Children's measurement thinking in the context of length

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This paper is concerned with the qualitative investigation of the pre- and post-conceptual knowledge of grade-2 students with respect to their ruler concepts and conceptions of the centimetre unit. The analysis of children's drawings of rulers and centimetres and their supporting explanations indicates that, by using a mental ruler, they already have concrete ideas of linear measurement thinking with different ways of understanding prior to formal instruction. Their knowledge and conceptions changed in different paths within a school year. Indeed some children interpret the measurement scale in a familiar arithmetical framework simply as counting, even after measurement has been formally introduced.

Introduction

Measurement of length is an integral part of our lives and an understanding of the process is fundamental for developing other higher measurement concepts such as area or volume and for understanding scales. Compared with the large number of publications that examine students' arithmetical concepts there are relatively few studies analysing measurement thinking in the context of length.

We owe the understanding of the development of children's length concepts primarily to the work of Piaget, who is widely recognized as the pioneer of research on measurement (Piaget et al., 1974). He showed that the concept of length measurement depends on the comprehension of the construction and co-ordination of linear units. In his opinion children must develop universal cognitive insights into the invariance and transitivity of length before they are able to understand the process of measuring. The studies of Hiebert (1984) and Schmidt & Weiser (1986) demonstrated that children acquire measurement ideas even if they fail in traditional Piagetian tasks. Today we know that cognitive development is based more on specific acquisition of knowledge than on universal cognitive structures. However, little appears to be known about the development of measurement thinking.

Some writers have questioned the significance of measurement for mathematics and the difficulties in understanding the measurement process. Wilson and Osborne (1992, p. 89) pointed out that "measuring is such a familiar task for adults that it is difficult to identify misconceptions about measure that cause children to perform poorly on measurement tasks." Those misconceptions relate for example to the relationships between units and sub-units, between length and area or to the application of formulas. Students' conceptions of length measurement contain a lot of unexpected aspects for adults. Their thinking and learning is based on situative- and context-specific experiences. It seems appropriate to take a closer look at children's conceptions and knowledge of the measurement process and to approach their concepts from their perspective.

Andrea, a seven year and ten months old student, described the process of mental measurement in the following way: "Firstly one has to look how long something is, and then one has to think about, how much, how big it is." For Andrea, measurement is similar to making a qualitative comparison a process of visual perception of objects. With the help of a kind of "length glasses" you focus on the one-dimensionality of length and see parts or objects as concrete representations of abstract units while fading out all the other attributes of the objects. But the measurement process implies a different basic concept. It starts with a question about 'how many?' and it requires knowledge of numbers in connection with units.

One has to figure out how many units fill the object to determine the length in a quantitative way. The knowledge and understanding of units and numbers are integrated into the process of linear measuring (Hiebert, 1984).

These ideas of the measurement process are based on three aspects fundamental to an understanding of length measurement:

1. *Choose a constant and convenient unit:* The unit must reflect the property being measured and is conserved across time and space—for example an elastic band could not be used as a unit. A concrete object like a paper clip or a pencil can represent a unit. These are informal measuring tools. Body-objects like steps or feet represent non-standard units too, but can be called intuitive or historical measuring tools because of their significance for younger children or in the history of measuring. Standard units are nowadays represented by the distances between marks on a conventional measuring tool like a ruler or a measuring tape.
2. *The measurement process requires the iteration or subdivision of units:* The unit must be applied repeatedly to the object or the object must be subdivided into equal parts. When there is no whole number that can fully cover the object, the unit must be systematically subdivided, so that the rest can be measured with smaller units. Furthermore, when measuring, one has to make decisions about the level of precision with regard to the last selected unit as well as the accurate alignment of the objects. Students have many problems understanding the relationships between units and sub-units. Frequently they are able to estimate the length of a 2m-string when they have a 2m-string in their hand. Difficulties arise when students estimate shorter strings and do not have the vocabulary to describe the fractional parts. For example, Leon, a grade 2 student, estimated correctly a 2m-string, but commented about a 50cm-string: "When that is one metre, this must be zero metres." The interviewer next asked about the length of a 25cm-string. Leon answered: "Oh, it must be no metres at all." Leon concentrated only on the reduction of the number ending with zero without thinking of the units.
3. During the measurement process one has to pay attention to the *number of units and sub-units and to the addition of the results* with respect to the inverse relationship between the size of the unit chosen and the number of units needed to measure a given object.

