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## **Branching Rules revisited**

URL: <http://www.zib.de/projects/integer-optimization/MIP>

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# Branching Rules Revisited

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## Abstract

Mixed integer programs are commonly solved with linear programming based branch-and-bound algorithms. The success of the algorithm strongly depends on the strategy used to select the variable to branch on.

We present a new generalization called *reliability branching* of today's state-of-the-art *strong branching* and *pseudocost branching* branching strategies for linear programming based branch-and-bound algorithms. After reviewing commonly used branching strategies and performing extensive computational studies we compare different parameter settings and show the superiority of our proposed new strategy.

**Keywords:** Mixed-integer-programming, Branch-and-Bound, Variable selection, Pseudocost-Branching, Strong-Branching, Reliability-Branching

## 1 Introduction

In this paper we are dealing with *mixed integer programs* (MIPs), which are optimization problems of the following form:

$$c^* = \min c^T x, \quad Ax \leq b, \quad x \in \mathbb{Z}^I \times \mathbb{R}^{N \setminus I}, \quad (1)$$

where  $A \in \mathbb{R}^{m \times n}$ ,  $b \in \mathbb{R}^m$ ,  $c \in \mathbb{R}^n$  and  $I \subseteq N = \{1, \dots, n\}$ .

Among the most successful methods are currently linear programming based branch-and-bound (B&B) algorithms where the underlying linear programs (LPs) are possibly strengthened by cutting planes. Most commercial integer programming solvers, see [7], are based on this method. As we will see below, B&B algorithms leave two choices: how to split a problem (branching) and which (sub)problem to select next. In this paper we focus on the branching step and introduce a new generalization that contains

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many of the known branching rules as special cases. We show that specific choices of parameters for this new class of branching rules perform in most cases better than the current rules when tested on real-world instances.

In Section 2 we review current branching strategies from the literature and present our new generalization. In Section 3 the results of extensive numerical tests on specific parameter choices are presented.

We use the following notation.  $X_{\text{MIP}}$  denotes the set of feasible solutions of (1), and we set  $c^* = \infty$  if  $X_{\text{MIP}} = \emptyset$ . The *linear programming relaxation* of (1) is obtained by removing the integrality constraints:

$$\bar{c}_{P_{\text{LP}}} = \min \{c^T x \mid x \in P_{\text{LP}}\}, \quad (2)$$

where  $P_{\text{LP}} = \{x \in \mathbb{R}^n \mid Ax \leq b\}$ . We also denote  $\bar{c}_{P_{\text{LP}}} = \infty$  if  $P_{\text{LP}} = \emptyset$ . Obviously,  $\bar{c}_{P_{\text{LP}}} \leq c^*$ , since  $P_{\text{LP}} \supseteq X_{\text{MIP}}$ . A typical LP based B&B algorithm for solving (1) looks as follows:

**Algorithm 1 (LP based branch-and-bound)**

*Input:* A MIP in the form (1).

*Output:* An optimal solution  $x^* \in X_{\text{MIP}}$  and its value  $c^* = c^T x^*$  or the conclusion that  $X_{\text{MIP}} = \emptyset$ , denoted by  $c^* := \infty$ .

1. Initialize the problem set  $S := \{P_{\text{LP}}\}$  with the LP relaxation of the MIP. Set  $c^* := \infty$ .
2. If  $S = \emptyset$ , exit by returning the value  $c^*$  (with an optimal solution  $x^*$ ).
3. Choose a problem  $Q \in S$  and delete it from  $S$ .
4. Solve the linear program  $\bar{c}_Q = \min\{c^T x \mid x \in Q\}$  with optimal solution  $\bar{x}_Q$ , where  $Q$  might have been strengthened by cutting planes.
5. If  $\bar{c}_Q \geq c^*$ , goto 2.
6. If  $\bar{x}_Q \in X_{\text{MIP}}$ , set  $c^* := \bar{c}_Q$  and  $x^* := \bar{x}_Q$ , and goto 2.
7. Branching: Split  $Q$  into subproblems, add them to  $S$  and goto 3.

If it is clear from the context we omit  $Q$  from all parameters and write  $\bar{c}$ ,  $\bar{x}$ , etc. instead of  $\bar{c}_Q$ ,  $\bar{x}_Q$ , etc.

## 2 Branching Rules

Since branching is in the core of any B&B algorithm, finding good strategies was important to practical MIP solving right from the beginning [3, 13]. We refrain from giving details of all existing strategies, but concentrate on the most popular rules used in today's MIP solvers. For a comprehensive study of B&B strategies we refer to [8, 10] and the references therein.

The only way to split a problem  $Q$  within an LP based B&B algorithm is to branch on linear inequalities in order to keep the property of having an LP

relaxation at hand. The easiest and most common inequalities are *trivial inequalities*, i. e., inequalities that split the feasible interval of a singleton variable. To be more precise, if  $i$  is some variable with a fractional value  $\bar{x}_i$  in the current optimal LP solution, we set  $f_i^+ = \lceil \bar{x}_i \rceil - \bar{x}_i$  and  $f_i^- = \bar{x}_i - \lfloor \bar{x}_i \rfloor$ . We obtain two subproblems, one by adding the trivial inequality  $x_i \leq \lfloor \bar{x}_i \rfloor$  (called the *left subproblem* or *left son*, denoted by  $Q_i^-$ ) and one by adding the trivial inequality  $x_i \geq \lceil \bar{x}_i \rceil$  (called the *right subproblem* or *right son*, denoted by  $Q_i^+$ ). This rule of branching on trivial inequalities is also called *branching on variables*, because it only requires to change the bounds of variable  $i$ . Branching on more complicated inequalities or even splitting the problem into more than two subproblems are rarely incorporated into general MIP solvers, even though it can be effective in special cases, see, for instance, [4], [5], or [15].

The basic algorithm for variable selection may be stated as follows:

**Algorithm 2 (generic variable selection)**

*Input:* Current subproblem  $Q$  with an optimal LP solution  $\bar{x} \notin X_{\text{MIP}}$ .

*Output:* An index  $i \in I$  of a fractional variable  $\bar{x}_i \notin \mathbb{Z}$ .

1. Let  $C = \{i \in I \mid \bar{x}_i \notin \mathbb{Z}\}$  be the set of branching candidates.
2. For all candidates  $i \in C$ , calculate a score value  $s_i \in \mathbb{R}$ .
3. Return an index  $i \in C$  with  $s_i = \max_{j \in C} \{s_j\}$ .

In the following we focus on the most common variable selection rules, which are all variants of Algorithm 2. The difference is how the score in Step 2 is computed.

The ultimate goal is to find a fast branching strategy that minimizes the number of B&B nodes that need to be evaluated. Since a global approach is unlikely, one tries to find a branching variable that is at least a good choice for the current branching. The quality of a branching is measured by the change in the objective function of the LP relaxations of the two children  $Q_i^-$  and  $Q_i^+$  compared to the relaxation of the parent node  $Q$ .

In order to compare branching candidates, for each candidate the two objective function changes  $\Delta_i^- := \bar{c}_{Q_i^-} - \bar{c}_Q$  and  $\Delta_i^+ := \bar{c}_{Q_i^+} - \bar{c}_Q$  are mapped on a single score value. This is typically done by using a function of the form (cf. [10])

$$\text{score}(q^-, q^+) = (1 - \mu) \cdot \min\{q^-, q^+\} + \mu \cdot \max\{q^-, q^+\}. \quad (3)$$

The *score factor*  $\mu$  is some number between 0 and 1. It is usually an empirically determined constant, which is sometimes adjusted dynamically through the course of the algorithm (in SIP we use  $\mu = 1/6$ ). Note that special treatment is necessary, if one of the subproblems  $Q_i^-$  or  $Q_i^+$  is infeasible.

In the forthcoming explanations all cases are symmetric for the left and right subproblem. Therefore we will only consider one direction, the other will be analogous.

## 2.1 Most Infeasible Branching

This still very common rule chooses the variable with fractional part closest to 0.5, i. e.,  $s_i = 0.5 - |\bar{x}_i - \lfloor \bar{x}_i \rfloor - 0.5|$ . The heuristic reason behind this choice is that this selects a variable where the least tendency can be recognized to which “side” (up or down) the variable should be rounded. Unfortunately, as the numerical results in Section 3 indicate, the performance of this rule is in general not better than selecting the variable randomly.

## 2.2 Pseudocost Branching

This is a sophisticated rule in the sense that it keeps a history of the success of the variables on which already has been branched. This rule goes back to [3]. In the meantime various variations of the original rule have been proposed. In the following we present the one used in SIP [12]. For alternatives see [10].

Let  $\varsigma_i^+$  be the objective gain per unit change in variable  $i$  at node  $Q$ , that is

$$\varsigma_i^+ = \Delta_i^+ / f_i^+. \quad (4)$$

Let  $\sigma_i^+$  denote the sum of  $\varsigma_i^+$  over all problems  $Q$ , where  $i$  has been selected as branching variable and  $Q_i^+$  has already been solved and was feasible. Let  $\eta_i^+$  be the number of these problems. Then the pseudocosts for the upward branching of variable  $i$  are

$$\Psi_i^+ = \sigma_i^+ / \eta_i^+. \quad (5)$$

Using  $s_i = \text{score}(f_i^- \Psi_i^-, f_i^+ \Psi_i^+)$  in Algorithm 2 yields what is called *pseudocost branching*.

Observe that at the beginning of the algorithm  $\sigma_i^+ = \eta_i^+ = 0$  for all  $i \in I$ . We call the pseudocosts of a variable  $i \in I$  *uninitialized for the upward direction*, if  $\eta_i^+ = 0$ . Uninitialized upward pseudocosts are set to  $\Psi_i^+ = \Psi_{\text{avg}}^+$ , where  $\Psi_{\text{avg}}^+$  is the average of the initialized upward pseudocosts over all variables. This average number is set to 1 in the case that all upward pseudocosts are uninitialized. The pseudocosts of a variable are called *uninitialized* if they are uninitialized in at least one direction.

## 2.3 Strong Branching

The idea of *strong branching*, introduced in CPLEX 7.5 (see also [2]), is to test which of the fractional candidates gives the best progress before actually branching on any of them. This test is done by temporarily introducing an lower bound  $\lfloor \bar{x}_i \rfloor$  and subsequently an upper bound  $\lceil \bar{x}_i \rceil$  for variable  $i$  with fractional LP value  $\bar{x}_i$ , and solving the linear relaxations.

If we choose as candidate set the full set  $C = \{i \in I \mid \bar{x}_i \notin \mathbb{Z}\}$  and if we solve the resulting LPs to optimality, we call the strategy *full strong branching*. In other words, *full strong branching* can be viewed as finding

the locally (with respect to the given score function) best variable to branch on. We will see in Section 3 that selecting this locally best variable usually works very well in practice w.r.t. the number of nodes needed to solve the problem instances.

Unfortunately the computation times per node of *full strong branching* are high. Accordingly most branching rules presented in literature may be interpreted as an attempt to find a (fast) estimate of what *full strong branching* actually measures.

One possibility to speedup *full strong branching*, is to restrict the candidate set in some way, e.g. by considering only a subset  $C' \subseteq C$  of the fractional variables. To estimate the changes in the objective function for a specific branching decision, often only a few simplex iterations are performed, because the change of the objective function in the simplex algorithm usually decreases with the iterations. Thus, the parameters of *strong branching* to be specified are the size of the candidate set  $C'$ , the maximum number  $\gamma$  of dual simplex iterations to be performed on each candidate variable, and a criterion according to which the candidate set is selected.

In SIP, the size of the candidate set is not fixed in advance to a specific value, but the candidates are evaluated with a “look ahead” strategy: if no new best candidate was found for  $\lambda$  successive candidates, the evaluation process is stopped. By evaluating variables with largest *pseudocost* scores first, only the most promising candidates are evaluated. The iteration limit for strong branching evaluations is set to  $\gamma = 2\bar{\gamma}$ , where  $\bar{\gamma}$  is the average number of simplex iterations per LP needed so far. Note that this number only protects from unexpected long simplex runs, on average the candidate LPs will be solved to optimality.

## 2.4 Hybrid Strong/Pseudocost Branching

Even with the speedups indicated at the end of Section 2.3, the computational burden of *strong branching* is high, and the higher the speedup, the less precise the decisions are.

On the other hand, the weakness of *pseudocost branching* is that at the very beginning there is no information available, and  $s_i$  is almost identical for all variables  $i \in C$ . Many of the early nodes are located in the upper part of the search tree where the decisions have the largest impact on the structure of the tree and the subproblems therein. With *pseudocost branching*, these decisions are taken with respect to pseudocost values that aren't useful yet.

To circumvent these drawbacks the positive aspects of *pseudocost* and *strong branching* are put together in the combination *hybrid strong/pseudocost branching*, where *strong branching* is applied in the upper part of the tree up to a given depth level  $d$ . For nodes with depth larger than  $d$ , *pseudocost branching* is used. This branching rule is available for example in LINDO [11].

## 2.5 Pseudocost Branching with Strong Branching Initialization

The decisions of *pseudocost* as well as the ones of *hybrid strong/pseudocost branching* in the lower part of the tree are potentially based on uninitialized pseudocost values, leading to an inferior selection of branching variables.

The idea to avoid this risk, which goes back to [10], is to use strong branching for variables with uninitialized pseudocosts and to use the resulting strong branching estimates to initialize the pseudocosts. In contrast to the fixed depth level of *hybrid strong/pseudocost branching*, this rule uses strong branching in a more dynamic way.

## 2.6 Reliability Branching

We generalize the idea of *pseudocost branching with strong branching initialization* by not only using strong branching on variables with uninitialized pseudocost values, but also on variables with *unreliable* pseudocost values. The pseudocosts of a variable  $i$  are called *unreliable*, if  $\min\{\eta_i^-, \eta_i^+\} < \eta_{\text{rel}}$ , with  $\eta_{\text{rel}}$  being the “reliability” parameter. We call this new branching rule *reliability branching*.

An outline of the selection of a branching variable with *reliability branching* is given in the following Algorithm 3 that replaces Step 2 of Algorithm 2.

### Algorithm 3 (Reliability branching)

2. For all candidates  $i \in C$ , calculate the score  $s_i = \text{score}(f_i^-, \Psi_i^-, f_i^+, \Psi_i^+)$  and sort them in non-increasing order of their pseudocost scores.

For all candidates  $i \in C$  with  $\min\{\eta_i^-, \eta_i^+\} < \eta_{\text{rel}}$ , do:

- (a) Perform a number of at most  $\gamma$  dual simplex iterations on each subproblem  $Q_i^-$  and  $Q_i^+$ . Let  $\tilde{\Delta}_i^-$  and  $\tilde{\Delta}_i^+$  be the resulting gains in the objective value.
- (b) Update the pseudocosts  $\Psi_i^-$  and  $\Psi_i^+$  with the gains  $\tilde{\Delta}_i^-$  and  $\tilde{\Delta}_i^+$ .
- (c) Update the score  $s_i = \text{score}(\tilde{\Delta}_i^-, \tilde{\Delta}_i^+)$ .
- (d) If the maximum score  $s^* = \max_{j \in C} \{s_j\}$  has not changed for  $\lambda$  consecutive score updates, goto 3.

## 2.7 Branching Rule Classification

Some of the proposed branching rules can be adjusted with parameter settings. All of the strategies using strong branching include the simplex iteration limit  $\gamma$  and the look ahead value  $\lambda$ . The *hybrid strong/pseudocost branching* exhibits an additional depth parameter  $d$ , while the *reliability branching* comes along with the reliability parameter  $\eta_{\text{rel}}$ .

It is interesting to note that depending on the parameter settings, the branching rules have interrelations as illustrated in Figure 1.

