Quarkonium at Collider Energies^{B+G}

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There are different time scales relevant for the production of quarkonium states:
1) the time needed to produce a heavy quark pair in a hard collision,
2) the time needed for a $Q\bar{Q}$ pair to form a bound state.

The production time of a $Q\bar{Q}$ pair in its rest frame is given by $\tau_p = \frac{1}{m_Q}$. This is 0.13 fm/c for $c\bar{c}$ and 0.05 fm/c for $b\bar{b}$. The Lorentz factor of the pair at midrapidity in the rest frame of the target is $\gamma \approx 10$, 100 and 3000 for a Quarkonium state at SPS, RHIC and LHC energies. At SPS fixed target energies $\gamma c \tau_p$ is smaller than the average internucleon distance in nuclei $r_{NN} \approx 1.8$ fm. Thus, the production of heavy quark pairs is incoherent. At RHIC the production distance of $c\bar{c}$ pairs is already as large as the diameter of a gold nucleus, and for $b\bar{b}$ pairs $c\tau_p > 1.8$ fm, but this is still small as compared to the nuclear radius. At LHC both production distances exceed the diameter of a lead nucleus by an order of magnitude. The hadronisation time t_H resp. the coherence length l_c of heavy Quarkonium is $l_c = c \cdot t_H = \frac{1}{\Delta E} \approx \frac{\gamma}{\Delta M}$ with:

$$\Delta E = \sqrt{p^2 + (M_{Q\bar{Q}} + \Delta M)^2} - \sqrt{p^2 + M_{Q\bar{Q}}^2} \approx \frac{(M_{Q\bar{Q}} + \Delta M)^2 - M_{Q\bar{Q}}^2}{2p} \approx \frac{M_{Q\bar{Q}} \Delta M}{p} = \frac{\Delta M}{\gamma}.$$

p is the momentum of the Quarkonium in the rest frame of the target, $M_{Q\bar{Q}}$ is the mass of the $Q\bar{Q}$-pair and $\Delta M = \int \psi^2(k) \frac{k^2}{M_Q} \mathrm{d}^3k / \int \psi^2(k) \mathrm{d}^3k$, $\psi(k)$ is the wavefunction of the Quarkonium state in momentum space, we use here the wave functions from the refs. [1, 2]. ΔM is the average kinetic energy of the $Q\bar{Q}$-pair in the bound state and $l_c/\gamma = 0.44(0.34)$ fm for the J/Ψ (Υ).

Thus, for charm and bottom production at RHIC and LHC $l_c > 2 \cdot R_A$ (R_A is the nuclear radius), $l_c < 2 \cdot R_A$ for fixed target SPS energies. The applicability of the approach developed in this paper requires that $l_c > 2 \cdot R_A$ which is fulfilled at RHIC and LHC.

We assume here that $Q\bar{Q}$ pairs are produced in AB collisions predominantly in hard collisions. The basic quantity is the cross section of production of $Q\bar{Q}$ pairs with light cone momenta z_i, k_i, which we parametrize as $\frac{d\sigma(AB \to Q\bar{Q}+X)}{\mathrm{d}^2k_1 \mathrm{d}z_1 \mathrm{d}z_2 \mathrm{d}^2k_2} = D_{AB}(z_1, z_2) \cdot \exp(-B(AB)(k_1^2 + k_2^2))$. Here k_i $\{i = 1, 2\}$ are the transverse momenta of the Q and the $\bar{Q}$ quark and z_i $\{i = 1, 2\}$ are the fractions of their longitudinal momenta. Such a factorization does not contradict the data in pp collisions [3].

To evaluate the suppression of hidden heavy flavour production resulting from the broadening of the transverse momentum distributions of Q quarks due to final state interaction, we deduce first a relationship between the slopes for the various processes of heavy quark production. In the following we use the relative transverse momentum $k_t = \frac{k_1 - k_2}{2}$ and the total transverse momentum $p_t = k_1 + k_2$ of the pair, writing $\frac{d\sigma(AB \to Q\bar{Q}+X)}{\mathrm{d}^2k_t \mathrm{d}^2p_t \mathrm{d}z_1 \mathrm{d}z_2} = D_{AB}(z_1, z_2) \exp\left(-B(AB)\left(-\frac{p_t^2}{2} - 2k_t^2\right)\right)$.

