Liberal Regulation: Privatization of Natural Monopolies with Adverse Selection

Emmanuelle Auriol* Pierre M. Picard†
University of Toulouse I University of Manchester

March 12, 2004

Abstract

This paper studies the effect of soft-budget constraints in a pure adverse selection model of monopoly regulation. We consider a government maximizing total surplus but incurring some cost of public funds à la Laffont Tirole (1993). We propose a regulatory set-up in which firms are free to enter natural monopoly markets and to choose their price and output levels as in the laisser-faire. In addition, the government proposes ex-post contracts to the private firms. We show that this regulatory set-up allows governments to avoid re-funding money-losing firms and that welfare is larger than under traditional regulation where governments commits to both investment and operation cash-flows.

Keywords: Privatization, Soft-budget contraint, adverse selection, regulation, natural monopolies
JEL Classification: L43, L51, D82, L33

*Correspondence: Emmanuelle Auriol: ARQADE and IDEI Université de Toulouse I, Place Anatole-France, 31042 Toulouse cedex, France. E-mail: eauriol@cict.fr
†University of Manchester, School of Economic Studies, Oxford Road, Manchester M13 9PL. E-mail: Pierre.Picard@man.ac.uk.
1 Introduction

The role of soft-budget constraints on efficiency has been extensively studied in moral hazard problems. As coined by Kornai’s (1980), soft budget constraints stems from the lack of government commitment not to bail out or subsidize money-losing firms\(^1\). Since less efficient firms are allowed to rely on the government for funding, they lack the financial discipline required for efficient management. In many cases, the poor economic performance of public enterprises associated to soft budget constraints has motivated the transfers of public ownership to private ownership. In Dewattripont and Maskin (1995), Schmidt (1996a, 1996b), Segal (1998) and Maskin (1999), soft-budget constraints are caused by the existence of contract incompleteness and time inconsistency between governments and firms.

By contrast to the existing literature, the present paper departs from moral hazard issues and focuses on the effect of soft-budget constraints in natural monopolies with pure adverse selection problems. There is indeed a widespread belief amongst economic theorists and practitioners that it is not socially optimal to leave monopoly profits to private managers or owners. Regulation of natural monopolies should be preferred even though managers of these firms may be privately informed and extract information rents. This paper challenges this view.

Yet, Auriol and Picard (2002) show that privatization can yield larger welfare than regulation when government’s budget problems are at stake. These authors indeed show that welfare can be improved with a privatization policy that forbids any price control and transfers between government and firms (laissez-faire). This indeed appears to be true when the shadow costs of public funds take high values as in many developing countries. Still, this privatization policy is not optimal since the government could improve welfare further by offering ex-post contracts or ‘bribes’ to private firms. In this paper, we consider this privatization regime combined with such bribes and we call it ‘liberal regulation’ because it embeds the features of traditional regulation theory under asymmetric information and the private firm’s freedom to participate in the subsidy program.

We consider a government maximizing total surplus but incurring some

---

\(^1\)Interesting surveys are available in Kornai (2000), Kornai, Maskin and Roland (2002) and Glaeser and Shleifer (2003).
cost of public funds à la Laffont Tirole (1993). We compare the ex-ante welfare of monopolies in three situations. In the laissez-faire regime, firms freely enter markets and set their prices and outputs whereas the government does never intervene. In the traditional regulation regime, the government decides to set up public firms that are run by public managers who benefit from private information about productivity; the government designs incentive contracts to entice the efficient level of production at some informational cost. Under liberal regulation, firms are free to enter markets and to choose their price and output levels as in the laissez-faire. However, ex-post, the government proposes bribes to the private firms. The latter get their laissez-faire profit plus some information rents.

The model yields the following results. The government provides more selective contracts under liberal regulation than under traditional regulation. High costs firms are not offered bribes for the following two reasons. First, under liberal regulation, the government has neither control or cash-flow rights over of private firms. So, it takes no responsibility over the losses of unproductive private firms and do not compensate the latter. Second, if the government subsidized these private firms, incentive considerations would require that it pays larger transfers to every more efficient firms. As a result, it would distort further output levels of low productivity firms, and, the output level of low productivity firms can become smaller than the level under laissez-faire. Considering only the high cost firms, the government is better-off with no bribe as in the laissez-faire. So the government choose not to intervene with high cost firms.

Liberal regulation allows governments to avoid re-funding money-losing firms. This actually is the traditional rationales of the soft-budget constraint argument. However, compared to traditional regulation, liberal regulation may also force the government to increase transfers to the most efficient firms. More importantly, we show that welfare is larger under liberal regulation in many situations. For instance, if the cost of public funds is small, liberal regulation is optimal as soon as the most inefficient firm makes a loss. Also, liberal regulation is optimal when the government is able to tap large enough revenues with ex-ante franchise fees. In particular, when the shadow cost of public funds is large, liberal regulation is optimal when the private monopoly can be franchised at the market value of the firm under laissez-faire. Examples with linear demand functions and uniform cost
distributions show that the set of economic parameters supporting liberal regulation is not negligible.

The paper relates to the existing literature in several ways. It studies the actual benefit of privatization of natural monopolies in the context of pure adverse selection, where many authors have advocated or assumed regulation (e.g. Baron and Myerson, 1982; Laffont and Tirole, 1993; and others). As already stated, the paper provides an approach complementary to most recent models that discuss privatization under the moral hazard issue or time inconsistency problem induced by the soft budget constraint (Dewatripont and Maskin, 1995; Schmidt, 1996a, 1996b; Segal 1998; Maskin, 1999). The paper also links to the literature of industry design under incomplete information (Dana and Spier, 1994; McGuire and Riordan, 1995) as it adds the possibilities of ex-post contracting. The paper is finally closely related to the literature on mechanism design under type-dependent utility (see Lewis and Sappington, 1989; Jullien 2000; Laffont and Martimort, 2002, Chap. 3).

The paper is structured as it follows. Section 2 describes the model. Section 3 presents the laissez-faire regime. Section 4 develops the traditional regulation regime. Section 5 derives the liberal regulation regime and it compares the levels of transfer and welfare with the other regimes. Section 6 concludes.

2 The Model

We consider a problem of industrial policy setting. The government has to decide whether an industry characterized by increasing returns to scale should be under public or private control. In line with Laffont and Tirole (1993), we call regulation regime the regime in which the government has control and cash-flow rights over a regulated firm. The government’s control rights are associated with accountability on profits and losses. That is, it must subsidize the firm in case of losses whereas it can tax the regulated firm in case of profits. In contrast, we call private regime the regime in which control and cash-flow rights belong to a private entrepreneur or, namely, a private firm. The government takes no responsibility for the firm’s profits or losses. Ex-ante and ex-post transfers are nevertheless possible between the private firm and the government. We call liberal regulation the context in which ex-post transfers are allowed between the government and the firm.
The *laissez-faire* situation occurs when ex-post transfers from/to the firm are forbidden. For simplicity, we focus on natural monopolies.

Consumers have the decreasing inverse demand function \( P(Q) \) and the gross surplus \( S(Q) = \int_0^Q P(x) \, dx \). As in Baron-Myerson (1982), we focus on industries characterized by increasing returns to scale. The firm makes an investment \( K > 0 \), and incurs an idiosyncratic marginal cost \( \beta \) and produces the output \( Q \). The investment is decided before the realization of \( \beta \). In contrast to Schmidt (1996a, 1996b), the model includes no time inconsistencies because committed investments must be paid and the realization of investments cannot be expropriated by the government. Hence, \( K \) is a fixed cost for the firm or the government, depending on who makes the investment decision. Finally, in contrast to Dewatripont and Maskin (1995) and others, the investment has a fixed size \( K \) so that no moral hazard issue emerges about the optimal size of investment (or manager’s effort).

