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INVESTIGATIONS INTO THE PRIOR KNOWLEDGE AND COGNITIVE STRATEGIES OF CHILDREN

STUDIES DONE DURING MATHEMATICS TEACHER TRAINING

Abstract:

This article reports on studies of the prior knowledge and cognitive strategies of children learning arithmetic. We view such studies as a necessary component of a teaching programme that focuses on the pupils' own activities. The article describes the four approaches used to investigate the children's prior knowledge and cognitive strategies: prior knowledge assessments, free productions, explorations of strategies, and clinical interviews. Student teachers have taken part in all of the studies, and their participation is an element of their own professional education.

For the last six years, two Swiss teacher education institutions, the pedagogical college of the canton of Aargau and the pedagogical institute of the city of Basel, have been conducting two types of empirical study. The first type concerns the testing and development of learning environments, primarily those that have arisen out of the "mathe 2000" project (WITTMANN and MÜLLER 1990, 1992, for example). Selected school classes have been made available to student teachers for this empirical study. These investigations have shown that there are some surprising discrepancies between the children's competencies on the one hand, and traditional teaching materials and official curricula on the other - the children are capable of more, and of different things, than are expected of them.

This motivated us to conduct a second type of empirical study focusing on assessing the prior knowledge already held by the children and their cognitive strategies. Two questions have been at the forefront:

a. What competencies do the children already have concerning a subject area that is about to be taught in school?

b. How have they obtained these competencies? Which cognitive and problem-solving strategies do they already possess?

The studies were initially conducted using eleven classes that were followed for the entire time that the children were in primary school i.e. four to five years. All of the investigations were planned, carried out and evaluated by the student teachers after they had been given an introduction to the subject matter. The research took the form of an elective study project to which they were able to devote one half-day a week for a year.

Four approaches

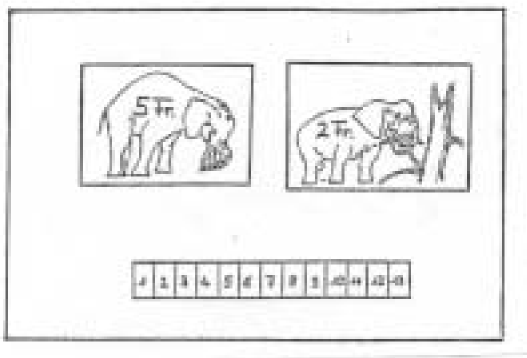
Four approaches for investigating the children's prior knowledge and cognitive strategies have arisen during the course of the research. These approaches differ mainly in what they target. They are:

- Prior knowledge assessments,
- Clinical interviews,
- Explorations focusing on strategies, and
- Free productions.

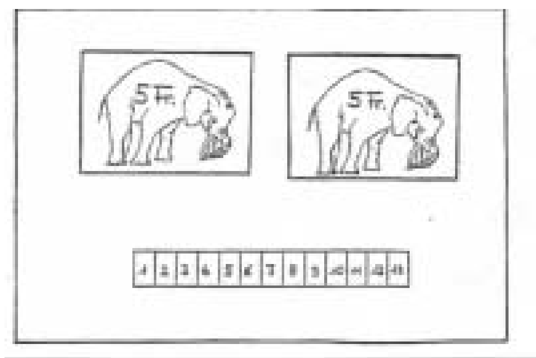
Two considerations were important in selecting these approaches. We wanted them to offer the student teachers optimal teacher training and qualification opportunities, and teachers working in schools should also be able to apply them. Most of the studies employed a combination of approaches.

1. Prior knowledge assessments show what knowledge and skills are already present in relation to subjects that have not yet been dealt with at school. For example, *before* formal schooling began we conducted clinical interviews and, in the aftermath, developed test tasks with the zoo as their theme which we then presented to 100 kindergarten children individually. The tasks concerned different aspects of numbers and various operations: quantity (the number of animals in an enclosure); ordinal numbers (applied to the ticket window line); calculating sums of money (the entry price); telling time (when are wild animals to be fed, opening hours); and doubling, addition and subtraction (elephant rides, animals who leave or enter their houses). Figure 1 shows four tasks arising out of this prior knowledge assessment that involve adding sums of money for entry tickets and doubling, taking elephant rides as an example. The results (the number of correct solutions expressed in percent) were compared with those from a different prior knowledge assessment conducted with kindergarten children who were asked to double numbers of animal legs (4 plus 4 and 6 plus 6). The results have implications for teaching introductory arithmetic. They suggest that greater use can be made of the prior mathematical knowledge already available from real-life situations, and they underscore the importance of doubling for a structured comprehension of quantities.