All in all, measurement relies upon the complex integration of space and number concepts with an understanding of iterative, dividable and countable units. These key ideas are employed when measuring tools like body-objects or informal tools and not on conventional tools. Measuring with a ruler requires *only* the technique of aligning the ruler and reading the scale. Bragg and Outhred (2000) as well as Hiebert (1984) emphasise that the correct use of the ruler does not indicate that students understand linear measurement in more complex tasks. Many students know the rules about using a ruler for simple measuring, but have obviously a poor understanding of the relationships between linear units, the measuring process and the formal scale of a ruler. A few studies indicate that children have already had experience with culturally developed measuring tools like a ruler and were more successful in using a ruler than in using informal tools like a piece of string. Furthermore, children apparently prefer to use a standard measuring device, if possible, even if they do not understand it fully or use it accurately. The attractiveness of the ruler leads to the assumption that children connect measuring with the use of a ruler because they possess one from their first school-day and usually see adults measuring length with rulers or measuring tapes (Boulton-Lewis et al., 1996, Nunes et al., 1993).

However, the ruler is more than a technical tool. It is a culturally developed tool with mathematical figures in an iconic illustration of the three fundamental ideas for the benefit of the user if he is able to read the signs on the measurement scale. Therefore a ruler has a concrete relationship to the reference context of measuring as well as a theoretical relationship to the representation of the measurement process (Steinbring, 1993).

As children use a ruler they construct individual understandings of length measuring and develop idiosyncratic concepts regarding the construction and co-ordination of marks, spaces and numbers. By investigating students' ruler concepts we can learn firstly about their measuring and drawing skills and secondly about their understanding of the concepts that underpin these procedures. For example, we can examine their knowledge and understanding of the relationship between the measurement of length and the number line represented on a ruler scale. This paper reports on an investigation into grade-2 students' understanding of the structural connections between linear units and the numbers used on a ruler. It examines how well the students interpret these symbols as a representation of the measuring system.

Methodology

The qualitative, longitudinal study underlying this paper is concerned with the investigation of the development of length measurement concepts of grade-2 students. Twelve children from an urban school in northern Germany were selected for the case study. Two girls and two boys considered being low, average and high achievers were interviewed three times during the school year. The first set of interviews was held at the beginning of the school year, the second a week later while the third set took place six months later. Classroom instruction followed the traditional sequence of measurement activities. The interview tasks were designed to include a variety of components of length measurement in different contexts. The interview items refer to three domains of measurement:

- Students' understanding of units like the centimetre or metre and the measurement process,
- Students' measurement strategies,
- Students' ruler concepts, their knowledge of ruler use and their drawings of the construction of the measurement scale represented on a ruler.

Instead of paper-and-pencil tests, practical tasks with measuring tools and objects were chosen because of their correspondence to measuring in real life contexts. The semi-standardized interviews were evaluated following the "interpretative paradigm" (Beck & Maier, 1993). The interview episodes referred to in this paper address the ways students think about the construction of a measuring scale and about their connections to conceptions of the centimetre unit. The two tasks reported here use students' iconic and verbal representations to answer the following question: How well do grade-2 students understand the measurement scale on a ruler as a symbolic representation of the measurement process? The tasks were:

- *Make a ruler picture and describe the numbers and marks*.
  The student was given a sheet of paper with the drawing of a rectangle—16.6 cm x 2.6 cm. The interviewer then said, "This is meant to be a ruler. Please fill in the parts that are missing in order to make it look like a typical ruler." After finishing the ruler picture the interviewer asked: "You have drawn numbers and marks. Can you tell me why you used those numbers and marks?"
- *Student understanding of a unit of measure- drawing a centimetre.*
  The student was told, "Please draw one centimetre on this sheet of paper."

