Repetitive Risk Aversion

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Abstract

This paper introduces and investigates the concept of repetitive risk aversion. The risk aversion of an increasing and concave utility function is repetitive if the fear of ruin, which measures agent's aversion to risking his entire income, is also increasing and concave. This is shown to be equivalent to the behaviorally meaningful condition that the risk premium is increasing at a non-increasing rate with the size of the bet. We find an additional justification for mixed risk aversion, which is known to be stronger than standard (and thus proper) risk aversion, in terms of this concept. We discuss several economic applications of repetitive risk aversion.

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1. Introduction

We introduce and investigate the concept of repetitive risk aversion. The risk aversion of an increasing and concave utility function is repetitive if the fear of ruin, which measures agent's aversion to risking his entire income, is also increasing and concave. We show that this is equivalent to the behaviorally meaningful condition that the risk premium is increasing at a non-increasing rate with the size of the bet.

The main motivation for this concept comes from the fact that in applications the assumption of decreasing absolute risk aversion (DARA) is often too week to obtain unambiguous comparative static results. We show that repetitive risk aversion is stronger than decreasing absolute risk aversion and has several economic applications. It is, however, not generally comparable to decreasing or increasing relative risk aversions. It is well accepted by now that DARA is also not sufficient for obtaining plausible behavior in models which unlike the Arrow-Pratt theory involve more than one risk. Additional restrictions need to be imposed so as to refine the set of vN-M utility functions. Thus, Pratt and Zeckhauser (1987) require the utility function to satisfy proper risk aversion (an undesirable risk must always remain undesirable in the presence of an independent undesirable risk). Similarly, Kimball (1993) introduces a stronger restriction, namely that of standard risk aversion (every risk that has negative interaction with a small reduction in wealth must also have a negative interaction with any undesirable, independent risk). More recently, Caballé and Pomansky (1996) propose mixed risk aversion, which is stronger than standard (and thus proper) risk aversion and equivalent to the condition that the marginal utility or the first derivative of the utility function is completely monotone over the interval $(0, \infty)$.[2] We show that the concept of repetitive risk aversion can also be seen as a step further in this refinement strategy and provide an additional justification for mixed risk aversion in terms of this concept. In the concluding section of their paper,

[2] This condition was introduced originally by Pratt and Zeckhauser (1987) as a sufficient condition for proper risk aversion. A real valued function $f(w)$ defined on $(0,\infty)$ is *completely monotone* if and only if its derivatives $f^n(w)$ of all orders exist and $(-1)^n \, f^n(w) \geq 0$, for all $w > 0$ and $n = 0,1,2, \cdots$.

Caballé and Pomansky (1996) also propose the refinement that the absolute risk aversion of the utility function is completely monotone. We show below that this too follows from infinitely repetitive risk aversion.

The paper is organized as follows. In the next section, we introduce the concept of repetitive risk aversion and establish its basic properties. In Section 3, we show that infinitely repetitive risk aversion implies both mixed risk aversion and completely monotone absolute risk aversion. Section 4 discusses some economic applications of repetitive risk aversion and draws the conclusion.

2. Repetitive Risk Aversion

As in Aumann and Kurz (1977), consider an agent with vN-M utility function u which is smooth (i.e. its derivatives of all orders exist) with $u'(w) > 0$ and $u''(w) < 0$ for all $w \geq 0$, and $u(0) = 0$. Suppose the agent is considering a bet in which he risks his entire wealth w against a possible gain of a small amount x. The probability p of ruin would have to be very small in order for him to be indifferent between such a bet and retaining his current wealth w. Moreover, the more unwilling he is to risk ruin, the smaller p will be. Thus p is an inverse measure of agent's aversion to risking ruin, and a direct measure of boldness; obviously p tends to zero as the potential winnings x shrink. Thus, boldness is the probability of ruin per dollar of potential winnings for small potential winnings, i.e., it is the limit of p/x as $x \to 0$. More formally, let

$$u(w) = (1-p)u(w+x) + p\,u(0) = (1-p)u(w+x), \tag{1}$$

since $u(0) = 0$. Hence

$$\frac{p}{x} = \frac{\dfrac{u(w+x)-u(w)}{x}}{u(w+x)}$$

and as $x \to 0,$ this tends to $u'(w)/u(w).$ Therefore, $u(w)/u'(w)$ is the *fear of ruin* at wealth level $w.$ Additional interpretations of this concept can be found in Aumann and Kurz (1977).

