Michèle Artigue, Paris

INSTRUMENTATION ISSUES AND THE INTEGRATION OF COMPUTER TECHNOLOGIES INTO SECONDARY MATHEMATICS TEACHING

Abstract

In France, for at least 20 years, the Ministry of Education has been actively supporting the integration of computer technologies into secondary mathematics teaching. Nevertheless, integration remains up to now a marginal phenomenon. Such resistance from the educational system, which is not a French particularity, is a matter of concern for didactic research and obliges us to question the research paradigms we have been using when investigating in that area. In this text, I would like to introduce and discuss some work developed by French didactic research in order to deal with these challenging issues.

Firstly, I will briefly review the current state of integration of computer technologies in secondary mathematics teaching in France, and articulate some hypotheses about possible sources for the observed resistance. I will then introduce the theoretical framework we have developed in order to approach the institutional and instrumental issues that seem to play a crucial role in integration, according to these hypotheses. Finally, I will focus on a national research project involving the TI92 calculator carried out within this theoretical perspective and present its main outcomes as regards instrumentation processes.

I. The current state of integration of computer technologies

The current state of integration is characterised by an evident discrepancy between official discourse and reality, and by a very poor return on the many institutional and political incentives. Let us mention some of these. Since 1980, the rational use of calculators is an explicit aim of the secondary mathematics curriculum and, some years ago, graphic calculators became compulsory at high school level. Every type of calculator, including symbolic calculators, is allowed in national and regional secondary mathematics examinations. As regards computers, the use of spreadsheets, dynamic geometrical software and, to a lesser extent, computer algebraic systems (CAS in the following), is explicitly integrated into the syllabus. A system of specific licences was developed, many years ago, in order to allow schools to buy selected software at a very reduced rate. The Ministry of Education supports innovative and research projects and finances the production of software which is then freely distributed to schools. It also supports the diffusion of innovative designs and research results through web sites, books, regional centres for computer resources, teacher training sessions during working time, summer schools…

However, in spite of these actions, the integration of computer technologies remains marginal at secondary level. At a time when great emphasis is put on new technologies such as Internet, calculators that have been introduced into the educational system more than 20 years ago are not yet fully integrated. Some recent surveys show that no more than 20% of secondary mathematics teachers fully integrate calculators into their teaching practices, thus helping students to develop the competencies necessary for their effective use. Calculators are allowed but they remain students' private tools. Despite official statements and the content of the mathematics syllabus, values and norms of mathematics learning and teaching are still defined with respect to the mathematical needs and practices associated with paper and pencil environments.

II. How to explain such a phenomenon ?

Reasons for such a state are certainly diverse and linked by complex systems of relations. I will focus on four of these which, in my opinion, require more attention from didactic research.

1. *The poor educational legitimacy of computer technologies as opposed to their high social and scientific legitimacy*

The resistance of the educational system to computer technologies cannot be analysed without addressing more general issues, such as the legitimacy of teaching means. Computer technologies obviously have a strong scientific and social legitimacy, but this is not sufficient to ensure their educational legitimacy. To gain such an educational legitimacy, computer technologies are mainly asked to prove that they can

- help teachers to face the recurrent difficulties met by mathematics teaching and learning better
- make teaching and learning easier and better
- help to disqualify teaching strategies which are too much orientated towards drill and practice and promote "conceptual learning".

This is not so easy to prove if the values and norms of mathematics teaching remain essentially shaped by the traditional values and norms of mathematical activity. This tends to generate some kind of vicious circle. Faced with legitimacy difficulties, innovators and promoters of computer technologies tend to minimise the cost of integration and overestimate its potential benefits. This does not help to address the complex issues of integration properly and, finally, acts against it. As was evidenced by a recent meta-study on the literature involving more than 600 publications from 1995 to 1998 in that area [1], research does not totally escape this general tendency.