The firm’s profit net of transfer is equal to

\[
\pi(\beta, Q) \equiv P(Q)Q - \beta Q - K
\]

with partial derivatives \( \pi_\beta(\beta, Q) \) and \( \pi_Q(\beta, Q) \). Its profit after transfer is equal to

\[
\Pi(\beta, Q, t, F) \equiv \pi(\beta, Q) + t - F
\]

where \( t \) is the ex-post transfer that the firm gets from the government and where \( F \) is a possible ex-ante franchise fee paid to the government. The transfer to the firm can either be positive (i.e., a subsidy), or negative (a tax). The marginal cost \( \beta \) is independently drawn from the support \( [\beta, \bar{\beta}] \) according to the density and cumulative distribution functions \( g(\cdot) \) and \( G(\cdot) \). The expectation operator is denoted \( E \) so that \( E[h(\beta)] = \int_\beta^{\bar{\beta}} h(\beta) \, dG(\beta) \). In the sequel, the terms ‘ex-post’ and ‘ex-ante’ correspond to the period before and after the realization of \( \beta \).

As in Laffont and Tirole (1993), the government is utilitarian and maximizes the sum of consumer’s and producer’s surpluses minus the social cost of transferring public funds to the firm. The government’s objective function is

\[
W(\beta, Q, t, F, \lambda) = S(Q) - \beta Q - K - \lambda t + \lambda F
\]

where \( \lambda \) is the shadow cost of public funds. Note that the surplus net of transfer is given by \( W(\beta, Q, 0, 0, \lambda) \).
The shadow cost of public funds, $\lambda$, drives the results of the paper. This shadow cost, which can be interpreted as the Lagrange multiplier of the government budget constraint, measures the social cost of the government’s economic intervention. For $\lambda$ close to 0, the government maximizes the net consumers' surplus; for larger $\lambda$, the government puts more weight on the social cost of transfers. The shadow cost of public funds is positive because transfers to regulated firms imply either a decrease in the production of public goods, such as schooling and health care, or an increase in distortionary taxation. Each dollar that is transferred to the regulated firm costs $1 + \lambda$ dollars to society. In developed economies, $\lambda$ is mainly equal to the deadweight loss accrued to imperfect income taxation. It is assessed to be around 0.3 (Snower & Warren, 1996). In developing countries, low income levels and difficulties in implementing effective taxation programs are strong constraints on the government’s budget, which leads to higher values of $\lambda$. In particular, the value is very high in countries close to financial bankruptcy. As a benchmark case the World Bank (1998) suggests a shadow cost of 0.9.

We study three configurations: *laissez-faire*, traditional regulation of public firm and liberal regulation of private firms.

### 3 Laissez-faire

Under *laissez-faire*, the production levels of private monopolies are not controlled by the government and firms receive no transfer: $t = 0$. The government can nevertheless raise money at the entry of private monopolies by auctioning the right to operate. Let $F \geq 0$ be the franchise fee that the private firm pays to the government in order to operate in the product market. The private monopoly contemplates the following sequential choices. First, it chooses to enter the market by paying the franchise fee $F$ and by committing to the investment $K$. If it enters, then nature chooses the marginal cost $\beta$ according to the distribution $G(\cdot)$. Then, the private firm learns $\beta$ and chooses a production level $Q$. The firm sells its output and pay its costs $\beta Q - K$. After the realization of $\beta$, the private firm never pays or receives a transfer from the government.

Ex-post, the private monopoly maximizes its profit $\pi(\beta, Q)$ and chooses the optimal production $Q(\beta)$ that satisfies

$$\pi_Q(\beta, Q) = P(Q) + P'(Q)Q - \beta = 0. \quad (3)$$
We assume interior maximum, which implies that the demand function is not too convex. Also, in order to rule out corner solutions, we assume that output always strictly positive under full information as well as under incomplete information (see next sections). This means that the willingness to pay of the first consumer must be sufficiently above marginal cost:

\[
\begin{align*}
A0 & \quad P''Q + 2P' < 0 \\
A1 & \quad P(0) > \frac{G(\beta)}{g(\beta)}
\end{align*}
\]

Substituting \( \tilde{Q} \) in equations (1) and (2), we get the ex-ante profit and welfare of a private monopoly

\[
\begin{align*}
E\tilde{\Pi} &= \int \left[ P(\tilde{Q}(\beta))\tilde{Q}(\beta) - \beta\tilde{Q}(\beta) \right] dG(\beta) - K - F \\
E\tilde{W}(\lambda) &= \int \left[ S(\tilde{Q}(\beta)) - \beta\tilde{Q}(\beta) \right] dG(\beta) - K + \lambda F
\end{align*}
\]

4 Traditional Regulation

Under traditional regulation, governments control the firm’s investment, its prices and its output while the public firms’ managers/owners control the firm’s input mix (technology, effort, labor, ...). However, in this paper, we abstract from the input mix decision and public managers have no real role in production. So, the public managers endorse no responsibility for profits and losses. This naturally leads to associate traditional regulation to public ownership where control and cash-flow rights fully accrue to the government.\(^2\)

In this model, the sole role of public managers is to acquire and disclose information about the firm’s productivity to the government. When the public managers have private information, they benefit from information rents. This creates a soft-budget constraint by which regulated firms get subsidized more often than necessary. Public managers of regulated monopolies always receive a positive utility. We acknowledge the relevant debate about whether rents flow to public managers or to employees. For simplicity

\(^2\)Grossman and Hart (1986) establish that control rights are what matters to define ownership and that the ownership structure does not matter if complete contracts can be written. Shleifer and Vishny (1994) and Roland (2000) show that the allocation of control right leads in economic efficiency when control rights are congruent to cash flows rights.
we assume that the public managers get the rents and that their reservation utility is zero, the key point of the paper being about who decides to invest $K$.

The timing is as it follows. The government firstly decides to commit the investment $K$; then, nature chooses the marginal cost $\beta$ according to the distribution function $G(\cdot)$; the regulated firm’s manager learns $\beta$, but the government does not; the government proposes a production and transfer scheme $(Q(\cdot), t(\cdot))$, finally the regulated firm reveals the information $\tilde{\beta}$ and production takes place according to the contract $(Q(\tilde{\beta}), t(\tilde{\beta}))$. The firms sells output and pays its cost $\beta Q + K$. In this regime, there is not franchise fee: $F = 0$.

By the revelation principle, the analysis can be restricted to direct truthful revelation mechanism ($\beta = \hat{\beta}$). The government maximizes welfare

$$\max_{\{Q(\cdot), t(\cdot)\}} EW (\beta, Q(\beta), t(\beta), 0, \lambda) = E [S(Q(\beta)) - \beta Q(\beta) - K - \lambda t(\beta)]$$

subject to

$$(d/d\beta)\Pi(\beta, Q(\beta), t(\beta), 0) = -Q(\beta) \quad (4)$$

$$(d/d\beta)Q(\beta) \geq 0 \quad (5)$$

$$\Pi(\beta, Q(\beta), t(\beta), 0) \geq 0 \quad (6)$$

where the first and second conditions are the first and second order incentive constraints and where the last condition is the public manager’s participation constraint. This program is a standard adverse selection problem as presented by Baron-Meyerson (1982). Its solution is standard. To avoid the technicalities of ‘bunching’, we make the classical monotone hazard rate assumption (see Guesnerie and Laffont, 1984; Jullien, 2002):

**A2** $G(\beta)/g(\beta)$ and $(G(\beta) - 1)/g(\beta)$ are non decreasing .

Then, the output in the traditional regulation regime with asymmetric information is given by $Q^*(\beta)$ which solves

$$P(Q) + \lambda \frac{1}{1+\lambda} P'(Q)Q - \beta = \lambda \frac{G(\beta)}{1+\lambda g(\beta)} \quad (7)$$

The output $Q^*(\beta)$ is non increasing in $\beta$. 
Note that it is readily established that, when the government has perfect information on the firm’s cost, the output level, say \( Q^o (\beta) \), solves this equality for \( G(\beta)/g(\beta) = 0 \). The latter output level is larger than under asymmetric information: \( Q^o (\beta) \geq Q^* (\beta) \). In order to reduce the firm’s incentive to inflate its cost report, the government asks high cost firms to set output levels that are smaller than it would ask under perfect information. For large shadow costs of public funds, the output level under symmetric information tends to that under \textit{laissez-faire}: \( \lim_{\lambda \to \infty} Q^o (\beta) = \bar{Q} (\beta) \). In this case, the objectives of the firm under \textit{laissez-faire} and the government under \textit{regulation} are congruent: the government puts no weight on consumers’ surplus and maximizes profits (see Auriol Picard 2002 for detailed analysis of this effect for natural monopolies and duopolies). Still, under asymmetric information the output level under traditional regulation can become smaller or larger than under \textit{laissez-faire}.