Results

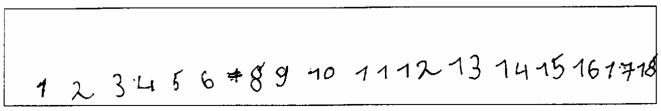
The analysis of ruler and centimetre pictures provides iconic evidence of students' interpretations of the structural relations of measuring and opens a window in students' measurement thinking. When children draw an object they consciously turn their attention to their mental images, organise their knowledge about the use and construction of the object and focus on those structural characteristics that are significant for them (Biester, 1991). The interpretation of the pictorial representations, which are supported by verbal explanations, offers the basis for the analysis of the ways in which students conceive and apply their length concept.

Examples of the construction and explanation of ruler pictures

All the interviewed students were able to construct ruler-pictures and to represent significant characteristics of the measurement scale. Before classifying the different ruler pictures, the construction of two ruler-pictures typical of student thinking at the beginning of grade 2 will be discussed.

- Regina (eight years four months) was a low achiever. She drew a *Number-Ruler*:

I Do you know what a ruler is?
R Yes.
I Where have you seen one before?
R My brother has one.
I What can you do with a ruler?
R You can, hmm, (*10 sec.*) if something is lying here, you could draw along it.
I This is meant to be a ruler *(the empty rectangle is placed on the table)*, but something is missing.
R The numbers *(looks towards I.)*
I Can you draw what is missing so that it looks like a proper ruler?
R *R. starts writing the following numbers – note the space left in the bottom left corner.*



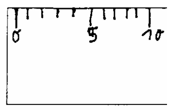
I Finished?
R *(shakes her head)* Still more, still more numbers.
I What could the numbers go up to on the ruler?
R *(6 sec.)* Up to 20.
I Does a ruler always go up to 20?
R *(nods her head)* Sometimes, my brother has one up to 30.
I Mmm, you have written numbers here *(points to the written numbers).* What are these numbers for?
R *(7 sec.)* You can use them for calculating.
I Mmm, what can you calculate with those numbers?
R *(12 sec.)* I don't know.

It can be seen that Regina had only vague, subjective domains of experiences (Bauersfeld, 1983) with a ruler. She knew about a ruler from her brother and connected the ruler with a drawing situation without any reference to measuring. She referred immediately to the numbers and constructed a connection to her arithmetical and counting skills. She interpreted the linearly ordered numbers in her conception of numbers and started the number line with "one" as the first counting number. She paid attention to the number aspect not to the space. It is interesting that she knew different rulers - the last number gives information on the quantity of numbers or on the size and the name of the ruler - but did not mention units at all. She could only see the significance of the numbers in a calculating context.

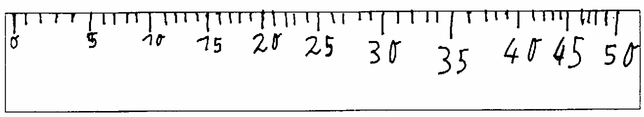
Rulers like this one are called *Number-Rulers* because of the exclusive interpretation of the scale in an arithmetical sense without any visible connection between numbers and units. Students connect the linearly ordered numbers of a ruler with their conceptualisation of numbers. This seems obvious because, in general, numbers are mentally internalised in an intuitive way as a number line from left to right (Lorenz 1998). Children in the first interview typically drew these types of rulers, (see Table 2).