2. *The underestimation of issues linked to the computerisation of mathematical knowledge*

Computerisation of mathematical knowledge in calculators and computer software complicates the already complex processes which govern the didactic transposition of mathematical knowledge initially described by Y. Chevallard [2]. When working with calculators or computers, teachers and students are faced with two slightly different transpositive worlds : the ordinary transpositive world associated with paper and pencil environments and the computer one. Research is more and more sensitive to this fact and to its cognitive and didactic effects [3]. These effects, as shown by research carried out around dynamic geometrical software, spreadsheets and CAS can be highly productive if the characteristics of the computer transposition of mathematical knowledge are seriously analysed and taken into account in engineering designs (see for instance [4] and [5]). Nevertheless, in the educational system, discrepancies with the standard mathematical world resulting from computer transpositive processes are often considered as parasitic phenomena which one has to minimise. Such an attitude, which certainly meets legitimacy issues, generates a lot of difficulties in the long term.

3. *The dominant opposition between the technical and conceptual dimensions of mathematical activity*

This opposition is not a recent one and can be seen as a consequence of a naïve vision of constructivism. However computer technologies tend to reinforce it. By freeing students from a lot of the technical burden, they a priori leave time for more reflective and conceptual work, and thus are generally considered as an ideal means for renewing teaching practices perceived as too narrow and technical. Such a fact certainly increases the educational legitimacy of computer technologies but it does not help us to consider and understand the dialectic interaction between the fundamental conceptual and technical facets of mathematical activity and the subtle ways by which technology modifies this dialectic by changing the means and economy of the mathematical work.

4. *The underestimation of the complexity of instrumentation processes*

Mathematics teaching and learning processes have been used to develop environments with reduced technological complexity. This does not make it easy to integrate the fact that by introducing complex technological tools, one introduces at the same time new mathematical and technological needs, which have to be fulfilled even if they go beyond ordinary mathematical needs. Once more, we meet legitimacy issues here: if mathematics legitimacy still remains attached to what is done and produced with standard tools, how can the price which needs to be paid in order to transform complex objects into efficient mathematical instruments be justified?

Within a few pages, it is not possible to elaborate more on these different points, but I hope to have made clear that the integration of computer technologies into mathematics education is far from being easy to achieve. Various characteristics of the mathematical culture and various constraints act as obstacles to integration and the strategies spontaneously developed by the educational system are not necessarily the most adequate. A better understanding of the way these characteristics and constraints shape teaching and learning processes in technological environments and the way they mutually intertwine, is today more than ever a necessity for research.

III. A theoretical approach based on anthropology and ergonomy

In order to approach integration issues as researchers, we need theoretical frameworks that allow us to approach the institutional and cultural dimensions of learning and teaching processes. At the same time we recognise the fact that teaching and learning mathematics in computer environments introduces a strong instrumental dimension to the corresponding processes. This is the reason why French researchers working in that area often rely both on the anthropological approach developed by Chevallard [6] and on approaches coming from cognitive ergonomy [7], [8][1]. The anthropological approach offers us a perspective where

[1] There is no doubt that these theoretical approaches were not the only ones offering support for such research perspectives. Our choices have been influenced by the didactic culture we are living in, the theoretical frameworks established within this culture, and the connections French didactic research has developed with cogni-

mathematical activity is conceived as human work strongly shaped by the cultural characteristics, constraints and norms of the institutions where it develops. Mathematical objects are thus conceived not as absolute entities but as cultural objects emerging from systems of practices. These practices or *praxeologies*, as they are called by Chevallard, are described in terms of:

- *tasks* in which the objects are embedded,
- *techniques* used to solve these tasks,
- *technology*, this term labelling here a discourse which explains and justifies the techniques, according to its etymology, and finally
- *theory* seen as a discourse justifying the technological discourse.

This approach leads to a more balanced vision between the conceptual and technical dimensions of mathematical activity, and to a strong sensitivity towards its semiotic and instrumental tools [9]. It is important to stress that the word *technique* has to be understood here in a very broad sense. A technique is a way of solving a task and techniques involved in the solving of mathematical tasks, except for routine ones, are a complex mixture of reasoning parts and routinised sub-techniques.