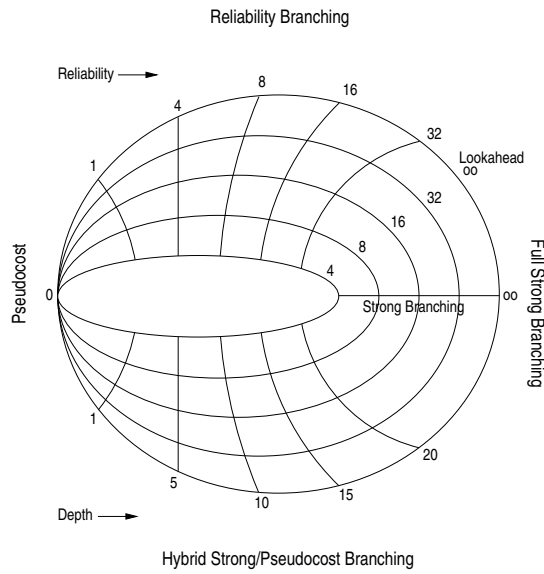


Figure 1. Interrelations between branching rules and their parameters

*Hybrid strong/pseudocost branching* with  $d = 0$  as well as *reliability branching* with  $\eta_{\text{rel}} = 0$  coincide with pure *pseudocost branching*. With  $\eta_{\text{rel}} = 1$ , *reliability branching* is equal to *pseudocost branching with strong branching initialization*. If the depth  $d$  and the reliability  $\eta_{\text{rel}}$  are increased, the number of strong branching evaluations also increases, and with  $d = \eta_{\text{rel}} = \infty$ , both strategies converge to pure *strong branching*. Additionally, if the look ahead parameter is set to  $\lambda = \infty$ , *strong branching* becomes *full strong branching*.

$$\text{Reliability} \left\{ \begin{array}{l} \eta_{\text{rel}} = 0 \\ \eta_{\text{rel}} = 1 \\ \eta_{\text{rel}} \in [4, 8] \\ \eta_{\text{rel}} = \infty \end{array} \right\} \text{ yields } \left\{ \begin{array}{l} \textit{pseudocost branching} \\ \textit{pseudocost branching with} \\ \textit{strong branching initialization} \\ \textit{best performing branching rules} \\ \textit{strong branching} \end{array} \right.$$



### 3 Computational Results

In this section we compare computational results for different branching rules and parameter settings on several MIP instances. All calculations were performed on a 833 MHz Alpha 21264 workstation with 4 MB Cache and 2 GB RAM.

#### 3.1 Test Set

Our test set consists of instances from MIPLIB 2003 [1] and instances used by [14]. We selected all problems where CPLEX 9.0 needs at least 5000 branching nodes and at most one hour CPU time for solving. (CPLEX was run with default settings, except that “absolute mipgap” was set to  $10^{-10}$  and “relative mipgap” to 0.0, which are the corresponding values in SIP.)

In all runs, we used a time limit of 3600 seconds. Note that the version of SIP used here utilizes CPLEX 9.0 as embedded LP solver. The strong branchings are performed using `CPXstrongbranch()`.

What makes benchmarking branching strategies difficult are the complex interrelations between cutting plane generation, primal heuristics, node selection, and branching variable selection. For example, it is possible that a “worse” branching rule results in less branching nodes and a smaller solution time for a specific instance, because the variable selection leads incidentally to an early discovering of a good or optimal primal solution. However, we dispensed with setting the optimal solution values beforehand, since leading towards feasible solutions fast is a desirable property of branching rules.<sup>1</sup>

Hence, for our comparison of the branching strategies we used the default parameter settings except that cuts are generated in the root node only. For this parameter setting, which is commonly known as cut-and-branch, the influence of the branching strategy is emphasized best.

To verify that the branch-and-bound environment we used with SIP is state-of-the-art, we also ran CPLEX 9.0 on our test set. To better compare

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<sup>1</sup>A better primal solution impacts the solution process in several different ways:

- Fewer nodes have to be evaluated, due to earlier cutoff.
- Less time per node is spend, because the dual simplex algorithm can stop when it reaches the upper bound. The same applies to the strong branching evaluations.
- The average depth of nodes that use strong branching decreases, because of the node selection algorithm. The bigger the gap between upper and lower bound the more the selection algorithm tries to find feasible solutions by selecting nodes in a depth-first fashion. If the gap becomes small it resembles more a best-first algorithm, and that causes *reliability branching* to resemble more a *hybrid strong/pseudocost branching*, which as you can see in Figure 2 is inferior.
- The behavior of the diving heuristic, that regularly “dives” down the tree to find feasible solutions is also influenced. This is important as pseudocosts are updated while diving.

the branching decisions we used SIP’s preprocessing and cutting plane generation also in CPLEX instead of CPLEX’ own routines. In this way both, CPLEX and SIP, operated with pure branch and bound on the same problems. In CPLEX we used the default branching strategy which to the best of our knowledge is some variant of pseudocost branching.

### 3.2 Description of Tables and Figures

In Table 1, the summary of all computational results is presented. Columns labeled “total” give the sums of the results over all instances, columns labeled “geom.” give the geometric mean over all instances. The last column lists how many instances were not solved to optimality because the limit of one hour CPU time was reached.

Tables 2 and 3 describe in detail some of our computational results. These tables show for each instance and each branching strategy the number of nodes explored and the time needed to solve all the instances. The number put in parentheses behind the branching rules is the reliability setting for *reliability branching* and the depth setting for *hybrid strong/pseudocost branching* (str/ps).

Numbers in bold face indicate the “winner” for a particular instance between *hybrid strong/pseudocost branching* with  $d = 10$ , *reliability branching* with  $\eta_{\text{rel}} = 1$  which is equal to *pseudocost branching with strong branching initialization*, and *reliability branching* with  $\eta_{\text{rel}} = 8$ . This is done separately for  $\lambda = 4$  and  $\lambda = 8$ .

We did not base our conclusions on performances of single instances and discuss those in detail.<sup>2</sup>We rather rely on average numbers over all instances. It is common sense that the geometric mean is a fair criterion for comparison and we used it as the basis for our conclusions in the next section.

One interesting observation in Table 3 is that *most infeasible branching* is basically as good as *random branching* showing that this rule is of no use. We refrain from considering both rules any further in the following discussion.

Table 4 gives the number of strong branching evaluations performed, i. e., the number of `CPXstrongbranch()` calls, which is the number of times a fractional variable was evaluated with strong branching by solving its two subproblems.<sup>3</sup>

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<sup>2</sup>The detailed results can be found starting at page 20 in Tables 5 to 44

<sup>3</sup>Every strategy employing strong branching can detect infeasibilities and cutoffs already in the strong branching evaluation of a fractional variable, if one or both of the two subproblems are infeasible or have dual bounds exceeding the current primal bound. These fathomed subproblems are actually not generated and therefore not counted as *nodes* in the tables. Therefore, *full strong branching* (and to a lesser extend *strong branching*) saves the generation of the final leaves, thus reducing the number of node evaluations by a factor of  $\frac{1}{2}$ . This reduction of node counts is neglectable for all other branching strategies.

The last column shows the average depth of the node tree as encountered while doing *full strong branching*. This gives an indication on how balanced the trees are, the smaller the number the more balanced the trees are. Examples can be seen in Figure 5 which display the trees of `vpm2` with a low average depth and `neos3` with a very high average depth.

### 3.3 Results

We have performed a comprehensive computational study of all variants of branching strategies discussed in this paper. For each parameter setting, reflected by an intersection point of two lines in Figure 1, we ran all instances. The total number as well as the geometric mean in terms of B&B nodes, time and strong branching evaluations is shown in Table 1 for all of these runs.

Figure 2 illustrates the geometric means of all runs. The circles, squares and stars indicate runs with *reliability branching*, *hybrid strong/pseudocost branching*, and *strong branching*, respectively. The CPLEX run is marked with an 'x'. The numbers give, depending on the branching strategy, information about the parameters  $\eta_{rel}$ ,  $d$ , and  $\lambda$ .

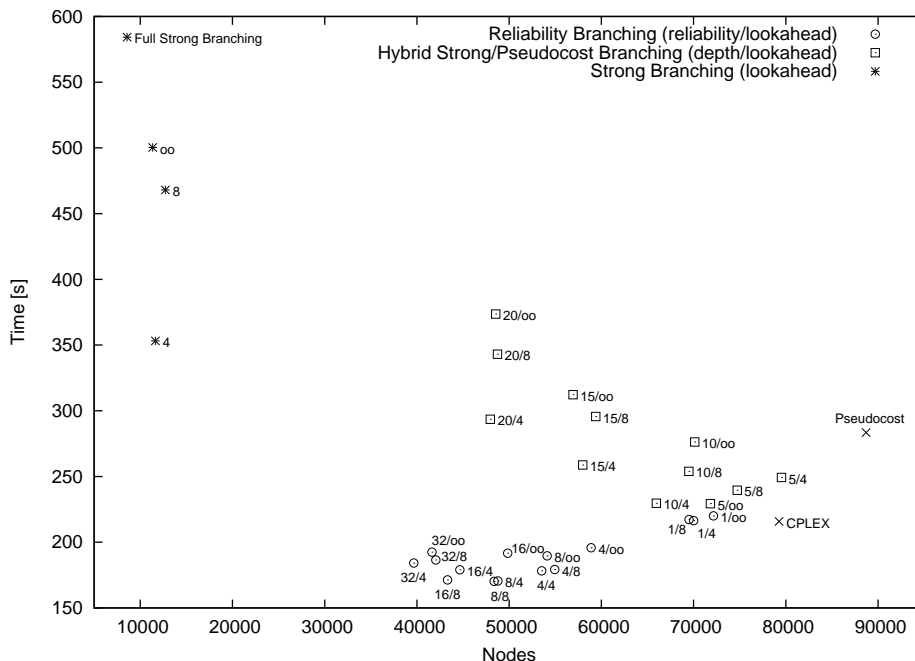


Figure 2. Nodes vs. time

The following conclusions may be drawn from these numbers:

- (i) *Strong branching* (stars in Figure 2) performs remarkably well with respect to the number of evaluated nodes, but, as expected, not with

respect to time. Only two instances could not be solved by *full strong branching* within 3600 seconds. Without time limit, `qiu` is solved in 15 659 nodes and 11 133 seconds, while `neos7` is solved in 476 601 nodes and 33 418 seconds.

The only instance where none of the variants of *strong branching* needs the least number of nodes is `neos7`. Especially unexpected is that *full strong branching* needs to evaluate considerably more nodes than *pseudocost branching*. This is due to the fact that *pseudocost branching* is (incidentally) able to find the optimal solution very early (after two minutes and 15277 nodes). Note that as a consequence *pseudocost branching* is more than twice as fast per node than *most infeasible branching*.

- (ii) With respect to time, regardless of the specific parameter setting, *reliability branching* always outperforms *hybrid strong/pseudocost branching*, as can be seen by comparing the circles with the rectangles in Figure 2.

At least one reason why *hybrid strong/pseudocost branching* performs worse than *reliability branching* can be seen in Figure 5. What is shown are the branch-and-bound trees generated by *full strong branching* visualized with VBCTOOL [9]. The first tree is from `vpm2` and is reasonably balanced. The second one is from `neos3` and looks like a path. The right branch (fixing a variable to one) is nearly always infeasible after two variables are fixed. Since *hybrid strong/pseudocost branching* uses a fixed depth for deciding where to do *strong branching*, only very few strong branching evaluations are performed, as shown in Table 4.

The last column of Table 4 gives the average depth of all nodes generated when using *full strong branching*. If one compares this to the depth setting of *hybrid strong/pseudocost branching* and looks at the number of strong branchings performed, see also Table 4, the difficulties with strategies based on some fixed setting become obvious.

- (iii) The lookahead  $\lambda$  from a certain value upwards doesn't seem to have much influence on the number of nodes.

The higher the settings the more time is spent per node evaluation, which seems not to pay off. This is not really a surprise recalling that a lookahead of four means, *no new best candidate found in four consecutive tries*. Since the candidates are already ordered by pseudocost value and the lookahead counter is reset with every new best candidate found, a setting of four turns out to be good enough to find the overall best candidate in most cases.

- (iv) Increasing the reliability  $\eta_{\text{rel}}$  in *reliability branching* or the depth  $d$  in *hybrid strong/pseudocost branching* decreases the number of evaluated nodes as expected.

See Figure 3, where the ratio of the number of nodes to strong branching evaluations is shown. With an increasing number of strong branching evaluations we are converging towards *strong branching* with respect to the number of evaluated nodes. The curve of *reliability branching* is always below the curve of *hybrid strong/pseudocost branching* indicating that the information provided by *strong branching* is used in a much better way.

Figure 4 on the other hand demonstrates the tradeoff between number of nodes and time per node. We again see that *reliability branching* is always better than *hybrid strong/pseudocost branching*. Figure 4 also nicely reflects that *pseudocost branching with strong branching initialization* ( $\eta_{\text{rel}} = 1$ ), see Section 2.5, is better than *pseudocost branching* itself ( $\eta_{\text{rel}} = 0$ ), but this is not the best choice. The performance increases up to  $\eta_{\text{rel}} = 8$  and decreases with larger values again.

Looking once again at Figure 2 we see that the setting ( $\eta_{\text{rel}} = 8, \lambda = 4$ ) for *reliability branching* marks a new sweet spot.

## 4 Conclusion

We presented a new generalization of today's state of the art *pseudocost branching* and *strong branching*, which we call *reliability branching*. This strategy was implemented in SIP, an LP based branch and bound solver for mixed integer programs. The superiority of the new branching rule was demonstrated in extensive computational experiments.

It was shown that a more intensive dynamic use of *strong branching* leads to significant improvements in both the number of B&B nodes and the time needed to solve the considered problem instances.

It also became evident, that there is still a gap to the number of nodes needed using *full strong branching*. The question is whether it is possible to bridge this gap without increasing the time spent per node too much.

