Hermann Maier, Regensburg

HOW MUCH TECHNICAL LANGUAGE DO PUPILS NEED IN MATHEMATICS EDUCATION?

Abstract:
In order to answer the question of the title it seems necessary to describe the character and the function of technical language. Subsequently it will be presented as a fundamental claim that pupils should not just use but really understand technical expressions, symbols, and the logic of propositions. Research reports about pupils' difficulties with this understanding contribute another aspect to answering the question. A balance between „**hyper**trophied“ (in this sense implying too many meanings) and „**hypo**trophied“ (in this sense implying an inadequate number of meanings) use of technical language elements in mathematics classrooms has to be found. This paper tries to contribute some ideas as to how this could be realised.

Technical language and everyday language should not be regarded in a dichotomous way as two different languages; every technical language is based on everyday language. What we really meet in mathematical classroom talk or in a mathematical paper is a more or less modified everyday language. The title question can, therefore, be formulated more precisely as follows: to what extent should the language by which the teacher addresses pupils and the language used by media be modified, and to what extent should pupils use elements of technical language when they talk or write about mathematics in the classroom?

1. About the character and the function of technical language

An adequate answer to the question of the title demands previous reflection on the character of technical language in the framework of classroom communication, and its function in the learning of mathematics. From that we gain our first criterion for a reasonable decision.

1.1 About the characteristics of technical language

As modifications of everyday language which are characteristic of technical language for mathematics the following may be considered:

- the use of specific expressions which do not appear in everyday language, and the use of words from everyday language in a more or less changed meaning;
- a specific logical structure of sentences;
- a particular form of defining concepts, representing sentences of general meaning (laws, theorems) and algorithms, and logical-deductive ways of argumentation (proofs).

In written technical language further modifications appear. The most important are:

- the use of symbols with constant meaning in order to abbreviate terms for objects and sets, for numerical, geometrical and logical relations or operations;
- the use of variables, i. e. of symbols rendering it possible to represent sets of objects and general expressions or statements.

1.2 About the function of technical language elements

What can be said about the function and the meaning of technical language elements for mathematical activities and the learning of mathematics?

- Firstly: Using technical expressions and symbols simplifies language performance, since they are able to bunch and condense information. An instruction like „*Connect a circle and a point P outside the circle by a line which has exactly one point in common with the circle*“ can be simplified by help of the word „tangent“ to „*Draw a tangent from P to the cir-*

cle." The formula „*The area of a triangle can be calculated by multiplying the length of one side of the triangle with the distance of the third vertex from this side and dividing the product by two*" can be abbreviated by „*The triangle area equals the half product of one side length and the length of its height* " or, by use of symbols, by „$A_\Delta = 1/2\ a \cdot h_a$".

- Secondly: Using mathematical symbols renders possible to operate without a conscious mental image of the information they contain. When someone wants to calculate the area of a triangle with a = 5 cm and h_a = 3.5 cm the activity can be restricted to replacing the variables a and h_a in the formula by 5 cm and 3.5 cm respectively, and carrying out the multiplication. In the same way it is possible to find the derivative $f'(x) = 6x^2 - 10x + 6$ of the function $f(x) = 2x^3 - 5x^2 + 6x - 4$ by the help of syntactic transformation rules alone. Besides their character of referring to objects, symbols possess a signal character, i. e. they demand acts. In other words, elements of technical language allow us to formalise mathematical operations and processes by using them according to certain rules linked to themselves and not necessarily to their meaning.
- Thirdly: The common characteristic of mathematical texts, to exactly define terms and symbols when they appear for the first time, is able to create a terminology whose meaning is clearly defined for every reader. Well-defined terms can contribute to the clear meaning of texts and they improve the probability that their readers would understand the meaning which had been intended by the author.
- Finally, elements of technical language can provide a rational order and structure of mathematical knowledge in which logical networks and conceptual hierarchies appear clearly. Statements are well ordered according to their character as definitions, axioms (propositions accepted as true) and derived propositions which are brought in a clear logical order of presupposition and conclusion.

1.3 The preference of understanding

Under which conditions can these functions really become effective for the learning of mathematics, the solving of mathematical problems, and the application of mathematics in real situations? The negative version of my answer is: -

These functions can become rather ineffective when unprepared pupils are confronted with technical terms and symbols, formulations, definitions and proofs, or if they are expected to make use of them as some norm they have to obey blindly.