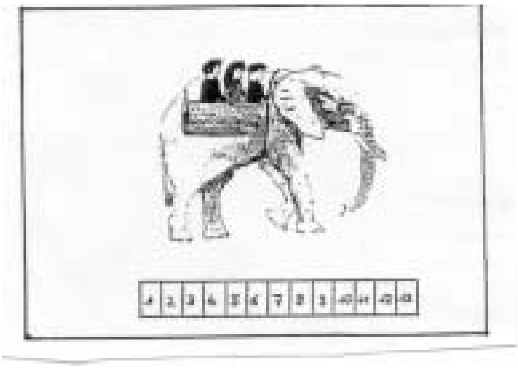
These and other investigations conducted throughout the primary school period show that, in addition to the differences between the individual children, there were also always sizeable differences between the classes. For this reason too, it is both reasonable and necessary to conduct prior knowledge assessments for all major subject areas.



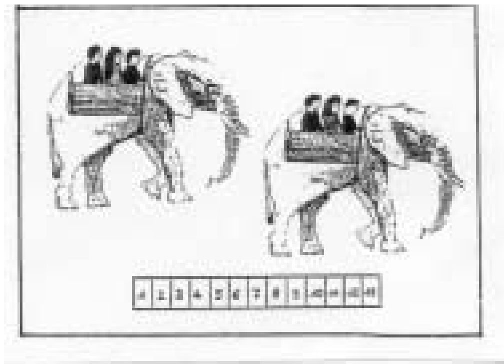
5 Fr. + 2 Fr. = 7 Fr. ***(59 %)***



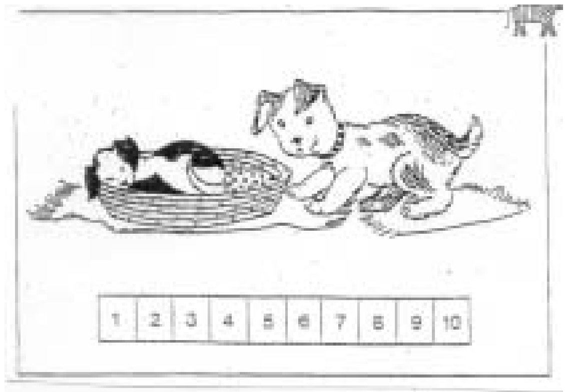
5 Fr. + 5 Fr. = 10 Fr. ***(80 %)***



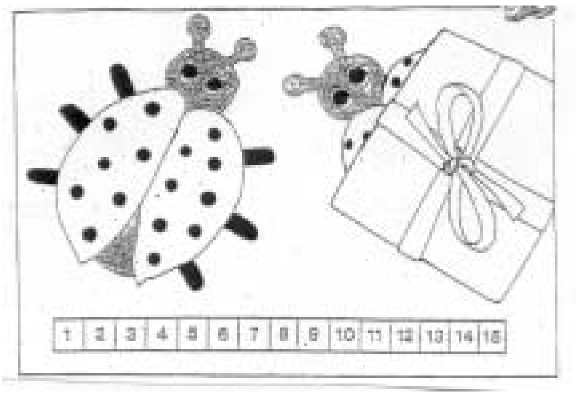
Doubling 3 + 3 = 6 ***(86 %)***



4 times 3 = 2 times 6 = 12 ***(64 %)***



Doubling 4 + 4 = 8 ***(78 %)***



Doubling 6 + 6 = 12 ***(53 %)***

Figure 1: Prior knowledge assessments of kindergarten children. Shown are four examples from the “Zoo” task series and, for comparison, two examples from the “Andrea’s birthday party” series. Each task was given to approximately 100 children individually.

2. Clinical interviews differ from the other approaches in their objectives and methods: here the focus is on investigating individual strategies. The interviews conducted with the individual children (more rarely with groups of pupils) incorporate a basic sequence of tasks but allow for deviations. Except for children in kindergarten, most of the interviews have been conducted as a follow-up to prior knowledge assessments made in class and carried out with either all of the children in the class or with a limited number thereof, with the children being selected by the teacher. It has normally been sufficient to interview about 20 children to gain an overview of their problem-solving methods and strategies. Conducting clinical interviews enables the student researchers to practice behaviours that are central to active discovery teaching. These include the interplay of listening and task performance, observation, letting pupils take their time, and expressing interest in the children’s individual cognitive strategies and letting them describe their strategies themselves.

