Arithmetic Disabilities of Children and Adults

- Neuropsychological approaches to mathematics teaching –

Marianne Nolte
Hamburg

Men and women, who are unable to calculate, experience great difficulties in coping with everyday life. Some people never learn to calculate and others lose their ability as a consequence of an illness (e.g. stroke). The problems of the former group can presumably be regarded as consequences of a developmental dyscalculia during childhood. Dyscalculia as consequence of an accident or an illness is termed 'acquired dyscalculia' (Shalev et al., 1995). In the field of neuroscience, cases of acquired dyscalculia are examined in order to develop theories about the brain's functional modes. We present neuroscientific results concerning acquired dyscalculia and then discuss their significance for theses about developmental dyscalculia.

Results from neuroscientific research

One research question is the localisation of psychic processes[1]. Nowadays in neuroscience it is stated that psychological processes are modular. Various neural part-functions collectively determine the execution of processes. Some local hemispheres of the brain contain not complex but elementary abilities. Kandel, a leading advocate of neuroscience, states that: "Complex functions are made possible through serial and parallel connections of different areas of the brain. " (Kandel 1996, p. 18).

Based on comparisons of competences before and after a brain has been injured, models were developed to demonstrate which modules can be associated with arithmetic competence. Most of the examinations were carried out as case studies with the results of single observations being discussed in relation to each other. Which kind of module found in the literature is related to arithmetic competences?

Cases

CasesAuthor/Case	Loss of	Available ability of
Warrington (1982): Dr. Constable (in (Butterworth 1999)	Number fact	Estimation: 5+7 is about 13 Comprehension of calculation Procedures
Carramazza, McCloskey (1987): Patient A	Procedures	Number fact
Carramazza, McCloskey (1987): Patient D	Comprehension of arithmetical operations	Number fact

[1] Following Lurija (1992), psychic processes are defined as emotional and cognitive neuronal processes.

Number fact in terms of Arabic numbers is understood by the authors to be the ability to recall the solution of a term immediately from memory. Comprehension of arithmetical operations is assumed, if the patient manages somehow to determine the result of a term. This may include processes of counting. With written methods, access to procedures may be assumed.

Whether these processes are modular ones or not is examined by double dissociation. The question is posed as to whether it is possible that abilities acquired in the past may get lost while, simultaneously, abilities acquired later remain unchanged. Therefore cases showing contrary or complementary results are viewed as a whole, for instance the loss of adding ability while multiplicative ability is preserved. From the above table one might therefore assume that (number) fact knowledge, comprehension of arithmetical operations and the ability to carry out procedures are part-functions of arithmetical abilities which may be isolated separately.

The interpretation of various case studies led to the theoretical conclusion that special mathematical knowledge is based on isolated neuronal part-functions. This thesis is widely recognised (e.g. Cipolotti et al. 1993; Cipolotti et al. 1995; Hittmair-Delazer, Semenza et al. 1994; McCloskey 1992; Dehaene 1992; Dehaene and Cohen 1991; Grafman et al. 1989). Isolated failure of the following abilities occurred[2]:

- Comprehension of numbers: meaning predominantly that a sequence of Arabic numbers can be assigned to a number word and that a number word can be assigned to an Arabic number (acquisition of information).
- Arabic numbers production: meaning the ability to pronounce and to write Arabic numbers (information output).
- Execution of arithmetical operations: meaning the ability to solve simple problems.
- Execution of algorithms: meaning the ability to carry out written/read arithmetical operations.
- Estimation.
- Quantifying.

  An isolated failure of the ability to quantify which, for small children, had already been proved possible (Butterworth 1999).
  Analyses will be carried out to determine whether neighbouring numbers can be determined and whether discovered numbers can be distinguished as being larger or smaller.
- Conceptual knowledge: does it become clear whilst working on the items that knowledge of operative relations is recalled, e.g. the commutative law?
- Multiplication fact retrieval.

Sokol, McCloskey et al. (1991) used this concept to help differentiate between different aspects of number fact knowledge. Thus, in one case, while checking knowledge of multiplication tables it became clear that different representations of rule-guided processes of recall – such as the product of zero or one as well as of processes of recall with isolated arithmetical fact retrieval like 7x8 or 6x8 – could be assumed.