In such cases there is a real danger that technical language makes understanding of mathematical concepts, statements and their justification, algorithms and procedures more difficult or even impossible for the pupils.

To say it positively:

If using elements of technical language is to become really effective with respect to the functions mentioned above and also for the learning of mathematics, it seems necessary that the pupils become able to link technical language with interpretations and meanings which are in sufficient congruence with the technical criteria listed above and with the teacher's context-specific intentions.

Someone who does not know what the concept „tangent" really means will not be able to follow the instruction „*Draw a tangent to the circle*", and someone who cannot link meaning to the words „area", „side" and „height", or to the symbols A_Δ, *g* and *h,* will not be able to use a formula for calculating areas in a sensible way.

Thus technical terms and symbols have to be used with some caution by both the teacher and the media used in the classroom until the pupils are sufficiently familiar with their meaning.

(This does not necessarily mean that they can only be introduced at the end of each learning sequence.) In addition, of course, the pupils ought not to be forced to apply technical terms and symbols before they have an adequate understanding of their meanings. Formulated as a rule it could be said that the understanding of technical language has absolute preference to its use merely as a norm to be followed.

2. About the understanding of technical language by pupils

In order to estimate what it means to learn elements of technical language in mathematics classroom under the premise of understanding, difficulties some pupils usually have with these elements must be discussed. Some of these difficulties are already well known from empirical research.

2.1 About the understanding of technical terms

There are only a few investigations into pupils' ability to understand technical terms and symbols according to their intended meaning, i. e. their passive competence in technical language. PATRONIS & SPANOS & GAGATSIS (1994) found that pupils, when reading geometrical problem texts, often attribute meanings to terms different from those intended by the author. They talk about semantic differences, when pupils, for example, interpret the word tangent-segment differently and use A and B twice to represent two different points.

SCHMIDT (1982) administered a test on reflections to 243 pupils of grade 5 (11 years old). 90% of the pupils tested attributed the word „perpendicular" only to lines which were parallel to the right and left margins. Only 65% of the pupils knew the concept of „right angle"; only 21% were able to determine the distance of a point to a straight line correctly (17% measured it by help of a line not perpendicular to the straight line). Research of VIET (1978), VOLLRATH (1978) and FRAUNHOLZ & MAIER &TROMMDORFF (1986) arrived at similar results.

We have some more knowledge about pupils' performance in technical language and their ability to make adequate use of terms and symbols in their own language i. e. about their active competence in technical language. MAIER asked pupils of grade 7 and 8 to describe complex geometrical figures in a text that could enable a classmate to reproduce the figure correctly (see MAIER & SCHWEIGER 1999). Some of the findings are:

- Only a few pupils applied technical terms which should be familiar to them from previous schooling. Instead they often used words from everyday language that made an exact and clear description more difficult.
- Many pupils had particular difficulties with describing the mutual position of points, segments and figures in an adequate way. Since they avoided the use of angles they did not arrive at a definite description of relations relative to the margins of their notebook.

GUILLERAULT & LABORDE (1981) and GALLO (1985) also analysed pupils' descriptions of geometrical figures and got similar results. GALLO realised in particular that pupils linked the images of standard models to the technical words for geometrical figures. If the sides of a square were not parallel to the margins this figure was represented as a diamond. If one of the legs of an isosceles right triangle was drawn horizontally many pupils thought that the triangle was rectangular. Only when the hypotenuse was drawn horizontally did they realise that the triangle was isosceles. JANVIER (1987) observed a remarkable lack of language competence when he let pupils interpret Cartesian graphs in higher grades of secondary school. Most pupils were not able to find adequate words when talking about intervals, maxima and minima, continuity and slope.

Evidently pupils are not able and not prepared to integrate the large number of technical terms they are offered by the teacher and the language of textbooks into their active language competence. With respect to their passive competence and performance it can be said that their image of the meaning of technical terms is often linked to visual impressions. They have only empirical, not theoretical concepts which are too narrow, not general enough, and often quite unclear. In some cases concepts of meaning are inadequate or missing completely.