The profit and transfer are equal to

\[
\Pi^* (\beta) = \int_{\beta} \bar{Q}^* (\beta) d\beta \\
t^* (\beta) = \int_{\beta} \bar{Q}^* (\beta) d\beta - \pi (\beta, Q^* (\beta))
\]

The firm with the lowest productivity \( \bar{\beta} \) gets zero information rent \( \Pi^* (\bar{\beta}) = 0 \) but it is associated with a transfer that compensates its loss or that taxes its net profit. Note that larger fixed costs raise the transfers to the regulated firm. The ex-ante welfare writes as

\[
EW^* = EW (\beta, Q^* (\beta), t^* (\beta), 0, \lambda) \\
= \int [S(Q^* (\beta)) - \beta Q^* (\beta) - K - \lambda t^* (\beta)] dG(\beta)
\]

We now turn the central issue of the paper.

5 \textbf{Liberal Regulation}

It is obviously not optimal for the government to let private monopolies operate at inefficient production levels in the \textit{laissez-faire} regime. The government can offer ex-post transfers to correct these levels. However, in contrast
traditional regulation of public firms, managers or owners of private firms must be given at least the profit they get under laissez-faire, \( \Pi(\beta) \). Hence, the government is obliged to leave large profit levels to some private firms but, in counter part, it is not obliged to compensate for the losses of very unproductive private firms. In particular, as the firm endorses the responsibility of its investment, the government is not obliged to reimburse the investment cost in case of bad productivity realizations. In the sequel we will make no distinction between the owner and the manager of the private firm. They are all residual claimants of the private firm’s profits.

Under liberal regulation, the sequence of choices is as follows. First, the monopoly chooses to enter the market by paying the franchise fee \( F \) and by committing the investment \( K \). If it enters, then nature chooses the marginal cost \( \beta \) according to the distribution \( G(\cdot) \). The private firm’s manager learns \( \beta \). Then the government proposes a set of ex-post contracts \( \{t^l(\beta), Q^l(\beta)\} \). The firm chooses a contract and implements its terms, or it chooses the production level in the laissez-faire. In any case, by Assumption A1, the firm sells its output and pays its cost \( \beta Q + K \). For the sake of presentation, we call such ex-post contracts 'bribes'. We firstly claim that only a fraction of firms receive a contract and that the number of firms receiving a bribe decreases with the shadow cost of public funds.\(^3\)

5.1 Selective Bribes

Let \( Q^l(\beta) \) and \( \Pi^l(\beta) \) denote the output and profit of private monopolies with bribes. After entry, the franchise is a sunk cost for the firm. So we temporarily set \( F \) to zero. The government solves

\[
\max_{\{Q(\cdot), t(\cdot)\}} \text{EW}(\beta, Q(\beta), t(\beta), 0, \lambda) = E[S(Q(\beta)) - \beta Q(\beta) - K - \lambda t(\beta)]
\]

subject to

\[
(\frac{d}{d\beta})\Pi(\beta, Q(\beta), t(\beta)) = -Q(\beta)
\]

\[
(\frac{d}{d\beta})Q(\beta) \geq 0
\]

\[
\Pi(\beta, Q(\beta), t(\beta), 0) \geq \hat{\Pi}(\beta)
\]

\(^3\)Note that we here voluntarily eliminates government’s re-nationalization strategies as in Schmidt (1996a, 1996b) and multi-principal issues such as in Laffont and Tirole (1991) where an additional agency issue arises between firm’s manager and owners.
where the participation constraint now states that the minimum profit acceptable is the profit under *laissez-faire*.

Under liberal regulation, the government may decide to grant no bribe to high cost firms for the following reason. High cost firms have huge losses and require high subsidies, which are socially costly. Furthermore, high subsidies entices low cost firms to cheat by reporting high cost. To alleviate this effect, the government imposes a penalty on high cost firms by forcing their outputs and thus their revenues further down. As a result, the subsidy may be too small and the penalty too high so that high cost firms prefer the *laissez-faire* situation. This is the idea of the following lemma.

**Lemma 1** Assume \( P'' + P'Q < 0 \). There exist a unique \( \beta_0 \in [\beta, \overline{\beta}] \) such that

\[
P(\hat{Q}(\beta_0)) - \beta_0 = \lambda \frac{G(\beta_0)}{g(\beta_0)}
\]  

(11)

Optimal output and profit are equal to

\[
Q^l(\beta) = \begin{cases} 
Q^*(\beta) > \hat{Q}(\beta) & \text{if } \beta < \beta_0 \\
\hat{Q}(\beta) & \text{if } \beta \geq \beta_0
\end{cases}
\]

(12)

\[
\Pi^l(\beta) = \begin{cases} 
\hat{\Pi}(\beta_0) + \int_{\beta_0}^{\hat{\Pi}} Q^*(\beta) d\beta > \hat{\Pi}(\beta) & \text{if } \beta < \beta_0 \\
\hat{\Pi}(\beta) & \text{if } \beta \geq \beta_0
\end{cases}
\]

(13)

**Proof.** See Appendix 1.

We have restricted our analysis for the case of not too convex demand \( (P'' + P'Q < 0) \). Suppose indeed, that the demand is very convex. For very low marginal costs \( \beta \), *laissez-faire* profits will increase very fast as \( \beta \) drops. Hence, the bribes to the firms with very low costs would be very large and would definitively attract firms with higher marginal costs. Incentive compatibility would then be difficult to maintain. Therefore, firms with very low marginal cost must also be excluded from bribes. Selectivity can thus affect both very high and very low types simultaneously (See for instance Jullien 2000, Proposition 2). For the sake of simplicity we do not focus on this case.

It is important to note that liberal and traditional regulations yield the same output levels when firms accepts incentive contracts and that output and profit are continuous variables at \( \beta_0 \): \( \hat{Q}(\beta_0) = Q^*(\beta_0) \) and \( \hat{\Pi}(\beta_0) = \hat{\Pi}(\beta_0) \).
Obviously, if \( \lambda \to \infty \), we have that \( \beta_0 \to \beta \). Let also \( \lambda_0 \) be the shadow cost which gives \( \beta_0 = \beta \); it is equal to

\[
\lambda_0 = g(\beta) \left[ P(\hat{Q}(\beta)) - \beta \right]
\]  

(14)

Then, we have the following.

**Proposition 2** Let \( \beta_0 \) and \( \lambda_0 \) be given by (11) and (14). There always exists a fraction \( G(\beta_0) \) of private firms that receives a bribe. This fraction decreases with larger \( \lambda \) and tends to zero when \( \lambda \to \infty \). All private firms will receive an ex-post bribe iff \( \lambda < \lambda_0 \).

Example: In a linear demand model where \( P(Q) = a - bQ \) and where types are uniformly distributed in \([0, a/2]\), we have that \( G(\beta)/g(\beta) = \beta \), \( \beta_0 = a/(1 + 2\lambda) \) and \( \lambda_0 = 1/2 \). All private firms will receive a bribe for \( \lambda \leq 1/2 \). For \( \lambda > 1/2 \), the fraction of bribed private firms is equal to \( G(\beta_0) = 2/ (1 + 2\lambda) \). At \( \lambda = \lambda_0 \approx 0.3 \), all private firms will receive an ex-post bribe. At \( \lambda = 0.9 \), 71% of firms receive a bribe. At \( \lambda = 2 \), 40% of firms receive a bribe.