Let $u_1(w) \equiv u(w)/u'(w).$ Since, unlike the Arrow-Pratt model, the amount risked or the size of the bet is also increasing with wealth, the fear of ruin must be increasing. Indeed, $u_1'(w) = 1 + (-u''(w)u(w)/(u'(w))^2) > 0,$ since $u''(w) < 0.$ Thus, the only question that remains is whether it is increasing at a non-increasing rate, i.e., whether $u_1''(w) \leq 0.$

We introduce the following definition. *The risk aversion of an increasing and concave utility function* u *is repetitive if the fear of ruin is also increasing and concave.*

Let $R(w) \equiv -u''(w)u(w)/(u'(w))^2.$ Then, given an increasing and concave utility function $u,$ $u_1''(w) \leq 0$ if and only if $R'(w) \leq 0,$ i.e., repetitive risk aversion is equivalent to the condition that $R'(w) \leq 0.$ Let $x(p,w)\,(>0)$ denote the solution to equation (1) and let $z(p,w) = w + \; x(p,w).$ Then, the risk premium $\pi(p,w) \equiv (1-p)z(p,w) - w.$ Clearly, $d\pi/dw = ((1-p)\,dz/dw) - 1 = (u'(w)/u'(z)) - 1 > 0,$ since $z > w$ and u is strictly concave. Thus the risk premium is increasing with the size of the *bet*. The implication that $d\pi/dw > 0$ (which however does not depend on the repetitiveness of the risk aversion) also means that the risk premium is increasing with *wealth* $w,$ which may appear to be counterintuitive. But it is not. The explanation is that, unlike the Arrow-Pratt model, the size of the bet is also increasing with, and equal to, the amount of wealth. Differentiating once more and rearranging,

$$\frac{d^2\pi}{dw^2} = \left(\frac{u'(w)}{u(w)}\frac{u'(w)}{u'(w+x)}\right)\left[\frac{-u''(w+x)}{u'(w+x)}\frac{u(w+x)}{u'(w+x)} - \frac{-u''(w)}{u'(w)}\frac{u(w)}{u'(w)}\right].$$

Since, as seen from equation (1), x can be made as close to zero as desired by choosing the probability p of the unfavorable outcome to be sufficiently small, $d^2\pi/dw^2 \leq 0$ for all p if and only if $R'(w) \leq 0$. Thus repetitive risk aversion is equivalent to the condition that the risk premium is increasing at a non-increasing rate with the amount risked or the size of the bet.

If $R'(w) = 0$, i.e. *the repetitive risk aversion is constant*, then $\pi(p,w)$ is linear in w. Furthermore, by integrating twice both sides of the equality $R'(w) = 0$ and using strict concavity of u and $u(0) = 0$, we obtain $u(w) = aw^{\alpha}$ with $0 < \alpha < 1$ and $a > 0$, i.e., the relative risk aversion is constant. Thus, constant repetitive risk aversion is equivalent to constant relative risk aversion. However, no further comparison between repetitive and relative risk aversions is possible and as seen below $R'(w) < 0$ does not imply either decreasing or increasing relative aversion. For now, we provide an example in which the repetitive risk aversion is indeed not constant, i.e., $R'(w) < 0$. One such utility function is $u(w) = w + w^{\alpha}$ with $0 < \alpha < 1$.

We offer an additional interpretation of RRA, which follows from a geometric interpretation of the fear of ruin. As shown in Fig. 1, the fear of ruin $u_1(w)$ is equal to the length of the subtangent at w, i.e., the length of the segment $[a,w]$.[3] Since $u_1'(w) = 1 + R(w)$ and $u_1''(w) = R'(w)$, it follows that risk aversion (or concavity) of u implies that the length of the subtangent is increasing and RRA implies that it is increasing at a non-increasing rate. Risk aversion $(u''(w) \leq 0)$ and RRA $(R'(w) \leq 0)$ can be therefore viewed as increasingly stringent refinements of the first derivative $u'(w)$ or the marginal utility of wealth. Our analysis below (see the first column of Table 1) confirms this intuition.

[3] For definition and more details concerning the early and classic concept of subtangent see, for example, Blakey (1962).