- Frank (eight years four months) was an average achiever. He drew a *Number-Mark-Ruler with a five-fold subdivision*:

I Do you know what a ruler is?
F Yes *(3 sec.)* from the work and, hmm, from home.
I What can you do with a ruler?
F Mmm, measuring, how big something is. I don't know more.
I This is meant to be a ruler, but something is missing. Can you draw what is missing so that it looks like a proper ruler?
F *F. starts writing the following numbers and marks, stopping before writing the number "2".*



No
He changes the number "1" into a "5" and writes a "10" at the final mark.
He draws the following picture with numbers increasing in size:



F. stops writing and looks at his ruler picture for 55 sec.
I Can you explain this to me, what are you thinking about?
F I'm thinking about how many, hmm, what a ruler looks like.
He looks at his ruler for another 4 sec., then at the Interviewer.
I You have drawn numbers and marks. What are these numbers for?
F 1, 2, 3, 4, <u>5</u>, 6, 7, 8, 9, <u>10</u>, 11, *(... up to 50)* <u>50</u> *(points at each mark)*
I Does a ruler always go up to 50?
F Hmm, sometimes up to 16, hmm, to 20, 30, my father has one up to 100.
I Mmm, and what are these marks on a ruler for?
F They say that there is a one, that one is, two, three, four and then there is a five.
I What does it mean, when there is a five *(points to"5")*.
F If you have to measure, if you want to measure a little stone, you know, how much it is, how big it is.
I Ok, and how big is the stone going up to five?
F Then it is five metres big.

Frank had a meaningful domain for understanding a ruler (Bauersfeld, 1983). He interpreted the ruler as a measuring tool similar to a folding rule or a measuring tape. His ruler picture begins with zero. This could indicate that he understood zero as the starting point of measuring. He focused, just as Regina did, on the last number of the ruler. He knew different rulers and it seems that he had a vague and qualitative understanding of different ruler lengths. It is possible that his lengthy pauses after finishing the picture are because his conceptualisation of a ruler up to 50 did not correspond with his ruler picture.

Although Frank's understanding of a ruler was limited, he was beginning to incorporate some of the fundamental ideas of length measurement. He started with two regular units with a five-fold subdivision, indicating that he was beginning to understand the subdivision between the 1-centimetre and 5-centimetre mark. He was also aware of the need for equal-sized intervals. Furthermore, he explained that the number of marks referred to a measurement. However, the development of his drawing and his explanations show the dominant interpretation of numbers as counting. Frank corrected the construction of the scale and counted each mark as a non-dividable unit. In this sense the number showed "how *much* it is". His understanding remained within an arithmetical framework.

Rulers like this one are called a *Number-Mark-Ruler* because each mark is counted without a deeper meaning of a subdivision of units. On the one hand this type of ruler picture is an illustration of an arithmetical interpretation of the ruler as a number line. However, on the other hand, the inclusion of zero and the spatial distribution of numbers and marks in particular, indicates an emerging interpretation of centimetre as a dividable unit. Some of the students counted a certain number of marks (or units) and then wrote numbers in the correct counting sequence. Therefore, few ruler pictures showed individual 'sense constructions' of unit subdivision on the scale. For example, Frank's ruler picture contained some elements of a measurement understanding. His use of the word "metre" reinforced this interpretation. He obviously knew that numbers are connected with unit-words, but he did not realize the quality of that connection—he used the word 'metre' just like a number companion that refers to length.

Classification of ruler pictures

It was found that children's drawings of rulers may be assigned to one of four categories. The classification is based on the construction and co-ordination of numbers and marks. They are the Number-Ruler, the Number-Intermarks-Ruler, the Number-Mark-Ruler and Unit-Ruler (Nührenbörger, 2002). No children drew a ruler picture using only the marks - that would be the opposite of a *Number-Ruler*.