Within this perspective, the understanding of teaching and learning processes, and their mutual relationships, requires the understanding of the associated mathematical praxeologies in their institutional and personal dimensions. In addition, reflecting on integration issues requires the analysis of the changes that computer technologies introduce or could introduce in these praxeologies. There is no doubt that computer technologies deeply modify the technical and technological level of praxeologies and, through these, the traditional balance which existed between conceptual and technical work.

Cognitive ergonomy, which also relies on anthropological perspectives, offers us complementary tools for approaching instrumentation issues. Indeed, contrasting with researchers in the didactic field, researchers in cognitive ergonomy are used to analysing learning processes in technologically complex environments, namely the workplace. Within this approach, artefacts (technical objects) are carefully distinguished from the instruments they can become through instrumental genesis. An instrument is thus seen as a mixed entity, constituted on the one hand of an artefact and, on the other hand, of the schemes that make it an instrument for a specific person. These schemes result from personal constructions but also from the appropriation of socially pre-existing schemes. Instrumental genesis works in two directions. In the first direction, instrumental genesis is directed towards the artefact, loading it progressively with potentialities, and eventually transforming it for specific uses; this is called by Verillon and Rabardel the *instrumentalisation* of the artefact. In the second direction, instrumental genesis is directed towards the subject, and leads to the development or appropriation of schemes of instrumented action which progressively constitute into techniques which allow us to solve given tasks efficiently. This is what is properly called *instrumentation*. In order to understand and eventually pilot this instrumental genesis, it is necessary to identify the constraints induced by the instrument and, especially for the type of instrument with which we are concerned here, two kinds of constraints: *command constraints* and *organisational con-*

tive ergonomy for many years through joint research projects founded by the CNRS (National Centre for Scientific Research).

straints[2]. These result from *internal* and *interface* constraints. It is also necessary, of course, to identify the new potentials offered by instrumented work. Research in ergonomy attests to the complexity of instrumental genesis when it deals with technologically complex environments as is the case with computer technologies.

I cannot expand more on this theoretical framework in this text but the reader can find extensive descriptions of it in the course on instrumentation given at the 10th Summer School in the Didactics of Mathematics in 1999 by J.B. Lagrange, L. Trouche and P. Rabardel, which is published in the proceedings of this Summer School [10].

IV. The national TI92 research project

This two-year project started in 1996. It aimed to:

1. Understand the potential of symbolic and geometric calculators such as the TI92 for high school mathematics education.
2. Reflect on viability conditions for an integration of such calculators at high school level through the conception and experimentation of didactic engineering products.

Four research teams were involved: Grenoble, Lyon, Montpellier and Paris-Rennes each dealing with specific aims and also with different levels of schooling. Grenoble worked with grade 10 students, mainly on geometry; Lyon with grade 11 and 12 scientific students, mainly on engineering designs organised around the solving of complex open problems (see for instance [11] for the philosophy of such engineering designs); Montpellier worked with grade 10 and 12 students, mainly on algebra, calculus and the ways the calculators could help to connect the different semiotic registers and frames at play in mathematical work; Paris-Rennes, finally, worked with grade 11 scientific students, mainly on instrumentation issues and calculus. In the following, I will focus on this last project as I was personally involved in it [12]. More information about the others can be found in the proceedings of the European conference we organised in 1998 [13].

The specific aims of our research project were the following:

- Understanding instrumentation processes in algebra and calculus and their relationships with mathematical teaching practices.
- Analysing the mathematical needs of an effective instrumentation of the TI92 in that area, and how these needs could be fulfilled.
- Investigating potential discrepancies and conflicts with institutional standards and norms, and how these were managed by teachers.
- Developing, and experimenting with, a first course in calculus fully integrating the TI92.