We now show that transfers under liberal regulation have properties very similar to those under traditional regulation.

### 5.2 Transfers

To compute the level of transfer under liberal regulation, we first note that by the envelop theorem, we have that \( (d/d\beta) \hat{\Pi}(\beta) = \pi_\beta(\beta, \hat{Q}(\beta)) = -\hat{Q}(\beta) \), so that we can write

\[
\hat{\Pi}(\beta) = \pi(\beta, \hat{Q}(\beta)) = \int_{\beta}^{\beta_0} \hat{Q}(\beta) \, d\beta + \hat{\Pi}(\beta_0) = \int_{\beta}^{\beta} \hat{Q}(\beta) \, d\beta + \hat{\Pi}(\beta_0)
\]

(15)

Combining this expression with (13) and the definition of ex-post profit, we easily get

\[
t^l(\beta) = \begin{cases} 
\int_{\beta}^{\beta_0} \left[ Q^*(\beta) - \hat{Q}(\beta) \right] \, d\beta + \pi(\beta, \hat{Q}(\beta)) - \pi(\beta, Q^*(\beta)) & \text{if } \beta < \beta_0 \\
0 & \text{if } \beta \geq \beta_0
\end{cases}
\]

(15)

The bribe consists of two terms: a positive information rent and a positive compensation for the loss in net profit that the private firm incurs when it accepts the output level specified in the bribe.
It is instructive to compare this bribe with the transfer for traditionally regulated firms, $t^* (\beta)$. Note that $t^l (\beta)$ is a difference between variables under traditional regulation and laissez-faire. Hence, in contrast with traditional regulation, fixed costs $K$ have no impact on the level of transfers. The government does not compensate for fixed costs. Note also that, by construction, $t^l (\beta)$ is non negative. The government is here not able to tap the revenues of the firm but it must offer positive monetary incentives to get socially optimal levels of output.

For any high cost firms such that $\beta > \beta_0$, we have $t^l (\beta) = 0$. The difference in transfer between bribed private and regulated public firms is equal to $-t^* (\beta)$ which can positive if the government taxes the traditionally regulated firm; negative if the government subsidizes the traditionally regulated firm. For any low cost firms, $\beta < \beta_0$, it turns out that the difference in transfers between traditional and liberal regulation is a constant.

**Proposition 3** For all $\beta < \beta_0$, bribes to private firms, $t^l (\beta)$, are equal to transfers to traditionally regulated public firms, $t^* (\beta)$, plus a constant, $\Delta t \equiv t^l (\beta) - t^* (\beta)$, that is equal to

$$\Delta t = \hat{\Pi} (\beta_0) - \Pi^* (\beta_0) = \int_{\beta_0}^{\beta} \left[ \hat{Q} (\beta) - Q^* (\beta) \right] d\beta + \hat{\Pi} (\beta)$$

**Proof.** See Appendix 2. ☐

Expression (16) includes the profit of the private firm $\hat{\Pi}$, which decreases with fixed costs, and the rent of the public manager $\Pi^*$, which is independent from fixed costs. Hence, larger fixed costs decrease the level of transfers under liberal regulation versus traditional regulation. Also, because $\hat{Q} (\beta) > Q^* (\beta)$ for all $\beta > \beta_0$ (see Lemma 1), the constant $\Delta t$ is positive if $\hat{\Pi} (\beta) > 0$. In this case, under liberal regulation, the government pays a bribe whereas it would collect a tax under traditional regulation (i.e. $t^l (\beta_0) = 0 > t^* (\beta_0)$). However, $\Delta t$ can be negative if $\hat{\Pi} (\beta)$ is sufficiently negative. In this case, the government pays a bribe to firms that would been subsidized under traditional regulation.

### 5.3 Welfare

The welfare with ex-post bribes is larger than that without bribes. Indeed, the government gives bribes only if it is welfare improving. For large $\lambda$,
paying bribes does not significantly improve welfare since government and firms almost share the same objective: as \( \lambda \to \infty \), government tends to maximize profits as the firm does.

Ex-ante, the welfare is equal to

\[
EW^t (\lambda) = \int_\beta^{\beta_0} W(\beta, Q^*(\beta), t^*(\beta), F, \lambda) dG(\beta)
+ \int_{\beta_0}^{\beta} W(\beta, \tilde{Q}(\beta), 0, F, \lambda) dG(\beta) + \lambda F
\]

Using (16), the linearity of the welfare function \( W(\cdot) \) in \( t(\beta) \) and \( F \) and Proposition (3), we rewrite the ex-ante welfare as

\[
EW^t (\lambda) = \int_{\beta_0}^{\beta} W(\beta, \tilde{Q}(\beta), 0, 0, \lambda) dG(\beta)
+ \int_\beta^{\beta_0} W(\beta, Q^*(\beta), t^*(\beta), 0, 0, \lambda) dG(\beta) - \lambda \Delta t G(\beta_0) + \lambda F
\]

The ex-ante welfare consists of four elements: the welfare accrued to traditionally regulated firms with low cost \( \beta \), the welfare accrued to the private firms with high cost \( \beta \) receiving no bribe, the social cost of additional rents \( \Delta t \) to the \( G(\beta_0) \) private firms receiving a bribe and finally the social value of the franchised fee.

Changing the integration boundaries, we can further write

\[
EW^t (\lambda) = EW^* (\lambda) - \lambda \Delta t G(\beta_0) + \lambda F
- \int_{\beta_0}^{\tilde{\beta}} [W(\beta, Q^*(\beta), t^*(\beta), 0, 0, \lambda) - W(\beta, \tilde{Q}(\beta), 0, 0, \lambda)] dG(\beta)
\]

Thus, liberal regulation is preferred to traditional regulation if and only if \( EW^t (\lambda) > EW^* (\lambda) \), or, using the welfare function linearity w.r.t. transfers,

\[
\left\{ -\lambda \Delta t G(\beta_0) + \lambda F + \lambda \int_{\beta_0}^{\tilde{\beta}} t^*(\beta) dG(\beta)
\right. \\
\left. + \int_{\beta_0}^{\tilde{\beta}} [W(\beta, \tilde{Q}(\beta), 0, 0, \lambda) - W(\beta, Q^*(\beta), 0, 0, \lambda)] dG(\beta) \right\} > 0
\]

In this expression, the difference between liberal and traditional regulation is decomposed between the social cost of additional rents \( \Delta t \), the social value of the franchised fee (as above), and between the rents to the public firm’s
manager (the first integral) and the gain in consumption’s (gross) surplus that is obtained through the reduction of distortion of the high cost firms that are not subsidized (the second integral).

We now analyze three situations: small shadow costs of public funds, large shadow cost of public funds and high franchise fees.

5.3.1 Small shadow costs of public funds

We firstly consider the case of small shadow costs of public funds: \( \lambda \leq \lambda_0 \). In this case, all private firms receive a bribe and the cut-off cost \( \beta_0 \) is set to its largest value \( \overline{\beta} \). It naturally comes that \( \Delta t = \Pi (\overline{\beta}) - \Pi^* (\overline{\beta}) = \overline{\Pi} (\overline{\beta}) \).