(We will refrain from presenting an example for reasons of space. Please see HENGARTNER 1999 for a related document.)

3. Focused explorations are also suitable for investigating the different strategies employed. In contrast to clinical interviews, we view such explorations in relation to existing curriculum designs: they serve to examine their basic underlying assumptions. These are exploratory tasks assigned to a class before the material in question is dealt with in school. To take one example, for the subtraction of three-digit numbers (met in the third grade) we analyzed the children's problem-solving methods in light of the informal strategies they utilized, as well as how frequently they used them and their rate of success (cf. HENGARTNER 1999).
(Figures 2 and 3)

Informal strategies of subtraction:

Partitioning (into hundreds, tens and ones):
853 - 689 = 200 - 30 - 6 = 164
800 - 600
50 - 80
3 - 9

Stepwise subtraction:
853 - 689 = 173 - 9 = 164
253 - 80 - 9

Calculating by simplifying:
853 - 689 = 164
854 - 690
864 - 700

Calculating by using an auxiliary task:
853 - 689 = 153 + 11 = 164
853 - 700 = 153

Adding up (bottom-up *or* top-down):
853 - 689 = 11 + 153 = 164
700

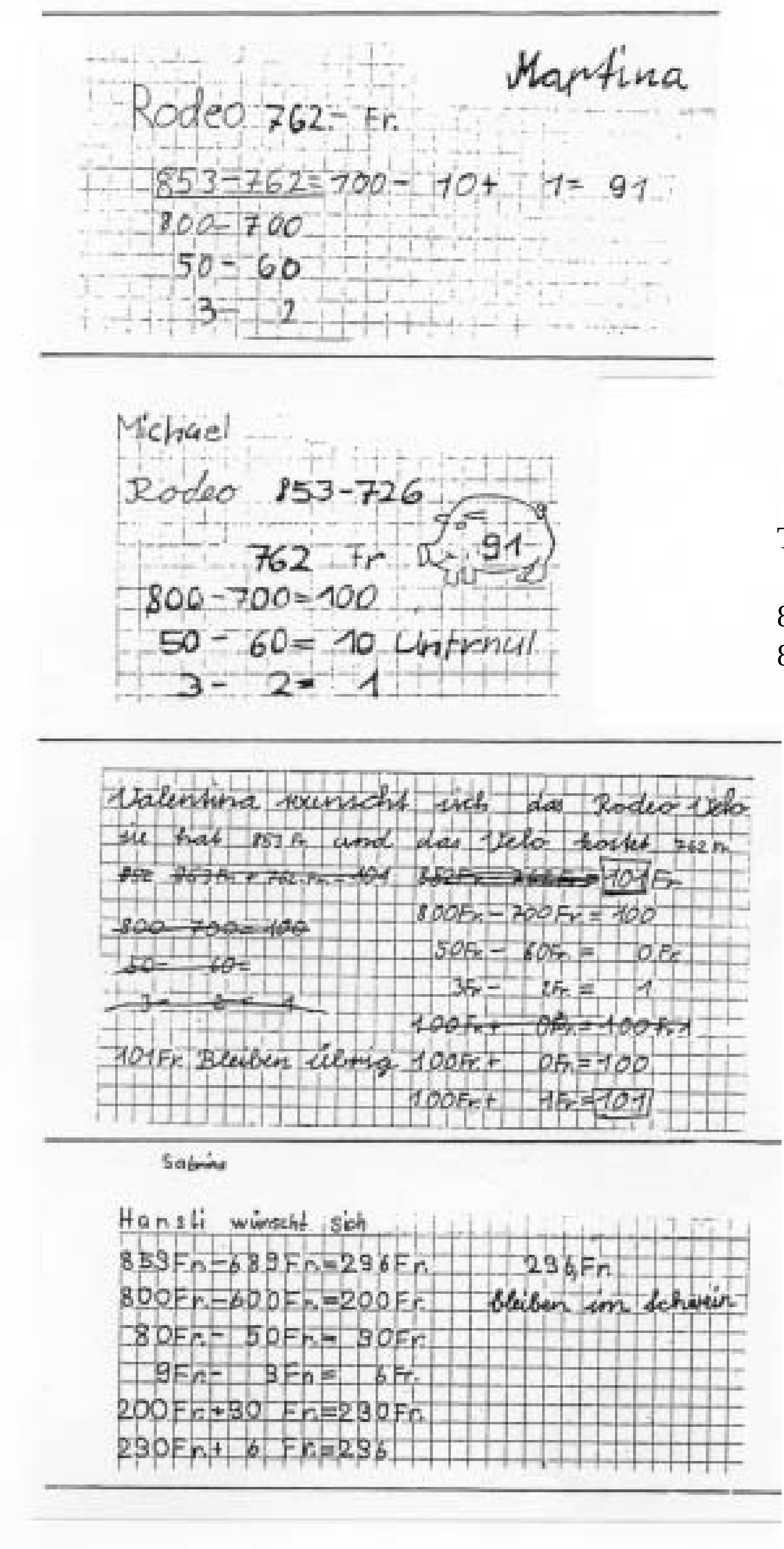
