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On the Disappearance of Mathematics from Societal Perception (*)

Abstract

Starting from the dilemma between the growing societal use of mathematics and the disappearance of mathematics from the attention of the "normal" worker, the paper offers a historical case study of the disappearance of mathematics by analysing the standard procedures of weighing in small and medium businesses. The study shows the growing implementation of mathematics into various workplace tools: The development of artefacts integrates and thus hides mathematical concepts and algorithms from the workers' perception. It is only in non-routine and „breakdown“ situations that mathematics shows up again and may be (re-)invented to cope with the „new“ situation. Consequences for learning mathematics at the workplace and in the classroom, for the use of information technology and for an appropriate research methodology for mathematics in vocational contexts finalise the paper.

Introduction

The starting point of this paper is a dilemma which can be observed when comparing two types of reactions around research on mathematics in workplace contexts and vocational training / education: On the one hand, politicians, managers and business administration people often speak of a growing societal use of mathematics. Part of this rhetoric is the often heard statement that application of mathematics is extended to more and more important fields of human activities. On the other hand, the researcher on mathematics in vocational, workplace contexts is used to face the comment of the "normal" worker saying that s/he is not using any mathematics (maybe apart from elementary arithmetic), or even worse: that mathematics gradually disappears from his/her workplace and/or her/his attention. The paper attempts to better understand at least one aspect of this dilemma.

A Case in Industrial and Social History

In order not to start with too global an approach, the paper first offers a historical case study of the disappearance of mathematics. To do this, it analyses the standard procedures of weighing and pricing in small and medium businesses showing the growing implementation of mathematics into various workplace tools (either material or organisational). It looks into a “standard” situation like weighing for instance some three kilos of potatoes and telling the price of this merchandise.

Weighing 1: traditionally

Once upon a time and nowadays in marketplaces and old fashioned shops, weighing was done with a pair of beam scales (see fig. 1 next page; all illustrations in this paper by courtesy of BIZERBA, one of the biggest producers of balances in Germany) and normed weights. The goods were placed in one, normed weights into the other scale (in complicated cases also into the one containing goods) to have the beam in complete balance. The weight of the goods can be read off the balance adding and/or subtracting the weight in the respective scales. Then the price was calculated separately (mentally / in writing on a sheet of paper) by multiplying the unit price of the merchandise with the weight read from the beam scales. As for the mathematics involved, we see the partitioning of the weight according to the weights available, addition (and subtraction) to calculate the overall weight and a proportional model for the pricing.

fig.1: traditional beam scales

fig.2: analogous weighing & pricing

Weighing 2: analogously

In Germany since 1924, this way of weighing & pricing was replaced by the slow introduction of a different analogous type of balance, in Germany called "Neigungs-schaltwaage" (also: "Fächerkopfwaage", a name related to the form of the balance; see figure 2 above). This type of balance still uses a simple proportional model, the partitioning and adding of weights was done „automatically“ by the balance because the hand of the balance would move to the right proportionally to the weight of the merchandise – indicating the total weight on top of the scale. In addition to that, the pricing by multiplication was taken over by this new artefact: the price could be read off the scale at the correct place of the hand indicated by the unit price. The correct reading of prices and eventually adding the price of whole units of weight were the essential competence the seller should have. Mathematical „interpolation“ was necessary in case of very large, very small or odd prices not on the hand and scale of the balance. Following information from BIZERBA, around 70% of the prices read off these balances were incorrect. Nevertheless, this type of artefact was widely used till the 80ies in Germany at least.

Weighing 3: digitally

Nowadays, especially in larger shops and supermarkets, you would find digital balances (see fig. 3 next page) which offer directly prices for the merchandise put onto them with printouts of prices to be pasted to the goods – if the shop ever sells goods which are not pre- packed. The proportional model is still in use but hidden from the perception of the buyer. Reading weights and prices has become easy, while interpolation of large and small prices is unnecessary, „odd“ prices (like the „famous“ 3.99 $ or DM) have only come into use with these balances or pre-packed merchandise.

fig. 3: weighing & pricing digitally

What is left to be done to the seller / the buyer him/herself is keying in either the unit price or an identification number or symbol for the goods to be purchased. Normally, addition of several goods and identification of the individual seller is done automatically. Mathematics travels up the professional hierarchy to managers who decide on (quantity) discounts and special offers while (programmers of) computerised systems for checking the flow of goods in a company are responsible for a constant and realistic flow of information on the cash balance and economic success of the company.