Children who drew *Number-Intermarks-Rulers* (see Table 1) understood the marks to be a key aspect of the measurement scale. They are only located between the numbers without any visible subdivision into units. Marks therefore have a personal meaning as some were counted and noted in the familiar arithmetical context. For example, 3 marks possibly stand for the 3 long marks around the cm and 4 marks possibly stand for the short marks between two longer marks. The intervals between the numbers were constantly equal because the marks were counted and noted rhythmically. It may be also possible that the students recognise that a ruler should have marks at regular intervals, but are unable to explain their structural significance in connection with conventional units.

Although the ruler pictures have the same intervals and look like a computer-ruler, the verbal explanations of the children indicate that they did not recognise units or the relationships between units and sub-units. The measurement scale is interpreted arithmetically as counting marks. The marks may be seen as an insignificant companion to numbers that are printed on the ruler. There is, however, another factor that should not be ignored. Many children started their number sequence with "1" and used the ruler in a measurement situation like a thermometer. They only paid attention to the last number without respect to the zero. But the role of zero is important for an understanding of rulers.

Students' drawings of *Unit-Rulers* (see Table 1) contained a connection between number and space concept that led to the idea of an iterated and subdivided unit. The students showed different facets of understanding of units, both for equal intervals and for rules of subdivisions. Most of the students drew a five-fold subdivision with regular intervals. The starting point of the scale was particularly important. A ruler with starting point '1' indicates an orientation on the sequence of number words. In contrast, a ruler beginning with '0' or starting right at the edge shows that the drawer recognizes that '1' represents the end of the first unit.

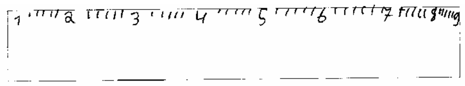
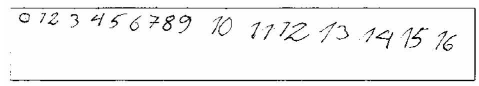
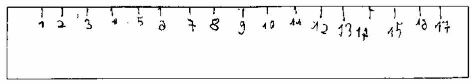
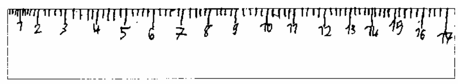
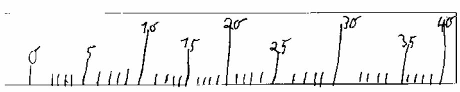
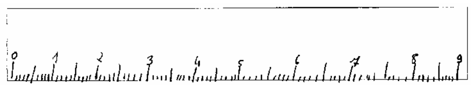
Number-Ruler	*Number-Intermarks-Ruler*
	
Number-Mark-Ruler	*Unit-Ruler*
	
	

Table 1: Examples of types of ruler pictures with different chief characteristics

The analysis of the ruler pictures suggests that at the beginning of grade 2, children have already constructed ruler concepts with different ways of understanding the key measurement ideas. Their ruler pictures indicate that:

- they are able to imagine a ruler and to perceive key aspects of measuring,
- they have had subjective experiences in daily life with standard units and tools,
- they have constructed different understandings of the key ideas of length concepts,
- some children interpret the measurement scale only in an arithmetical sense as an illustration of numbers, and
- each child drew both long and short marks instead of using single marks for 1 cm to divide the scale into equal units.

Paths of development of ruler pictures

In spite of the relatively small sample of 12 students there is a clear diversity of ruler concepts with respect to their drawings prior to formal instruction (see Table 2). It is remarkable, for grade-2, that four students could draw a Unit-Ruler, one of them with a ten-fold subdivision. This research shows that ruler concepts not only vary among children but also change and develop along different paths during a school year!

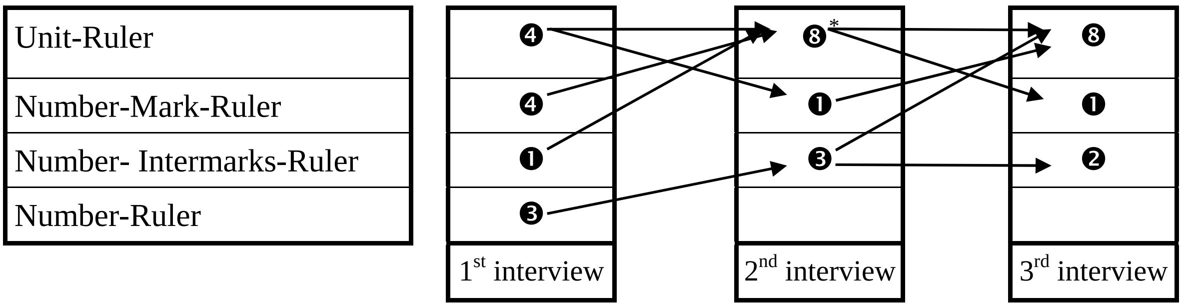


Table 2: Paths of development of the ruler pictures (❸ number of children–*one child was absent in the 3rd interview)

The development of the ruler pictures indicates a growing orientation towards a structural construction of the measurement scale. From the first interview to the second and third, two different paths of development may be recognized:

- from the Number-Ruler to the Number-Intermarks-Ruler and
- from the other three ruler types to the Unit-Ruler with a fivefold subdivision.

It is remarkable that even directly after the measurement unit none of the students was able to draw a correct ruler and that three ruler pictures still referred to an arithmetical number concept. The analysis of the third interview shows that the majority of children stabilized or developed further structural understanding of rulers in the absence of formal instruction. For example, two students show a development path from a five-fold subdivision Unit-Ruler to a ten-fold one, while two other students continued to draw a less sophisticated Number-Intermarks-Ruler.

It is amazing that after the formal instruction on the basic unit of measure, the children rarely used 1cm divisions on their scale.

Only two students in the third interview varied the successive drawing process and first represented the unit centimetre. For example, Franz noted after “3” firstly “4” and a mark - see Table 3a and 3b - before he subdivided the unit - see Table 3c.

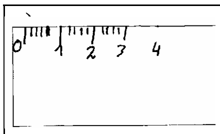
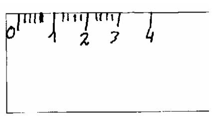
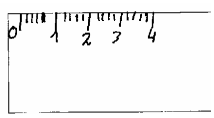
	b) 	c) 

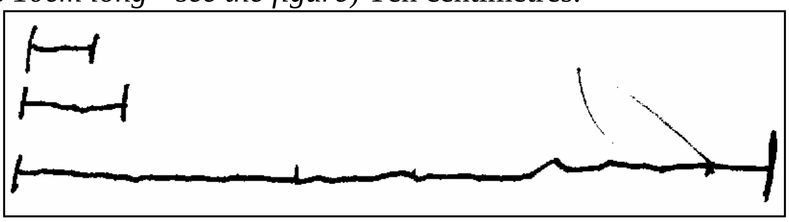
Table 3: Students drawing process using conceptions of 1cm

Examples of students’ drawings of a centimetre and their explanations

Before classifying the different drawings, two examples, typical of students at the beginning of second grade, will be discussed.

- Leon (eight years) was an average achiever. He thought of one centimetre as the linear distance you get when you count the five short gaps between the millimetre-marks of a measurement scale:

I Please draw one centimetre on this sheet of paper.
L One centimetre? (*L. looks for a moment to the I. and starts drawing a segment*) One, two, three, four (*he connects 5 segments to a line 0.9cm long—see the figure*).
I I heard you talking quietly. What did you say?
L I counted.
I What did you count?
L Very small lines, how many lines there are between.
I Please draw a line that is two centimetres long.
L One, two, three, four, five, six, seven, eight, nine (*L. draws a line with five and four segments—the line is 1.4cm long—see the figure*). I have counted again.
I Mmm, please draw a line that is ten centimetres long.
L Ooh, one, two, three, four, five (*1 sec.*) one, two, three, four, five, one, two, three, four, five, mmm? Three (*stressed*) one, two, three, four, five, one, two, three, four, five. (*L. draws a line with short segments*) How many have I? Four? One, two, three, four, five … (*L constructs his line with six short lines, each one with five segments—the line is 10cm long—see the figure*) Ten centimetres.

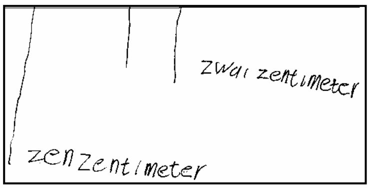


I Can you draw a line that is exactly one millimetre long?
L Millimetre? I don't know.

Leon used his mental picture of the measurement scale and counted each gap between the millimetre-marks. His drawing of a centimetre was mostly related to his personal experience of a "ruler". For this reason he was unable to apply an appropriate strategy for the construction of a 10-centimetre line. It is interesting that even though he used a five-fold subdivision, he did not understand each segment to be a linear representation of either a millimetre or a centimetre. In fact he considered that each segment was a "distance-holder" ensuring that every centimetre stays the same length.

- Mats (eight years three months) was a low-achiever in mathematics. He thought of a centimetre in an arithmetical framework. He had no conventional representation of centimetres.

I Please draw one centimetre on this sheet of paper.
M One centimetre?
I A line which is one centimetre long.
M OK. I will try it (*1 sec.*) one centimetre (*15 sec.*) No, I don't know how big a one centimetre line is.
I What do you think approximately?
M *M. starts drawing a line 3cm long—see the reduced figure.*
I Please draw a line that is two centimetres long.
M Ooh! (*He draws a line 3.6cm long—see the figure*)
I Please write the length of the line.
M *M. writes the words "two centimetres"—see the figure.*
I Please try to draw a line that is ten centimetre long.
M Ooh! (*He draws a line 7.5cm long—see the figure*)
I Please write the length of the line again.
M *M. writes the words "ten centimetres"—see the figure.*



Mats' drawing is similar to that of three other children in the first interview. Although he was uncertain about drawing lines of a given length, he was able to construct vague qualitative drawings with the help of the interviewer. Mats did not understand the key ideas of length measurement. The relative size of the number determined the length of the line so, in effect, the number did not refer to the number of units. In this sense the unit played only the role of a 'number companion' referring to length. In spite of this, Mats showed that he understood the significance of the number both in a length context and also in an arithmetical context by writing the number word.

Classifications of the conceptions of the unit centimetre

In the first interview there were no children using a concrete referent such as a fingernail or pencil-width for a centimetre. Although two students were able to draw a centimetre line, they were unable to demonstrate an understanding of what the line really was. They had a qualitative *conceptualisation* of a centimetre that refers to a *short line*.

Most of the students used a *mental ruler* as an internal measurement tool. “This is not a static image, but a mental process of moving along an object, segmenting it and counting the segments. … This is a critical point in their development of measurement sense” (Clements, 1999, p.8). Obviously, the subjective domain experience of a “ruler” is largely based on a “centimetre” (Bauersfeld, 1983). However, the use of the ruler-like representations varies between the children:

- Most of the students focused on the distance between the two longer marks and drew an equivalent line.
- Few students focused on the linear distance of the number of intermarks as did Leon.
- Only one child represented 1cm similar to a scale (see Figure 1). This form of drawing appears to an “area representation” (Bragg & Outhred, 2001, p. 214): The student indicated the marks delineating a unit similar to a measurement scale but had a representation of the unit itself that could be two-dimensional.

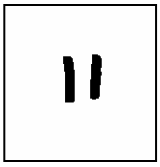


Figure 1

Four students were not able to construct a picture referring to length. Like Mats, their conceptualisation of linear measurement was based on an arithmetical interpretation – the size of the number referring to the size of the line.

Paths of development in the drawings of the unit centimetre

Children’s ideas about the centimetre are just as varied as their ruler pictures (see Table 4) and also change and develop in different ways during a school year.

No connection concretely to a referent representation	- precise
Concrete connection to a referent representation	- precise - vague
Connection to a ruler-like representation	- precise - vague
No conventional representation	- vague - no idea

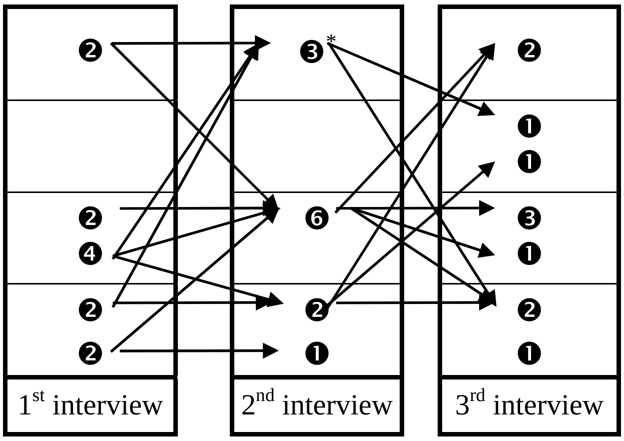


Table 4: Paths of development of the cm-conceptions (❸ number of children–*one child was absent in the 3rd interview)

The development shows an improvement in accuracy. This means that lengths of the drawings differed not more than 30% from 1cm. The quality of the precision varied within the school year whereas the quality of the representation was largely stable. It is amazing to note that directly after the measurement unit none of the students used a concrete referent in a representation except in the third interview. Moreover there were three children who were still unable to produce a correct ruler and three ruler pictures were based on an arithmetical interpretation. It is significant that many children used their mental ruler. Obviously, the ruler has a central and a stable significance to the children in grade-2 as a reference for a representation of centimetres. Therefore it is remarkable that all students were able to use a ruler both as a concrete model and also as the preferred tool for measurement and drawing prior to formal instruction (Nührenbörger, 2002).

Conclusion

The analysis of the interviews offers insights into the variety of students' measurement thinking and the diversity of the paths of development. At the beginning of grade-2, children already have a subjective understanding of the measurement scale. They are able to imagine a ruler and to perceive key aspects of measuring. Their everyday experiences with measurement not only promote an intuitive approach to the technical use of rulers, but also influence the development of individual ruler concepts.

However, this diversity of understandings does not necessarily indicate a deeper understanding of the relations between linear units and the measurement scale. Although students are able to measure with a ruler, they often do not possess structured insights into the key ideas of measurement. Instead they connect their beliefs about measuring with their number concept and interpret some of the measuring aspects in the context of their arithmetical understanding of numbers and counting. These results support the findings of Hiebert (1984) and Bragg and Outhred (2000, 2001) that many children are learning the procedures of ruler use but not the underlying relations. They do not have a clear understanding of the measurement scale and representation of a centimetre.

The study has shown that grade-2 students have idiosyncratic conceptualisations of rulers as concrete and mental tools in the context of length. According to this, rulers can be used as a tool for measuring and as a visualization for the *key ideas* of length measurement (Nührenbörger, 2001). Tools are an essential resource and support for building measurement understanding. The tools children use influence the kinds of understanding they develop (Hiebert et al., 1997). Understanding of the construction of the measurement scale is a basis for the knowledge of the relation between measurement of length and number lines. In addition, Bragg and Outhred (2000) pointed out:

> If students do not understand how scales are constructed, they will not have the basic knowledge to relate measurement of length and number lines, nor have the foundation to develop area, volume and other higher order mathematical applications (p. 103).

Ruler pictures could be used in classrooms as:

- a *diagnostic tool* for the teacher to identify students' pre-conceptions,
- a *starting point* for classroom discussions in order to reflect upon subjective measurement experiences and ideas about how scales are constructed and the importance of zero (e.g. its varying position on different measurement tools) and,
- a help for constructing *mental rulers* that can be used for estimating.

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