[2] How these terms should be interpreted can best be understood if, in the literature, it is shown from which examples of tasks the theses were developed. In the field of mathematics teaching the points mentioned here have a more extensive meaning.

Cipolotti and de Lacy Costello (1995) postulated that each kind of calculation is represented separately in the memory. In relation to adding, subtraction and multiplication they refer to various studies which prove that even the ability to divide may fail separately.

Hittmair-Delazer et al. (1994) even postulate that each number rule within multiplication tables is memorized in isolated modules.

Format dependent failure

In connection with the ability to recognize and produce numbers, format dependent failures referring to different ways of representation were observed. These included the sum of patterns of points, representation of Arabic and Roman numbers, verbal representation (oral and written) etc. Furthermore, failures in connection with the meaning of numbers could also be observed (Cipolotti et al. 1995). This is well illustrated by one of McCloskey's cases: "For the problem 4 + 5, e.g., he said "eight" and wrote "5" (but chose "9" from a multiple-choice list)" (McCloskey, 1992, p. 116).

Is the representation of numbers abstract or dependent on the context?

Cohen et al. (1994) report on the case of a man who had lost the ability to deal with arbitrary numbers but who could still handle historical and biographical data meaningfully. This leads to the conclusion that specific processes of memorizing correspond to distinct neuronal functions

Do we have a computer centre in our head?

The possibility of defining the locations of module functions raises the question of whether it is possible to assign complex abilities to specific areas of the brain, and even whether a kind of computer centre might be found there.

Because it was observed that arithmetical abilities were no longer available after various areas of the brain had been injured, neuro-psychologists surmise that besides looking at the modular processes, our arithmetical competence should not be viewed separately from other abilities, such as general intelligence, reasoning and three-dimensional imagination (Kahn/Whitaker,1991, Butterworth, 1999). Such abilities cannot therefore be limited to a specific area.

More and more precise information about the locations of neuronal functions lends support to this position. For instance, in the cortex of monkeys, 25 to 32 different areas [3] have been found which all contribute to the visual functions (Kolb and Whishaw, 1996, S. 139).

Contrary to the opinion that one cannot uncover a computer centre in our brain, Butterworth (1999) tried to find such a location. He collected cases of patients who had lost their knowledge of numbers and calculation in various ways. Their injuries all concerned the left parietal lobe. Butterworth therefore speaks of a "mathematical brain" which he expects to be located there. In order to prove his hypothesis he analysed several cases by the technique of double dissociation. From this he concluded that knowledge of language, memory processes and logical thinking do not depend on arithmetical abilities.

For such analysis the following three aspects must be taken into consideration:

- The area of the left parietal lobe has been related to mathematical abilities for quite a long time (Dehaene 1999; Temple 1998). Damage in this area may lead to the Gerstmann-syndrome, which includes finger-agnosis, confusion of right and left side and disturbances

[3] Depending on the corresponding modelling.

in writing and calculating (Kolb and Whishaw 1996). From this it can be concluded that the left parietal lobe is also responsible for other functions.

- The patients' partial functions, described as the so-called "loss syndrome" (Verlustsyndrom) which are related to this area, make up just a small part of the functions that are needed for mathematical activities. Thus among others, dissociation of logical thinking is not consistent with the term "mathematical brain" for this area.
- With regard to arithmetical functions, there are already proven activities in other areas of the brain (Grafman et al. 1989).

With the help of image-producing processes, Dehaene and his team demonstrated that for multiplication and for comparing two magnitudes or numbers, further brain areas in the right and left cortex are activated (Neumärker 2000, S.4). Dehaene (1999) believes that the right part of the brain is able to carry out simple operations and estimation.