We note some basic implications of RRA of a utility function u, that is,

$$R'(w) = \frac{u''(w)}{u'(w)}\left[\left(\frac{u(w)}{u'(w)}\right)\left(\frac{-u'''(w)}{u''(w)} - 2\,\frac{-u''(w)}{u'(w)}\right) - 1\right] \le 0\,. \tag{2}$$

The necessary condition $-u'''(w)/u'(w) \ge -2u''(w)/u'(w)$ for this inequality, which is a unique characteristic of RRA, is stronger than the requirement of decreasing absolute risk aversion, i.e. $-u'''(w)/u'(w) \ge -u''(w)/u'(w)$. The relative magnitudes of $-u''(w)/u'(w)$ and $-u'''(w)/u''(w)$ are known to play an important part in many applications. For example, Drezè and Modigliani (1972, Theorem 3.1) implicitly use the condition $-u'''(w)/u''(w) \ge -2u''(w)/u'(w)$, which is necessary for RRA but not for decreasing absolute or relative risk aversions, to sign the precautionary saving effect when preferences admit a separable representation between initial and future consumption. Similarly, Sinclair-Desgagné and Gabel (1997) use this condition to characterize the optimal audit rule.

RRA is however not generally comparable to non-increasing relative risk aversion, that is

$$rr'(w) = \frac{u''(w)}{u'(w)}\left[w\left(\frac{-u'''(w)}{u''(w)} - \frac{-u''(w)}{u'(w)}\right) - 1\right] \le 0\,. \tag{3}$$

Since u is concave, that is, $u'(x)$ is non-increasing in x and $u(0) = 0$, $u(w)/u'(w) = (1/u'(w))\int_0^w u'(x)dx \ge w$. Therefore, inequalities (2) and (3) are not generally comparable. We provide an example of a utility function such that $rr'(w) < 0$ but $R'(w) > 0$. Let $u(w) = w/(w^{\alpha}+1)$, $\frac{1}{2} < \alpha < 1$. It is seen that $rr'(w) < 0$ for w sufficiently large but $R'(w) > 0$ for all w. A convenient utility function for which $R'(w) < 0$ but $rr'(w) > 0$ is not easy to find. Inequalities (2) and (3) suggest that such a utility function must be such that the fear of ruin $u(w)/u'(w)$ is very high compared to w and the absolute risk aversion $-u''(w)/u'(w)$ is very low compared to the absolute prudence $-u'''(w)/u''(w)$.

3. Infinitely Repetitive and Mixed Risk Aversions

Besides the obvious requirement of risk aversion (or concavity), both Pratt (1964) and Arrow (1971) also emphasized the property of DARA so as to obtain plausible comparative static results about the relation between wealth and risk taking by an investor. Pratt and Zeckhauser (1987) introduced afterwards the family of proper utility functions which constitute a strict subset of the functions satisfying DARA. The purpose of this section is to show that RRA can also be viewed as part of this process of refining the set of risk averse utility functions.

Let $\boldsymbol{u}$ denote the set of all smooth utility functions u, which satisfy $u(0)=0, u'(w)>0,$ and $u''(w)\le 0$ for all $w\in[0,\infty]$. Let T denote the operator $T(u)=u/u'$. Then RRA of $u\in\boldsymbol{u}$ is equivalent to $T(u)\in\boldsymbol{u}$. Let $T^{n+1}(u)=T(T^n(u))$, and $u_n=T^n(u), n=1,2,\cdots$ with the convention that $T^1=T$. We introduce the following definitions:

A vN-M utility function $u\in\boldsymbol{u}$ satisfies $R^{(n)}RA$ if $\mathrm{T}^{\mathrm{k}}(u)\in\boldsymbol{u}$ for all $k=1,2,\cdots,n,$ where $R^{(n)}RA$ means *repetitive repetitive* $\cdots(n$ times$)$ *risk aversion.*[4]

By definition $R^{(n)}RA$ implies $R^{(k)}RA$ for all $k=1,2,\cdots,n$ and $RRA=R^{(1)}RA$. Table 1 below summarizes the implications and the relationship between these increasingly stronger concepts of decreasing risk aversion. The arrows indicate what implies what. The proofs can be seen from the proof of Theorem 1 below.

[4] Without having said it, we have been following the convention that decreasing means non-increasing unless specifically stated to be strictly decreasing.

u	u_1	u_2	. . .
$u' > 0$	$u_1' > 0$	$u_2' > 0$	. . .
↗	↗		
$u'' \leq 0$	$u_1'' \leq 0$	$u_2'' \leq 0$	. . .
↙	↙		
$u''' \geq 0$	$u_1''' \geq 0$		
↗			
$u'''' \leq 0$	.		
.	.		
.	.		
.			