Two classes were involved with each student being given a TI92 for the whole academic year.

The research methodology was a qualitative one, based on the triangulation of multiple sources of data : students' attitudinal questionnaires (one each term), regular classroom observations, a selection of students' written productions including specific assessments with the TI92, and following about 12 students, selected according to their sex, mathematical and

[2] *Command constraints* are those generated by the commands available, their range of efficiency…, *organisational constraints* are linked to the fact that working with a specific instrument influences the way we plan and organise our mathematical work, taking into account its specific ergonomy and ways of functioning.

technological profiles, through regular interviews and specific observation in classroom sessions.

V. Some results about instrumentation processes

Research clearly showed that the instrumentation of the TI92 is a very complex process, even if some first level of instrumentation seems easily accessible. It also showed that instrumentation requires specific mathematical knowledge, especially in order to:

- master the complex game between the exact and approximate modes offered by the TI92,
- efficiently deal with the diversity of algebraic forms provided by the calculator and relate these with the standard institutional forms,
- master semantic and formal equivalence between such expressions,
- correctly interpret the graphical productions of the calculator and the different perceptive phenomena induced by discretisation processes,
- efficiently manage the interplay between applications and commands when solving mathematical tasks.

It also becomes clear that the mathematical knowledge needed to instrument the calculator goes beyond what is considered as being taught by teachers and learnt by students at secondary level, according to the norms and values of usual mathematics teaching.

The analysis of collected data also confirmed our initial hypothesis that, due to legitimacy and cultural constraints, an efficient institutional treatment of these mathematical needs as well as of *instrumented techniques* is not easy to achieve, even for expert teachers. Moreover, it requires substantial changes in the content and management of the tasks given to students. Finally, research showed that instrumental genesis depends on students' profiles and that mathematical and technological competencies strongly intertwine in it.

These are general results. I will use a specific example in order to illustrate them, namely "variation tasks", which are prototypical of the first contact with calculus in France and have been extensively analysed by B. Defouad in the framework of a PhD related to this research project [14]. This analysis took place through the regular observation of classroom sessions but also through regular interviews with the selected students. During these interviews, the students were given a variation problem involving a function that was not yet familiar to them. They were asked to formulate conjectures about the variations of the function, and then to try to prove these. They could use their calculator freely and this calculator was connected to a computer thus allowing the interviewer to have access to their interaction with the calculator without disturbing them. The mathematical work was organised in two phases: a first phase of autonomous work, and a second phase where the interviewer tried to figure out what was accessible to the student through appropriate discussions. The specific characteristics of this situation, i.e. a familiar task, but dealing with non-familiar objects, which is solved individually and outside the classroom, thus having less pressure from the didactic contract, allowed us to understand the complexity of instrumental genesis better and observe phenomena which tended to remain nearly invisible in the classroom observations.

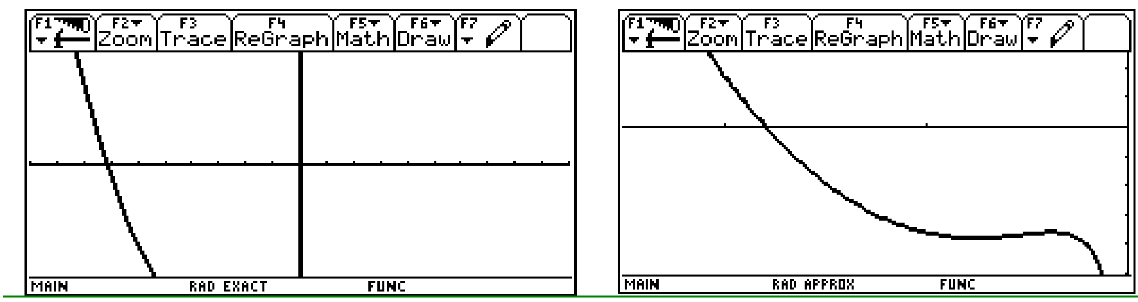
For this kind of task, we observed a global evolution at three levels:

1. The pre-calculus level where numerical and graphical approaches to variation, initially introduced at grade 10 without calculus resources, are still predominant since the symbolic application is poorly understood.
2. The intermediate level where symbolic manipulations begin to enter the mathematical scene and be connected with the dominant numerical and graphical approaches.
3. The calculus level, where symbolic manipulations become predominant tools for proof with the numerical and graphical approaches now being engaged essentially in heuristic and control phases.