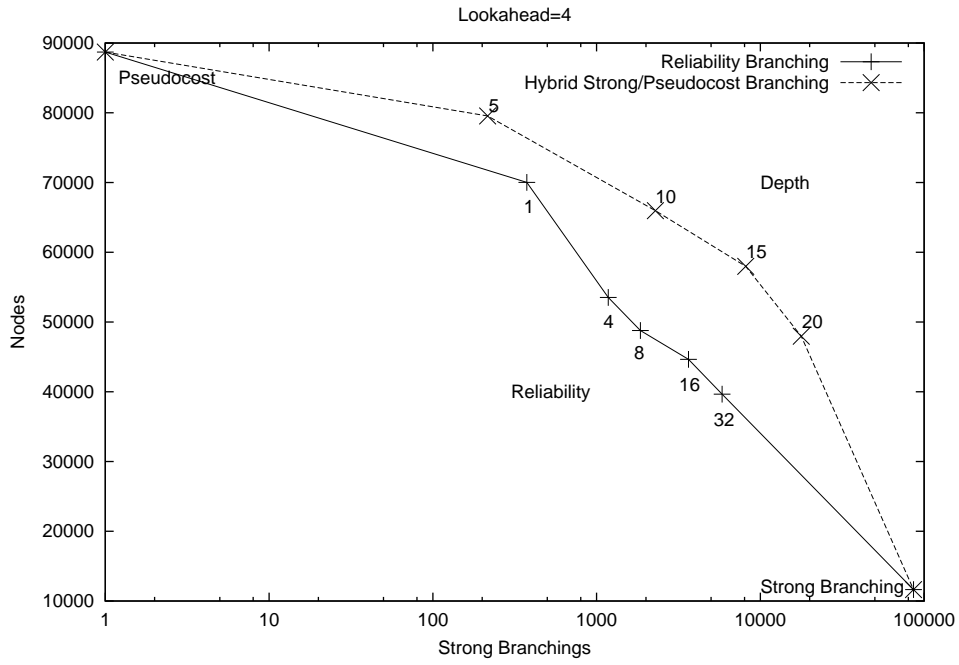


Figure 3. Nodes vs. strong branching evaluations

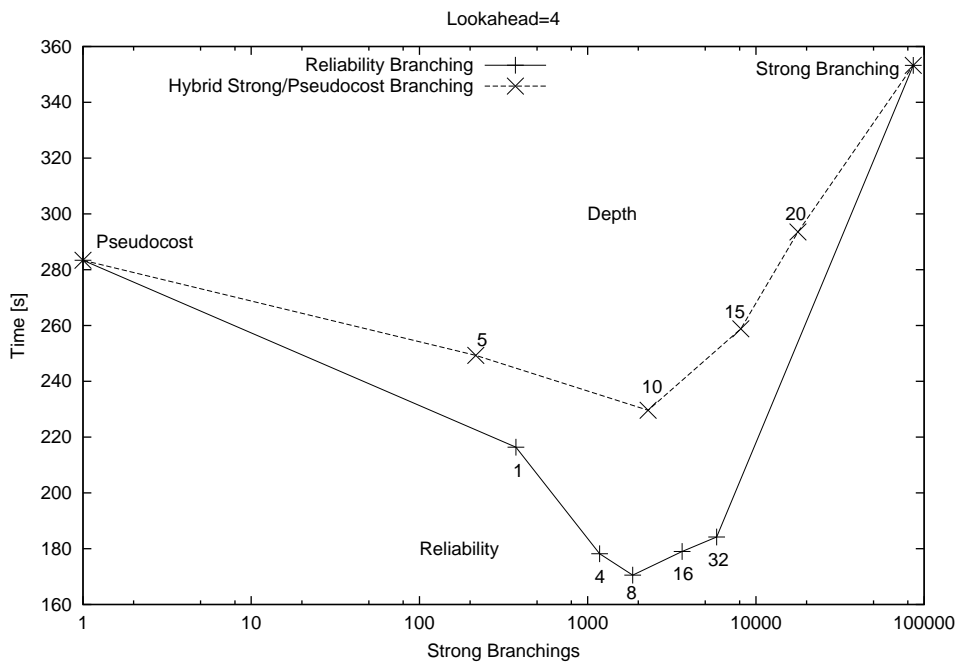


Figure 4. Time vs. strong branching evaluations

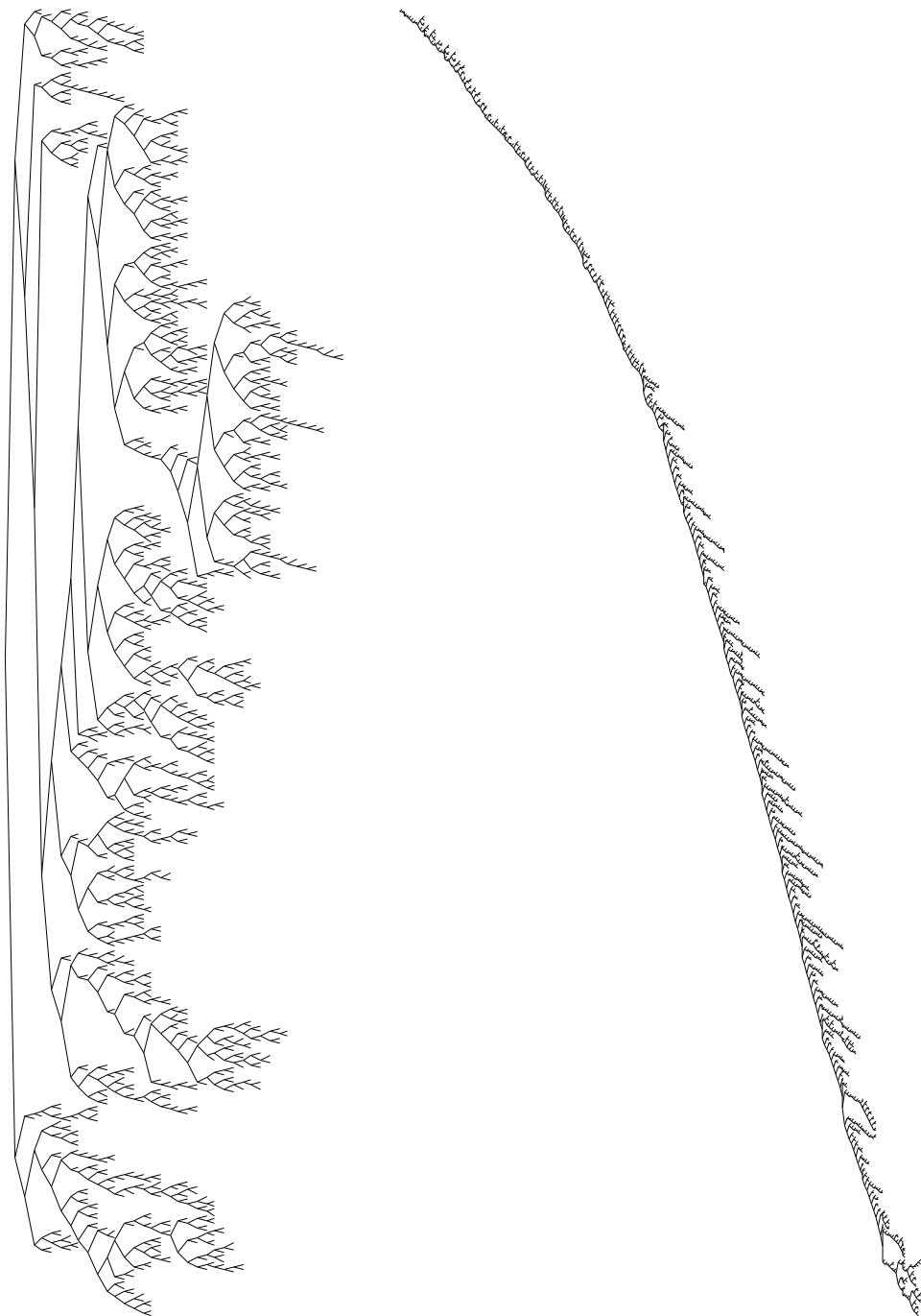


Figure 5. Comparison of node trees resulting from *full strong branching* for **vpm2** and **neos3**.

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Table 1. Summary of all considered strategies

Strategy	B&B nodes		time (sec)		strong branchings		fails
	total	geom.	total	geom.	total	geom.	
random	21 246 277	275 789.4	46 202.2	923.6	0	0.0	11
most infeasible	19 421 589	262 368.9	48 037.5	938.0	0	0.0	11
pseudocost	9 381 001	88 706.8	19 945.8	283.4	0	0.0	2
full strong	929 347	7 241.7	26 397.2	504.4	15 569 652	146 784.2	2
lookahead = 4							
strong/pscost (5)	9 698 397	79 535.5	19 487.8	249.3	5 792	216.2	2
strong/pscost (10)	8 251 942	65 966.3	16 499.4	229.6	74 812	2 284.2	2
strong/pscost (15)	7 982 847	57 976.8	17 855.3	258.8	523 377	8 137.8	2
strong/pscost (20)	7 890 374	47 958.5	19 175.6	293.5	2 825 100	17 780.2	2
reliability (1)	9 000 334	70 013.6	17 199.6	216.4	39 126	374.8	1
reliability (4)	6 906 698	53 522.9	13 402.9	178.2	110 628	1 176.7	0
reliability (8)	7 937 968	48 772.8	11 132.7	170.5	117 643	1 850.3	0
reliability (16)	6 022 024	44 649.9	10 782.6	179.0	187 578	3 640.6	0
reliability (32)	7 940 797	39 655.2	11 103.0	184.2	253 014	5 837.8	0
strong branching	1 325 589	11 639.5	20 427.2	353.2	9 965 454	86 188.6	3
lookahead = 8							
strong/pscost (5)	8 653 318	74 730.8	17 389.5	239.6	7 397	268.6	1
strong/pscost (10)	8 332 543	69 489.0	18 362.5	253.9	102 082	2 523.8	2
strong/pscost (15)	7 456 685	59 398.8	20 479.8	295.7	750 577	9 983.1	2
strong/pscost (20)	7 551 419	48 736.5	22 388.8	343.1	3 695 577	20 837.9	3
reliability (1)	8 663 537	69 501.0	16 753.8	217.2	53 557	429.0	1
reliability (4)	8 338 386	54 937.7	12 497.2	179.2	74 906	1 104.6	0
reliability (8)	7 813 409	48 377.3	12 380.2	170.2	133 545	1 998.0	0
reliability (16)	7 579 400	43 311.9	11 946.7	171.3	185 136	3 589.7	0
reliability (32)	7 207 836	42 047.5	11 835.7	186.5	259 482	5 913.2	0
strong branching	1 294 569	12 714.7	25 619.4	468.0	12 651 504	119 799.1	4
lookahead = $\infty$							
strong/pscost (5)	8 498 292	71 817.4	18 116.9	229.3	14 675	489.9	2
strong/pscost (10)	9 247 636	70 125.8	20 472.1	276.2	154 458	3 870.1	2
strong/pscost (15)	6 670 440	56 926.6	19 907.4	312.2	890 187	13 127.2	3
strong/pscost (20)	7 627 640	48 547.0	23 538.5	373.6	3 842 516	26 557.4	3
reliability (1)	7 747 290	72 159.1	15 825.8	220.0	48 162	408.4	1
reliability (4)	9 068 723	58 886.4	14 258.1	195.8	78 625	1 096.6	2
reliability (8)	8 551 045	54 118.3	13 563.0	189.6	135 541	2 042.9	1
reliability (16)	6 567 432	49 839.9	12 766.4	191.6	196 220	3 601.0	0
reliability (32)	7 502 942	41 636.1	12 393.6	192.5	281 822	6 000.7	0
strong branching	1 163 822	11 355.7	26 176.4	500.3	12 793 364	127 737.1	4
CPLEX/SIP cuts	10 467 429	79 269.0	18 617.4	215.8	—	—	1

Table 2. B&amp;B nodes needed to solve each problem instance

Example					lookahead = 4				lookahead = 8				CPLEX 9.0 (SIP cuts)
	random	most inf.	pscost	full str.	str/ps(10)	reli(1)	reli(8)	strong	str/ps(10)	reli(1)	reli(8)	strong	
aflow30a	>1 248 221	>1 025 389	276 002	46 649	<b>164 112</b>	203 544	181 397	44 455	210 127	<b>170 547</b>	171 909	55 478	105 204
cap6000	7 454	6 717	6 791	3 402	4 957	5 076	<b>4 253</b>	3 452	4 957	5 076	<b>4 253</b>	3 452	4 982
gesa2-o	>1 023 765	>1 150 438	126 459	13 743	75 141	57 534	<b>53 881</b>	15 831	74 673	79 841	<b>50 078</b>	21 988	37 293
mas74	>4 662 991	>4 725 208	5 160 828	551 780	<b>5 163 685</b>	5 478 830	5 521 061	>790 014	<b>4 886 438</b>	5 296 806	5 425 001	>645 364	5 617 512
mas76	2 935 689	2 038 945	603 683	114 228	587 813	496 370	<b>482 122</b>	125 516	666 398	496 370	<b>336 826</b>	188 882	507 536
misc07	47 941	17 955	19 407	2 713	49 822	<b>35 187</b>	54 932	5 228	50 893	<b>41 902</b>	55 740	5 394	86 825
pk1	1 064 666	850 198	437 758	56 546	<b>294 469</b>	367 763	331 339	128 299	<b>298 771</b>	367 763	311 611	143 206	366 340
pp08aCUTS	1 877	1 740	673	124	<b>355</b>	673	489	201	<b>318</b>	651	464	148	663
qiu	>163 998	>165 538	15 471	>2 974	25 180	<b>16 479</b>	18 405	6 178	23 974	16 148	<b>14 847</b>	>6 416	9 443
rout	>714 812	>702 392	309 779	2 836	45 457	44 232	<b>13 883</b>	4 179	39 212	39 192	<b>19 743</b>	11 305	>1 467 395
vpm2	92 880	40 258	22 568	1 457	11 710	13 780	<b>9 648</b>	1 974	10 826	17 409	<b>10 054</b>	1 812	4 306
ran8x32	321 933	674 613	40 069	4 902	29 957	26 360	<b>17 668</b>	6 824	30 526	31 697	<b>21 092</b>	4 249	45 143
ran10x26	>1 637 058	>1 500 171	128 327	8 520	54 965	64 640	<b>48 065</b>	8 279	56 448	68 237	<b>48 626</b>	11 112	56 548
ran12x21	>1 667 430	>1 471 340	234 915	13 420	<b>126 153</b>	172 161	135 037	12 185	135 165	159 383	<b>124 455</b>	18 577	196 478
ran13x13	907 443	758 186	149 239	9 147	<b>86 648</b>	109 241	95 288	16 033	<b>86 566</b>	97 195	93 939	20 516	97 325
mas284	162 255	123 569	21 586	3 226	<b>18 700</b>	24 383	21 179	4 564	<b>17 799</b>	21 217	20 360	5 580	21 472
prod1	>2 854 213	2 317 393	89 293	10 644	72 890	<b>63 674</b>	64 186	14 670	69 992	65 679	<b>62 689</b>	15 609	106 369
bc1	33 631	34 127	40 781	2 981	42 882	35 132	<b>25 196</b>	3 666	40 212	35 132	<b>25 196</b>	3 603	18 463
bienst1	66 099	58 841	19 418	3 687	13 594	<b>10 427</b>	13 911	5 274	13 602	<b>9 248</b>	9 951	5 340	12 963
neos2	>640 712	>703 794	>609 109	618	195 730	187 331	<b>22 742</b>	4 405	244 292	83 146	<b>30 790</b>	2 685	158 240
neos3	>403 674	>393 853	>446 035	1 402	>505 184	>737 254	<b>556 835</b>	13 215	>536 938	>736 396	<b>626 894</b>	15 306	606 168
neos7	>471 073	>339 479	390 910	>44 061	>546 623	535 586	<b>202 482</b>	>55 369	>598 209	498 573	<b>252 766</b>	>55 799	422 264
swath1	33 021	81 309	19 924	8 285	64 320	36 264	<b>35 161</b>	11 268	73 512	66 995	<b>10 615</b>	16 471	128 017
swath2	83 441	>240 136	211 976	22 002	71 595	278 413	<b>28 808</b>	>44 510	162 695	258 934	<b>85 510</b>	>36 277	390 480
Total (24)	21 246 277	19 421 589	9 381 001	929 347	8 251 942	9 000 334	7 937 968	1 325 589	8 332 543	8 663 537	7 813 409	1 294 569	10 467 429
Geom. Mean	275 789	262 369	88 707	7 242	65 966	70 014	48 773	1 163 9.5	69 489	69 501	48 377	1 271 4.7	79 269