Dehaene and Cohen (1995) showed that both hemispheres are able to read numbers and can carry out magnitude comparison, but the left hemisphere alone controls the linguistic sequence of number words. Mental calculation is bound to linguistic processes and the recall of factual knowledge is not exclusive to the right hemisphere. For written arithmetical operations there are relations to visual-spatial arrangements which are dependent on abilities within the right hemisphere. Within the left hemisphere visual, verbal and quantitative contents are linked to each other. In the right hemisphere there exist connections only between visual and magnitude representations. Information is transferred between both hemispheres of the brain through the corpus callosum. While dealing with numbers and words several parallel neuronal circuits are activated in various areas of the cortex. Different circuits are connected to each other, dependent on the demands of each task (Dehaene, 1999).

Models for representation of mathematical knowledge

From various case descriptions, models were developed which refer to internal representations of arithmetic knowledge. The best known are those of McCloskey and his team (1992), of Campbell and Clark (1992) and those of Dehaene (1999).

McCloskey et al. (1992) proceed from a modal representation of numbers. For each incoming and outgoing piece of information there exists one way to translate it. A number like 64 is stored according to its grouping qualities 6x10 + 4, rather than its quantitative aspects. In addition to the components of calculation there are independent models which are related to the recall of factual knowledge and the carrying out of operations. All further aspects, such as knowledge of process and relation and operation signs, are part of it. With their model the authors intended to develop a basis for discussion and to display process mechanisms which could be repeatedly verified.

Various studies refer to this model (e.g. Cipolotti et al. 1994) and confirm its applicability. However the amodal representation of number is regarded as being critical. Is such a representation always undertaken while processing numbers? What kind of role do individual experiences play, bearing in mind the variations due to cultural differences? (see e.g. Campbell and Clark 1992; Cohen et al. 1994; Deloche and Willmers 2000; Seron et al. 1992).

Campbell and Clark (1992) predict modality-specific representations depending on one's learning background and therefore also on culture. Campbell (1994) believes that different ways of representing a task have direct impacts on the way the task is processed. Therefore he rejects the idea of direct information input by amodal representation as a specific way of processing a task. "Number meaning is not limited to quantity knowledge. We may know, for instance, that 16 is a power of 2 and that 17 is a prime number. We also possess encyclopaedic knowledge of some numbers such as 1914 or 1789. This suggests that the semantic representation in our model should eventually be completed by non-quantitative semantic features such as “power of 2”, “prime”, “famous date”, and so on (Dehaene and Cohen 1995, p. 86). Aspects like this are not considered in sufficient detail in McCloskey’s model.

Dehaene and Cohen (1995) proceed from the assumption that numbers can be represented in three different codes: a visual code related to Arabic numbers, a sound and verbal code referring to a pronounced or written representation of numbers, and a code connected with the magnitude of numbers. There are parallel ways of processing information. An input of information may cause a recall of meanings, but not necessarily in all cases, because there are relations between all the different kinds of representations.

According to this model, numbers can be processed or read without recalling the concept of magnitude.

Subtypes of arithmetic disabilities during the development of children

Classification tests have been used to ascertain whether special disabilities of neuronal functions are related to specific forms of arithmetic disabilities. Through various studies by Rourke (1993), groups of children with only arithmetic disabilities were distinguished from other groups which performed weakly in both arithmetic and language. Within both groups different neuronal disorders could be found. Different studies lead to the same observation: among children with arithmetic disabilities there is quite a large group having problems with arithmetic combined with linguistic deficiencies, and a large group with various neuropsychological problems. In addition there exists a small group of children showing exclusively arithmetic disabilities resulting from difficulties in processing visual information (Silver et al. 1999).

Classification tests criticized

It is claimed that classification using neuropsychological criteria means that even group classification with different methods of data collection leads to comparable classes (Fuerst and Rourke 1991). However, a study by Silver et al. (1999) throws doubt on this. They observed that after 19 months only a third of the children were still to be classified in the same group.

Attempts to teach mathematics

For adults a loss of factual knowledge and other mathematical competences are tested on a symbolic level. The analysis of children’s disorders deals with the construction of cognitive structures which model disorder in the acquisition of concepts.

Acquisition of mathematical knowledge is dependent on a variety of factors. Correspondingly, mathematics teaching favours a definition which attempt to understand arithmetic disabilities as a multi-causal phenomenon (e.g. Lorenz and Radatz 1993; Schmassmann 1990). Disorders of neuronal part-functions is only one field of problems among many others.