This global evolution generally took a very long time to stabilise and, even at the end of the first experimental year, most students could still be easily destabilised by rather small perturbations.

Let us give an example namely the case of Frederic, a student with a standard mathematics level and a positive relationship to technology. At the first interview, about two months after the introduction of the derivative, he is asked to graph the function f(x)=x(x+7)+9/x, stop when satisfied, make conjectures about variation and try to prove these. It is the first time that Frederic has met a function with a critical point. He firstly defines the function in the symbolic application (called HOME on this calculator), then enters it in the Y= application and asks for the graph in the standard window: [-10,10]x[-10,10]. This strategy leads to a very partial picture (see below). Of course, the function was chosen because it produces such a phenomenon.

His interpretation is that the graph is included in the half plane corresponding to negative x. Without checking this interpretation by looking at the algebraic expression, he decides to reduce the window to negative x and then tries to adjust the vertical interval in order to make at least one extremum point visible. This is done by a pure trial and error strategy, guided by some idea of reasonable form, without any connection with the algebraic expression of the function. The graph obtained in the standard window, the graph he finds adequate, and the two graphs he could obtain from the first one by simply using the command *zoomout* or the command *zoomfit* are shown in figure 1.



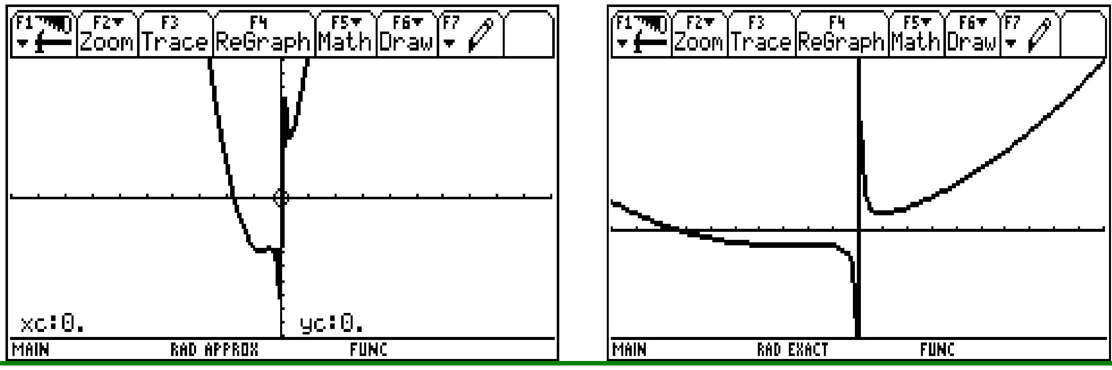


Figure 1

Frederic then jumps to the symbolic application, asks for the derivative and for its factorisation but is apparently unable to use it. He is visibly puzzled by the complexity of the expression he obtains and does not understand that the factorisation gives him the sign of the derivative. Hence he quickly comes back to the graph application, graphs the derivative and uses both the information given by the two graphs, and the table application, in order to conclude about variation. He then checks his conjecture by using the math-menu of the graph application for finding the extremum and, by zooming on the apparently flat part of the graph, this looks satisfactory (see figure 2).