2.2 About the understanding of technical symbols

An extended use of technical symbols can become, for many pupils, an impediment to understanding. Even in the process of language perception they run into difficulties with the task of decoding and unfolding the condensed content of meaning. But using symbols in their own text production may even be more difficult for them. In MAIER's investigation mentioned above (see MAIER & SCHWEIGER 1999) most pupils' descriptions contained no symbols at all. The pupils had no idea how to label points and segments by characters; they did not use signs like $\|$ or $\perp$ as abbreviations for parallel or perpendicular. Only a few pupils wrote the symbols l for length, h for height, r for radius and d for diameter. Another investigation by MAIER about the relationship of concept understanding and errors in the calculation of area in geometry (see again MAIER & SCHWEIGER 1999) revealed pupils' difficulties with technical symbolism as well. Asked to express the circumference and the area of a rectangle with s and t as the lengths of the sides, many of them wrote the common $a \cdot b$ instead of $s \cdot t$. Further typical wrong answers were $s \cdot s + t \cdot t$ or $s + s \cdot t + t$. ARZARELLO (1998) found that pupils see algebraic expressions as a formal instrument. They do not link semantic ideas to variable expressions and do not use them in specific situations as a method of proof and generalisation. They run into big problems when asked to make use of symbolism in order to express general solutions and proofs. MALLE (1993) and TIETZE (1988) report on difficulties found by university students with the interpretation of quite simple algebraic terms or equations.

2.3 About the understanding of sentences and texts in technical language

Pupils' difficulties with technical language are not restricted to the understanding of technical terms and symbols. Such difficulties recur when pupils deal with the grammar and logic of mathematical discussions and texts. Again they can be confused by the difference to the language customs of their everyday experience. Some of the problems are as follows:

- In everyday language the indefinite article „a“ and the number word „one“ are not clearly distinguished. The existential quantifier is normally interpreted in a meaning which is represented in technical language by the expression „one and only one“ or „exactly one“. A sentence like „There are two points on a straight line“ may confuse pupils, for whom it excludes the existence of more than two points.
- The different use of logical operators in technical and everyday language is well known. The usual meaning of the conjunctive operator „and“ does not correspond with its use in technical language unless complete sentences are linked. In the case of the disjunctive operator „or“ (in its non-excluding meaning) pupils have problems in distinguishing it from the excluding disjunction, which in technical language is only expressed by „either – or“.
- It seems to be difficult for many pupils to escape the usual identification of implication and equivalence in everyday language, both attributed to the pattern „if...then“. For that reason they often cannot understand why sophisticated formulations like „only when...then“ are necessary.

2.4 About the reasons for difficulties

Thinking about reasons for the difficulties mentioned above we can follow two kinds of hypotheses. The first refers to the frequently discussed problem that mathematics education is concentrated too much on algorithmic training and too little on understanding. It fails to pay enough attention to conceptual knowledge i.e. it makes too little effort to build up images of meaning linked to technical terms and symbols. Teachers do not pay the necessary attention to frequently appearing expressions. They do not ask about pupils' misconceptions, or else they suppose that frequent use alone should make the meaning clear for them.

In addition there is not enough care taken about the problem of interference of meaning with everyday language, not only in the case of phrases but also in reference to a lot of technical and didactical terms. In classroom language there are, indeed, only a few technical terms that do not appear in everyday language as well. Most even belong to its basic vocabulary, e.g. „circle“, „triangle“, „height“ and the number words. Many mathematical terms have found their way into everyday language, e.g. „tangent“, „parallel“, „perpendicular“, „equation“, „ellipse“, „cone“, and expressions for measure units like m, cm^2, dm^3, etc. On the other hand, some expressions of everyday language were introduced into technical language, e.g. „plane“, „net“, „similar“, „group“, „ring“**,** „field“, etc. However, only a few words of everyday language are used there with exactly the same meaning. In some cases the technical meaning is narrower – one can think of words like „similar“ or „circumference“; in other cases it is broader, e.g. for words like „quadrangle“, „square“, „perpendicular“. Sometimes it also follows another kind of system – one can think of words like „height“ or „length“. Finally, the meaning can be completely different from everyday meaning, which may be true for words like „figure“, „vertex“, „absolute value“, „group“, „ring“, „field“**.** In addition the interpretation of composite words according to rules in everyday language can be misleading e.g. the contrary of „common fractions“ is not „uncommon fractions“ but „decimal numbers“.