853 - 689 = 53 + 100 + 11 = 164
800
700

Figure 2: An exploration focusing on individual strategies of third-grade pupils asked to subtract three-digit numbers (the "bicycle" task) and an overview of the informal strategies anticipated.

Figure 2 shows the task "Buying and comparing prices of children's bicycles" and presents an overview of the informal strategies that the children's problem-solving attempts could be associated with. Figure 3 presents examples of this broken down into the three strategies most frequently employed. The frequency and success rates of these three strategies were as follows:

Partitioning (into hundreds, tens and ones)	22% frequency	1 out of 5 correct
Stepwise subtraction	14% frequency	About half correct
Adding up (bottom-up *or* top-down)	51% frequency	3 out of 5 correct

Strategy: Partitioning (into hundreds, tens and ones)

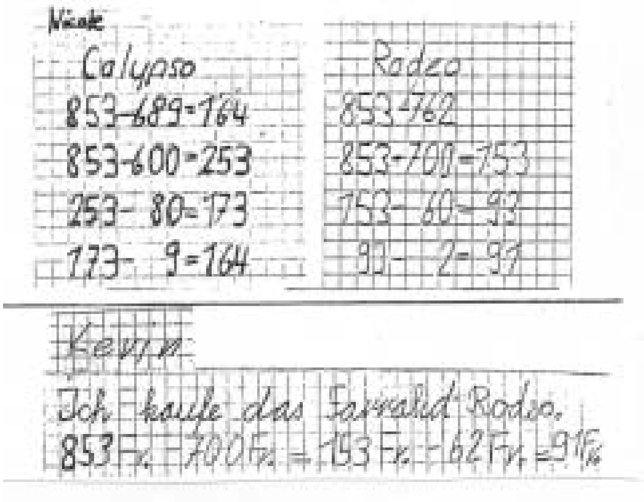


The tasks were:

853-762 = 91
853-689 = 164

Figure 3a: Explorations focusing on individual strategies of third-grade pupils asked to subtract three-digit numbers – examples of the three strategies most frequently employed.

Strategy: Stepwise subtraction



The tasks were:

-853762= 91
853-689=164
525-389=136

Strategy: Adding up (bottom-up *or* top-down)