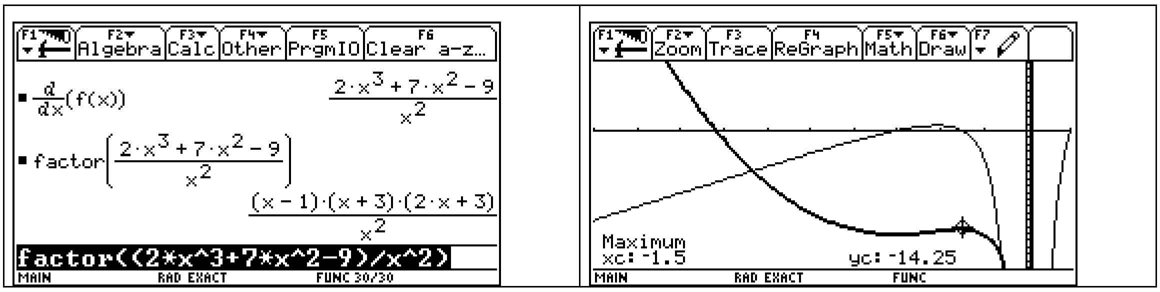


Figure 2

At the second interview, two months later, there is an evident evolution but Frederic is still in the intermediate phase and does not easily make sense of the graphical phenomena associated with vertical asymptotes. At the actual assessment, in June, he is clearly in the calculus phase and has developed specific and efficient instrumented schemes for framing and variation analysis, by connecting the symbolic and graphical applications of the calculator. However, soon after, at the third interview, he is faced with a new type of function which mixes square roots and trigonometric functions hence generating new phenomena linked to discretisation processes. This perturbation is a bit too much for him. He spends a lot of time trying to produce a graph that exactly touches the x-axis (see figure 3), doubts about the periodicity of the function, and when he gets the formal expression of the derivative, he is completely stuck. He asks for particular values of this derivative but needs help in order to prove the conjecture he has made about its sign from the graph.