To take into account possible nuclear effects on the longitudinal momentum distribution we make the ansatz $D_{AB}(z_1, z_2) = D(AB) \cdot f_{AB}(p_z) \cdot \exp\left(-\frac{k_z^2}{C_{AB}^2}\right)$, where p_z and k_z are the total and relative longitudinal momentum. We further assume that $f_{AB}(p_z) = f_{pp}(p_z)$, which means that we neglect parton energy losses of the pair, this effect will be discussed later on. The normalization condition follows from the QCD factorization theorem [4] for the total cross section: $\frac{D(AB)}{B(AB)^2 \cdot C_{AB}} = \frac{AB \cdot D(pp)}{B(pp)^2 \cdot C_{pp}}$ The differential cross sections are proportional to the square of the two body wave functions ϕ.

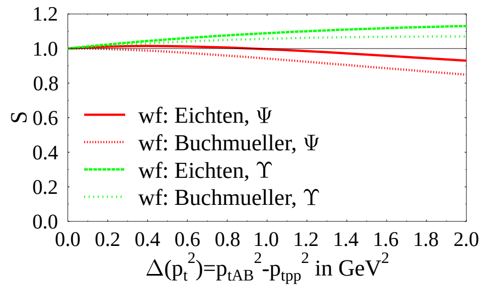

Figure 1: The righthandside of eq.(1) vs. the transverse momentum broadening for the J/Ψ and the Υ is plotted.

The production cross section of bound states of heavy quarks is proportional to the overlap integral of the two-body wave function and the wave function of the bound $\psi(k_t)$ state to get $\frac{\mathrm{d}^3\sigma(AB \to Quarkonium+X)}{\mathrm{d}^2p_t \mathrm{d}p_z} \propto |\langle \psi(k_t, k_z) | \phi_{AB}(k_t, p_t, k_z, p_z) \rangle|^2$. Here we neglected the difference between the current quark mass in the two body wave function and the constituent quark mass in the wave function of the bound state. With this one can evaluate the survival probability: $S \equiv \frac{\sigma(A+B \to Quarkonium+X)}{AB \cdot \sigma(p+p \to Quarkonium+X)}$. Our final result is then

$$S = \frac{B(AB)C_{AB}}{B(pp)C_{pp}} \left| \frac{\int \mathrm{d}^3k \psi(k) \exp(-B(AB)k_t^2) \exp\left(-\frac{k_z^2}{2C_{AB}^2}\right)}{\int \mathrm{d}^3k \psi(k) \exp(-B(pp)k_t^2) \exp\left(-\frac{k_z^2}{2C_{pp}^2}\right)} \right|^2 \quad (1)$$

up to nuclear effects in the parton distribution functions.

Note that if one defines the survival probability as the ratio of the differential cross section $\frac{\mathrm{d}^2\sigma}{\mathrm{d}^2p_t}$ for nuclear and nucleon targets their p_t dependence would be a factor $\exp\left(-\frac{B(AB)-B(pp)}{2}p_t^2\right)$. That means that the p_t dependence of J/Ψ suppression is due to the broadening of the transverse momentum distribution as a result of the final state interactions of the Q quarks in the nuclear medium.

In fig. 1 the result of eq. (1) is plotted versus the transverse momentum broadening of the J/Ψ's: $\Delta p_t^2 = 2/B(pp) - 2/B(AB)$. E.g. $\Delta p_t^2 = 0.48$ GeV^2 was found at Fermilab energies in pAu. $C_{AB}^2 = C_{pp}^2$ is used as a first approximation. $S \approx 1$ for the J/Ψ. That means there is practically no change due to the broadening of the transverse momentum distribution. The parameters used for the calculation are explained in ref. [5]. As one can see we obtain even a slight enhancement for the Υ meson production. This model neglects a lot of effects that might occur in AB-collisions, but predicts that the J/Ψ is less suppressed in pA collisions at collider energies than at fixed target energies.

References

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