Table 3. Time in seconds needed to solve each problem instance

Example					lookahead = 4				lookahead = 8				CPLEX 9.0 (SIP cuts)
	random	most inf.	pscost	full str.	str/ps(10)	reli(1)	reli(8)	strong	str/ps(10)	reli(1)	reli(8)	strong	
aflow30a	>3600.0	>3600.0	582.5	2212.6	<b>364.6</b>	403.6	416.4	704.7	457.6	391.7	<b>368.2</b>	1522.8	330.6
cap6000	57.1	50.3	52.6	47.4	41.1	38.6	<b>36.1</b>	58.9	40.7	37.8	<b>36.3</b>	59.5	18.8
gesa2-o	>3600.0	>3600.0	442.7	864.2	275.0	203.2	<b>199.1</b>	463.7	272.5	284.3	<b>195.7</b>	926.6	93.9
mas74	>2651.9	>2683.5	2734.2	3087.1	<b>2750.3</b>	3015.9	2807.4	>3600.0	<b>2902.2</b>	3066.4	3104.2	>3600.0	1995.3
mas76	1307.1	916.0	291.7	486.6	274.5	240.6	<b>225.1</b>	420.2	315.3	242.2	<b>161.2</b>	713.9	136.8
misc07	123.4	54.8	54.6	299.3	141.6	<b>97.5</b>	150.2	213.3	143.2	<b>113.2</b>	150.2	267.4	225.6
pk1	501.3	483.7	249.9	488.5	<b>154.3</b>	199.2	193.3	658.1	<b>157.2</b>	200.8	170.3	885.0	161.7
pp08aCUTS	6.8	7.7	3.4	12.4	7.8	<b>4.5</b>	5.4	10.3	10.3	<b>4.4</b>	5.5	9.9	2.4
qiu	>3600.0	>3600.0	332.3	>3600.0	927.4	<b>369.5</b>	419.1	1901.9	1213.9	371.2	<b>341.9</b>	>3600.0	152.2
rout	>3600.0	>3600.0	976.9	621.2	150.0	135.7	<b>58.4</b>	254.7	165.2	122.2	<b>81.3</b>	785.8	>3600.0
vpm2	124.5	62.6	30.6	43.5	18.2	18.6	<b>15.7</b>	25.4	19.1	25.0	<b>16.3</b>	30.9	4.1
ran8x32	592.4	1790.9	78.7	212.1	62.4	54.7	<b>38.3</b>	141.8	59.3	63.7	<b>47.2</b>	133.0	76.2
ran10x26	>3600.0	>3600.0	192.5	463.0	89.0	99.5	<b>86.5</b>	183.1	100.3	104.7	<b>82.0</b>	294.2	113.0
ran12x21	>3600.0	>3600.0	412.4	757.1	<b>208.9</b>	272.5	214.5	317.4	222.2	250.8	<b>199.0</b>	529.3	419.1
ran13x13	1189.5	1168.8	217.7	287.4	<b>132.8</b>	159.9	136.9	233.7	<b>134.4</b>	144.8	136.1	355.0	126.9
mas284	146.4	116.5	24.6	59.5	28.5	28.8	<b>25.9</b>	48.5	27.8	<b>23.7</b>	24.1	71.0	15.4
prod1	>3600.0	1741.5	151.0	405.1	112.9	<b>94.6</b>	100.8	287.1	101.1	<b>96.2</b>	97.8	359.3	185.7
bc1	1243.8	1317.7	1218.5	1180.5	1337.0	1272.9	<b>1120.0</b>	1108.2	1282.2	1295.2	<b>1153.0</b>	1071.5	1947.4
bienst1	868.2	749.2	230.9	243.0	121.3	<b>107.9</b>	115.3	256.8	138.4	<b>83.3</b>	91.0	270.3	194.1
neos2	>3600.0	>3600.0	>3600.0	872.8	693.6	675.6	<b>206.7</b>	277.2	883.6	385.5	<b>222.2</b>	216.9	671.4
neos3	>3600.0	>3600.0	>3600.2	2882.3	>3600.0	>3600.0	<b>2707.2</b>	1284.6	>3600.0	>3600.0	<b>3145.8</b>	1550.2	2513.8
neos7	>3600.0	>3600.0	2009.4	>3600.0	>3600.0	2661.3	<b>1039.8</b>	>3600.0	>3600.0	2377.6	<b>1350.3</b>	>3600.0	1831.7
swath1	330.3	894.3	173.4	795.2	566.0	<b>335.3</b>	407.3	777.6	668.6	604.6	<b>166.2</b>	1166.1	693.1
swath2	1059.4	>3600.0	2285.3	2876.3	842.3	3109.8	<b>407.2</b>	>3600.0	1847.6	2864.5	<b>1034.6</b>	>3600.0	3108.2
Total (24)	46202.2	48037.5	19945.8	26397.2	16499.4	17199.6	11132.7	20427.0	18362.5	16753.8	12380.2	25618.6	18617.4
Geom. Mean	923.6	938.0	283.4	504.4	229.6	216.4	170.5	353.2	253.9	217.2	170.2	468.0	215.8

Table 4. strong branching evaluations performed and maximal depth of random branching

Example	full str.	lookahead = 4				lookahead = 8				avg. depth full str.
		str/ps(10)	reli(1)	reli(8)	strong	str/ps(10)	reli(1)	reli(8)	strong	
aflow30a	972871	3625	396	2847	239787	5604	397	2865	495934	38.7
cap6000	4602	762	92	597	4683	762	92	597	4683	16.7
gesa2-o	401182	2231	435	2331	114464	3373	1841	2178	235608	28.8
mas74	8773236	6609	98	649	6094848	10531	88	673	6617267	23.3
mas76	1508479	6307	69	530	861005	9080	69	558	1551253	21.2
misc07	85451	3120	379	2471	65988	2943	266	3262	84665	29.9
pk1	897902	7008	54	391	879768	9334	54	402	1256800	24.6
pp08aCUTS	2227	954	61	433	1498	1531	60	438	1530	7.3
qiu	79685	6879	48	363	47170	11732	48	370	87289	11.6
rout	114383	3004	573	2703	58916	5554	426	3473	210641	22.1
vpm2	21665	1948	165	881	12712	2971	141	1089	15435	18.0
ran8x32	129698	1770	570	2657	63719	2765	667	2534	58732	24.8
ran10x26	267261	3038	536	1985	88240	3771	586	3417	144009	30.2
ran12x21	456400	5414	593	2675	141880	6588	940	2591	269767	25.5
ran13x13	210862	5304	167	1276	144070	7920	311	1521	234413	26.3
mas284	54311	4350	110	462	32685	5010	85	477	47686	14.9
prod1	209540	151	153	1202	118575	335	158	1173	152860	32.9
bc1	17413	804	3189	5092	16891	787	3189	5092	16810	35.9
bienst1	31222	4150	27	211	27841	4380	27	206	30331	13.8
neos2	115097	1076	9998	15893	44008	267	13311	13994	37362	121.6
neos3	305325	1264	11730	45143	170679	270	19218	54459	223816	134.6
neos7	475374	1058	6170	11256	344673	171	5944	17062	417934	130.2
swath1	111797	1682	1598	8170	80944	3235	2088	6474	129431	28.1
swath2	323669	2304	1915	7425	310410	3168	3551	8640	327248	38.0
Total (24)	15569652	74812	39126	117643	9965454	102082	53557	133545	12651504	—
Geom. Mean	146784	2284	375	1850	86189	2524	429	1998	119799	28.2

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
afLOW30a	1248221	22	0	1121.680021	1180	3600.0	5.199
cap6000	7454	3	0	-2451377	-2451377	57.1	0.000
gesa2-o	1023765	171	0	25636165.22	25804886.59	3600.0	0.658
mas74	4662991	0	0	11316.99594	12156.87628	2651.9	7.421
mas76	2935689	0	0	40005.05414	40005.05414	1307.1	0.000
misc07	47941	71	0	2810	2810	123.4	0.000
pk1	1064666	0	0	11	11	501.3	0.000
pp08aCUTS	1877	494	0	7350	7350	6.8	0.000
qiu	163998	1	0	-183.4303822	-132.8731378	3600.0	27.562
rout	714812	68	0	1026.780486	1079.19	3600.0	5.104
vpm2	92880	214	0	13.75	13.75	124.5	0.000
ran8x32	321933	93	0	5247	5247	592.4	0.000
ran10x26	1637058	56	0	4191.797533	4290	3600.0	2.343
ran12x21	1667430	62	0	3626.01421	3677	3600.0	1.406
ran13x13	907443	78	0	3252	3252	1189.5	0.000
mas284	162255	0	0	91405.72368	91405.72368	146.4	0.000
prod1	2854213	133	0	-60.80537903	-56	3600.0	7.903
bc1	33631	0	0	3.338362548	3.338362548	1243.8	0.000
bienst1	66099	23	0	46.75	46.75	868.2	0.000
neos2	640712	29	0	-192.8700538	486.920203	3600.1	352.460
neos3	403674	42	0	-2300.094909	480.6941847	3600.0	120.899
neos7	471073	112	0	666536.746	727934	3600.0	9.211
swath1	33021	59	0	379.0712957	379.0712957	330.3	0.000
swath2	83441	80	0	385.1996929	385.1996929	1059.4	0.000
Total (24)	21246277	1811	0			46202.3	540.167
Geom. Mean	275789.4		0.0			923.6	

Table 5: random branching

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	1025389	22	0	1128.084312	1182	3600.0	4.779
cap6000	6717	3	0	-2451377	-2451377	50.3	0.000
gesa2-o	1150438	171	0	25695246.64	25799147.33	3600.0	0.404
mas74	4725208	0	0	11328.45882	12199.33728	2683.5	7.688
mas76	2038945	0	0	40005.05414	40005.05414	916.0	0.000
misc07	17955	71	0	2810	2810	54.8	0.000
pk1	850198	0	0	11	11	483.7	0.000
pp08aCUTS	1740	494	0	7350	7350	7.7	0.000
qiu	165538	1	0	-173.0330337	-132.873137	3600.0	23.209
rout	702392	68	0	1018.80106	1083.54	3600.0	6.354
vpm2	40258	214	0	13.75	13.75	62.6	0.000
ran8x32	674613	93	0	5247	5247	1790.9	0.000
ran10x26	1500171	56	0	4197.825257	4281	3600.0	1.981
ran12x21	1471340	62	0	3633.577589	3664	3600.0	0.837
ran13x13	758186	78	0	3252	3252	1168.8	0.000
mas284	123569	0	0	91405.72368	91405.72368	116.5	0.000
prod1	2317393	133	0	-56	-56	1741.5	0.000
bc1	34127	0	0	3.338362548	3.338362548	1317.7	0.000
bienst1	58841	23	0	46.75	46.75	749.2	0.000
neos2	703794	29	0	-349.441453	513.2718699	3600.0	246.884
neos3	393853	42	0	-1511.754003	490.5778177	3600.0	132.451
neos7	339479	112	0	650826.8103	733934	3600.0	12.769
swath1	81309	59	0	379.0712957	379.0712957	894.3	0.000
swath2	240136	80	0	382.724	385.1996929	3600.0	0.647
Total (24)	19421589	1811	0			48037.5	438.004
Geom. Mean	262368.9		0.0			938.0	

Table 6: most infeasible branching

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
afLOW30a	46649	22	973061	1158	1158	2229.9	0.000
cap6000	3402	3	4602	-2451377	-2451377	47.1	0.000
gesa2-o	13743	171	437133	25779856.37	25779856.37	856.6	0.000
mas74	551780	0	9853577	11801.18573	11801.18573	3306.4	0.000
mas76	114228	0	1686361	40005.05414	40005.05414	489.4	0.000
misc07	2713	71	95342	2810	2810	292.4	0.000
pk1	56546	0	996973	11	11	505.2	0.000
pp08aCUTS	124	494	2309	7350	7350	12.0	0.000
qiu	15659	1	361712	-132.8731853	-132.8731853	11133.0	0.000
rout	2836	68	126592	1077.56	1077.56	625.7	0.000
vpm2	1457	214	23490	13.75	13.75	44.1	0.000
ran8x32	4902	93	134873	5247	5247	214.4	0.000
ran10x26	8520	56	283797	4270	4270	471.1	0.000
ran12x21	13420	62	486298	3664	3664	780.0	0.000
ran13x13	9147	78	226258	3252	3252	289.5	0.000
mas284	3226	0	57953	91405.72368	91405.72368	61.2	0.000
prod1	10644	143	226101	-56	-56	401.9	0.000
bc1	2981	0	18028	3.338362548	3.338362548	1194.1	0.000
bienst1	3687	23	32384	46.75	46.75	237.8	0.000
neos2	618	32	115613	454.864697	454.864697	872.3	0.000
neos3	1402	42	309856	368.842751	368.842751	2967.0	0.000
neos7	476601	112	4079187	721934	721934	33417.8	0.000
swath1	8285	59	122776	379.0712957	379.0712957	801.7	0.000
swath2	22002	80	361767	385.1996929	385.1996929	2852.2	0.000
Total (24)	1374572	1824	21016043			64102.9	0.000
Geom. Mean	8570.1		181371.7			584.2	

Table 7: full strong branching

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	276002	22	0	1158	1158	582.5	0.000
cap6000	6791	3	0	-2451377	-2451377	52.6	0.000
gesa2-o	126459	171	0	25779856.37	25779856.37	442.7	0.000
mas74	5160828	0	0	11801.18573	11801.18573	2734.2	0.000
mas76	603683	0	0	40005.05414	40005.05414	291.7	0.000
misc07	19407	71	0	2810	2810	54.6	0.000
pk1	437758	0	0	11	11	249.9	0.000
pp08aCUTS	673	494	0	7350	7350	3.4	0.000
qiu	15471	1	0	-132.873137	-132.873137	332.3	0.000
rout	309779	68	0	1077.56	1077.56	976.9	0.000
vpm2	22568	214	0	13.75	13.75	30.6	0.000
ran8x32	40069	93	0	5247	5247	78.7	0.000
ran10x26	128327	56	0	4270	4270	192.5	0.000
ran12x21	234915	62	0	3664	3664	412.4	0.000
ran13x13	149239	78	0	3252	3252	217.7	0.000
mas284	21586	0	0	91405.72368	91405.72368	24.6	0.000
prod1	89293	133	0	-56	-56	151.0	0.000
bc1	40781	0	0	3.338362548	3.338362548	1218.5	0.000
bienst1	19418	23	0	46.75	46.75	230.9	0.000
neos2	609109	29	0	131.4396174	499.0922328	3600.0	279.712
neos3	446035	42	0	-1525.807871	505.4519055	3600.2	133.127
neos7	390910	112	0	721934	721934	2009.4	0.000
swath1	19924	59	0	379.0712957	379.0712957	173.4	0.000
swath2	211976	80	0	385.1996929	385.1996929	2285.3	0.000
Total (24)	9381001	1811	0			19945.8	412.839
Geom. Mean	88706.8		0.0			283.4	

Table 8: pseudocost branching



Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	203544	22	396	1158	1158	403.6	0.000
cap6000	5076	3	92	-2451377	-2451377	38.6	0.000
gesa2-o	57534	171	435	25779856.37	25779856.37	203.2	0.000
mas74	5478830	0	98	11801.18573	11801.18573	3015.9	0.000
mas76	496370	0	69	40005.05414	40005.05414	240.6	0.000
misc07	35187	71	379	2810	2810	97.5	0.000
pk1	367763	0	54	11	11	199.2	0.000
pp08aCUTS	673	494	61	7350	7350	4.5	0.000
qiu	16479	1	48	-132.8731612	-132.8731612	369.5	0.000
rout	44232	68	573	1077.56	1077.56	135.7	0.000
vpm2	13780	214	165	13.75	13.75	18.6	0.000
ran8x32	26360	93	570	5247	5247	54.7	0.000
ran10x26	64640	56	536	4270	4270	99.5	0.000
ran12x21	172161	62	593	3664	3664	272.5	0.000
ran13x13	109241	78	167	3252	3252	159.9	0.000
mas284	24383	0	110	91405.72368	91405.72368	28.8	0.000
prod1	63674	167	153	-56	-56	94.6	0.000
bc1	35132	0	3189	3.338362548	3.338362548	1272.9	0.000
bienst1	10427	23	27	46.75	46.75	107.9	0.000
neos2	187331	29	9998	454.864697	454.864697	675.6	0.000
neos3	737254	42	11730	229.611825	414.5387268	3600.0	80.539
neos7	535586	112	6170	721934	721934	2661.3	0.000
swath1	36264	59	1598	379.0712957	379.0712957	335.3	0.000
swath2	278413	80	1915	385.1996929	385.1996929	3109.8	0.000
Total (24)	9000334	1845	39126			17199.6	80.539
Geom. Mean	70013.6		374.8			216.4	

Table 9: reliability branching ( $\eta_{\text{rel}} = 1, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	180019	22	1453	1158	1158	357.6	0.000
cap6000	4474	3	331	-2451377	-2451377	35.1	0.000
gesa2-o	55888	171	1486	25779856.37	25779856.37	203.2	0.000
mas74	3797950	0	316	11801.18573	11801.18573	1841.2	0.000
mas76	487445	0	302	40005.05414	40005.05414	234.5	0.000
misc07	54541	71	1178	2810	2810	141.5	0.000
pk1	345480	0	204	11	11	197.9	0.000
pp08aCUTS	434	494	232	7350	7350	4.0	0.000
qiu	18619	1	187	-132.8731399	-132.8731399	432.2	0.000
rout	25038	68	1615	1077.56	1077.56	88.5	0.000
vpm2	12502	214	394	13.75	13.75	17.5	0.000
ran8x32	26659	93	1660	5247	5247	53.9	0.000
ran10x26	52969	56	1227	4270	4270	89.2	0.000
ran12x21	137797	62	2717	3664	3664	238.1	0.000
ran13x13	94288	78	842	3252	3252	120.2	0.000
mas284	21146	0	244	91405.72368	91405.72368	24.5	0.000
prod1	61355	167	699	-56	-56	94.4	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1127.6	0.000
bienst1	12603	23	108	46.75	46.75	131.5	0.000
neos2	14779	29	8088	454.864697	454.864697	131.0	0.000
neos3	820037	42	47044	368.842751	368.842751	3573.0	0.000
neos7	574309	112	24899	721934	721934	3329.3	0.000
swath1	22776	59	5016	379.0712957	379.0712957	257.4	0.000
swath2	57431	80	5465	385.1996929	385.1996929	679.8	0.000
Total (24)	6906698	1845	110628			13402.9	0.000
Geom. Mean	53522.9		1176.7			178.2	

