Exact Conservation Laws in the Gradient Expanded Kadanoff–Baym Equations[G]

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One of the challenging problems in quantum many-body physics is the appropriate inclusion of resonances or particles with broad damping width into a self-consistent non-equilibrium dynamics. Such kind of description can be constructed on the basis of the so called Kadanoff–Baym equations (KBE) derived within the non-equilibrium Green function technique [1]. While the KBE are exact, in actual calculations one has to rely on further approximations. They provide (a) a truncated self-consistent scheme and (b) through the gradient approximation they lead to transport type of equations of motion. Interested in the dynamics of particles with broad mass width one has to avoid the quasi-particle approximation, and solely rests on the first-order gradient approximation of the KBE. This concept was first addressed by Kadanoff and Baym [1] and recently reconsidered in the context of hadronic matter and heavy ion collisions [2–7]. As for any such approximation, however, symmetries and conservation laws as well as detailed balance and thermodynamic consistency may no longer a priori be guaranteed.

In ref. [8] we re-investigated a generalization of the Φ derivable method of Baym [9] to the real-time Green function technique which provides truncated self-consistent approximations which are conserving and thermodynamically consistent at the level of KBE. In particular a conserved energy-momentum tensor could be derived for local field couplings. The subsequent gradient approximation leads to two coupled equations: a quantum transport equation, which governs the four-phase distribution functions $f(\vec{x},t,p)$, and a retarded equation, which determines the time evolution of the spectral function $A(\vec{x},t,p)$. For this approximate set of equations the conservation laws generally are expected to become only *approximate*. Such approximate nature of conservation laws may be well acceptable theoretically as its accuracy precisely corresponds to that of the approximation. Nevertheless, both from a principle perspective and also from a practical point of view this situation is less satisfactory.

In this work [10] we investigated the quantum kinetic equations in the form originally derived by Kadanoff and Baym. The key point is to do a systematic first-order gradient expansion of all gradient terms even those internally present in the self-energies. Through a careful investigation of all gradient terms we could in fact prove that the quantum kinetic equations possess the generic feature of exact conservation laws at the expectation value level.

The conserved currents and the energy-momentum tensor take the original Noether form [8] ($X=(\vec{x},t)$)

$$J^\mu(X) = \int \frac{\mathrm{d}^4p}{(2\pi)^4} p^\mu f(X,p) A(X,p), \qquad (1)$$

$$\Theta^{\mu\nu}_{\rm loc}(X) = \int \frac{\mathrm{d}^4p}{(2\pi)^4} v^\mu p^\nu f(X,p) A(X,p) + g^{\mu\nu}\left(\mathcal{E}^{\rm int}_{\rm loc}(X) - \mathcal{E}^{\rm pot}_{\rm loc}(X)\right) \qquad (2)$$

now however in the so called local form, i.e. void of any gradient corrections. For the energy–momentum tensor the first term accounts for the single particle part which by itself overcounts the interaction energy. This is compensated by gradient terms which assemble to the difference between interaction energy density and single-particle potential energy density, $\mathcal{E}^{\rm int}_{\rm loc}(X) - \mathcal{E}^{\rm pot}_{\rm loc}(X)$, both obtained from the same Φ-functional in the local approximation as the self-energies driving the gradient expanded KBE.

In order to preserve the exact conserving property, a few conditions have to be met. First, the original KBE should be based on a Φ-derivable approximation scheme that guarantees that the KBE themselves are conserving [8, 9]. Second, all possible memory effects due to internal vertices within the self-energy diagrams are also consistently expanded to first-order gradients. Finally it is important that after the gradient expansion no further approximations are applied that violate the balance between different first-order gradient terms.

The presence of exact conservations puts the Kadanoff–Baym formulation of quantum transport on the level of generic phenomenological equations. They offer a phenomenological approach to the dynamical description of particles with broad damping widths, such as resonances, with built-in consistency and exact conservation laws. For practical simulations of complex dynamical systems this approach may even be applied in cases, where the smallness of the gradients can not always be guaranteed.

References

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