Disappearance and (Re-)Discovery of Mathematics

Looking back to the case described above, the disappearance of mathematics from societal perception – at least from the perception of the actual buyers and sellers – can be illustrated: In fact, mathematics (addition/subtraction of weights, calculation and addition of prices) is progressively turned into algorithms, automated and integrated in machines (into „artefacts“, see below) and thus hidden from the notice of those involved in the activity. If the job runs smoothly and routinely without unfamiliar and unforeseen events (the worst case would be the breakdown of the supply of electricity), practitioners tend to rely on well-known routines for repetitive problems. These routines are often implemented in tools (like machines for calculating, scales to read, charts to fill etc., i.e. "primary" artefacts sensu Wartofsky 1979, p. 201ff). Difficulties when using mathematics tend to be simplified, if not totally avoided by algorithms and routine flows of activities. Bookkeeping with its since long formalised set of concepts and practices (like discount and increase, recording of transactions by means of accounts, book-keeping by double entry etc.) can serve as an additional illustration how complicated workplace practices are routinised by "simple" algorithms which do not call for mathematical competencies. As long as the workplace does not present unexpected situations, these tools (for Rabardel 1995: “instruments”, i.e. “artefacts” together with schemata of activities – “schèmes d’utilisation”) go unrecognised and "hide" the mathematics they incorporate. Nevertheless it would be wrong stating mathematics disappears altogether or becomes less important socially. To the contrary: the third phase of the weighing clearly shows the growing social importance of mathematics – thus „explaining“ the dilemma described in the introduction of this paper.

Is there a chance of „rediscovering“ mathematics in vocational situations? Recent research on mathematics in vocational contexts offers a somewhat deceiving answer to this question: It is only in non-routine and non-standard situations, when usual practices fail or do not cover the situation to be faced at the workplace (the "breakdowns" or unfamiliar situations), that (even qualified) practitioners go back to unfamiliar, maybe innovative procedures: "(they) apply a fragment of professional knowledge, a half-remembered rule from school mathematics or a novel, though generally unsuccessful, use of a familiar tool" (Noss et al. 1998, p. 14; for the same finding also Magajna 1998; for the non-understanding of the workplace mathematics cf. also Hogan 1996, p. 288). Here again, the artefacts show up as one way to somehow manage non-routine problems - and the "banking mathematics study" shows that computers and finely tuned software can even be used to offer a micro-world for exploring non-routine, unusual situations (cf. Noss&Hoyles 1996; for a detailed discussion of computer technology as a special type of artefact see below in this paper).

If we take “artefacts” in the broad sense of Wartofsky, we can describe the trend of hiding mathematics in algorithms and routines in an easy way: By integrating mathematical concepts, relations and procedures into various types of “primary artefacts” (be it rules to follow, charts to fill in, computer technology to handle or other machines to use) mathematics tends to gradually disappear from the attention of the worker. Even if mathematics is increasingly used on a global level, the individual professional does not notice this development, s/he tends to describe the ongoing process as a gradual disappearance of mathematics from her/his workplace. It is only in non-routine and „breakdown“ situations that mathematics shows up again and may be (re-)invented to cope with the „new“, non-tool-governed situation.

In more general terms: the development of artefacts or “instruments” (Rabardel) integrates and thus hides mathematical concepts and algorithms from the workers’ perception in standard workplace situations. In situations like these, vocational training and education will be of value as well as the use of artefacts or tools. Consequently, research on the use of mathematics at the workplace has to analyse the artefacts used at the workplace and should look into the non-standard, non-routine, maybe even disastrous situations to (re)discover the authentic use of mathematics in vocational contexts.

Consequences *for Learning Mathematics*

(Most of the following is from Straesser at the 8th International Congress on Mathematics Education ICME 8).

If we look into organisational features of vocational education and training around the world, we find two extremes of learning principles: learning at the workplace versus classroom instruction (and the standard oscillation and insecurity of political decisions on this alternative). Classroom type of vocational mathematics education tends to present mathematics as a separate body of knowledge, sometimes even structured along a disciplinary system from mathematics. In this case, mathematics has to be linked to work and workplace practice by building mathematical models and applying mathematics by the well-known modelling cycle of "situation - (mathematical) model - interpretation of the situation". The situation is to come from the workplace, the mathematical model rests upon mathematical structures and algorithms known before or taught on the spot and the solution of the model hopefully can be interpreted in a way to cope with the given professional situation (for a summary of this approach see Blum 1988). In this pedagogy, mathematics can come first and can be taught / learned along its own, disciplinary structure while applying it to work via modelling may come second, sometimes never or inappropriately. As can be seen from this description, the modelling approach clearly distinguishes two types of knowledge - namely professional and mathematical knowledge. In this respect, it does not respect the unitary type of knowledge of the workplace, but it supposes the integration of different types of knowledge from the individual (future) worker. In most cases, modelling vocational problems by applying mathematics is a major difficulty for the (future) worker. As is known from research on examinations (e.g. the rather “old” study by Ploghaus 1967), the extraction of the mathematical model from a professional situation is especially difficult for (future) workers.