Table 10: reliability branching ( $\eta_{\text{rel}} = 4, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	181397	22	2847	1158	1158	416.4	0.000
cap6000	4253	3	597	-2451377	-2451377	36.1	0.000
gesa2-o	53881	171	2331	25779856.37	25779856.37	199.1	0.000
mas74	5521061	0	649	11801.18573	11801.18573	2807.4	0.000
mas76	482122	0	530	40005.05414	40005.05414	225.1	0.000
misc07	54932	71	2471	2810	2810	150.2	0.000
pk1	331339	0	391	11	11	193.3	0.000
pp08aCUTS	489	494	433	7350	7350	5.4	0.000
qiu	18405	1	363	-132.8731399	-132.8731399	419.1	0.000
rout	13883	68	2703	1077.56	1077.56	58.4	0.000
vpm2	9648	214	881	13.75	13.75	15.7	0.000
ran8x32	17668	93	2657	5247	5247	38.3	0.000
ran10x26	48065	56	1985	4270	4270	86.5	0.000
ran12x21	135037	62	2675	3664	3664	214.5	0.000
ran13x13	95288	78	1276	3252	3252	136.9	0.000
mas284	21179	0	462	91405.72368	91405.72368	25.9	0.000
prod1	64186	167	1202	-56	-56	100.8	0.000
bc1	25196	0	5092	3.338362548	3.338362548	1120.0	0.000
bienst1	13911	23	211	46.75	46.75	115.3	0.000
neos2	22742	29	15893	454.864697	454.864697	206.7	0.000
neos3	556835	42	45143	368.842751	368.842751	2707.2	0.000
neos7	202482	112	11256	721934	721934	1039.8	0.000
swath1	35161	59	8170	379.0712957	379.0712957	407.3	0.000
swath2	28808	80	7425	385.1996929	385.1996929	407.2	0.000
Total (24)	7937968	1845	117643			11132.7	0.000
Geom. Mean	48772.8		1850.3			170.5	

Table 11: reliability branching ( $\eta_{\text{rel}} = 8, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	175649	22	5554	1158	1158	426.2	0.000
cap6000	4998	3	1099	-2451377	-2451377	46.6	0.000
gesa2-o	51495	171	3800	25779856.37	25779856.37	203.7	0.000
mas74	3803361	0	1173	11801.18573	11801.18573	1848.8	0.000
mas76	497653	0	1042	40005.05414	40005.05414	246.0	0.000
misc07	38680	71	6352	2810	2810	115.1	0.000
pk1	313777	0	756	11	11	176.5	0.000
pp08aCUTS	324	494	896	7350	7350	7.0	0.000
qiu	13101	1	723	-132.8731399	-132.8731399	323.7	0.000
rout	19412	68	5339	1077.56	1077.56	90.2	0.000
vpm2	9870	214	1974	13.75	13.75	17.4	0.000
ran8x32	18728	93	5116	5247	5247	48.3	0.000
ran10x26	39207	56	7289	4270	4270	76.3	0.000
ran12x21	112818	62	6068	3664	3664	192.1	0.000
ran13x13	97849	78	4815	3252	3252	155.1	0.000
mas284	20815	0	931	91405.72368	91405.72368	25.8	0.000
prod1	65130	167	2375	-56	-56	105.4	0.000
bc1	24188	0	7414	3.338362548	3.338362548	1204.0	0.000
bienst1	14586	23	409	46.75	46.75	139.9	0.000
neos2	9370	29	14558	454.864697	454.864697	144.2	0.000
neos3	301563	42	56269	368.842751	368.842751	1928.9	0.000
neos7	283419	112	22956	721934	721934	1789.2	0.000
swath1	16857	59	13103	379.0712957	379.0712957	297.9	0.000
swath2	89174	80	17567	385.1996929	385.1996929	1174.3	0.000
Total (24)	6022024	1845	187578			10782.6	0.000
Geom. Mean	44649.9		3640.6			179.0	

Table 12: reliability branching ( $\eta_{\text{rel}} = 16, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	235478	22	11184	1158	1158	562.5	0.000
cap6000	4010	3	1695	-2451377	-2451377	42.2	0.000
gesa2-o	40561	171	6902	25779856.37	25779856.37	175.5	0.000
mas74	5961790	0	2447	11801.18573	11801.18573	3370.3	0.000
mas76	501169	0	1940	40005.05414	40005.05414	241.1	0.000
misc07	46618	71	11622	2810	2810	154.4	0.000
pk1	292458	0	1494	11	11	157.6	0.000
pp08aCUTS	280	494	1422	7350	7350	9.9	0.000
qiu	14163	1	1470	-132.8731399	-132.8731399	382.1	0.000
rout	19346	68	9680	1077.56	1077.56	114.3	0.000
vpm2	8959	214	3171	13.75	13.75	19.4	0.000
ran8x32	12726	93	7783	5247	5247	42.9	0.000
ran10x26	42754	56	8036	4270	4270	96.2	0.000
ran12x21	116887	62	8973	3664	3664	207.8	0.000
ran13x13	81012	78	5962	3252	3252	122.0	0.000
mas284	20839	0	1770	91405.72368	91405.72368	27.2	0.000
prod1	67782	167	4442	-56	-56	113.4	0.000
bc1	20416	0	10331	3.338362548	3.338362548	1214.2	0.000
bienst1	10126	23	831	46.75	46.75	103.6	0.000
neos2	6843	29	23911	454.864697	454.864697	188.3	0.000
neos3	167218	42	58432	368.842751	368.842751	1310.0	0.000
neos7	221647	112	31955	721934	721934	1489.7	0.000
swath1	20639	59	19722	379.0712957	379.0712957	416.1	0.000
swath2	27076	80	17839	385.1996929	385.1996929	542.2	0.000
Total (24)	7940797	1845	253014			11103.0	0.000
Geom. Mean	39655.2		5837.8			184.2	

Table 13: reliability branching ( $\eta_{\text{rel}} = 32, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	144280	22	3268	1158	1158	297.9	0.000
cap6000	4474	3	331	-2451377	-2451377	35.7	0.000
gesa2-o	40741	171	1816	25779856.37	25779856.37	152.6	0.000
mas74	5348980	0	659	11801.18573	11801.18573	2859.8	0.000
mas76	464712	0	514	40005.05414	40005.05414	217.6	0.000
misc07	49930	71	1590	2810	2810	134.2	0.000
pk1	367800	0	463	11	11	229.8	0.000
pp08aCUTS	324	494	524	7350	7350	5.1	0.000
qiu	14663	1	358	-132.8731399	-132.8731399	357.0	0.000
rout	24331	68	3397	1077.56	1077.56	101.0	0.000
vpm2	11174	214	954	13.75	13.75	17.3	0.000
ran8x32	25860	93	2425	5247	5247	57.1	0.000
ran10x26	47026	56	2316	4270	4270	88.1	0.000
ran12x21	120986	62	3124	3664	3664	195.1	0.000
ran13x13	83928	78	1975	3252	3252	123.1	0.000
mas284	19724	0	377	91405.72368	91405.72368	25.3	0.000
prod1	60992	167	686	-56	-56	99.3	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1167.8	0.000
bienst1	10917	23	274	46.75	46.75	123.2	0.000
neos2	45534	29	16141	454.864697	454.864697	254.8	0.000
neos3	760920	42	39282	368.842751	368.842751	3228.7	0.000
neos7	332730	112	7631	721934	721934	1743.7	0.000
swath1	13953	59	7755	379.0712957	379.0712957	214.9	0.000
swath2	12363	80	7087	385.1996929	385.1996929	227.8	0.000
Total (24)	8034501	1845	107868			11956.9	0.000
Geom. Mean	47082.6		1775.0			169.3	

Table 14: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 16, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	149993	22	6162	1158	1158	317.2	0.000
cap6000	4552	3	326	-2451377	-2451377	34.8	0.000
gesa2-o	55411	171	2256	25779856.37	25779856.37	213.0	0.000
mas74	4290897	0	1092	11801.18573	11801.18573	2094.2	0.000
mas76	484027	0	824	40005.05414	40005.05414	217.3	0.000
misc07	48698	71	2356	2810	2810	140.6	0.000
pk1	386040	0	769	11	11	239.8	0.000
pp08aCUTS	301	494	766	7350	7350	6.2	0.000
qiu	14177	1	554	-132.8731517	-132.8731517	339.2	0.000
rout	14769	68	5986	1077.56	1077.56	86.5	0.000
vpm2	10698	214	1607	13.75	13.75	18.8	0.000
ran8x32	19384	93	4884	5247	5247	48.6	0.000
ran10x26	42580	56	4466	4270	4270	85.0	0.000
ran12x21	126350	62	4788	3664	3664	211.6	0.000
ran13x13	82494	78	3580	3252	3252	110.1	0.000
mas284	21934	0	571	91405.72368	91405.72368	28.4	0.000
prod1	63495	167	755	-56	-56	100.2	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1177.5	0.000
bienst1	13603	23	463	46.75	46.75	149.5	0.000
neos2	24153	29	13343	454.864697	454.864697	192.1	0.000
neos3	546419	42	56572	368.842751	368.842751	2602.5	0.000
neos7	252863	112	18920	721934	721934	1513.8	0.000
swath1	18164	59	14170	379.0712957	379.0712957	337.6	0.000
swath2	41030	80	16642	385.1996929	385.1996929	663.2	0.000
Total (24)	6740191	1845	166773			10927.8	0.000
Geom. Mean	46664.5		2757.3			178.0	

Table 15: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 32, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	170547	22	397	1158	1158	391.7	0.000
cap6000	5076	3	92	-2451377	-2451377	37.8	0.000
gesa2-o	79841	171	1841	25779856.37	25779856.37	284.3	0.000
mas74	5296806	0	88	11801.18573	11801.18573	3066.4	0.000
mas76	496370	0	69	40005.05414	40005.05414	242.2	0.000
misc07	41902	71	266	2810	2810	113.2	0.000
pk1	367763	0	54	11	11	200.8	0.000
pp08aCUTS	651	494	60	7350	7350	4.4	0.000
qiu	16148	1	48	-132.873137	-132.873137	371.2	0.000
rout	39192	68	426	1077.56	1077.56	122.2	0.000
vpm2	17409	214	141	13.75	13.75	25.0	0.000
ran8x32	31697	93	667	5247	5247	63.7	0.000
ran10x26	68237	56	586	4270	4270	104.7	0.000
ran12x21	159383	62	940	3664	3664	250.8	0.000
ran13x13	97195	78	311	3252	3252	144.8	0.000
mas284	21217	0	85	91405.72368	91405.72368	23.7	0.000
prod1	65679	163	158	-56	-56	96.2	0.000
bc1	35132	0	3189	3.338362548	3.338362548	1295.2	0.000
bienst1	9248	23	27	46.75	46.75	83.3	0.000
neos2	83146	29	13311	454.864697	454.864697	385.5	0.000
neos3	736396	42	19218	323.1427543	378.749656	3600.0	17.208
neos7	498573	112	5944	721934	721934	2377.6	0.000
swath1	66995	59	2088	379.0712957	379.0712957	604.6	0.000
swath2	258934	80	3551	385.1996929	385.1996929	2864.5	0.000
Total (24)	8663537	1841	53557			16753.8	17.208
Geom. Mean	69501.0		429.0			217.2	

Table 16: reliability branching ( $\eta_{\text{rel}} = 1, \lambda = 8$ )



Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	231706	22	1467	1158	1158	452.6	0.000
cap6000	4474	3	331	-2451377	-2451377	37.1	0.000
gesa2-o	53054	171	1287	25779856.37	25779856.37	189.8	0.000
mas74	5771953	0	341	11801.18573	11801.18573	3173.7	0.000
mas76	427671	0	267	40005.05414	40005.05414	188.4	0.000
misc07	42609	71	1394	2810	2810	117.1	0.000
pk1	318561	0	204	11	11	186.6	0.000
pp08aCUTS	421	494	260	7350	7350	3.9	0.000
qiu	21411	1	188	-132.8731565	-132.8731565	445.4	0.000
rout	28642	68	1853	1077.56	1077.56	90.8	0.000
vpm2	10494	214	544	13.75	13.75	15.4	0.000
ran8x32	27887	93	2164	5247	5247	58.1	0.000
ran10x26	51712	56	1612	4270	4270	87.5	0.000
ran12x21	127131	62	1243	3664	3664	203.2	0.000
ran13x13	101209	78	925	3252	3252	147.0	0.000
mas284	20266	0	271	91405.72368	91405.72368	24.2	0.000
prod1	58136	163	624	-56	-56	89.9	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1139.7	0.000
bienst1	10303	23	104	46.75	46.75	104.9	0.000
neos2	50790	29	13680	454.864697	454.864697	292.5	0.000
neos3	347769	42	26934	368.842751	368.842751	1712.9	0.000
neos7	477884	112	5148	721934	721934	2345.2	0.000
swath1	20170	59	4174	379.0712957	379.0712957	218.3	0.000
swath2	105974	80	4970	385.1996929	385.1996929	1173.0	0.000
Total (24)	8338386	1841	74906			12497.2	0.000
Geom. Mean	54937.7		1104.6			179.2	

Table 17: reliability branching ( $\eta_{\text{rel}} = 4, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	171909	22	2865	1158	1158	368.2	0.000
cap6000	4253	3	597	-2451377	-2451377	36.3	0.000
gesa2-o	50078	171	2178	25779856.37	25779856.37	195.7	0.000
mas74	5425001	0	673	11801.18573	11801.18573	3104.2	0.000
mas76	336826	0	558	40005.05414	40005.05414	161.2	0.000
misc07	55740	71	3262	2810	2810	150.2	0.000
pk1	311611	0	402	11	11	170.3	0.000
pp08aCUTS	464	494	438	7350	7350	5.5	0.000
qiu	14847	1	370	-132.8731516	-132.8731516	341.9	0.000
rout	19743	68	3473	1077.56	1077.56	81.3	0.000
vpm2	10054	214	1089	13.75	13.75	16.3	0.000
ran8x32	21092	93	2534	5247	5247	47.2	0.000
ran10x26	48626	56	3417	4270	4270	82.0	0.000
ran12x21	124455	62	2591	3664	3664	199.0	0.000
ran13x13	93939	78	1521	3252	3252	136.1	0.000
mas284	20360	0	477	91405.72368	91405.72368	24.1	0.000
prod1	62689	163	1173	-56	-56	97.8	0.000
bc1	25196	0	5092	3.338362548	3.338362548	1153.0	0.000
bienst1	9951	23	206	46.75	46.75	91.0	0.000
neos2	30790	29	13994	454.864697	454.864697	222.2	0.000
neos3	626894	42	54459	368.842751	368.842751	3145.8	0.000
neos7	252766	112	17062	721934	721934	1350.3	0.000
swath1	10615	59	6474	379.0712957	379.0712957	166.2	0.000
swath2	85510	80	8640	385.1996929	385.1996929	1034.6	0.000
Total (24)	7813409	1841	133545			12380.2	0.000
Geom. Mean	48377.3		1998.0			170.2	

Table 18: reliability branching ( $\eta_{\text{rel}} = 8, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	163331	22	5690	1158	1158	364.2	0.000
cap6000	4998	3	1099	-2451377	-2451377	49.1	0.000
gesa2-o	39751	171	3874	25779856.37	25779856.37	160.6	0.000
mas74	5387375	0	1316	11801.18573	11801.18573	3068.4	0.000
mas76	304694	0	1029	40005.05414	40005.05414	136.1	0.000
misc07	52422	71	5859	2810	2810	144.7	0.000
pk1	323364	0	793	11	11	179.6	0.000
pp08aCUTS	397	494	909	7350	7350	7.7	0.000
qiu	12550	1	736	-132.8731399	-132.8731399	332.8	0.000
rout	19531	68	5356	1077.56	1077.56	93.2	0.000
vpm2	10714	214	2040	13.75	13.75	18.6	0.000
ran8x32	22464	93	5366	5247	5247	54.2	0.000
ran10x26	33990	56	8612	4270	4270	74.5	0.000
ran12x21	129279	62	4940	3664	3664	216.4	0.000
ran13x13	84623	78	3222	3252	3252	116.5	0.000
mas284	19818	0	995	91405.72368	91405.72368	26.2	0.000
prod1	60253	163	2310	-56	-56	97.2	0.000
bc1	23936	0	7496	3.338362548	3.338362548	1203.7	0.000
bienst1	12210	23	398	46.75	46.75	106.7	0.000
neos2	16624	29	16704	454.864697	454.864697	187.8	0.000
neos3	435159	42	42205	368.842751	368.842751	2208.7	0.000
neos7	390198	112	41645	721934	721934	2508.0	0.000
swath1	13339	59	11324	379.0712957	379.0712957	241.1	0.000
swath2	18380	80	11218	385.1996929	385.1996929	350.9	0.000
Total (24)	7579400	1841	185136			11946.7	0.000
Geom. Mean	43311.9		3589.7			171.3	