In fact, bribes differ from transfers under traditional regulation only by the additional transfer \( \Delta t \) that is equal to the laissez-faire profit under the worst cost realization. When it is positive, this additional transfer corresponds to a rent given to all firms under liberal regulation: firms must at least keep the profit they have in the worst realization under laissez-faire. When it is negative, this corresponds to a rent given to the government: the latter must at least avoid the social cost of operating the monopoly in the worst cost realization. Liberal regulation will be preferred if the social cost of this additional transfer is balanced by the social benefit of franchise fee. Indeed, since the second integral in expression (17) cancels out, the ex-ante welfare can be written as

\[
EW^L (\lambda) = EW^* (\lambda) - \lambda \overline{\Pi} (\overline{\beta}) + \lambda F
\]

At \( \lambda = 0 \), liberal regulation is equivalent to laissez-faire: \( EW^{PM*} (0) = EW^* (0) \). For larger \( \lambda \), we get the following proposition.

**Proposition 4** For small shadow cost public funds (\( \lambda \leq \lambda_0 \)), liberal regulation is preferred to traditional regulation of monopolies if and only if \( F > \overline{\Pi} (\overline{\beta}) \).

The attractiveness of this proposition is that it is easy to assess: it only requires to know the franchise fee required by the government and the worst profit realization of the monopoly. For such values of shadow cost of public funds, the government is interested by liberal regulation as soon as it is able to recoup the smallest monopoly’s laissez-faire profit through a franchise fee. As a consequence, it necessarily chooses liberal regulation.
when it is able to sell the firm at its expected market price \( F = E\Pi \) because \( E\Pi > \Pi (\beta) \). One would expect that this situation is likely to occur in many markets. Furthermore, when the government is not be able/allowed to raise a franchise fee \( (F = 0) \), liberal regulation is still preferred as long as the firm makes a loss with a positive probability \( (\Pi (\beta) < 0) \). Industries with large uncertainties and large fixed costs are likely to give rise to such a situation. The proposition mostly applies for developed economies where estimates of shadow costs of public funds yields small values (around 0.3). However, in many developing countries, the shadow costs of public funds are likely to be very large. We now focus on the extreme but instructive case where \( \lambda \to \infty \).

5.3.2 Large shadow costs of public funds

We now consider very large shadow costs of public funds: \( \lambda \to \infty \). In this case, the government puts no (relative) weight on consumer’s surplus and it is principally interested in collecting funds. No firms get ex-post bribes because \( \beta_0 \to \beta \) and \( G (\beta_0) \to 0 \) when \( \lambda \to \infty \). The government actually sells the business to an entrepreneur and does not intervene afterwards. Since welfare functions tend to infinity when \( \lambda \to \infty \), we need to consider the slopes of these functions in order to compare welfare under liberal and traditional regulation. Under liberal regulation, the ex-ante welfare (17) tends to infinity with the following slope:

\[
\lim_{\lambda \to \infty} EW^l (\lambda) / \lambda = \int_{\lambda} \lim_{\lambda \to \infty} \left[ W (\beta, \hat{Q} (\beta), 0, 0, \lambda) / \lambda \right] dG (\beta) + F = F
\]

Under traditional regulation, the ex-ante welfare (10) tends to infinity with a slope that reflects the transfers that the government expects to collect or pay. When \( \lambda \to \infty \), the first order condition (9) yields an output level \( Q^* (\beta) \) that is equal to \( \hat{Q} (v (\beta)) \) where \( v (\beta) \equiv \beta + G (\beta) / g (\beta) > \beta \) is called ‘virtual cost’. Then, we successively get

\[
\lim_{\lambda \to \infty} EW^* / \lambda = \int_{\beta} \lim_{\lambda \to \infty} [-t^* (\beta)] dG (\beta)
\]

\[
= \int_{\beta} \left[ \int_{\beta} \lim_{\lambda \to \infty} -Q^* (\beta) d\beta + \lim_{\lambda \to \infty} \pi (\beta, Q^* (\beta)) \right] dG (\beta)
\]

\[
= \int_{\beta} \left[ \int_{\beta} -\hat{Q} [v (\beta)] d\beta + \pi (\beta, \hat{Q} [v (\beta)]) \right] dG (\beta) \quad (19)
\]
The expected welfare under traditional regulation includes the cost of information rent to the public manager (the first term in this expression) and the average profit tapped from the public firm (the second term). Therefore, we can write the following proposition.

**Proposition 5** For very large shadow cost public funds \((\lambda \to \infty)\), liberal regulation is preferred to traditional regulation of monopolies if and only if \(F > \lim_{\lambda \to \infty} EW^*/\lambda\) which is given by (19).

The condition (19) involves more information than the condition presented in Proposition 4. It requires to know the exact distribution of cost as well as the profit and output functions for all cost values. In practice, fewer information may be available to government and some conditions based on a smaller set of statistics is welcome. The following corollary provides a sufficient and a necessary condition based on smaller information requirements.

**Corollary 6** Suppose \(\lambda \to \infty\). Then, (i) liberal regulation is preferred to traditional regulation if \(F \geq \widehat{E\Pi} \beta \beta \left[ v(\beta) \right] (\beta - E\beta)\). (ii) Traditional regulation is preferred to liberal regulation if \(F \leq \pi(\beta, Q \left[ v(\beta) \right]) \leq \widehat{E\Pi}(\beta)\).

**Proof.** See Appendix 3. 

The information required in this corollary involves the expected values of laissez-faire profit \(E\widehat{\Pi}\) and costs \(E\beta\) and it uses the values of virtual cost, output and profit at the largest possible cost \(\beta\).

The ability to extract high franchise fees obviously fosters the case for liberal regulation. Still, liberal regulation does not require to extract the full amount of expected market value of the firm under laissez-faire, \(E\widehat{\Pi}\). Moreover, the franchise fee that is needed to sustain liberal regulation decreases with higher likelihood of losses under laissez-faire; that is, when higher fixed costs are large (through smaller \(E\widehat{\Pi}\)) and when the cost distribution is skewed toward high costs (i.e. larger difference \((\beta - E\beta)\)). Also, liberal regulation is choosen for small franchise fees if the magnitude of production at the largest virtual cost \(v(\beta)\) is sufficiently large (it is assumed to be positive by Assumption A1). It is easy to construct examples in which liberal regulation is preferred over traditional regulation with zero franchise fee (that is, where \(\widehat{Q} \left[ v(\beta) \right] (\beta - E\beta) > E\widehat{\Pi} > 0\)).
5.3.3 High franchise fees

In this section, we show that liberal regulation is preferred by the government when the latter is able to tap a sufficiently large share of the firm’s expected profit through the ex-ante franchise.

We first need to compute the value of the maximal franchise fee, i.e. the monopoly’s expected profit when no franchise fee is asked. Using (13) and (15), we know that the profit under liberal regulation is equal to the profit under laissez-faire plus a positive information rent that is equal to 

$$R \beta \left[ Q^* (z) - \hat{Q} (z) \right] dz$$

for each firm with $\beta < \beta_0$. Using (8) and (9), this information rent can be written as

$$\Pi^* (\beta) - \Pi^* (\beta_0) - [\Pi (\beta) - \Pi (\beta_0)]$$

by (16) is equal to

$$\Pi^* (\beta) - \Pi (\beta) + \Delta t.$$ Hence, the ex-ante profit with zero franchise fee is equal to

$$E \Pi^*_0 \equiv E \Pi + \int_{\beta}^{\beta_0} \int_{\beta}^{\beta_0} [Q^* (z) - \hat{Q} (z)] dz dG (\beta)$$

$$= E \Pi + G (\beta_0) \Delta t + \int_{\beta}^{\beta_0} [\Pi^* (\beta) - \Pi (\beta)] dG (\beta)$$

(20)

We now show that liberal regulation is preferred when franchise fees are sufficiently close to the latter value of ex-ante private profits. To do this, we add $\lambda E \Pi^*_0$ on both sides of condition (18). After some substitutions and simplifications (see Appendix 4), condition (18) becomes

$$\lambda \left[ E \Pi^*_0 - F \right] < \left\{ \begin{array}{l}
\lambda \int_{\beta}^{\beta_0} \Pi^* (\beta) dG (\beta) \\
+ \lambda \int_{\beta}^{\beta_0} [\Pi (\beta) - \pi (\beta, Q^* (\beta))] dG (\beta) \\
+ \int_{\beta}^{\beta_0} [W (\beta, \hat{Q} (\beta), 0, 0, \lambda) - W (\beta, Q^* (\beta), 0, 0, \lambda)] dG (\beta)
\end{array} \right.$$  

(21)

where the RHS includes three positive terms. Indeed, we have that $\hat{Q} (\beta) \geq Q^* (\beta)$ for $\beta > \beta_0$, $\Pi^* (\beta) \geq 0$ and $\Pi (\beta) = \max_Q \pi (\beta, Q) \geq \pi (\beta, Q^* (\beta))$. These terms respectively denote the expected value of the public manager’s information rent, the additional profit and the additional (gross) surpluses that high cost firms generate under liberal regulation.