In all these cases the pupils and the teacher as well may lack knowledge of the fact that something has yet to be learnt in the mathematics classroom. In fact the pre-knowledge from everyday language is often more a barrier than a help for the understanding of technical expressions, since it can interfere unfavourably with the technical meaning and make its understanding more difficult. However it can also cover or dispel already achieved understanding. (About the problem of interference see MAIER & SCHWEIGER 1999.)

3. Consequences of using technical language in the classroom

Which consequences can be drawn from the reflections and analyses in the previous sections on using technical language in the mathematics classroom? With how many elements of technical language can the pupils be confronted and how much technical language should be obligatory for them? My discussion of this question shall cover three topics, namely the use of technical terms, the use of technical symbols, and the textual form of definitions and proofs.

3.1 About the use of technical terms

Enabling pupils to attribute adequate or intended meaning to terms (and symbols) used by the teacher or by classroom media, moreover their use in a sensible manner, must be seen as a demanding and time consuming task. The greater the number of these terms (and symbols) the less time remains available for building up the required understanding of meaning for each of them. If the number is large, there is a big danger that efforts for making the pupils understand this meaning are curtailed, and that the pupils may well attribute a meaning to expressions which is too narrow, too broad or even erroneous. In addition, they will easily mix up the meaning of different expressions. For that reason it would be expected that the teacher restricts the unavoidable introduction of technical terms and symbols to those

(1) which seem indispensable as an effective simplification of classroom language;
(2) which are wide ranging, i. e. can be used frequently or even permanently;
(3) which are not just negligible synonyms of terms and symbols already introduced.

These three criteria should coincide and they should be taken into consideration for the introduction of new notions and symbols. On the other hand the teacher must not avoid or unnecessarily delay the introduction of ideas which meet these criteria.

Investigations of mathematical textbooks for secondary school show that the number of new technical terms the pupils are expected to understand and to use comes up to in between 120 and 200 per school year. Many of them appear on not more than one page or in one school year and never again. Such a great number of new terms can give pupils the impression of learning a difficult foreign language (see LÖRCHER 1976 and MAIER & SCHWEIGER 1999).

Where can possibilities be found to avoid hypertrophy and to reduce the amount of necessary technical vocabulary to a reasonable level? What can be done by the teacher and the textbook author?

a) Refrain from complete classifications

In mathematical classrooms and textbooks it sometimes can be seen that mathematical objects and procedures are classified in quite a differentiated way, and that all objects are labelled with a particular term. (This observation is at least true for Germany.) Angles are classified not only as acute, right and obtuse, but also in terms which literally may be translated as „straight angles“, „reflex angles“, „zero angles“, and „revolutions“. In addition pupils learn names for different angles at two parallel lines; they hear about „vertical“ or „opposite“**,** „interior“ and „exterior“ angles (of a triangle). A German textbook for grade 7 classifies straight lines in relation to a circle as „secants“, „tangents“, „central lines“ and „passing lines“. It describes propositional functions (open sentences) as „partly true“ (in a set), „generally true“ and „unattainable“. I doubt whether all these expressions meet the criteria mentioned above.

b) Admit polysemiotics

A motivation which often leads to a remarkable increase in technical vocabulary in the mathematics classroom is the attempt to avoid polysemiotics (i.e. multiple relationships between words and the concepts to which they refer). In Germany teachers may ask pupils to distinguish between „Kreislinie“ (circle as a line = the circumference) and „Kreisfläche„ (circle as a plane figure), „Winkel“ (= angle) and „Winkelmass“ (= angular measure), „Höhenlinie“ (lit. height line as a line segment) and „Höhe“ (= height as a length), shape and area, etc. The chord theorem is formulated in a textbook for grade 9 in a way which can be roughly translated as follows: *The product of length numbers of the parts of one chord equals the product of length numbers of the parts of the other chord*. LÖRCHER (1976) found distinctions in primary school textbooks between number and number name, number word and number sign, and he complains about a „unreasonable“ precision of mathematical terminology.

An unambiguous use of terms is not only unavoidable in many cases but, with regard to language economy, often not desirable as well. The words „circle“, „triangle“ and „square“ can be used to label a line or a sequence of straight line segments on the one hand and a shape on the other. The words „radius“, „diameter“, „height“ and „surface“ (and the relative abbreviations r, d, h, S as well) are able to represent a chord of the circle or a straight line segment, but at the same time the length of the respective line.