Table 19: reliability branching ( $\eta_{\text{rel}} = 16, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	145980	22	11147	1158	1158	305.3	0.000
cap6000	4010	3	1695	-2451377	-2451377	42.1	0.000
gesa2-o	40222	171	6292	25779856.37	25779856.37	168.6	0.000
mas74	4999610	0	2480	11801.18573	11801.18573	2823.9	0.000
mas76	454361	0	2234	40005.05414	40005.05414	205.6	0.000
misc07	48338	71	11508	2810	2810	155.7	0.000
pk1	352716	0	1551	11	11	204.2	0.000
pp08aCUTS	317	494	1599	7350	7350	10.8	0.000
qiu	12777	1	1476	-132.8731595	-132.8731595	366.7	0.000
rout	20773	68	11017	1077.56	1077.56	122.6	0.000
vpm2	8698	214	3315	13.75	13.75	18.8	0.000
ran8x32	18155	93	6352	5247	5247	51.6	0.000
ran10x26	39713	56	8177	4270	4270	91.7	0.000
ran12x21	114304	62	9862	3664	3664	207.0	0.000
ran13x13	84047	78	6529	3252	3252	135.8	0.000
mas284	18623	0	1773	91405.72368	91405.72368	24.9	0.000
prod1	59335	163	4284	-56	-56	103.6	0.000
bc1	18385	0	10011	3.338362548	3.338362548	1213.1	0.000
bienst1	10229	23	736	46.75	46.75	90.6	0.000
neos2	9823	29	20941	454.864697	454.864697	177.6	0.000
neos3	430739	42	65191	368.842751	368.842751	2559.9	0.000
neos7	256009	112	29359	721934	721934	1621.4	0.000
swath1	25453	59	20225	379.0712957	379.0712957	456.0	0.000
swath2	35219	80	21728	385.1996929	385.1996929	678.2	0.000
Total (24)	7207836	1841	259482			11835.7	0.000
Geom. Mean	42047.5		5913.2			186.5	

Table 20: reliability branching ( $\eta_{\text{rel}} = 32, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	147325	22	3236	1158	1158	290.9	0.000
cap6000	4474	3	331	-2451377	-2451377	35.5	0.000
gesa2-o	49412	171	1597	25779856.37	25779856.37	187.3	0.000
mas74	5328123	0	709	11801.18573	11801.18573	3077.8	0.000
mas76	374046	0	545	40005.05414	40005.05414	171.1	0.000
misc07	58624	71	1474	2810	2810	160.4	0.000
pk1	365257	0	491	11	11	214.8	0.000
pp08aCUTS	541	494	521	7350	7350	6.4	0.000
qiu	15641	1	365	-132.8731601	-132.8731601	370.1	0.000
rout	84775	68	3820	1077.56	1077.56	236.2	0.000
vpm2	9272	214	1100	13.75	13.75	15.3	0.000
ran8x32	18916	93	2772	5247	5247	44.3	0.000
ran10x26	47409	56	2381	4270	4270	82.9	0.000
ran12x21	118905	62	2678	3664	3664	189.0	0.000
ran13x13	90486	78	1898	3252	3252	132.4	0.000
mas284	21470	0	405	91405.72368	91405.72368	28.1	0.000
prod1	61400	163	686	-56	-56	97.2	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1154.7	0.000
bienst1	11446	23	271	46.75	46.75	124.2	0.000
neos2	39797	29	17509	454.864697	454.864697	315.9	0.000
neos3	676114	42	43070	368.842751	368.842751	2918.1	0.000
neos7	317136	112	18765	721934	721934	1619.1	0.000
swath1	12704	59	7784	379.0712957	379.0712957	204.5	0.000
swath2	22656	80	7533	385.1996929	385.1996929	347.3	0.000
Total (24)	7904088	1841	124862			12023.3	0.000
Geom. Mean	50937.3		1882.9			178.9	

Table 21: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 16, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	162318	22	6400	1158	1158	389.1	0.000
cap6000	4552	3	326	-2451377	-2451377	35.6	0.000
gesa2-o	45052	171	2292	25779856.37	25779856.37	174.2	0.000
mas74	5519583	0	1181	11801.18573	11801.18573	3084.8	0.000
mas76	382316	0	804	40005.05414	40005.05414	174.5	0.000
misc07	48927	71	2664	2810	2810	137.5	0.000
pk1	330389	0	861	11	11	172.3	0.000
pp08aCUTS	385	494	852	7350	7350	7.1	0.000
qiu	12348	1	602	-132.8731458	-132.8731458	306.5	0.000
rout	25409	68	6910	1077.56	1077.56	123.2	0.000
vpm2	10858	214	1626	13.75	13.75	18.3	0.000
ran8x32	23086	93	4176	5247	5247	57.1	0.000
ran10x26	44861	56	4301	4270	4270	87.0	0.000
ran12x21	110413	62	4515	3664	3664	195.6	0.000
ran13x13	82666	78	3051	3252	3252	125.8	0.000
mas284	18756	0	577	91405.72368	91405.72368	23.7	0.000
prod1	60658	163	773	-56	-56	95.5	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1195.4	0.000
bienst1	12295	23	421	46.75	46.75	108.0	0.000
neos2	25223	29	15719	454.864697	454.864697	177.9	0.000
neos3	660840	42	41400	356.1420421	370.6295699	3600.0	4.068
neos7	235967	112	11209	721934	721934	1086.8	0.000
swath1	11936	59	11453	379.0712957	379.0712957	235.1	0.000
swath2	14473	80	11245	385.1996929	385.1996929	307.9	0.000
Total (24)	7871470	1841	138279			11918.9	4.068
Geom. Mean	44566.0		2637.9			168.4	

Table 22: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 32, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	225745	22	13683	1158	1158	488.8	0.000
cap6000	4345	3	355	-2451377	-2451377	33.5	0.000
gesa2-o	42066	171	3426	25779856.37	25779856.37	168.1	0.000
mas74	5512555	0	2145	11801.18573	11801.18573	3346.6	0.000
mas76	498782	0	1558	40005.05414	40005.05414	238.2	0.000
misc07	39334	71	4535	2810	2810	121.4	0.000
pk1	357602	0	1381	11	11	195.4	0.000
pp08aCUTS	365	494	1084	7350	7350	8.8	0.000
qiu	12767	1	975	-132.8731517	-132.8731517	339.1	0.000
rout	26396	68	11477	1077.56	1077.56	159.1	0.000
vpm2	10531	214	2762	13.75	13.75	21.2	0.000
ran8x32	20511	93	7465	5247	5247	59.5	0.000
ran10x26	41057	56	8448	4270	4270	94.9	0.000
ran12x21	120191	62	8681	3664	3664	213.3	0.000
ran13x13	78051	78	6255	3252	3252	116.1	0.000
mas284	21132	0	897	91405.72368	91405.72368	26.9	0.000
prod1	65618	163	940	-56	-56	100.7	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1128.2	0.000
bienst1	10173	23	633	46.75	46.75	105.8	0.000
neos2	60902	29	19507	454.864697	454.864697	338.4	0.000
neos3	552853	42	45623	368.842751	368.842751	3020.7	0.000
neos7	278237	112	17159	721934	721934	1359.9	0.000
swath1	11209	59	16460	379.0712957	379.0712957	277.1	0.000
swath2	21260	80	20233	385.1996929	385.1996929	491.2	0.000
Total (24)	8039841	1841	200603			12452.9	0.000
Geom. Mean	47137.5		4108.1			189.3	

Table 23: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 64, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	188650	22	9662	1158	1158	425.4	0.000
cap6000	4474	3	331	-2451377	-2451377	35.6	0.000
gesa2-o	48071	171	2604	25779856.37	25779856.37	184.6	0.000
mas74	5062536	0	6405	11801.18573	11801.18573	2904.2	0.000
mas76	498594	0	5312	40005.05414	40005.05414	229.1	0.000
misc07	56391	71	2755	2810	2810	147.9	0.000
pk1	386217	0	4443	11	11	234.3	0.000
pp08aCUTS	225	494	2067	7350	7350	13.0	0.000
qiu	18001	1	369	-132.8731399	-132.8731399	430.9	0.000
rout	13521	68	16178	1077.56	1077.56	125.1	0.000
vpm2	6664	214	6588	13.75	13.75	22.9	0.000
ran8x32	16388	93	11606	5247	5247	59.6	0.000
ran10x26	42209	56	11470	4270	4270	92.9	0.000
ran12x21	113008	62	12514	3664	3664	209.9	0.000
ran13x13	76934	78	9969	3252	3252	124.3	0.000
mas284	19827	0	386	91405.72368	91405.72368	24.6	0.000
prod1	62556	163	2997	-56	-56	103.9	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1133.8	0.000
bienst1	11118	23	1148	46.75	46.75	99.0	0.000
neos2	37892	29	11182	454.864697	454.864697	209.3	0.000
neos3	347769	42	26934	368.842751	368.842751	1708.5	0.000
neos7	477884	112	5148	721934	721934	2299.4	0.000
swath1	20170	59	4174	379.0712957	379.0712957	218.9	0.000
swath2	105974	80	4970	385.1996929	385.1996929	1165.3	0.000
Total (24)	7643232	1841	164133			12202.3	0.000
Geom. Mean	48268.9		4214.2			194.9	

Table 24: dynamic reliability branching ( $\eta_{\text{rel}} = 4 - 128, \lambda = 8$ )



Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	184984	22	407	1158	1158	414.9	0.000
cap6000	5076	3	92	-2451377	-2451377	37.6	0.000
gesa2-o	66583	171	627	25779856.37	25779856.37	248.3	0.000
mas74	4207847	0	96	11801.18573	11801.18573	2020.7	0.000
mas76	496370	0	69	40005.05414	40005.05414	239.5	0.000
misc07	41535	71	387	2810	2810	119.6	0.000
pk1	367763	0	54	11	11	193.7	0.000
pp08aCUTS	651	494	60	7350	7350	4.5	0.000
qiu	16148	1	48	-132.873137	-132.873137	367.7	0.000
rout	45318	68	821	1077.56	1077.56	121.6	0.000
vpm2	13124	214	122	13.75	13.75	17.8	0.000
ran8x32	46039	93	1577	5247	5247	83.8	0.000
ran10x26	63050	56	692	4270	4270	94.7	0.000
ran12x21	208719	62	633	3664	3664	353.9	0.000
ran13x13	115793	78	187	3252	3252	166.5	0.000
mas284	21964	0	106	91405.72368	91405.72368	25.2	0.000
prod1	64193	156	153	-56	-56	98.0	0.000
bc1	35132	0	3189	3.338362548	3.338362548	1285.6	0.000
bienst1	8703	23	28	46.75	46.75	81.6	0.000
neos2	178106	29	12153	454.864697	454.864697	645.5	0.000
neos3	817180	42	19520	363.852925	368.842751	3600.0	1.371
neos7	417083	112	1502	721934	721934	2121.0	0.000
swath1	66995	59	2088	379.0712957	379.0712957	609.9	0.000
swath2	258934	80	3551	385.1996929	385.1996929	2874.2	0.000
Total (24)	7747290	1834	48162			15825.8	1.371
Geom. Mean	72159.1		408.4			220.0	

Table 25: reliability branching ( $\eta_{\text{rel}} = 1, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	128910	22	1463	1158	1158	296.7	0.000
cap6000	4474	3	331	-2451377	-2451377	35.1	0.000
gesa2-o	48818	171	1354	25779856.37	25779856.37	184.8	0.000
mas74	6290002	0	374	11794.87806	11801.18573	3600.0	0.053
mas76	459034	0	286	40005.05414	40005.05414	214.4	0.000
misc07	54790	71	1053	2810	2810	145.2	0.000
pk1	328700	0	205	11	11	187.4	0.000
pp08aCUTS	570	494	230	7350	7350	4.9	0.000
qiu	22147	1	188	-132.8731517	-132.8731517	478.6	0.000
rout	37624	68	1803	1077.56	1077.56	149.9	0.000
vpm2	13934	214	604	13.75	13.75	20.3	0.000
ran8x32	29101	93	2645	5247	5247	64.9	0.000
ran10x26	51788	56	1511	4270	4270	82.2	0.000
ran12x21	130173	62	1413	3664	3664	218.6	0.000
ran13x13	89596	78	591	3252	3252	126.4	0.000
mas284	22051	0	264	91405.72368	91405.72368	26.5	0.000
prod1	69166	156	525	-56	-56	108.1	0.000
bc1	28159	0	4921	3.338362548	3.338362548	1123.8	0.000
bienst1	13752	23	108	46.75	46.75	140.2	0.000
neos2	38746	29	9972	454.864697	454.864697	221.9	0.000
neos3	718479	42	32418	354.3425985	369.4101772	3600.0	4.252
neos7	357385	112	5643	721934	721934	1690.7	0.000
swath1	29728	59	5713	379.0712957	379.0712957	318.2	0.000
swath2	101596	80	5010	385.1996929	385.1996929	1219.4	0.000
Total (24)	9068723	1834	78625			14258.1	4.306
Geom. Mean	58886.4		1096.6			195.8	

Table 26: reliability branching ( $\eta_{\text{rel}} = 4, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	144418	22	2880	1158	1158	315.6	0.000
cap6000	4253	3	597	-2451377	-2451377	36.1	0.000
gesa2-o	51454	171	2435	25779856.37	25779856.37	211.3	0.000
mas74	6094785	0	675	11801.18573	11801.18573	3313.9	0.000
mas76	380456	0	506	40005.05414	40005.05414	162.4	0.000
misc07	55242	71	3064	2810	2810	152.9	0.000
pk1	237780	0	399	11	11	119.2	0.000
pp08aCUTS	400	494	453	7350	7350	5.3	0.000
qiu	19736	1	373	-132.8731462	-132.8731462	456.5	0.000
rout	88970	68	3399	1077.56	1077.56	227.6	0.000
vpm2	15286	214	1182	13.75	13.75	23.8	0.000
ran8x32	28033	93	2823	5247	5247	60.8	0.000
ran10x26	43295	56	4589	4270	4270	76.2	0.000
ran12x21	129531	62	3020	3664	3664	213.3	0.000
ran13x13	101314	78	1492	3252	3252	146.2	0.000
mas284	22644	0	486	91405.72368	91405.72368	29.0	0.000
prod1	62944	156	1092	-56	-56	103.5	0.000
bc1	25196	0	5092	3.338362548	3.338362548	1113.2	0.000
bienst1	11677	23	209	46.75	46.75	115.1	0.000
neos2	26049	29	15229	454.864697	454.864697	193.4	0.000
neos3	582896	42	57473	368.6645774	368.842751	3600.0	0.048
neos7	359672	112	9819	721934	721934	1997.9	0.000
swath1	32431	59	8610	379.0712957	379.0712957	384.5	0.000
swath2	32583	80	9644	385.1996929	385.1996929	505.4	0.000
Total (24)	8551045	1834	135541			13563.0	0.048
Geom. Mean	54118.3		2042.9			189.6	