The other extreme and contrasting pedagogy is training on the job, where learning takes place at work whenever it is needed by the workplace practice and its problems. The focus is on coping with the situation at hand - and mathematics may come in or not when solving a workplace problem. An apprenticeship may offer a chance to legitimately participate in the workplace activities. At the beginning participation in work activities may be only peripheral (e.g. starting with minor preparatory or cleaning up activities). The duties of the apprentice will gradually shift from unimportant to activities essential for production or distribution. Without an explicit reference to the skills and knowledge necessary for coping with workplace requirements, without too formal or institutionalised teaching/learning, the new participant of the “community of practice” gradually develops from a beginner to an expert at the workplace. With this approach, learning may be identified with taking part in the community of practice and gradually developing from a beginner to an experienced, full practitioner by means of situated learning (for a thorough discussion of the underlying concept of "legitimate peripheral participation" see Lave&Wenger 1991). This pedagogy starts from a uniform concept of knowledge present in a community of practice (not in individual workers), "knowing is inherent in the growth and transformation of identities and it is located in relations among practitioners, their practice, the artefacts of that practice and the social organisation and political economy of communities of practice" (Lave&Wenger 1991, p. 122). As a consequence, mathematics can continue to go invisibly, embedded in the workplace practice and serving as a tool used to cope with professional problems if needed. A problem-oriented integration of concepts tends to hide mathematical relations under the uniform workplace practices. Following this approach, studies on "street mathematics" (like Nunes et al. 1993) had to detect and bring back to light the mathematical procedures in workplace activities, to describe them and to show the competence of the practitioners in using mathematics.

As a consequence of a preference for the situated learning and community of practice approach, why not dissolve any classroom type of training at least in vocational mathematics and totally rely on training on the job for vocational mathematics? I want to draw the attention to a finding which might be forgotten when closing vocational / technical colleges: In the study on Brazilian bookies, the protagonists of street mathematics state: "... the influence of schooling is not limited to topics explicitly taught in classrooms but ... school experience provides a different way of analysing and understanding everyday activities. ... Schooled bookies ... seem to have a different attitude toward procedures for solving problems as a result of their schooling. ... school experience has an effect on how people deal with more academic questions, such as explaining their everyday procedures or making explicit the mathematical structures implicit in their everyday activities. School experience is also related to better performance on solving problems that differ from those usually encountered at work" (Schliemann&Acioly 1989, p. 216 ff.).Obviously, classroom type of activities can offer an opportunity to broaden the perspective of the future worker, to empower her/him with solving problems not common to workplace practice and to foster understanding of the workplace procedure. Classroom type of activities can offer an understanding which goes beyond the narrow confines of the actual situation, which transcends the situation and the problem where and when knowledge is developed. Classroom type activities in schools or colleges can show mathematics as a way to transcend the context with more general problem solving strategies and structures. But how to cope with the transfer problem?

As a way out the following “solution” may be available: Modelling with the help of mathematics should not be taken as a means to get rid of the dirty specialities of the concrete workplace to solve the abstracted problem by means of pure mathematics. It is by exploiting the interplay of the professional, concrete situation and the structural, mathematical model that one can cope with the given professional problem. In doing so, one can develop a mathematical structure maybe adaptable to a variety of different problems linked to the initial professional situation (to set up a "domain of abstraction" where the "dialectic between concrete and abstract" closely ties together mathematical ideas and practical knowledge of the professional domain, cf. Noss&Hoyles 1996, p. 27). In doing so, mathematics is not reduced to the general type of activity of theorising, analysing language and seeing structures implicitly devaluing situated learning as learning no mathematics.

If mathematics is taught as a bridge between the concrete, maybe vocational situation and the abstract, maybe systematic structure, even classroom vocational education can show mathematics as a "general" tool which is of larger an importance than just coping with the narrow tasks of the everyday work practice or the inculcation of algorithms. If college type education aims at presenting (vocational) mathematics in this way, one condition for success seems to be that mathematics is taught in a way it is "meaningful to the individual" who is learning. Technical and vocational colleges then have to strive for problems from the workplace which are as realistic as possible. And the problems should be taught in a way as close as possible to the actual concerns of the students and using artefacts as near as possible to the workplace situation (for an elaboration of this cf. Boaler 1993).