A rigorous use of distinctive vocabulary in order to avoid polysemiotics leads in most cases to a more clumsy language. Pupils are familiar with polysemiotics from everyday language, and they should be able to deal with the fact that technical terms also can have different meanings in different contexts. They should be made aware of polysemiotics in technical language rather than avoiding it altogether. They must become able to distinguish different meanings of terms and even explain them. In this way sometimes different words may be seen as a help but subsequent attempts to arrange and keep to a one-to-one correspondence between technical terms and concepts may, by increase of vocabulary, do more harm than good.

c) Avoid synonyms

A third reason for an unnecessary increase in technical terms can be seen in the practice of labeling the same mathematical object in different contexts with different expressions. Point sets in geometry are sometimes called „geometrical loci“ as well as „locus lines“ and „locus areas“. Variables are, in some contexts, called „empty places“, „placeholders“, „unknowns“ or „parameters“. Alternatively a function can be described by the words „mapping“ or „relation“, etc. It should be questioned whether the introduction of other expressions for an already known concept is really necessary. Quite often the answer can be negative.

d) Refrain from unnecessary theorising

A further motive for an increase in the technical register can be provided by the following examples. In order to introduce the pupils to equations they have to experience the theory of propositions and propositional functions, so that subsequently the equations can be explained as a particular kind of open statement and their solutions as elements of a set which make the statement true. Alternatively the introduction to functions crosses relations of particular properties. While normally the way of introducing concepts goes from more special to more general ones (e. g. from square to rectangle, parallelogram and trapezium to irregular polygons), we meet, in the cases mentioned above, the reverse methodological order i.e. from general to more specific concepts. We can ask if specialising the order of topics in this way does not often lead to an exaggerated theorising of mathematical topics. This enlarges the technical register without real need, and confronts pupils with terms whose meaning is difficult to understand.

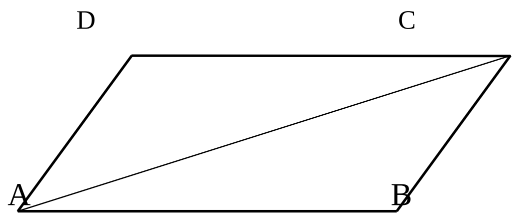
It should be made clear that, besides hypertrophies in the use of technical words discussed so far, we also find in the classroom the contrary phenomenon, namely a lack of usage of technical vocabulary. (Imitating medical doctors we can talk of a „hypotrophy“.) A few examples are:

- In primary school, the technical word plus, minus, equals, add, subtract, multiply, divide, etc. are replaced by ambiguous words of everyday language which may aggravate the problem of interference of meaning.
- In secondary education, expressions like commutative or distributive law are replaced by „exchange-law“ or „distribution law“ respectively. Instead of using x-co-ordinate and y-co-ordinate German textbooks speak about „right value“ and „high value“. In geometry, words like „column“ or „roller“ replace the word „cylinder“. Such kind of so-called „pupil-adequate“ words can seriously hinder the understanding of a general mathematical concept like co-ordinates (think of negative co-ordinates) or cylinder (think of thin disks).
- When pupils transform terms and solve equations in algebra they are often not made familiar with the concept of equivalence, which justifies the transformations carried out. Also,

they often miss the distinction between the equivalence of terms (same value for all replacements) and the equivalence of equations (same solution set for all transformations).

3.2 About the use of symbols, particularly variables

Is it possible to limit the use of symbolism in the mathematics classroom as well? To begin with an example I will confront two versions of a geometrical proof. Given the task to prove that the opposite sides of a parallelogram are of the same length, on the basis of its definition as a quadrangle with parallel sides, I present different versions in adjacent columns:



Assumption: $AB \parallel CD$ and $AD \parallel BC$

Proof:

1. $\angle BAC \cong \angle ACD$ (Z-angles)
2. $\angle ACB \cong \angle CAD$ (Z-angles)

$\Delta ABC \cong \Delta ACD \Rightarrow AB = DC,\ AD = BC$

or

$\Delta ABC \cong \Delta ACD \Rightarrow$

$l(AB) = l(DC) \wedge l(AD) = l(BC)$

Let the parallelogram be defined as a quadrangle with parallel opposite sides. I am going to prove that its opposite sides have the same length.
I join the vertices A and C by the diagonal and get the triangles ABC and ACD. If they were congruent their relative sides would be of the same length, what means that this would also be true for the opposite sides of the parallelogram, namely for AB/DC and AD/BC respectively.
I prove the congruence of the triangles with the help of the equivalence theorem „angle-side-angle“. They have the side AC in common. The angles BAC and ACD are Z-angles on parallel lines (see assumption) and therefore of equal measure. The same is true for the angles ACB and CAD.