Table 27: reliability branching ( $\eta_{\text{rel}} = 8, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	144316	22	5506	1158	1158	344.2	0.000
cap6000	4998	3	1099	-2451377	-2451377	46.6	0.000
gesa2-o	46488	171	3745	25779856.37	25779856.37	181.1	0.000
mas74	3990024	0	1222	11801.18573	11801.18573	1954.9	0.000
mas76	439700	0	997	40005.05414	40005.05414	201.7	0.000
misc07	53696	71	5570	2810	2810	150.0	0.000
pk1	366697	0	776	11	11	219.8	0.000
pp08aCUTS	355	494	878	7350	7350	7.2	0.000
qiu	33771	1	738	-132.873154	-132.873154	697.6	0.000
rout	21702	68	6949	1077.56	1077.56	109.5	0.000
vpm2	12972	214	2561	13.75	13.75	22.4	0.000
ran8x32	18647	93	4109	5247	5247	47.4	0.000
ran10x26	50813	56	4297	4270	4270	99.2	0.000
ran12x21	115856	62	6644	3664	3664	204.8	0.000
ran13x13	79323	78	3509	3252	3252	122.7	0.000
mas284	20083	0	951	91405.72371	91405.72371	25.8	0.000
prod1	60939	156	1987	-56	-56	95.8	0.000
bc1	23936	0	7496	3.338362548	3.338362548	1201.2	0.000
bienst1	9458	23	403	46.75	46.75	81.6	0.000
neos2	24640	29	18651	454.864697	454.864697	233.4	0.000
neos3	576820	42	54772	368.842751	368.842751	3080.0	0.000
neos7	391371	112	35066	721934	721934	2437.4	0.000
swath1	16303	59	11900	379.0712957	379.0712957	280.0	0.000
swath2	64524	80	16394	385.1996929	385.1996929	922.0	0.000
Total (24)	6567432	1834	196220			12766.4	0.000
Geom. Mean	49839.9		3601.0			191.6	

Table 28: reliability branching ( $\eta_{\text{rel}} = 16, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	151594	22	11085	1158	1158	367.8	0.000
cap6000	4010	3	1695	-2451377	-2451377	42.1	0.000
gesa2-o	37936	171	6668	25779856.37	25779856.37	168.4	0.000
mas74	5365880	0	2437	11801.18573	11801.18573	3006.0	0.000
mas76	452126	0	2002	40005.05414	40005.05414	217.7	0.000
misc07	53100	71	11269	2810	2810	172.0	0.000
pk1	298672	0	1560	11	11	165.7	0.000
pp08aCUTS	227	494	1413	7350	7350	9.6	0.000
qiu	25413	1	1476	-132.873155	-132.873155	661.3	0.000
rout	16069	68	9722	1077.56	1077.56	112.8	0.000
vpm2	10261	214	3324	13.75	13.75	20.9	0.000
ran8x32	12774	93	6518	5247	5247	41.9	0.000
ran10x26	37628	56	10732	4270	4270	88.4	0.000
ran12x21	109430	62	10258	3664	3664	200.9	0.000
ran13x13	86213	78	5551	3252	3252	145.6	0.000
mas284	19414	0	1775	91405.72368	91405.72368	25.4	0.000
prod1	63365	156	3836	-56	-56	105.7	0.000
bc1	18385	0	10011	3.338362548	3.338362548	1148.8	0.000
bienst1	9377	23	740	46.75	46.75	89.6	0.000
neos2	7598	29	21802	454.864697	454.864697	181.5	0.000
neos3	289503	42	73905	368.842751	368.842751	1893.4	0.000
neos7	357720	112	37595	721934	721934	2146.8	0.000
swath1	20805	59	22796	379.0712957	379.0712957	448.6	0.000
swath2	55442	80	23652	385.1996929	385.1996929	933.2	0.000
Total (24)	7502942	1834	281822			12393.6	0.000
Geom. Mean	41636.1		6000.7			192.5	

Table 29: reliability branching ( $\eta_{\text{rel}} = 32, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	198089	22	247	1158	1158	380.1	0.000
cap6000	5795	3	64	-2451377	-2451377	43.1	0.000
gesa2-o	75541	171	224	25779856.37	25779856.37	266.1	0.000
mas74	5617233	0	226	11801.18573	11801.18573	2884.3	0.000
mas76	578986	0	212	40005.05414	40005.05414	276.7	0.000
misc07	43418	71	380	2810	2810	111.8	0.000
pk1	414067	0	288	11	11	261.4	0.000
pp08aCUTS	244	494	332	7350	7350	3.4	0.000
qiu	23227	1	271	-132.8731399	-132.8731399	532.9	0.000
rout	207437	68	266	1077.56	1077.56	490.1	0.000
vpm2	12462	214	250	13.75	13.75	15.9	0.000
ran8x32	32549	93	192	5247	5247	62.7	0.000
ran10x26	78545	56	240	4270	4270	120.6	0.000
ran12x21	162745	62	247	3664	3664	250.0	0.000
ran13x13	101572	78	202	3252	3252	129.9	0.000
mas284	21437	0	253	91405.72368	91405.72368	24.9	0.000
prod1	64223	167	43	-56	-56	98.0	0.000
bc1	40237	0	137	3.338362548	3.338362548	1234.2	0.000
bienst1	9047	23	240	46.75	46.75	85.2	0.000
neos2	408051	29	542	454.864697	454.864697	1871.5	0.000
neos3	568518	42	137	30.55608862	422.2833322	3600.0	1281.994
neos7	746440	112	395	720799.3672	721934	3600.0	0.157
swath1	68506	59	191	379.0712957	379.0712957	599.9	0.000
swath2	220028	80	213	385.1996929	385.1996929	2545.2	0.000
Total (24)	9698397	1845	5792			19487.8	1282.151
Geom. Mean	79535.5		216.2			249.3	

Table 30: hybrid strong/pseudocost branching ( $d = 5, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	164112	22	3625	1158	1158	364.6	0.000
cap6000	4957	3	762	-2451377	-2451377	41.1	0.000
gesa2-o	75141	171	2231	25779856.37	25779856.37	275.0	0.000
mas74	5163685	0	6609	11801.18573	11801.18573	2750.3	0.000
mas76	587813	0	6307	40005.05414	40005.05414	274.5	0.000
misc07	49822	71	3120	2810	2810	141.6	0.000
pk1	294469	0	7008	11	11	154.3	0.000
pp08aCUTS	355	494	954	7350	7350	7.8	0.000
qiu	25180	1	6879	-132.8731547	-132.8731547	927.4	0.000
rout	45457	68	3004	1077.56	1077.56	150.0	0.000
vpm2	11710	214	1948	13.75	13.75	18.2	0.000
ran8x32	29957	93	1770	5247	5247	62.4	0.000
ran10x26	54965	56	3038	4270	4270	89.0	0.000
ran12x21	126153	62	5414	3664	3664	208.9	0.000
ran13x13	86648	78	5304	3252	3252	132.8	0.000
mas284	18700	0	4350	91405.72368	91405.72368	28.5	0.000
prod1	72890	167	151	-56	-56	112.9	0.000
bc1	42882	0	804	3.338362548	3.338362548	1337.0	0.000
bienst1	13594	23	4150	46.75	46.75	121.3	0.000
neos2	195730	29	1076	454.864697	454.864697	693.6	0.000
neos3	505184	42	1264	-318.0996496	448.4167142	3600.0	240.967
neos7	546623	112	1058	717934	721934	3600.0	0.557
swath1	64320	59	1682	379.0712957	379.0712957	566.0	0.000
swath2	71595	80	2304	385.1996929	385.1996929	842.3	0.000
Total (24)	8251942	1845	74812			16499.4	241.525
Geom. Mean	65966.3		2284.2			229.6	

Table 31: hybrid strong/pseudocost branching ( $d = 10, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	141456	22	18452	1158	1158	355.3	0.000
cap6000	3955	3	2393	-2451377	-2451377	47.5	0.000
gesa2-o	66221	171	7344	25779856.37	25779856.37	257.1	0.000
mas74	4770449	0	166155	11801.18573	11801.18573	2445.9	0.000
mas76	577955	0	90864	40005.05414	40005.05414	300.4	0.000
misc07	47707	71	7569	2810	2810	147.7	0.000
pk1	309525	0	54336	11	11	197.4	0.000
pp08aCUTS	217	494	1380	7350	7350	9.6	0.000
qiu	12202	1	32215	-132.8731475	-132.8731475	1712.9	0.000
rout	21694	68	8874	1077.56	1077.56	125.1	0.000
vpm2	6814	214	5182	13.75	13.75	18.4	0.000
ran8x32	25919	93	7499	5247	5247	63.8	0.000
ran10x26	46276	56	8639	4270	4270	88.0	0.000
ran12x21	113911	62	26007	3664	3664	229.6	0.000
ran13x13	78959	78	25368	3252	3252	149.3	0.000
mas284	15250	0	17266	91405.72368	91405.72368	41.8	0.000
prod1	73672	167	367	-56	-56	113.4	0.000
bc1	35911	0	1919	3.338362548	3.338362548	1175.4	0.000
bienst1	13775	23	17996	46.75	46.75	224.2	0.000
neos2	188779	29	1507	454.864697	454.864697	755.9	0.000
neos3	512536	42	1237	-430.8470911	455.5620833	3600.0	205.736
neos7	726355	112	1019	720052.1143	721934	3600.0	0.261
swath1	65350	59	9115	379.0712957	379.0712957	642.1	0.000
swath2	127959	80	10674	385.1996929	385.1996929	1554.6	0.000
Total (24)	7982847	1845	523377			17855.3	205.998
Geom. Mean	57976.8		8137.8			258.8	

Table 32: hybrid strong/pseudocost branching ( $d = 15, \lambda = 4$ )



Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	118518	22	46475	1158	1158	367.3	0.000
cap6000	3803	3	3476	-2451377	-2451377	52.7	0.000
gesa2-o	47928	171	17595	25779856.37	25779856.37	233.3	0.000
mas74	4883991	0	1796527	11744.31645	11893.87257	3646.8	1.273
mas76	419280	0	404979	40005.05414	40005.05414	342.9	0.000
misc07	42348	71	15302	2810	2810	157.3	0.000
pk1	310978	0	169842	11	11	286.3	0.000
pp08aCUTS	254	494	1953	7350	7350	12.5	0.000
qiu	6934	1	44470	-132.8731482	-132.8731482	1799.6	0.000
rout	13497	68	26518	1077.56	1077.56	177.9	0.000
vpm2	3793	214	8869	13.75	13.75	20.9	0.000
ran8x32	19105	93	18153	5247	5247	75.1	0.000
ran10x26	43054	56	20283	4270	4270	109.2	0.000
ran12x21	82121	62	59947	3664	3664	259.5	0.000
ran13x13	66732	78	74717	3252	3252	214.9	0.000
mas284	8219	0	32171	91405.72368	91405.72368	50.1	0.000
prod1	62571	167	1017	-56	-56	93.0	0.000
bc1	33729	0	3338	3.338362548	3.338362548	1186.7	0.000
bienst1	5677	23	28052	46.75	46.75	248.7	0.000
neos2	427739	29	1185	454.864697	454.864697	1430.8	0.000
neos3	611026	42	1692	-351.7966207	420.0806155	3600.0	219.410
neos7	464298	112	1768	721934	721934	2059.9	0.000
swath1	45858	59	17054	379.0712957	379.0712957	532.9	0.000
swath2	168921	80	29717	385.1996929	385.1996929	2217.6	0.000
Total (24)	7890374	1845	2825100			19175.6	220.684
Geom. Mean	47958.5		17780.2			293.5	

Table 33: hybrid strong/pseudocost branching ( $d = 20, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	44455	22	239787	1158	1158	704.7	0.000
cap6000	3452	3	4683	-2451377	-2451377	58.9	0.000
gesa2-o	15831	171	114464	25779856.37	25779856.37	463.7	0.000
mas74	790014	0	6094848	11548.32371	11801.18573	3600.0	2.190
mas76	125516	0	861005	40005.05414	40005.05414	420.2	0.000
misc07	5228	71	65988	2810	2810	213.3	0.000
pk1	128299	0	879768	11	11	658.1	0.000
pp08aCUTS	201	494	1498	7350	7350	10.3	0.000
qiu	6178	1	47170	-132.8731482	-132.8731482	1901.9	0.000
rout	4179	68	58916	1077.56	1077.56	254.7	0.000
vpm2	1974	214	12712	13.75	13.75	25.4	0.000
ran8x32	6824	93	63719	5247	5247	141.8	0.000
ran10x26	8279	56	88240	4270	4270	183.1	0.000
ran12x21	12185	62	141880	3664	3664	317.4	0.000
ran13x13	16033	78	144070	3252	3252	233.7	0.000
mas284	4564	0	32685	91405.72368	91405.72368	48.5	0.000
prod1	14670	167	118575	-56	-56	287.1	0.000
bc1	3666	0	16891	3.338362548	3.338362548	1108.2	0.000
bienst1	5274	23	27841	46.75	46.75	256.8	0.000
neos2	4405	29	44008	454.864697	454.864697	277.2	0.000
neos3	13215	42	170679	368.842751	368.842751	1284.6	0.000
neos7	55369	112	344673	708069.7944	721934	3600.2	1.958
swath1	11268	59	80944	379.0712957	379.0712957	777.6	0.000
swath2	44510	80	310410	376.4462593	387.6584702	3600.0	2.978
Total (24)	1325589	1845	9965454			20427.2	7.126
Geom. Mean	11639.5		86188.6			353.2	

Table 34: hybrid strong/pseudocost branching ( $d = \infty, \lambda = 4$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	217744	22	384	1158	1158	478.8	0.000
cap6000	5795	3	64	-2451377	-2451377	41.0	0.000
gesa2-o	66015	171	356	25779856.37	25779856.37	233.6	0.000
mas74	5236627	0	337	11801.18573	11801.18573	2964.2	0.000
mas76	543507	0	325	40005.05414	40005.05414	255.9	0.000
misc07	41476	71	449	2810	2810	111.0	0.000
pk1	341929	0	398	11	11	184.4	0.000
pp08aCUTS	466	494	309	7350	7350	4.8	0.000
qiu	20408	1	524	-132.8731473	-132.8731473	501.9	0.000
rout	95870	68	462	1077.56	1077.56	264.0	0.000
vpm2	19796	214	389	13.75	13.75	26.0	0.000
ran8x32	38242	93	366	5247	5247	76.3	0.000
ran10x26	59476	56	331	4270	4270	95.3	0.000
ran12x21	140140	62	386	3664	3664	238.5	0.000
ran13x13	93526	78	381	3252	3252	137.3	0.000
mas284	22188	0	398	91405.72368	91405.72368	26.4	0.000
prod1	66116	163	88	-56	-56	101.7	0.000
bc1	40237	0	137	3.338362548	3.338362548	1252.8	0.000
bienst1	8277	23	377	46.75	46.75	80.4	0.000
neos2	529779	29	152	454.864697	454.864697	2125.7	0.000
neos3	463388	42	113	-254.4223987	457.6576489	3600.0	279.881
neos7	342560	112	84	721934	721934	1657.4	0.000
swath1	62909	59	274	379.0712957	379.0712957	559.1	0.000
swath2	196847	80	313	385.1996929	385.1996929	2372.9	0.000
Total (24)	8653318	1841	7397			17389.5	279.881
Geom. Mean	74730.8		268.6			239.6	

Table 35: hybrid strong/pseudocost branching ( $d = 5, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	210127	22	5604	1158	1158	457.6	0.000
cap6000	4957	3	762	-2451377	-2451377	40.7	0.000
gesa2-o	74673	171	3373	25779856.37	25779856.37	272.5	0.000
mas74	4886438	0	10531	11801.18573	11801.18573	2902.2	0.000
mas76	666398	0	9080	40005.05414	40005.05414	315.3	0.000
misc07	50893	71	2943	2810	2810	143.2	0.000
pk1	298771	0	9334	11	11	157.2	0.000
pp08aCUTS	318	494	1531	7350	7350	10.3	0.000
qiu	23974	1	11732	-132.8731746	-132.8731746	1213.9	0.000
rout	39212	68	5554	1077.56	1077.56	165.2	0.000
vpm2	10826	214	2971	13.75	13.75	19.1	0.000
ran8x32	30526	93	2765	5247	5247	59.3	0.000
ran10x26	56448	56	3771	4270	4270	100.3	0.000
ran12x21	135165	62	6588	3664	3664	222.2	0.000
ran13x13	86566	78	7920	3252	3252	134.4	0.000
mas284	17799	0	5010	91405.72368	91405.72368	27.8	0.000
prod1	69992	163	335	-56	-56	101.1	0.000
bc1	40212	0	787	3.338362548	3.338362548	1282.2	0.000
bienst1	13602	23	4380	46.75	46.75	138.4	0.000
neos2	244292	29	267	454.864697	454.864697	883.6	0.000
neos3	536938	42	270	-98.44534014	463.9424776	3600.0	571.269
neos7	598209	112	171	720799.3672	721934	3600.0	0.157
swath1	73512	59	3235	379.0712957	379.0712957	668.6	0.000
swath2	162695	80	3168	385.1996929	385.1996929	1847.6	0.000
Total (24)	8332543	1841	102082			18362.5	571.427
Geom. Mean	69489.0		2523.8			253.9	

