pereatedly seen on television,
- Number patterns on clothings or on posters,
- Point images on dice,
- Number arrangements of the school marks.

With the fancy arrangements the children ordered the numbers according to subjective signifiance, for example their favorite number was placed big in the middle and the zero as "a worthless number" put into the corner.

As a special mathematical criterium for subjective number orderings I found a classification after even and odd numbers (8 times), a pair arrangement of 2 succeding numbers (2 times), the emphasis of the tenth-numbers as the subjectively most important numbers (6 times), the numbers located as in of the well known 100-field (1 time), special addition patterns (2 times) and special the multiplication structure patterns (1 time).

5. Outcome:
During the whole elementary schooling almost all children had their favorite and their evil numbers. Many childen connected the special persons, objects, actions and colours with numbers. These conceptions were often very unstable.

Of the 27 children that were questioned more than once the answers were as follows
- 8 children (30%) the same favorite number according to their inclination,
- 4 children (15%) the same evil number according to their inclination,
- no child the same arrangement between numbers and an object, a person or an incident.

The relationships of content between the number on one side and the object on the other, a person or an incident were very different. The refer to
- the age or the birthday of a child or another person,
- a special event, like Christmas,
- an address (house number),
- special objects, which carry numbers (dice, a case...),
- the number of a football idol,
- a special animal or plant of which the children possess associations in form of numbers,
- a fancy figure which was connected with mystic aspects.

Remarkable is also that in 45% of the 170 interviews the children associated colours with the numbers. Of the 27 more than once questioned children once 4 children through the first to the fourth class repeatedly assigned the same colour. Dahaene also referring to this phenomenon said that 5 to 10 per cent of all people are convinced that "numbers have colours and oc-

cupy a very clearly outlined area" (Dehaene 1999, p. 100). Dehaene assumes that such subjective emotionally influenced number conceptions "have something to do with the brain development of maps, as it were, of area and number, and likewise colour, forms and places of objects are encoded in cortical maps which overlap the map of the growing messages of the numerical system (Dehaene 1999, p. 102). For such an assumption there habe been several empirical findings in the meantime.

6. Outcome:
Subjective number conception of elementary school children influence the motivation of many children when learning in math class. The use of the favorite number can enhance the child's pleasure and interest while writing figures in the first class . Or it can improve doing sums in all school years. Calculating with a bad luck number can hamper motivation.
This assessment was confirmed while I attended math classes and also during my interviews of individual children. At the same time more than half of the children found themselves influenced of their favorite number or their bad luck number, but not in such a serious manner that the liking of special numbers was related to their computing abilities in generell.

7. Outcome:
There are no peculiaritieses depending on the sex of the children and there is only a partial relationship between subjective number conceptions and the mathematical efficiency of the children. The investigations show that achieving children preferred mathematical non-standard number orders in their associations (which does not meet 100% of the results of Gray and Tall).

There are clear clues for the different quality in the individual treatment of subjective number conceptions. For slow-learning children subjective conceptions which differ from the usual standard in math classes lead to additional difficulty when studying mathematics. On the other hand bright elementary school children can easily discriminate between their individual and subjective number conceptions and the general knoweledge of correctness in class. They like to "play" with their individual "number patterns", use them frequently for advantageous computing or those idiosyncratic approaches trigger original problem solving ideas (a function of the subjective number conception stimulates creativity).

5. Conclusions for the organisation of teaching-learning-processes in elementary math classes

The present investigations emphasis that diverse subjective number conceptions are an important starting point and an immediate factor of math learning processes with elementary school children. Therefore the teachers should give the pupils the possibility to grapple with different subjective number conceptions instead of restricting the children to the step by step progress or leaving the children alone with their different subjective number conceptions.
Subjective number conceptions could even sensibly be used for triggering and organising learning processes in math class. In my opinion, the subjective knowledge and significance of numbers should be talked about - with the intention that children recognize their own subjectice number conceptions and come to an understanding of them. With the result that they also learn to discriminate between mathematical and other views. The children could, for example, do a project on the theme "My favorite number" during their first class (Käpnick 1997).

Examples of the children's own works on the subject of "my favorite number"

To counteract the tendency that children stick to the norm of standard orders and thus experience mathematics as "one - sided", "rigid", "boring" , or "foreign", they should generally be offered the opportunity of playtime for their development of their (still) undefined fancy. One could give the children, for example, blank paper sheets with 100-squares and ask them to write the numbers according to their options of ordering. Then all children should work together and understand peculiarities of individual number orders and the special thinking of the single children. This would be a good opportunity in math class to experience mathematics as a structured science and also as a "free play of thought" for the children.
With the organisation of such a learning sequence I could always notice the great activity of the children, their interest and their well defined fancy in putting together number arrangements and in discovering, formulating and imagining relationships of numbers. The free and playful treatment of numbers and of number arrangements corresponded with the open, spontaneous and creative behavior of elementary school children and it helped to correct their often one-sided picture of mathematics.