The two versions, both correct and complete, differ dramatically in the amount of mathematical symbolism used. Certainly intermediate steps can be found between these two extremes. However is it not a hypertrophied use of technical language elements if pupils are pushed to formulate, as far as possible, geometrical proofs from the beginning with help of symbols, or if the transition from a strictly everyday language version to a strongly formalised version becomes obligatory too early and too quickly? Could the fact that pupils generally often have difficulties with proofs and dislike them not be due to such kinds of hypertrophy? Is there not also a danger that pupils run into difficulties of understanding when they are not able to decode either hypertrophied descriptions of geometrical objects and constructions but also formulations of theorems and problems? Thus when a German textbook for grade 7 describes the Thales circle as follows: $k_T = \{P \mid \angle APB = 90^0 \vee \angle BPA = 90^0\}_{A,B \notin k_T} = \{P \mid |\angle APB| = 90^0$, I dare to ask if the whole analytical or vector geometry – with respect to the age level of the pupils to which it is being introduced – could not be regarded as a dangerous hypertrophy in the use of technical symbolism.

Certainly, hypertrophied use of symbols is found mainly in geometry, but we find it also in arithmetic and algebra. Although these areas seem almost defined by symbolism, the teacher could often find ways of reducing formal representation in signs. This shall be demonstrated by reformulating part of a German textbook into more everyday language. See the textbook version the left and my version on the right side of the following table:

Slope of a linear function: $m=\frac{y_2-y_1}{x_2-x_1}=\frac{\Delta y}{\Delta x}$; $x_1 \neq x_2$ bzw. $\Delta x \neq 0$ Parallel straight lines: For two straight lines g_1 with $y = m_1 x + t_1$ and g_2 with $y = m_2 x + t_2$ is true: $g_1 \parallel g_2 \Leftrightarrow m_1 = m_2$	Be $A(x_1;y_1)$ and $B(x_2;y_2)$ two points on the graph of a linear function, we get the gradient by dividing the difference $y_2 - y_1$ of their y-co-ordinates by the difference $x_2 - x_1$ of their x-co-ordinates. Parallel straight line have equal gradients.

This example may also show that algebraic definitions and axioms can be formulated in a language with should be easier to understand for the pupils. In addition it must be possible to avoid exaggerated complexity when working with terms and equations. The transformation of long and complicated terms and the solution of highly complex equations often require more endurance and care than mathematical knowledge and competence.

Quite often symbols in a mathematical text require virtually the same amount of space as the corresponding words. We can think of symbols for logical quantifiers and logical operators (e. g. $\wedge$ instead of „for all“ or $\Leftrightarrow$ instead of „only if“), although the signal character of such kinds of mathematical symbol can be useful as well. On the other hand there are symbols which provide different densities of information for the same concept and it can be asked if, or when, the pupils should be forced to make use of these more concise forms (e. g. of the sign $\sum$ for sum or $\prod$ for product, the apostrophe for derivation and the stylised S for integration). Mathematicians of former generations have demonstrated impressively how possible it is to refrain from hypertrophied use of mathematical symbolism and, at the same time, define, formulate theorems and arguments or prove in a strict logical manner.

3.3 About formulating definitions, logical reasoning and proving

A typical way of fixing the meaning of expressions in mathematics is to formulate definitions. Therefore pupils should become able to define, in the sense of integrating the concept to be defined in an adequate generic term, and to state all the specifying properties. They should also become conscious of the fact that each mathematical object can be defined in different ways, e. g. the parallelogram

- as a quadrangle the opposite sides of which are parallel,
- as a quadrangle opposite sides of which have the same length,
- as a quadrangle in which opposite angles have the same size,
- as a quadrangle in which the diagonals bisect each other, or
- as a central-symmetrical quadrangle;

but also (by change of the generic term)

- as a trapezium with parallel sides, or

– as a trapezium in which the parallel sides have the same length.