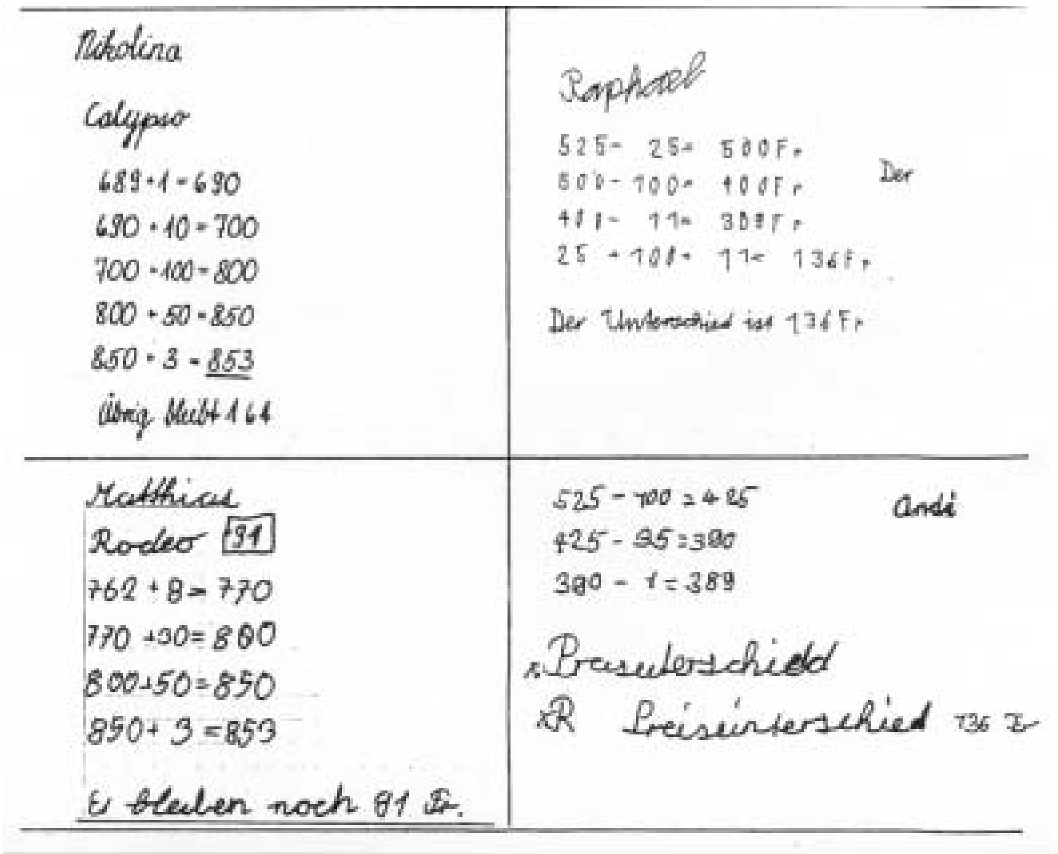


Figure 3b: An exploration focusing on individual strategies of third-grade pupils asked to subtract three-digit numbers – examples of the three strategies most frequently employed.

Adding up (in its different variations) was the most frequently employed and the most successful strategy. Only a few children attempted stepwise subtraction (of hundreds, tens and ones), but those who did were often successful. Partitioning seems to be the strategy which is most prone to error. Such focused explorations are useful for a form of instruction that attributes greater importance to informal strategies and which helps children to go their own individual ways.

4. The free productions of children assigned open-ended tasks have often surprised both the teachers and the student researchers, and opened their eyes to the knowledge already at hand and ways of thinking which they can explore further in studies of their own. Sometimes open-ended tasks are also a response to prior knowledge assessments if one wants to meet the differences they bring to light by opening up the space available for the children's own productions. Exercise patterns (number walls, for example) and open-ended situations have a special value that provides leeway for different task concepts and problem-solving strategies.