Examples of the children's subjective number conceptions on the 100-field

References

BAUER, L. A. (1988): Mathematik und Subjekt. - Wiesbaden: Dt. Universitätsverlag

BAUERSFELD, H. (1983): Subjektive Erfahrungsbereiche als Grundlage einer Interaktionstheorie des Mathematiklernens und -lehrens. - In: Bauersfeld, H. u.a.: Lernen und Lehren von Mathematik. -Reihe: Untersuchungen zum Mathematikunterricht, Bd. 6, IDM. - Köln, S. 1-56

BUSSMANN, H. (1997): Mathematiklernen - lehrbar? : Die mathematiklernenende Person im Zyklus ihrer Lebensumwelt. - Frankfurt a. M.; Berlin; Bern; New York; Paris; Wien: Lang

COPP, P. (1996): Theories of Mathematical Learning and Constructivism: A personl View. - In: Trends und Perspektiven: Beiträge zum 7. Internationalen Symposium zur „Didaktik der Mathematik“ in Klagenfurt vom 26.-30.9.1994/ G. Kadunz ... (Hrsg.). - Wien: Hölder-Pichler-Tempsky. - p. 61-84

DEHAENE, S.; MEHLER, J. (1992): „Cross-linguistic regularities in the frequenc of number words.“. - In: Cognition 43. - p. 1-29

DEHAENE, S.; BOSSINI, S. ET AL. (1993): „The mental representation of parity and number magnitude.“ - In: Journal of Experimental Psychology: General 122. - p. 371-396

DEHAENE, S.; COHEN, L. (1995): Towards an Anatomical and Functional Model of Number Processing. - In: Mathematical Cognition, Vol. 1

DEHAENE, S. (1999): Der Zahlensinn. – Basel; Boston; Berlin: Birkhäuser

ELLROTT, D. (1995): Förderdiagnostik Mathematik Primarstufe. – Offenburg: Mildenberger

GRASSMANN, M.; KLUNTER, M.; MIRWALD, E.; VEITH, U. (1995): Was können Schulanfänger bereits vor ihrer ersten Mathematikstunde? – In: Grundschulunterricht. – Berlin 42/6. – S. 23-49

GRAY, E. M.; TALL, D. O. (1994): Duality, Ambiguity and Flexibility: A „Proceptual“ View of Simple Arithmetic. - In: Journal for Research in Mathematics Education, Volume 25, Number 2. - p. 116-140

HASEMANN, K. (1998): Die frühe mathematische Kompetenz von Kindergartenkindern und Schulanfängern – Ergebnisse einer empirischen Studie. - In: Beiträge zum Mathematikunterricht 1998. – Hildesheim: Franzbecker. – S. 263-266

HEUVEL-PANHUIZEN, M. VAN DEN; New Chances for Paper – and – Pencil – Tests; Paper presented at CIAEM. – Cagliari, Italy, 5. – 10.Juli 1993

KÄPNICK, FRIEDHELM (1997): „Meine Lieblingszahl“ - Anregungen zu einem Projekt über subjektive Zahlauffassungen von Grundschulkindern. - In: Grundschulunterricht. - Berlin 44/1. - S. 22-24

KÄPNICK, FRIEDHELM (1998): On the subjective conception of numbers in elementary school children. – In: Selected Papers from the Annual Conference of Didactics of Mathematics 1996. – Edited by E. Cohors-Fresenborg, H. Maier, K. Reiss, G. Toerner, H-G. Weigand. – Osnabrueck. – p. 51-62

LIETZMANN, W. (1919): Methodik des mathematischen Unterrichts (1. Teil). – Leipzig: Verl. von Quelle und Meyer. – S. 142-143

LORENZ, J. H. (1992): Anschauung und Veranschaulichungsmittel im Mathematikunterricht. - Göttingen, Toronto, Zürich: Hogrefe

LORENZ, J. H. (1998): Das arithmetische Denken von Grundschulkindern. – In: Das besondere Kind im Mathematikunterricht der Grundschule (Hrsg. von A. Peter-Koop). – Offenburg: Mildenberger. – S. 59-80

MC CLOSKEY, M. (1992): Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia. - In: Cognition 44. - p. 107-157

MÄHLER, C. (1995): Weiß die Sonne, daß sie scheint? Eine experimentelle Studie zur Deutung animistischen Denkens bei Kindern. – Münster, New York: Waxmann

RESTLE, F. (1970): Speed of adding and comparing numbers. - In: Journal of Experimental Psychology, N. 91. - p. 191-205

SCHÜTTE, S. (1994): Mathematiklernen in Sinnzusammenhängen. – Stuttgart, Düsseldorf, Berlin, Leipzig: Klett

SCHÜTTE, S. (1995): Subjektive Zugänge zu Zahlen und individuelle Rechenstrategien am Schulanfang. – In: Beiträge zum Mathematikunterricht 1995. – Hildesheim: Franzbecker. – S. 424-427

SERON, X.; PESENTI, M.; NOEL, M.-P.; DELOCHE, G.; CORNET, J.-A. (1992): Images of numbers or „When 98 is upper left and 6 sky blue“. - In: Cognition 44. - S. 159-196

Sfard, A.: The Development of the Concept of Concept Development: From God’s Eye View to what can be seen with Mind’s Eye. - In: Trends und Perspektiven: Beiträge zum 7. Internationalen Symposium zur „Didaktik der Mathematik“ in Klagenfurt vom 26.-30.9.1994/ G. Kadunz ... (Hrsg.). - Wien: Hölder-Pichler-Tempsky. - p. 327-352

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