Definitions do not only clarify conceptual hierarchies; they are also the basis for deriving other properties of figures according to technical norms.

In my opinion it means a hypotrophic restriction of geometrical education if it is limited to rational aesthetics or techniques of calculation. At least addition should be seen as an area of practice for defining, formulating theorems, argumentation (i.e. sensible and methodical reasoning) and proving. Activities of that kind should not be restricted to „big events“ such as the Pythagorean theorems or theorems about angles in the circle. They can be cultivated and trained with more hope of success in the case of „smaller events“ as shown in the examples above. Thus they can be introduced by less demanding ways of argumentation, like plausible argumentation, dialogical or pre-formal proving. They should not, however, be made more difficult for the pupils by hypertrophied use of technical terms and symbols. The quality of pupils' arguments must not be decided by relating to technical standards but rather to their communicative function. Arguments have to convince an addressee, and this requires that they are able to understand the language. Defining and logical argumentation should begin as early in the school career as possible, and it must not be limited to geometry education. Arithmetic and algebra too should not be reduced to mere algorithmic techniques and instruments for problem solving. The pupils should learn

– to give reasons for ways and strategies of mental calculation in primary school,
– to define rational numbers (as classes of equivalent fractions), lengths (as classes of congruent segments), areas (as classes of shapes which are congruent or can be transformed into congruent shapes by decomposition or addition of appropriate pieces), etc.

3.4 About the learning of technical language elements

Certainly a direct transfer of mathematical knowledge, and meaning of words, to the pupils by means of language alone is generally not possible. The teacher needs their active participation in constructing their own knowledge and cognition. However this construction may not succeed on the basis of active involvement of the pupils with adequate situations alone, as constructivism suggests. I would reject the claim of Vygotsky that it is not possible or even not allowed to intervene into the mysterious process of concept forming, rather that it has to be left to its own rules. I would suggest as well that verbal explanations of new terms by the teacher can make sense, even if its effect should not be overestimated. I appreciate the interactive proposition that individual pupils interpret not only visual models and related acts, but also verbal discussions in a different way. It therefore demands intensive communication to guide pupils carefully to an intended interpretation, in order to arrive at consent with the teacher on an adequate understanding of meaning.

The development of such an understanding of meaning demands, in my mind, at least the measures and pupils' activities as follows:

- ♦ relating terms and symbols to concrete or iconic models in a reflective way;
- ♦ (previously) describing their meaning in everyday language and extended paraphrases;
- ♦ communicating about understanding of meaning in an intensive way (with the teacher and classmates);
- ♦ contrasting different meanings of terms in everyday language and in mathematical context and making the differences conscious. In this way mathematical meanings should be added to meaning of everyday language without dispelling or even extinguishing them;
- ♦ comparing concepts on the basis of technical terms and symbols with relative concepts and learning to deal with polynomy (i.e. multiple meanings) and synonyms as well;

- ♦ repeatedly challenging the understanding of meaning by spontaneous verbal statements and production of written texts.

It has to be realised that, normally, technical terms and symbols will be integrated into the active language competence and performance of pupils later than into the passive one. For that reason they should be allowed to use everyday language in their own discussions and texts – even after the teacher has introduced related terms and symbols – as long as they do not decide to make use of these elements of mathematical language by themselves. In any case pupils should always be encouraged to speak and to write authentically. They should not feel pressured to make use of language elements they have not yet understood or learnt properly.

In order to find the right balance between hyper- and hypotrophy when using technical language in mathematics education, it may be helpful to quote a linguist who has produced criteria for determining what makes language learning easy and hard (GOODMAN 1986). He says

it is easy when ...	*it is hard when ...*
it is real and natural	it is artificial
it is whole	it is broken into bits and pieces
it is sensible	it is nonsense
it is interesting	it is dull and uninteresting
it belongs to the learner	it belongs to someone else
it is relevant	it is irrelevant to the learner
it is part of a real event	it is out of context
it has a purpose for the learner	it has no discernible purpose
it is accessible to the learner	it is inaccessible to the learner

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Prof. Dr. Hermann Maier
Naturw,Fak.I-Mathem.
Universitätsstr. 31
93040 Regensburg