Table 36: hybrid strong/pseudocost branching ( $d = 10, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	160260	22	24185	1158	1158	413.8	0.000
cap6000	3955	3	2393	-2451377	-2451377	46.0	0.000
gesa2-o	42668	171	12242	25779856.37	25779856.37	203.3	0.000
mas74	4032872	0	228138	11801.18573	11801.18573	2096.5	0.000
mas76	673256	0	115749	40005.05414	40005.05414	354.9	0.000
misc07	48670	71	8643	2810	2810	160.0	0.000
pk1	395401	0	102137	11	11	303.2	0.000
pp08aCUTS	237	494	1938	7350	7350	12.8	0.000
qiu	11253	1	75265	-156.7635313	-132.8731682	3600.0	15.240
rout	20435	68	15955	1077.56	1077.56	171.1	0.000
vpm2	6964	214	7652	13.75	13.75	25.9	0.000
ran8x32	24083	93	8584	5247	5247	60.9	0.000
ran10x26	48889	56	14049	4270	4270	116.1	0.000
ran12x21	108774	62	29130	3664	3664	232.7	0.000
ran13x13	77781	78	33649	3252	3252	166.4	0.000
mas284	13657	0	20339	91405.72368	91405.72368	44.5	0.000
prod1	67428	163	949	-56	-56	105.1	0.000
bc1	35537	0	1936	3.338362548	3.338362548	1171.6	0.000
bienst1	16257	23	17816	46.75	46.75	224.1	0.000
neos2	247754	29	637	454.864697	454.864697	897.6	0.000
neos3	579422	42	449	-8.446167301	464.5073112	3600.0	5599.622
neos7	532834	112	740	721934	721934	2741.9	0.000
swath1	72628	59	14180	379.0712957	379.0712957	803.8	0.000
swath2	235670	80	13822	385.1996929	385.1996929	2927.6	0.000
Total (24)	7456685	1841	750577			20479.8	5614.862
Geom. Mean	59398.8		9983.1			295.7	

Table 37: hybrid strong/pseudocost branching ( $d = 15, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	156348	22	63565	1158	1158	505.6	0.000
cap6000	3803	3	3476	-2451377	-2451377	52.4	0.000
gesa2-o	45713	171	23564	25779856.37	25779856.37	260.7	0.000
mas74	4549153	0	2165284	11783.43461	11801.18573	3600.0	0.151
mas76	586552	0	591079	40005.05414	40005.05414	515.8	0.000
misc07	42015	71	18401	2810	2810	179.0	0.000
pk1	301968	0	352125	11	11	413.7	0.000
pp08aCUTS	148	494	1530	7350	7350	10.1	0.000
qiu	6371	1	84603	-195.1941608	-132.8731568	3600.1	31.928
rout	23134	68	46443	1077.56	1077.56	331.2	0.000
vpm2	3709	214	13554	13.75	13.75	31.5	0.000
ran8x32	19377	93	16810	5247	5247	73.7	0.000
ran10x26	45080	56	28137	4270	4270	136.2	0.000
ran12x21	85741	62	63641	3664	3664	280.9	0.000
ran13x13	66793	78	75150	3252	3252	217.1	0.000
mas284	9416	0	43047	91405.72368	91405.72368	67.9	0.000
prod1	61573	163	3224	-56	-56	102.7	0.000
bc1	35167	0	3373	3.338362548	3.338362548	1228.4	0.000
bienst1	5907	23	30012	46.75	46.75	277.4	0.000
neos2	215631	29	608	454.864697	454.864697	695.8	0.000
neos3	549304	42	515	-80.06097644	484.0603447	3600.0	704.615
neos7	444334	112	1217	721934	721934	2329.4	0.000
swath1	57789	59	26643	379.0712957	379.0712957	735.9	0.000
swath2	236393	80	39576	385.1996929	385.1996929	3143.4	0.000
Total (24)	7551419	1841	3695577			22388.8	736.693
Geom. Mean	48736.5		20837.9			343.1	

Table 38: hybrid strong/pseudocost branching ( $d = 20, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	55478	22	495934	1158	1158	1522.8	0.000
cap6000	3452	3	4683	-2451377	-2451377	59.5	0.000
gesa2-o	21988	171	235608	25779856.37	25779856.37	926.6	0.000
mas74	645364	0	6617267	11513.08574	12292.91697	3600.0	6.773
mas76	188882	0	1551253	40005.05414	40005.05414	713.9	0.000
misc07	5394	71	84665	2810	2810	267.4	0.000
pk1	143206	0	1256800	11	11	885.0	0.000
pp08aCUTS	148	494	1530	7350	7350	9.9	0.000
qiu	6416	1	87289	-193.5047713	-132.8731475	3600.4	31.333
rout	11305	68	210641	1077.56	1077.56	785.8	0.000
vpm2	1812	214	15435	13.75	13.75	30.9	0.000
ran8x32	4249	93	58732	5247	5247	133.0	0.000
ran10x26	11112	56	144009	4270	4270	294.2	0.000
ran12x21	18577	62	269767	3664	3664	529.3	0.000
ran13x13	20516	78	234413	3252	3252	355.0	0.000
mas284	5580	0	47686	91405.72368	91405.72368	71.0	0.000
prod1	15609	163	152860	-56	-56	359.3	0.000
bc1	3603	0	16810	3.338362548	3.338362548	1071.5	0.000
bienst1	5340	23	30331	46.75	46.75	270.3	0.000
neos2	2685	29	37362	454.864697	454.864697	216.9	0.000
neos3	15306	42	223816	368.842751	368.842751	1550.2	0.000
neos7	55799	112	417934	709654.4626	721934	3600.1	1.730
swath1	16471	59	129431	379.0712957	379.0712957	1166.1	0.000
swath2	36277	80	327248	373.4967249	389.5079992	3600.3	4.287
Total (24)	1294569	1841	12651504			25619.4	44.124
Geom. Mean	12714.7		119799.1			468.0	

Table 39: hybrid strong/pseudocost branching ( $d = \infty, \lambda = 8$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	152417	22	1003	1158	1158	368.8	0.000
cap6000	5795	3	64	-2451377	-2451377	42.2	0.000
gesa2-o	48481	171	1484	25779856.37	25779856.37	174.2	0.000
mas74	4817986	0	372	11801.18573	11801.18573	2526.9	0.000
mas76	487621	0	341	40005.05414	40005.05414	222.2	0.000
misc07	52755	71	585	2810	2810	138.6	0.000
pk1	376201	0	442	11	11	202.8	0.000
pp08aCUTS	446	494	363	7350	7350	4.7	0.000
qiu	45147	1	1054	-132.8731782	-132.8731782	875.3	0.000
rout	106110	68	1039	1077.56	1077.56	300.6	0.000
vpm2	12637	214	718	13.75	13.75	18.6	0.000
ran8x32	21126	93	754	5247	5247	41.0	0.000
ran10x26	65019	56	956	4270	4270	110.6	0.000
ran12x21	138391	62	1159	3664	3664	221.4	0.000
ran13x13	109899	78	792	3252	3252	165.5	0.000
mas284	18944	0	544	91405.72368	91405.72368	22.1	0.000
prod1	67884	156	323	-56	-56	103.4	0.000
bc1	40237	0	137	3.338362548	3.338362548	1250.6	0.000
bienst1	7207	23	500	46.75	46.75	67.2	0.000
neos2	666315	29	202	454.864697	454.864697	2702.4	0.000
neos3	441612	42	228	-589.5108992	511.5284678	3600.0	186.772
neos7	682983	112	910	720460.9034	721934	3600.0	0.204
swath1	51424	59	316	379.0712957	379.0712957	438.0	0.000
swath2	81655	80	389	385.1996929	385.1996929	919.7	0.000
Total (24)	8498292	1834	14675			18116.9	186.976
Geom. Mean	71817.4		489.9			229.3	

Table 40: hybrid strong/pseudocost branching ( $d = 5, \lambda = \infty$ )



Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
afLOW30a	147810	22	8409	1158	1158	383.9	0.000
cap6000	4957	3	762	-2451377	-2451377	40.9	0.000
gesa2-o	62134	171	13652	25779856.37	25779856.37	260.1	0.000
mas74	5815301	0	12226	11801.18573	11801.18573	3556.4	0.000
mas76	484042	0	9273	40005.05414	40005.05414	202.5	0.000
misc07	49998	71	3582	2810	2810	143.0	0.000
pk1	376305	0	11658	11	11	206.8	0.000
pp08aCUTS	280	494	1599	7350	7350	10.8	0.000
qiu	31784	1	25993	-132.8731384	-132.8731384	2177.4	0.000
rout	48147	68	7291	1077.56	1077.56	202.8	0.000
vpm2	11167	214	4659	13.75	13.75	25.6	0.000
ran8x32	20044	93	3327	5247	5247	46.9	0.000
ran10x26	74615	56	7234	4270	4270	135.1	0.000
ran12x21	124672	62	10512	3664	3664	234.0	0.000
ran13x13	103787	78	11794	3252	3252	172.8	0.000
mas284	19852	0	6421	91405.72368	91405.72368	35.8	0.000
prod1	65005	156	785	-56	-56	100.5	0.000
bc1	40212	0	787	3.338362548	3.338362548	1292.7	0.000
bienst1	13421	23	4634	46.75	46.75	135.0	0.000
neos2	482124	29	398	454.864697	454.864697	1783.5	0.000
neos3	496138	42	439	-265.6892095	471.2518713	3600.0	277.370
neos7	576776	112	1693	719949.0129	721934	3600.0	0.276
swath1	75401	59	3623	379.0712957	379.0712957	683.7	0.000
swath2	123664	80	3707	385.1996929	385.1996929	1442.0	0.000
Total (24)	9247636	1834	154458			20472.1	277.645
Geom. Mean	70125.8		3870.1			276.2	

Table 41: hybrid strong/pseudocost branching ( $d = 10, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	147861	22	31388	1158	1158	452.3	0.000
cap6000	3955	3	2393	-2451377	-2451377	45.4	0.000
gesa2-o	61403	171	37626	25779856.37	25779856.37	350.7	0.000
mas74	3293309	0	253958	11801.18573	11801.18573	1647.5	0.000
mas76	685295	0	120302	40005.05414	40005.05414	355.8	0.000
misc07	49496	71	9884	2810	2810	164.4	0.000
pk1	386692	0	119283	11	11	295.6	0.000
pp08aCUTS	185	494	1568	7350	7350	10.1	0.000
qiu	3922	1	72833	-297.9264066	-119.2175182	3601.6	59.984
rout	31930	68	27218	1077.56	1077.56	286.8	0.000
vpm2	7791	214	10279	13.75	13.75	32.2	0.000
ran8x32	20310	93	10437	5247	5247	62.9	0.000
ran10x26	69082	56	21054	4270	4270	154.7	0.000
ran12x21	111742	62	41826	3664	3664	279.1	0.000
ran13x13	84200	78	47311	3252	3252	200.8	0.000
mas284	14532	0	22922	91405.72368	91405.72368	51.9	0.000
prod1	63355	156	2176	-56	-56	104.1	0.000
bc1	35537	0	1936	3.338362548	3.338362548	1176.5	0.000
bienst1	15376	23	20999	46.75	46.75	260.2	0.000
neos2	217194	29	569	454.864697	454.864697	680.0	0.000
neos3	508961	42	647	-144.8889495	484.9386703	3600.0	434.697
neos7	648644	112	2731	720322.1301	721934	3600.0	0.224
swath1	74370	59	15123	379.0712957	379.0712957	788.5	0.000
swath2	135298	80	15724	385.1996929	385.1996929	1706.3	0.000
Total (24)	6670440	1834	890187			19907.4	494.905
Geom. Mean	56926.6		13127.2			312.2	

Table 42: hybrid strong/pseudocost branching ( $d = 15, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	108726	22	79374	1158	1158	496.4	0.000
cap6000	3803	3	3476	-2451377	-2451377	52.2	0.000
gesa2-o	58602	171	63948	25779856.37	25779856.37	431.3	0.000
mas74	4468326	0	2095085	11801.18573	11801.18573	3260.7	0.000
mas76	735551	0	709846	40005.05414	40005.05414	639.8	0.000
misc07	42486	71	19377	2810	2810	173.9	0.000
pk1	331170	0	308821	11	11	388.7	0.000
pp08aCUTS	219	494	2173	7350	7350	13.5	0.000
qiu	3171	1	74723	-314.9907564	-119.2175182	3600.0	62.152
rout	29159	68	64331	1077.56	1077.56	434.4	0.000
vpm2	4539	214	14127	13.75	13.75	34.7	0.000
ran8x32	14316	93	19612	5247	5247	70.5	0.000
ran10x26	57480	56	45191	4270	4270	194.7	0.000
ran12x21	84428	62	92214	3664	3664	345.6	0.000
ran13x13	78723	78	93836	3252	3252	269.7	0.000
mas284	6489	0	34688	91405.72368	91405.72368	56.6	0.000
prod1	66236	156	6237	-56	-56	118.5	0.000
bc1	35167	0	3373	3.338362548	3.338362548	1200.5	0.000
bienst1	5810	23	30335	46.75	46.75	283.1	0.000
neos2	153173	29	747	454.864697	454.864697	535.0	0.000
neos3	566287	42	896	27.06697074	410.2985392	3600.0	1415.864
neos7	498056	112	4205	712542.8525	721934	3600.0	1.318
swath1	65320	59	29483	379.0712957	379.0712957	801.1	0.000
swath2	210403	80	46418	385.1996929	385.1996929	2937.6	0.000
Total (24)	7627640	1834	3842516			23538.5	1479.334
Geom. Mean	48547.0		26557.4			373.6	

Table 43: hybrid strong/pseudocost branching ( $d = 20, \lambda = \infty$ )

Example	B & B	Cuts	StrBra	Dual Bound	Primal Bound	Time	Gap %
aflow30a	34855	22	602929	1158	1158	1669.9	0.000
cap6000	3452	3	4683	-2451377	-2451377	61.7	0.000
gesa2-o	17294	171	335723	25779856.37	25779856.37	1161.4	0.000
mas74	615493	0	6812753	11513.4239	11801.18573	3600.0	2.499
mas76	164161	0	1406438	40005.05414	40005.05414	670.3	0.000
misc07	4861	71	89217	2810	2810	286.8	0.000
pk1	113940	0	1034246	11	11	716.8	0.000
pp08aCUTS	199	494	2144	7350	7350	13.1	0.000
qiu	3152	1	75517	-314.0791134	-119.2175182	3600.1	62.042
rout	7512	68	190851	1077.56	1077.56	801.9	0.000
vpm2	2502	214	27728	13.75	13.75	55.3	0.000
ran8x32	3674	93	61342	5247	5247	135.4	0.000
ran10x26	12080	56	207370	4270	4270	408.6	0.000
ran12x21	19194	62	362574	3664	3664	718.5	0.000
ran13x13	12048	78	161604	3252	3252	248.2	0.000
mas284	4710	0	43870	91405.72368	91405.72368	66.3	0.000
prod1	15895	156	172732	-56	-56	401.5	0.000
bc1	3603	0	16810	3.338362548	3.338362548	1073.0	0.000
bienst1	5548	23	30317	46.75	46.75	278.2	0.000
neos2	2683	29	45243	454.864697	454.864697	276.4	0.000
neos3	14641	42	251155	368.842751	368.842751	1732.2	0.000
neos7	53276	112	423037	709085.0815	721934	3600.0	1.812
swath1	13465	59	110397	379.0712957	379.0712957	1000.8	0.000
swath2	35584	80	324684	375.8885115	388.970984	3600.1	3.480
Total (24)	1163822	1834	12793364			26176.4	69.834
Geom. Mean	11355.7		127737.1			500.3	

Table 44: hybrid strong/pseudocost branching ( $d = \infty, \lambda = \infty$ )