Proposition 7 Liberal regulation is preferred to regulation of public firms for any shadow cost of public funds for any franchise fee $F$ that is sufficiently close to $E \Pi^*_0$. 

18
In particular, liberal regulation is preferred to traditional regulation for any shadow cost of public funds if the government is able to tap the whole ex-ante profit through the ex-ante franchise fee required to operate in the monopoly market. This situation can occur when the government is able to successfully auction the market to private investors and entrepreneurs. This occurs when the number of investors is large, when investors do not collude and when there exist no information asymmetry in the ex-ante stage (for instance the value of the fixed cost $K$ is common knowledge). Otherwise, the government is not able to tap the full amount of expected profit and it may ultimately have recourse to traditional regulation. Also, the possibility to auction markets strongly depends on the government’s ability to forecast the emergence of a market.

We finally note that whereas the LHS of (21) decrease with fixed costs (through the term $E\Pi$ in $E\Pi_0^l$), the RHS is independent of them. Higher fixed costs will thus diminish the franchise fee that would make the government prefer liberal regulation. Hence, when the ex-ante profit is sufficiently small, a franchise fee may not be needed to yield liberal regulation.

The three last propositions rest on two key assumptions: risk neutral entrepreneurs have access to financial markets and government cannot auction the right to run the public firm to potential public managers. This last assumption is justified because public managers usually lack financial resources and because information rents usually take the form of in-kind benefits that can hardly be auctioned. But, for the sake of the argument, suppose that the government is able to auction the right to operate a privately-regulated firm to some risk neutral public manager. The government is then able to recoup the public manager’s expected earnings,

$$\int_{\beta} \Pi^* (\beta) \, dG (\beta)$$

The ex-ante welfare in the public regulated firm is increased by this amount. This is equivalent to cancel the second term in (21). The above proposition is still valid. Liberal regulation is preferred to traditional regulation for any franchise fee $F$ that is sufficiently close to $E\Pi^l$. So, the crucial assumption is zero cost access to financial market by entrepreneurs, which makes private investment less costly than public investment.
5.3.4 Calibrated Example

The welfare benefits from liberal regulation can be highlighted by the following example. Consider a linear inverse demand function $P = a - bQ$ and a uniform distribution of cost $\beta$. Because what really matters is the net surplus from consumption by the first consumer $(a - \beta)$, we can normalize $\beta = 0$ without loss of generality. We also index the spread of the distribution by $\alpha \in [0,1]$ so that $\beta = \alpha * a/2$. Under linear utility function, profits, rents and welfare are proportional to the term $a^2/b$ and it is convenient to relate profits and welfare to the *ex-ante* value of operation profits:

$$V \equiv (12 - 6\alpha + \alpha^2) a^2/(48b).$$

Hence, the functions $E\Pi, EW^* \text{ and } EW^l$ are multiplicatively separable in $V$ and in some functions of $\lambda, \alpha, F/V$ and $K/V$. This allows us to normalize the fixed cost to its share in the *ex-ante* operating profit: let indeed $k \equiv K/V$. Obviously, a firm makes zero ex-ante profit when $k = 1$ and it makes $E\Pi = (1 - k)V$ when $0 < k < 1$.

It is shown in Appendix 5 that under linear demand and uniform distribution, liberal regulation is always preferred when the government is able to recoup the *ex-ante laissez-faire* profit, $F = E\Pi$. When franchise fees are smaller, liberal regulation is preferred only for sufficiently large levels of fixed costs. Figure 1 shows the normalized levels of fixed costs, $k$, above which liberal regulation is preferred in the case of a zero franchise fee and in the case of a franchise fee equal to half the *ex-ante laissez-faire* profit. Several indices of cost spreads $\alpha$ are shown. It is readily observed that liberal regulation is more likely to be preferred for larger cost spreads i.e. for larger business uncertainty. Also, liberal regulation is more likely to be preferred when the government is able to tap larger franchise fees from the monopoly. The reader will not that the set of parameters supporting liberal regulation is not negligible.

INSERT FIGURE 1 HERE

6 Conclusion

This paper studies the effect of soft-budget constraints in a pure adverse selection model of monopoly regulation. We consider a government maximizing total surplus but incurring some cost of public funds à la Laffont.
Tirole (1993). We compare three regulation regimes. First, under traditional regulation, the government delegates the firm’s operation to a public manager who reports cost realization. Second, under *laissez-faire*, the government lets firms to enter natural monopoly markets and to choose their price and output levels as in the *laissez-faire*. Third, under liberal regulation, the government also lets firms to enter and operate but in addition, it proposes ex-post contracts or bribes to the private firms. The two last regimes correspond to two privatization policies in natural monopoly markets. In these regimes, the government does not take any responsibility over firms’ cash flows. Privatized firms’ owners pay for their losses. In fact, privatization solves the soft budget constraint that takes place under traditional regulation where public firms’ managers are able to tap revenues from the government in exchange for the information they disclose about their firms.

Liberal regulation is obviously welfare improving compared to laissez-faire. Indeed, the government can always implement *laissez-faire* within the liberal regulation regime by setting bribes to zero. The issue is then whether liberal regulation is to be preferred to traditional regulation. Liberal regulation indeed allows the government to avoid to offer high subsidies to high cost firms. This relaxes the incentive constraint and liberal regulation then permits to reduce output distortions in high cost firms. However, liberal regulation forbids the government to tap revenues from low cost, profitable firms.

In this paper, we show that, under liberal regulation, the government offers bribes - i.e. ex-post contracts - to a group of low cost firms only. These firms are asked output and price schedules that are similar to those under traditional regulation. By contrast, high cost firms are not offered any ex-post contracts or bribes and they operate exactly as in the *laissez-faire* regime. The set of high cost, non-bribed firms grow with larger shadow cost of public funds. We also show that transfers under liberal and traditional regulation are equal up to a constant. Still, transfers may be smaller or larger under liberal regulation. For instance, bribes paid to the managers of private firms under liberal regulation are larger than transfers paid to the managers of public firms under traditional regulation if firms always make positive profits. But, if large enough losses occur, bribes under liberal regulation are lower than the transfer under traditional regulation.

Obviously, the benefit of liberal regulation depends on the shadow costs
of public funds and on the size or the franchise fees that the government is able to extract ex-ante. We show that liberal regulation is clearly preferred by the government in three distinct situations. When shadow costs of public funds are small, all private firms are bribed and liberal regulation is always preferred when the franchise fee is large than the laissez-faire profit in the worst case. In fact, bribes differ from transfers under traditional regulation only by an additional transfer that is equal to the laissez-faire profit under the worst cost realization. Liberal regulation will be preferred as soon as the social cost of this additional transfer is balanced by the franchise fee. We show that the same is true when shadow costs of public funds are large enough. Finally, we show that liberal regulation is also preferred when the government is able to set a franchise fee that is sufficiently large but that is still acceptable for the private entrepreneur. These results remain true even if one imagines that positions as public managers under traditional regulation are auctioned to potential public entrepreneurs. The crucial point lies in the free access to financial market by private entrepreneurs, which makes private investment less costly than public investment.