If one understands arithmetic disabilities as consequences of disorder in mathematical learning processes, it becomes clear that it is impossible to refer here to every important aspect. Questions about causes, diagnostic possibilities and attempts at help are central to most mathematics teaching problems (Lorenz 1991).

About causes

What kinds of disorder may occur during acquisition of mathematical knowledge? An analysis of the mathematical content is required. According to neuro-scientific models a child who mixes up one and two digit numbers cannot adequately understand or produce numbers. From the viewpoint of mathematics didactics this is insufficiently differentiated. Rather one should ask whether a child is able to group numbers, to put them together to form a new unit and to regard their order as a relevant fact. Changing the perspective by regarding a number as both a single and a composite entity is a cognitive prerequisite for the place value concept. Comparable considerations concerning various mathematical contents were collected by Gerster and Schulz (1998).

About diagnostics

Attempts at teaching mathematics are focused on children's cognitive competences which are examined in relation to requests dependent on age groups. To state arithmetic disabilities the children's complex learning conditions must be considered. Standardized performance or intelligence measuring as demanded for instance by the ICD-10 (WHO) are regarded very critically in the mathematics teaching debate (e.g. Lorenz and Radatz, 1993).

Example:

At the end of the first school year Anna's teacher was afraid that Anna would not be able to cope with the second school year's demands. Anna is quite withdrawn. Very often she seems to be absent-minded and not able to concentrate. Anna calculates up to 20 predominantly by counting very slowly and makes a lot of mistakes.

In this case neuropsychological approaches would ask whether there are disorders because of functional disabilities. Starting out from observable phenomena, mathematics teaching approaches first define the knowledge and perceptions of a child. What kind of previous knowledge does this child have? Does Anna know number word sequences? How does she handle materials? Does she know that the number of some given objects does not depend on their spatial arrangement? Which ideas of calculation operations did she develop? Does she recognise operative relations and is she able to use them? Subsequently it will be asked why certain knowledge was not acquired, and ways for initiating learning processes will be sought.

About supporting attempts

In mathematics didactics, children like Anna provoke questions such as how materials and representations must be put together to provide good preconditions for adequate learning processes (Lorenz 1992; Schipper 1982). For example, we ask them for their own interpretations of mathematical objects in order to get access to their thinking processes. However this is recognised as being interrelated with the individual preconditions existing in both the family and the school environment (Nolte 2000).

Can mathematics teaching use neuropsychological findings?

1. Modularity of psychic functions

The attempt to define the modularity of neuronal functions proves to be helpful in working with children. Knowledge about disabilities of partial functions may contribute to producing possibilities for compensation. For instance, children with weak visual perception often need special support to process visual information. The analysis of mathematical working processes also provide modular ways of viewing. If calculation operations are difficult to solve, it is necessary to find out exactly what a child can master and what it cannot. A child who has difficulties only with number orders but knows about addition and subtraction operations, needs special help only in distinguishing orders to be able to solve such tasks without mistakes.

2. Classification of children with arithmetic disabilities

The most important reason for classifying disorders is that it is possible to develop support strategies. However, classifications are also used for school career decisions. If there are simultaneous linguistic and arithmetic disabilities, this is frequently understood as a general learning disability. However, this position is disproved by the examinations of Rourke (1993).

Further problems:

1. Children may show arithmetic disabilities without underlying neuro-physiologic problems (v. Aster, 2000).
2. The existence of neurophysiological weaknesses does not mean that children develop arithmetic disabilities (v. Aster, 2000).
3. Depending on the extent of the reduction in the brain's plasticity, neuronal functions can be stimulated (Kolb and Whishaw, 1996).
4. The diagnosis of arithmetic disability in a person does not mean it will be a permanent personal characteristic (Silver et al. 1999).

3. Modelling representations of mathematical knowledge in relation to a child's learning process

Sabrina

Sabrina showed difficulties in reading and writing numbers, especially if they contained a zero. Numbers like 15 and 50 or 105 and 150 sound very similar and make high demands on her language processing (Nolte 2000).