An additional case for learning mathematics not in a too narrow workplace context is expressed in a reminder I would like to place at the end of this section: "Mathematics in vocational education is serving more as a background knowledge for explaining and avoiding mistakes, recognising safety risks, judicious measurement and various forms of estimation. ... Not practice at the workplace but deepening of the professional knowledge, education to a responsible use of tools and machines and the understanding of and coping with everyday mathematical problems legitimise mathematics in vocational education" (Appelrath 1985, p. 133/139; translation R.S.).

Consequences for learning with information technology

Obviously, a very versatile and widely useful artefact can be modern information technology: computers, appropriate software and other electronic devices. Nevertheless, this technology can have ambivalent effects in the workplace and in vocational training and education.

Using finely tuned information technology can further hide the mathematics incorporated in the software. Noss&Hoyles 1996 precisely describe the effects of banking software which has been developed by highfly specialists and completely hides the inherent functional, mathematical relations between the numbers processed. Even in the number driven world of banking, numbers and commercial arithmetic disappear from the consciousness of the average employee. Mathematics hide in computer algorithms which are applied without paying attention to the underlying mathematical model of the banking process. Even somewhat complicated procedures (like calculating the present value of a treasury bill by discounting from face value in dependence of the day of maturity) go unrecognised by the average employee who relies on the programs designed by an unknown specialist in an unknown software house or department. The concepts of the users at the workplace reduce these numbers to mere indices of banking information, the underlying mathematics is totally blended out and ignored. "... these models were almost entirely hidden from view. Understanding and reshaping them was the preserve of the rocket scientists; the separation between use and understanding was absolute and the models' structures were obscured by the data-driven view encouraged by the computer screens" (cf. Noss&Hoyles 1996, p. 17). Additional examples could be the multitude of functional and numerical relations which usually go unrecognised when a spreadsheet is used. Most computer based accounting systems act in the same way and do not ask for a deep understanding of the double entry approach. CAD software may automatically change the perspective drawing of a 3D-object or a virtual reality by trial-and-error control without even asking for an understanding of the underlying concepts like axial versus central perspective and / or different types of modelling 3D-objects (for an interesting case of computer use at the workplace see Magajna&Monaghan 1998). In most workplaces, the use of modern (computer) technology implies the use of sophisticated mathematical models - but these models go without recognition by the average employee.

On the other hand, the same paper by Noss&Hoyles shows that the same technology can be used to de-grey the black boxes, to show the mathematical relations and offer an opportunity to explore the inherent, implemented relations in a way, workplace reality would never allow because of the risk of material, financial and time losses. The practice of using sophisticated mathematics can be brought to the foreground and consciousness of the user by appropriate software and vocational training / education. It is modern computer technology and appropriate software which can be successfully used to really explore and understand the underlying banking mathematics.

To sum up: Modern computer technology itself has an ambivalent role in the process of using mathematics at the workplace: It can be used as a way to hide mathematics in sophisticated software. Mathematics as a tool, a man-made artefact disappears in workplace routines - and modern technology can speed up this disappearance. On the other hand, the very same technology can be used to foster understanding of the professional use of mathematics by explicitly modelling the hidden mathematical relations and offering software tools to explore and better understand the underlying mathematical models.

Consequences for an appropriate research methodology

If the description of the use of mathematics in vocational contexts given above is appropriate, obvious consequences for research in this area are to be drawn:

(i) Survey and interview studies into the use of mathematics in workplace contexts tend to come up with no valuable information on the actual use of mathematics because mathematics is hidden from the perception of the interviewee by artefacts – be it material tools, workplace procedures or organisational features like distributing knowledge in a special way across a company's work hierarchy.

(ii) Research into the use of mathematics in workplace contexts has to choose between participant observation in an ethnographic style (a more or less "passive" methodology; various examples can be identified, especially in the Australian research community) or research has to create opportunities for the qualified worker to show her/his workplace practice including the mathematics therein (the more "active methodology; see for instance the examples in Noss&Hoyles 1996 or the study by Straesser&Bromme 1992 into technical drawing). Both, active as well as passive research methods need a thorough and intensive analysis of data after the collecting of information to enable the researcher to (re)discover the mathematics hidden in the workplace practice.

As a final remark I would like to point to the fact that the widespread preference for ethnographic studies in research in workplace contexts thus does not come from an a-priori preference of this methodology. In the perspective of this paper, a certain superiority of ethnographic studies and stimulated response type of research over traditional interview and survey studies is deeply rooted in the characteristics of research on the use of mathematics at the workplace. Nevertheless, the active research methodology seems also appropriate – especially if there is already knowledge available about workplace situations which seem to involve some mathematics.

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