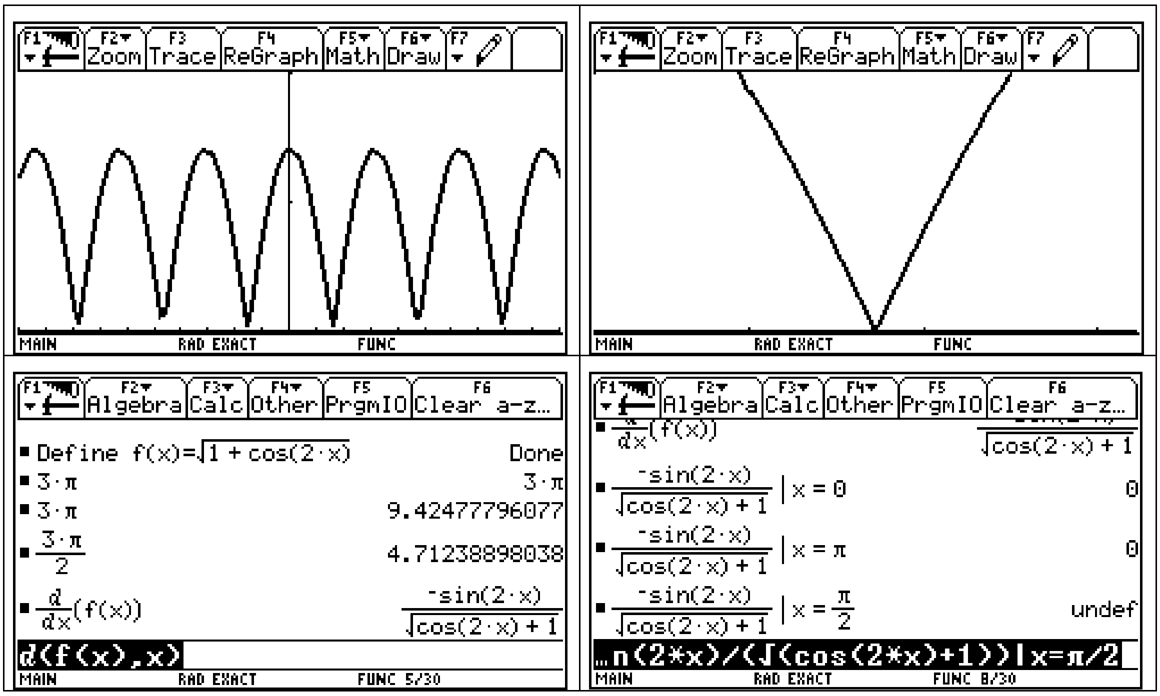


Figure 3

The detailed observation of students' instrumented work in the non-familiar situations allowed by these interviews helped us to identify some specific phenomena arising in the instrumental genesis.

The first one was the succession between what B. Defouad called *bursting* and *condensation* phases in the instrumental genesis. Bursting phases seemed to result from the great number of possible actions offered by the TI92 at a very low cost, and from the difficulties students had in fixing a strategy when they were not mathematically able to structure their choices. This led to what we called *zapping* (quick change between commands and applications similar to the behaviour they often have with TV channels). Trying to make sense of this zapping behaviour, we became more sensitive to the institutional life of instrumented techniques in the classroom, and to the characteristics of this institutional life which could contribute to reinforce zapping. By coming back to the data, we thus noticed that during the first experimental year, instrumented techniques had not been managed by teachers as paper / pencil techniques were. Technological and theoretical discourse did not really integrate them. They mostly escaped the *routinisation* and *institutionalisation* processes attached to official paper / pencil techniques. Selection within their diversity and technical training in their use was left to the individual responsibility of students. The first analysis carried out made teachers sensitive to these differences. The second year, they tried to correct the observed discrepancies and help the teachers involved in the experiment to give a more adequate status to the instrumented techniques. This resulted in an obvious gain in the development of instrumentation.

The second point I would like to make deals with the changes we observed in the relationships to proof. In this environment, students, if the pressure of the didactic contract is not too strong (as for instance during the interviews), tend firstly to search for coherence between the information provided by the different applications at the expense of more decisive proofs. Frederic's behaviour in the first interview is symptomatic of such a tendency. Obviously, there is a change in the economy of validation processes. Such a change, certainly, has positive effects: students, by learning to connect data coming from different frameworks and expressed in different semiotic registers, develop essential forms of cognitive flexibility. Nevertheless, this tendency has to be controlled by teachers as, if not, it can seriously affect students' relationships to mathematical rationality.

In such a short paper I cannot enter into the details of the engineering we developed and experimented with during the three years for a first calculus course, fully integrating the TI92. We tried to avoid the formal trap evidenced by several researchers i.e. using the calculator as a source of phenomena (numeric, algebraic and graphic) and mathematics knowledge as a means for understanding, foreseeing, reproducing and controlling such phenomena [12], [15]. The results we obtained showed some nice achievements. At the end of the second experimental year, tests showed that about 75% of students had reached the main aims we had set in terms of mathematical and instrumental achievements.

VI. Conclusion

The results obtained through this national research project are internally coherent and also coherent with those obtained by different researchers (see for instance [16], [17], [18], [19]). They clearly prove the complexity of instrumental genesis and the necessity to take it into consideration in teaching processes. They show not only the positive outcomes of such an integration if adequately managed but also illustrate the fact that such an efficient management requires specific didactical knowledge on the part of the teachers and may necessitate some substantial changes. Coming back to our initial concerns about the poor state of integration of computer technologies into secondary mathematics teaching, we are once more faced with the recurrent issue of relationships between research and practice. How can we make our results useful outside the community of educational researchers and experimental settings? How can we induce the necessary questioning of standard norms and values of mathematics teaching? How can we make engineering products i.e. resources that mathematics teachers would be able to adapt efficiently to their teaching style and to the specific constraints they meet, without losing their essence? We are well aware of the difficulties of the task and of the poor efficiency of the training strategies developed up to now. Developing and evaluating pre-service and in-service teacher training designs, taking into account the present culture of the educational system and the economy of changes, is certainly now the key issue we have to face.