References


Economy, 94, p. 691-719.


Appendix 1: Proof of Lemma 1

The proof is an application of Jullien (2000) in the particular case of common value adverse selection. We make it here explicit the dynamic programming approach. The program can be written as

$$\max_{Q, \Pi, \mu, \gamma} \int_{\beta} H_1(\beta, Q, \Pi, \mu, \gamma) \, d\beta$$

where

$$H_1(\beta, Q, \Pi, \mu, \gamma) = [S(Q) + \lambda P(Q) - (1 + \lambda) \beta Q - \lambda \Pi] g(\beta)$$

$$+ \mu Q + \lambda \gamma(\beta) \left( \Pi - \hat{\Pi}(\beta) \right)$$

where \(\mu(\beta)\) and \(\gamma(\beta)\) are the co-state variable of the incentive constraint and the Lagrange multiplier of the participation constraints. Integrating by part this is equivalent to

$$\max_{Q, \Pi, \mu, \gamma} \int_{\beta} H_2(\beta, Q, \Pi, \mu, \gamma) \, d\beta$$

where

$$H_2(\beta, Q, \Pi, \mu, \gamma) = [S(Q) + \lambda P(Q) - (1 + \lambda) \beta Q - \lambda \Pi] g(\beta)$$

$$- \mu(\beta) \Pi + \mu(\beta) Q + \lambda \gamma(\beta) \left( \Pi - \hat{\Pi}(\beta) \right)$$

with \(\Pi(\beta) = \Pi(\beta) \mu(\beta) = 0\). It is easy to check that concavity conditions are respected. So, the following necessary condition is also sufficient:

$$P(Q) + \frac{\lambda}{1 + \lambda} P'(Q) Q = \beta + \frac{\lambda}{1 + \lambda} G(\beta) - \Gamma(\beta)$$

(22)

where \(\Gamma(\beta) = \int_0^\beta \gamma(\beta) \, d\beta\) is an increasing function such that \(\Gamma(\beta) = 0\) and \(\Gamma(\beta) = 1\). By assumption A2, the solution \(t^*(\beta, \Gamma)\) of condition (22) is a non increasing function of \(\beta\) and a non decreasing function of \(\Gamma\) (see ‘potential separation’ in Jullien (2000)). The solution is displayed as the bold curve of Figure 2.
A binding participation constraint implies that $\Pi(\beta) = \widehat{\Pi}(\beta)$ and that $\gamma(\beta) = 0$; the solution of the program is then the function $l^*(\beta, \Gamma) = \widehat{Q}(\beta)$. Comparing this condition with (3), we find that $l^*(\beta, 1) > \widehat{Q}(\beta)$. Hence, $\Gamma(\beta) = 1$ for some $\beta$ is impossible. Thus, if $l^*(\beta, 0)$ cross $\widehat{Q}(\beta)$, it must cross at some $\beta_0$ that satisfies both conditions (3) and (22), which yields the expression (11) that is re-written here:

$$P(\widehat{Q}(\beta_0)) - \beta_0 = \lambda \frac{G(\beta_0)}{g(\beta_0)}$$

The intersection between $l^*(\beta, 0)$ and $\widehat{Q}(\beta)$ at $\beta_0$ is unique if $P''Q + P' < 0$. Indeed, under assumption A2, the RHS of the last expression increases in $\beta_0$. The LHS is positive and it decreases in $\beta_0$ iff $P'(\widehat{Q}(\beta))\widehat{Q}(\beta)$ is increasing. Since $\widehat{Q}$ is non increasing in $\beta$, this is true if $P''Q + P' < 0$.

We thus have for $\beta < \beta_0$, the solution of the program is $Q^l = l^*(\beta, 1) = Q^*(\beta)$ and $\gamma(\beta) = 0$, and, for $\beta \geq \beta_0$, $Q^l = \widehat{Q}(\beta) = l^*(\beta, \gamma(\beta))$ and $\gamma(\beta) > 0$. Also, one can check that $\widehat{Q}(\beta) > Q^*(\beta)$ iff $\beta > \beta_0$.

**Appendix 2**

For any $\beta < \beta_0$,

$$\Delta t = t^l(\beta) - t^*(\beta)$$

$$= \int_{\beta}^{\beta_0} (Q^*(\beta) - \widehat{Q}(\beta))d\beta + \pi(\beta, \widehat{Q}(\beta)) - \pi(\beta, Q^*(\beta))$$

$$- \left[ \int_{\beta}^{\beta_0} Q^*(\beta) d\beta - \pi(\beta, Q^*(\beta)) \right]$$

$$= \int_{\beta}^{\beta_0} (Q^*(\beta) - \widehat{Q}(\beta))d\beta + \pi(\beta, \widehat{Q}(\beta)) - \int_{\beta}^{\beta_0} Q^*(\beta) d\beta$$

$$= \int_{\beta}^{\beta_0} Q^*(\beta) d\beta + \widehat{\Pi}(\beta_0) - \int_{\beta}^{\beta_0} Q^*(\beta) d\beta$$

$$= \int_{\beta}^{\beta_0} Q^*(\beta) d\beta + \widehat{\Pi}(\beta_0) = \widehat{\Pi}(\beta_0) - \Pi^*(\beta_0)$$

where we used (15) in the third and fourth equalities and (8) in the last equality. This expression can be re-written as $\int_{\beta_0}^{\beta} [\widehat{Q}(\beta) - Q^*(\beta)] d\beta + $
\( \hat{\Pi}(\eta) \) where the first term is positive because \( \hat{Q}(\beta) > Q^*(\beta) \) for all \( \beta > \beta_0 \) (see proof of the Lemma 1).

**Appendix 3**

For the sake of clarity, let us denote \( v(\beta) \) and \( v(\eta) \) by \( v \) and \( \bar{v} \).

(i) Note that \( E\hat{\Pi} = \int_{\beta}^0 \pi[\beta, \hat{Q}(\beta)]dG(\beta) \). Adding the LHS and subtracting the RHS of this equality in (19) as well as adding and subtracting \( \int_{\beta}^0 \hat{Q}(\bar{v})d\beta dG(\beta) \) successively yield

\[
\lim_{\lambda \to \infty} EW^*/\lambda = E\hat{\Pi} + \int_{\beta}^0 \int_{\beta}^0 \left[ \int_{\beta}^0 -\hat{Q}(\bar{v})d\beta + \pi[\beta, \hat{Q}(\bar{v})] - \pi[\beta, \hat{Q}(\beta)] \right] dG(\beta) \\
= E\hat{\Pi} + \int_{\beta}^0 \int_{\beta}^0 -\hat{Q}(\bar{v})d\beta + \int_{\beta}^0 \left[ \hat{Q}(\bar{v}) - \hat{Q}(\beta) \right] d\beta dG(\beta) \\
+ \int_{\beta}^0 \pi[\beta, \hat{Q}(\bar{v})] - \pi[\beta, \hat{Q}(\beta)]dG(\beta) \\
= E\hat{\Pi} - \hat{Q}(\bar{v}) \int_{\beta}^0 (\bar{v} - \beta)d\beta \\
+ \int_{\beta}^0 \left[ \int_{\beta}^0 \left[ \hat{Q}(\bar{v}) - \hat{Q}(\beta) \right] d\beta + \pi[\beta, \hat{Q}(\bar{v})] - \pi[\beta, \hat{Q}(\beta)] \right] dG(\beta)
\]

where the last integral in the RHS of the last equality is negative because \( \bar{v} \geq v \) and thus \( \hat{Q}(\bar{v}) < \hat{Q}(v) \) and because \( \pi[\beta, \hat{Q}(v)] \leq \pi[\beta, \hat{Q}(\beta)] = \max_Q \pi(\beta, Q) \). Hence, \( \lim_{\lambda \to \infty} EW^*/\lambda > E\hat{\Pi} + \hat{Q}(\bar{v}) \int_{\beta}^0 (\bar{v} - \beta)d\beta = E\hat{\Pi} - \hat{Q}[v(\bar{v})](\bar{v} - E\beta) \).