Table 1

In order to assure the reader that these definitions are not vacuous, we note that the class of utility functions $u(w) = aw^{\alpha}, a > 0$ and $0 < \alpha \leq 1$, which are most commonly used in financial economics, satisfy $R^{(n)}RA$ for all $n \geq 1$, i.e., *infinitely repetitive risk aversion.*

The convexity of absolute risk aversion means that the higher the wealth, the smaller the reduction in risk premium of a small risk for a given increase in wealth. It is a natural condition and known to be sufficient for risk vulnerability (Gollier and Pratt (1996)).[5] It is easily seen that RRA joint with decreasing prudence is a sufficient condition for convexity of absolute risk aversion as well as for the substitutability of independent risky assets (see Gollier (2001, Proposition 35). However, as noted earlier, some even more stringent refinements of the set of vN-M utility functions have been considered in the literature, namely mixed risk aversion (see Caballé and Pomansky (1996); and Pratt and Zeckhauser (1987) who show that mixed risk aversion is a sufficient condition for properness). We show that the infinitely repetitive risk aversion implies mixed risk aversion, which by definition is equivalent to the condition that the first derivative of the utility function is completely monotone over the interval $(0, \infty)$.

Theorem 1: A vN-M utility function $u \in \boldsymbol{u}$ satisfies infinitely repetitive risk aversion only if its first derivative u' is completely monotone on $(0, \infty)$.

Caballé and Pomansky (1996) also propose the refinement that the absolute risk aversion of the utility function is completely monotone. We show that this too follows from infinitely repetitive risk aversion.

Corollary 1: A vN-M utility function $u \in \boldsymbol{u}$ satisfies infinitely repetitive risk aversion only if its absolute risk aversion $-u''/u'$ is completely monotone on $(0, \infty)$.

[5] Risk vulnerability means that adding an unfair background risk to wealth makes risk averse individuals more risk averse.

Note that the absolute risk aversion is completely monotone for the class of utility functions $u(w) = aw^{\alpha}$, $0 < \alpha \leq 1$. Another example is the class of utility functions that have the first derivative $u'(w) = e^{-w^{1+\alpha}}$, $0 < \alpha \leq 1$.[6]

A function f is *operator monotone* on $(0, \infty)$ if $(-1)^{n-1} f^{n}(w) \geq 0$ for $n = 1, 2, \cdots$. Theorem 1 and Corollary 1 can be thus rephrased as follows: if u satisfies infinitely repetitive risk aversion then u and $-\log u'$ are both operator monotone on $(0, \infty)$.[7]

Proof of Theorem 1: Let $D^n f$ denote the n^{th} derivative of *f*, i.e. $D^n f(w) = \frac{d^n}{dw^n} f(w)$.

A little reflection on Table 1 and the fact that $u_1 = u / u'$ show that we need to prove only the following:

Given any $u \in \boldsymbol{u}$, let $m \geq 2$ be some integer. Then, $(-1)^n D^n (u/u') \leq 0$ for each n with $2 \leq n \leq m$ implies $(-1)^n D^n (u') \geq 0$ for each n with $2 \leq n \leq m$.

The proof for this assertion has two parts:

Claim 1: If $(-1)^n D^n (u / u') \leq 0$ for each n with $2 \leq n \leq m$, then $(-1)^n D^{n-1} (-u'' / u') \leq 0$ for each n with $2 \leq n \leq m$.

The claim is clearly true for $n = 2$. From induction in n and the identity

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[6] The integral of $u'(w) = e^{-w^{1+\alpha}}$ exists because it is bounded above by the integrable function e^{-w} over the interval $[0, \infty)$.

[7] Operator monotone functions have been widely used in matrix analysis (see e.g. Bhatia (1996)). Very few functions have been identified to be operator monotone, a canonical example is the familiar class of functions $f(w) = w^{\alpha}$, $0 < \alpha \leq 1$. It is known that if f is operator monotone on $(0, \infty)$, then f has a Taylor expansion $f(x) = \sum_{n=0}^{\infty} a_n (x-1)^n$ in which the coefficients a_n are positive for all odd n and negative for all even n. Clearly, the first derivative of an operator monotone function is completely monotone.

$$D^n\left((u/u')(-u''/u')\right)=\sum_{r=0}^{n}\frac{n!}{(n-r)!r!}D^r\,(u/u')\,D^{n-r}\,(-u''/u'),$$

it is seen that it also holds for all n with $2\le n\le m$.

Next, define $f(x)=-\log u'(x)$. