Chiral Phase Transition in the scaled $O(4)$-Model

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Due to the non-Abelian character of QCD gluons self-interact and form bound states, so-called glueballs. Such glueballs have been seen in recent lattice simulations and are actively searched for in experiment. Glueballs can be used to construct effective models of QCD which respect the symmetries and anomaly structure of the theory.

At the classical level and in the limit of vanishing quark masses QCD for n flavors exhibits a global chiral $U(n)_L \times U(n)_R$ symmetry and is in addition invariant under scale transformations. Due to anomalies not all of the associated currents are conserved and the symmetry is broken down to $SU(n)_L \times SU(n)_R$ which, in the case of two flavors, is isomorphic to $O(4)$. This symmetry is spontaneously broken to $SU(n)_{L+R}$. The divergence of the anomalous scale current is given by the trace of the energy-momentum tensor which, in the limit of massless quarks, is given by

$$\langle \theta^{\mu}_{\mu} \rangle = \langle \frac{\beta(g)}{2g} G^a_{\mu\nu}(x) G^{a\mu\nu}(x) \rangle \ , \tag{1}$$

where $G^a_{\mu\nu}(x)$ denotes the gluonic field-strength tensor and $\beta(g)$ is the usual QCD beta function. An effective realization of the scale anomaly can be achieved by adding to the classical Lagrangian a scalar color singlet dilaton field χ with an interaction potential of the form

$$V(\chi) = h \left(\frac{\chi}{\chi_0} \right)^4 \left(\ln \frac{\chi}{\chi_0} - \frac{1}{4} \right) \ , \tag{2}$$

where h is a constant that is related to the vacuum energy density ε_{vac} via $h = -4\varepsilon_{vac}$, when there are no quarks. The potential has a minimum at $\chi = \chi_0$.

In a previous work [?] we have tested a novel renormalization group approach to investigate chiral symmetry restoration at finite temperature and could analyze the critical behavior at the chiral phase transition of the $O(N)$-model. In this work we investigate the influence of the additional dilaton field on the chiral phase transition and the critical behavior. Therefore we couple a massive scalar dilaton field, which breaks the scale invariance, to the $O(4)$-model. Here we follow here the work in [?] and consider the following Lagrangian

$$\begin{aligned} \mathcal{L} &= \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma + \frac{1}{2}\partial_\mu \vec{\pi} \partial^\mu \vec{\pi} + \frac{1}{2}\partial_\mu \chi \partial^\mu \chi \\ &\quad - \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - \frac{\chi^2}{\zeta^2} \right)^2 - V(\chi) \ , \end{aligned} \tag{3}$$

where σ, $\vec{\pi}$ denote the sigma- and the pion fields respectively.

Lattice calculations hint that the lightest glueball has a mass of 1.3 - 1.6 GeV. We use this mass as a constraint to fix the parameters of the model at $T = 0$. We then perform a finite-temperature calculation, where we calculate the vacuum expectation value (VEV) of the meson fields and the critical exponents of the chiral phase transition. From a comparison with the $O(4)$-model calculation without the dilaton field we can then estimate the influence of the dilaton field on the chiral phase transition.

The temperature dependence of the scalar mesonic VEV $\langle \phi \rangle$ (cf. Fig. 1) is very similar to the temperature dependence in the pure $O(4)$-model calculation without the dilaton field. Around T_c we again obtain a scaling behavior of the VEV $\langle \phi \rangle$ and of the mesonic coupling constant λ with critical exponents $\beta = 0.39$ and $\nu = 0.79$.

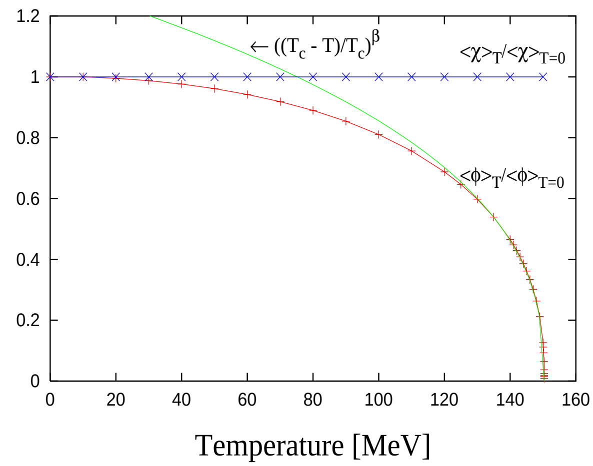

Figure 1: The temperature evolution of the vacuum expectation values of the dilaton field $\langle \chi \rangle$ and the scalar meson field $\langle \phi \rangle$ in the chiral limit.

These values of the exponents coincide within the estimated numerical error bars with the pure $O(4)$-model values.

On the other hand the glueballs themselves change very little in the temperature region up to the chiral phase transition. The change of the mass as well as of the VEV of the dilaton field (cf. Fig. 1) is less then 0.1% in this region. Calculations within the framework of a pure dilaton model show that the glueballs begin to be modified considerably at temperatures around 250 MeV.

In summary we can conclude that the glueballs, due to their high mass of ≈ 1.5 GeV, have very little influence on the temperature evolution in the mesonic sector where we still find a second order chiral phase transition with $O(4)$ critical exponents.

References

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[2] H. Gomm, J. Schechter, Phys. Lett **B158** (1985) 449; E. K. Heide, S. Rudaz, P. J. Ellis, Phys. Lett. **B293** (1992) 259.