Following the model of Dehaene (1999) this would be termed a disorder of number production. On a cognitive level this leads to the question whether the child has understood the construction of our number words. On a neuronal level it must be ascertained whether the assimilation of information is disturbed.

Conquering new number spaces aims to integrate numbers and their relations into a complex network. This includes quantitative imagination, imagination of order of numbers and their relations, operational possibilities of numbers, as well as reading and writing numbers.

A child who cannot always distinguish exactly between spoken number words like 15 and 50 will not be able to use pictures to develop quantitative imagination in the intended way. Sometimes they will assign the quantity of 15 objects and sometimes that of 50 to both number words. Hence the child does not obtain clear information and contents can have several meanings, although in mathematical terms the definitions are precise.

In accordance with the model this implies a disorder of quantitative imagination in the case of a special number word. The consequences for the production of numbers are as follows: Which number shall be written, 15 or 50? One sound in the child's experience will sometimes be assigned to the former, sometimes to the latter.

The child concerned then develops internal representations, based on their experiences, which cannot be restricted to a single place in the model, and which, with reference to mathematical contents, we must label as being inadequate.

This example demonstrates that when these problems are encountered by children they can be illustrated by the model. However, the interrelations between the processes of obtaining, giving out and processing, prevent disorders being assigned to specific areas of the model.

Concluding remarks

The results of neuroscientific research produce interesting models for representing mathematical knowledge in the brain. To some degree the idea of modules which can be isolated by neuroscientists give a strange impression to mathematicians because most of the differentiated actions of our brain are executed unconsciously.

It is not yet clear whether the findings described above from research with adults can be transferred to work with children, especially if, from a neuroscientific perspective, there are established conclusions. Thus, in mathematics teaching research, more profound analysis of the demands present in a learning process must be made.

References

Butterworth, B. (1999). The Mathematical Brain. London, Papermac.

Campbell, J. I. D. (1994). “Architectures for numerical cognition.” Cognition 53: 1-44.

Butterworth, B. (1999). The Mathematical Brain. London, Papermac.

Campbell, J. I. D. (1994). “Architectures for numerical cognition.” Cognition 53: 1-44.

Campbell, J. I. D. / Clark, J. M. (1992). Cognitive number processing: an encoding-complex perspektive. The Nature and Origins of Mathematical Skills. J. I. D. Campbell. Amsterdam, Elsevier Science Publishers B.V.

Cipolotti, L., Butterworth, B., et al. (1994). “From "One thousand nine hundred and forty-five" to 1000,945.” Neuropsychologia 32: 503-509.

Cipolotti, L. / de Lacy Costello, A. (1995). “Selective Impairment for simple division.” Cortex 31: 433-449.

Cohen, L., Dehaene, S., et al. (1994). “Number words and numer non-words. A case of deep dyslexia extending to arabic numerals.” Brain 117: 267-279.

Dehaene, S. (1992). “Varieties of numerical abilities.” Cognition 44: 1- 42.

Dehaene, S. (1999). Der Zahlensinn oder warum wir rechnen können. Basel, Boston, Berlin, Birkhäuser.

Dehaene, S. / Cohen, L. (1991). “Two mental calculation systems: a case study of severe acalculia with preserved approximation.” Neuropsychologia 29: 1045-1074.

Dehaene, S. / Cohen, L. (1995). “Towards an Anatomical and Funktional Model of Number Processing.” Mathematical Cognition I: 83-120.

Deloche, G. / Willmers, K. (2000). “Cognitive neuropsychological models of adult calculation and number processing: the role of the surface format of numbers.” European Child & Adolescent Psychiatry Supplement 2: 27-40.

Fuerst, D. R. / Rourke, B. P. (1991). Validation of Psychosocial Subtypes of Children with Learning Disabilities. Neuropsychological Validation of Learning Disability Subtypes. B. P. Rourke. New York, The Guilford Press.

Gerster, H.-D. / Schulz, R. (1998). Schwierigkeiten beim Erwerb mathematischer Konzepte im Anfangsunterricht. Freiburg im Breisgau, Pädagogische Hochschule Freiburg. Institut für Mathematik und Informatik und ihre Didaktik.