(ii) Note that \( \pi(\beta, \hat{Q}(v)) = \pi(\bar{v}, \hat{Q}(\bar{v})) - \int_{\beta}^0 \{ \pi_\beta[\beta, \hat{Q}(\bar{v})] + \pi_Q[\beta, \hat{Q}(\bar{v})] \hat{Q}'(v)v' \} d\beta \)

where \( \pi_\beta[\beta, \hat{Q}(v)] = -\hat{Q}(v) \). Adding the RHS of this equality in (19), subtracting the LHS and simplifying yield

\[
\lim_{\lambda \to \infty} EW^*/\lambda = \int_{\beta}^0 \left[ \pi[\beta, \hat{Q}(\bar{v})] + \int_{\beta}^0 \pi_Q[\beta, \hat{Q}(v)][-\hat{Q}'(v)v']d\beta \right] dG(\beta) \\
\]

where the last term in the integral is positive because \( \hat{Q}'(v) < 0, v' = 1 + (G(\beta))/g(\beta) \)' > 0, \( \hat{Q}(v) < \hat{Q}(\beta) \) and thus \( \pi_q[\beta, \hat{Q}(v)] > \pi_q[\beta, \hat{Q}(\beta)] = 0 \).
Hence, \( \lim_{\lambda \to \infty} E W^*/\lambda > \pi[\beta, \tilde{Q}(\pi)] \). Since \( \tilde{Q}(v) < \check{Q}(\beta), \pi(\beta, \tilde{Q}[v(\beta)]) \leq \hat{\Pi}(\tilde{\beta}) \).

**Appendix 4**

We here detail the proof of condition (21). Adding \( \lambda (E\Pi_0' - F) \) on both sides of condition (18) we get

\[
-\lambda \Delta tG(\beta_0) + \lambda E\Pi_0' + \lambda \int_{\beta_0}^{\beta} t^*(\beta) dG(\beta) + \int_{\beta_0}^{\beta} \left[W(\beta, \tilde{Q}(\beta), 0, 0, \lambda) - W(\beta, Q^*(\beta), 0, 0, \lambda)\right] dG(\beta) > \lambda \left(E\Pi_0' - F\right)
\]

Replacing \( E\Pi_0' \) by its value in (20) and using the identity \( \int_{\beta_0}^{\beta} t^*(\beta) dG(\beta) = \int_{\beta_0}^{\beta} \left[\Pi^*(\beta) - \pi(\beta, Q^*(\beta))\right] dG(\beta) \), the first line on the LHS becomes

\[
\lambda E\tilde{\Pi} + \lambda \int_{\beta}^{\beta_0} \left[\Pi^*(\beta) - \hat{\Pi}(\beta)\right] dG(\beta) + \lambda \int_{\beta_0}^{\beta} \left[\Pi^*(\beta) - \pi(\beta, Q^*(\beta))\right] dG(\beta)
\]

Adding the term \( \lambda E\Pi^* - \lambda E\tilde{\Pi} - \lambda \int_{\beta_0}^{\beta} \left[\Pi^*(\beta) - \hat{\Pi}(\beta)\right] dG(\beta) = 0 \), this expression becomes

\[
\lambda E\Pi^* + \lambda \int_{\beta_0}^{\beta} \left[\hat{\Pi}(\beta) - \pi(\beta, Q^*(\beta))\right] dG(\beta)
\]

which yields the condition (21).

**Appendix 5**

We consider the linear inverse demand function \( P = a - bQ \) a uniform distribution of cost \( \beta \). Without loss of generality, we normalize \( \beta = 0 \) and we set \( \tilde{\beta} = \alpha * a/2 \) where \( \alpha \in [0, 1] \) is a parameter that index the spread of cost distribution. Under linear utility function, profits and welfare are proportional to \( a^2/b \). This allows to normalize the fixed cost and the franchise fee such that \( K = kV \) and \( F = fV \) where \( V \) is the value of operational profit when the laissez-faire monopoly makes zero ex-ante profit. That is, we have that \( E\tilde{\Pi} = 0 \iff K = V \equiv (12 - 6\alpha + \alpha^2) a^2/(48b) \). Hence, \( k = 1 \) implies zero \emph{ex-ante} profit under \emph{laissez faire}. We can compute
that $E/P/V = (1 - k)$, that $\lambda_0 = \frac{2 - \alpha}{2\alpha}$ and that welfare under traditional regulation is equal to

$$EW^*/V = 2 \frac{12(1 + \lambda)^2 + \alpha^2(1 + 2\lambda)^2 - 6\alpha(\lambda + 1)(2\lambda + 1)}{(12 - 6\alpha + \alpha^2)(1 + 2\lambda)} - (1 + \lambda)k$$

Welfare under liberal regulation is equal to

$$EW^l/V = \begin{cases} \frac{2(3-\alpha)\lambda}{12 - 6\alpha + \alpha^2} - k + \lambda f & \text{if } \lambda < \lambda_0 \\ \frac{8 - 12\alpha(1+2\lambda)+6\alpha^2(1+2\lambda)^3-\alpha^3(1+2\lambda)^2(1+6\lambda)}{2\alpha(12-6\alpha+\alpha^2)(1+2\lambda)^2} - k + \lambda f & \text{if } \lambda > \lambda_0 \end{cases}$$

where the numerator of the second item in this expression is a cubic expression. It can be readily be numerically shown that the latter is always positive. Hence liberal regulation is always preferred when the franchise fee extracts the whole laissez-faire profit: $f = (1 - k)$. Then

$$EW^l/V - EW^*/V = \begin{cases} \frac{2(3-\alpha)\lambda}{12 - 6\alpha + \alpha^2} - k + \lambda f & \text{if } \lambda < \lambda_0 \\ \frac{8 - 12\alpha(1+2\lambda)+6\alpha^2(1+2\lambda)^3-\alpha^3(1+2\lambda)^2(1+6\lambda)}{2\alpha(12-6\alpha+\alpha^2)(1+2\lambda)^2} - k + \lambda f & \text{if } \lambda > \lambda_0 \end{cases}$$

Note first the case in which the franchise fee extracts the whole laissez-faire profit: $f = (1 - k)$. Then

$$k(\lambda, f, \alpha) = \begin{cases} \frac{3(2-\alpha)^2}{(12 - 6\alpha + \alpha^2)} - f & \text{if } \lambda < \lambda_0 \\ \frac{-8 + \alpha^3(1+2\lambda)^2(1+8\lambda) - 6(1+4\lambda)\alpha^2(1+2\lambda)^2 + 12\alpha(2\lambda+1)(4\lambda^2+2\lambda+1)}{2\alpha(12-6\alpha+\alpha^2)\lambda(1+2\lambda)^2} - f & \text{if } \lambda > \lambda_0 \end{cases}$$

When the franchise fee is set to zero we get

$$k(\lambda, 0, \alpha) = \begin{cases} \frac{3(2-\alpha)^2}{(12 - 6\alpha + \alpha^2)} & \text{if } \lambda < \lambda_0 = \frac{1}{2} \\ \frac{-8 + \alpha^3(1+2\lambda)^2(1+8\lambda) - 6(1+4\lambda)\alpha^2(1+2\lambda)^2 + 12\alpha(2\lambda+1)(4\lambda^2+2\lambda+1)}{2\alpha(12-6\alpha+\alpha^2)\lambda(1+2\lambda)^2} & \text{if } \lambda > \lambda_0 = \frac{1}{2} \end{cases}$$

This yields the curves displayed in the figure.
Figure 1: Liberal v/s Traditional Regulation for Linear Demand Functions (P=a-bQ) and Uniform Cost Distributions (0, \beta); k=K/E\Pi.
Figure 2: Output levels and shadow value of participation constraint $\Gamma(\beta)$