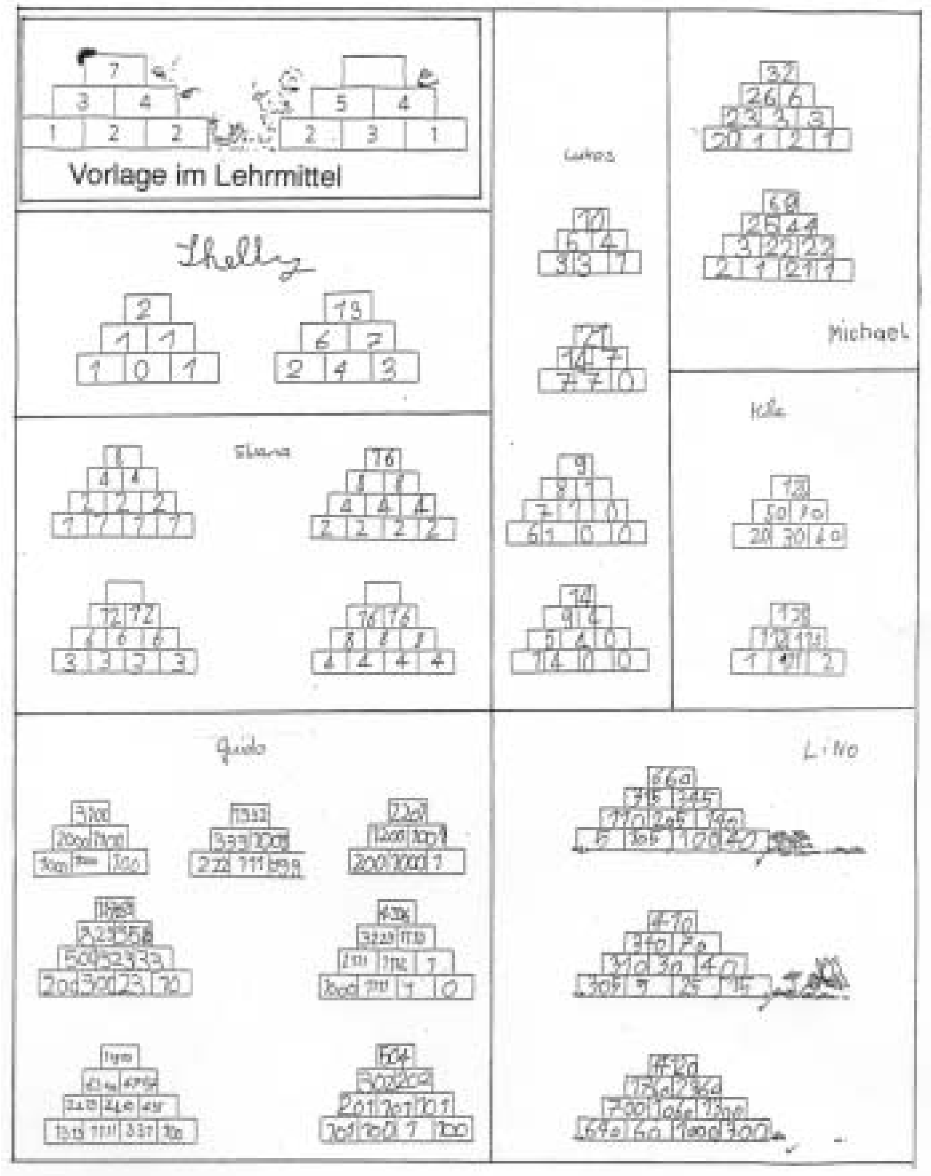


Figure 4: Free productions of number walls by first-grade pupils.

Figure 4 shows examples of number walls created by children in the first grade. In a number wall, a “stone” will always show the sum of the stones it rests on. After working through the example presented in the teaching material, the pupils are given empty number walls to fill in themselves. Their products range from simple examples containing numbers up to 20, through experiments with the same number throughout the first row, multiples of ten, one-, two- and three-digit numbers, and the sophisticated calculations of fourth graders.

Significance of the studies for teacher training and practice

These investigations into prior knowledge and cognitive strategies are being carried out in a teacher-training context in that they are designed to be part of a programme for a professional educational qualification, but they also have special value for the teachers and children who participated. Based on observations, the feedback received and written documents, the significance of the studies can be described as follows.

For the student teachers, these investigations have brought them face to face with the very different and, in part, surprising levels of prior knowledge already held by the children, and with a wide variety of problem-solving and cognitive strategies. This has made the student teachers sceptical of a policy of small-scale, step-by-step guidance and early standardization. It favours an understanding of teaching that provides the children with space for their own structures, and opens the student teachers’ eyes as to how pupils see things.

Participating teachers, in addition, have been able to integrate several of the studies into their curriculum planning. Prior knowledge assessments, for example, belong in the planning of a curriculum that builds on the knowledge that is already present and assists in developing it further. The studies can also help provide a basis for teaching processes: prior knowledge assessments can, for example, lay the groundwork for instruction that uses rich learning environments and more complex tasks to develop a holistic understanding of mathematics, while the results of focused explorations of strategies and clinical interviews can pave the way for a liberalization of methods.

For the children, the following effect can be assumed: they experience teacher actions that help them to learn efficiently. Among these is that the teachers ask them about their problem-solving strategies and thought processes and attentively try to follow their thinking. The children are required to describe their reflections and explain them to one another using their own means. As they do this, misconceptions come to light and can be discussed. Errors are viewed as being a part of the learning process. Children see that much is expected of them, and this can strengthen their self-confidence.

Our studies of prior knowledge and cognitive strategies can be viewed as providing practice in a way of teaching mathematics. This approach assumes that children have a sense of the overall structures and believes that they are capable of discovering things on their own. It opens up free space for them to use their own strategies and think independently, and is home to a living culture of mutual understanding.

References

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