Grafman, J., Kampen, D., et al. (1989). “The Progressive Breakdown of Number Processing and Calculation Ability: A Case Study.” Cortex 25: 121-133.

Hittmair-Delazer, H., Sailer, U., et al. (1995). “Impaired arithmetic facts but intact conceptual knowledge - a single-case study of dyscalculia.” Cortex 31: 139-147.

Hittmair-Delazer, M., Semenza, C., et al. (1994). “Concepts and facts in calculation.” Brain 117: 715-728.

Kandel, E. (1996). Gehirn. Neurowissenschaften: eine Einführung. E. Kandel. Heidelberg, Spektrum.

Kolb, B. / Whishaw, I. Q. (1996). Neuropsychologie. Heidelberg.

Lorenz, J. H. (1991). “Rechenschwache Schüler in der Grundschule - Erklärungsversuche und Förderstrategien - Teil I.” Journal für Mathematikdidaktik 12: 3-34.

Lorenz, J. H. (1992). Anschauung und Veranschaulichungsmittel im Mathematikunterricht. Mentales visuelles Operieren und Rechenleistung. Göttingen, Hofgrefe.

Lorenz, J. H. / Radatz, H. (1993). Handbuch des Förderns im Mathematikunterricht. Hannover, Schroedel.

Lurija, A. R. (1992). Das Gehirn in Aktion. Einführung in die Neuropsychologie. Reinbek.

McCloskey, M. (1992). “Cognitive mechanisms in numerical processing: Evidence from acquired dyscalculia.” Cognition 44: 107-157.

McCloskey, M., Macaruso, P., et al. (1992). The Functional Architecture of Numerical Processing Mechanisms: Defending the Modular Model. The Nature and Origins of Mathemaitcal Skills. J. I. D. Campbell. Amsterdam, Elsevier Science Publishers B. V.

Neumärker, K.-J. (2000). “Mathematics and the brain: uncharted territory?” European Child & Adolescent Psychiatry 9: II/2-II/10.

Nolte, M. (2000). Fallbericht: Sascha - ein Kind mit Rechenschwächen. Integrative Lerntherapie. Ein Reader. M. Nolte, C. L. Naumann and i. K. m. d. F.-F. f. i. Lerntherapie. Hannover, Fachbereich Erziehungswissenschaften der Universität Hannover. Band 74.

Nolte, M. (2000). Rechenschwächen und gestörte Sprachrezeption. Beeinträchtigte Lernprozesse im Mathematikunterricht und in der Einzelbeobachtung. Bad Heilbrunn, Julius Klinkhardt.

Rourke, B. P. (1993). “Arithmetic disabilities, specific and otherwise: a neuropsychological perspective.” Journal of Learning Disabilities 26, No. 4: 214-226.

Schipper, W. (1982). “Stoffauswahl und Stoffanordnung im mathematischen Anfangsunterricht.” Journal für Mathematikdidaktik Heft 3.

Schmassmann, M. (1990). Dyskalkulie-Prävention im schulischen und ausserschulischen Alltag. Teilleistungsschwächen. Prävention und Therapie

HPS-Reihe 2. M. Brunsting, H.-J. Keller and J. Steppacher. Luzern.

Seron, X., Pesenti, M., et al. (1992). “Images of numbers. Or "When 98 is upper left and 6 sky blue".” Cognition 44: 159-196.

Silver, C. H., Pennett, H. D.-L., et al. (1999). “Stability of Arithmetic Disability Subtypes.” Journal of Learning Disabilities 32: 108-119.

Sokol, S. M., McCloskey, M., et al. (1991). “Cognitive Representations and Processes in Arithmetic: Inferences From the Performance of Brain-Damaged Subjects.” Journal of Experimental Psychology: Learning, Memory and Cognition 17: 355-376.

Temple, C. (1998). Developmental Cognitive Neuropsychology. Hove,UK, Psychology Press.

.von Aster, M. (2000). ”Developmental cognitive neuropsychology of number processing and calculation: varities of developmental dyscalculia.” European Child & Adolescent Psychiatry **9**: II/41-II/57.