References:

LAGRANGE J.B. & al. (2001). A meta study on IC technologies in education. Towards a multidimensional framework to tackle their integration, in M. van den Heuvel-Panhuizen (ed), *Proceedings of the 25th Conference on the International Group for the Psychology of Mathematics Education*, vol. I, 111-122. Utrecht : Freudenthal Institute.

CHEVALLARD Y. (1985). *La transposition didactique*. Grenoble : La Pensée Sauvage.

BALACHEFF N. (1994). Didactique et intelligence artificielle. *Recherches en Didactique des Mathématiques*, vol. 14/1.2, 9-42.

LABORDE C. (1999). Vers un usage banalisé de Cabri-géomètre avec la TI92 en classe de seconde : analyse des facteurs de l'intégration, in D. Guin (ed), *Calculatrices symboliques et géométriques dans l'enseignement des mathématiques*, 79-94. IREM de Montpellier.

ARTIGUE M. (1997). Le logiciel DERIVE comme révélateur de phénomènes didactiques liés à l'utilisation d'environnements informatiques pour l'apprentissage. *Educational Studies in Mathematics*, 33/2, 133-169.

CHEVALLARD Y. (1992). Concepts fondamentaux de la didactique: perspectives apportées par une approche anthropologique. *Recherches en Didactique des Mathématiques*, vol. 12.1, 73-111.

VERILLON P. & RABARDEL P. (1995). Cognition and Artifacts: a contribution to the study of thought in relation to instrumental activity. *European Journal of Psychology of Education* X(1), 77-101.

RABARDEL P. (1996). *Les hommes et les technologies : une approche cognitive des instruments contemporains*. Paris : Armand Colin.

BOSH M., CHEVALLARD Y. (1999) La sensibilité de l'activité mathématique aux ostensifs. *Recherches en Didactique des Mathématiques*, vol. 19.1, 77- 123.

BAILLEUL M. & ROBERT A. (2000). *Actes de la Xème Ecole d'Eté de Didactique des Mathématiques*. IUFM de Caen.

ALDON G. (1999). Démarche scientifique et calculatrices symboliques, in D. Guin (ed), *Calculatrices symboliques et géométriques dans l'enseignement des mathématiques,* 165-175. IREM de Montpellier.

ARTIGUE M. & al. (1998) *L'intégration de calculatrices complexes à l'enseignement des mathématiques au lycée.* Cahier DIDIREM spécial n°3. IREM Paris 7.

GUIN D. (ed). (1999*) Calculatrices symboliques et géométriques dans l'enseignement des mathématiques*. IREM de Montpellier.

DEFOUAD B. (2000). *Etude de genèses instrumentales liées à l'utilisation d'une calculatrice symbolique en classe de première S*. Thèse de doctorat. Université Paris 7.

LAGRANGE J.B. 1999. Complex calculators in the classroom: theoretical and practical reflections on teaching pre-calculus. *The International Journal of Computers for Mathematical Learning*, 4, 51-81.

GUIN D. & TROUCHE L. (1999). The complex process of converting tools into mathematical instruments : the case of calculators. *The International Journal of Computers for Mathematical Learning,* 3(3), 195-227.

SCHNEIDER E. (2000). Teacher experiences with the use of a CAS in a mathematics classroom. *The International Journal for Computer Algebra in Mathematics Education,* 7/2,119-141.

DRIJVERS P. (2000). Students encountering obstacles using CAS. *The International Journal of Computers for Mathematical Learning,* 5/3, 189-209.

KENDAL M. & STACEY K. (2001). The impact of teacher privileging on learning differentiation with technology. *The International Journal of Computers for Mathematical Learning,* 6/2, 143-165.

Dr. Michèle Artigue
IREM, Université Paris 7
Case 7018, 2 place jussieu
75251 Paris Cedex 05 ;   E-mail : artigue@math.jussieu.fr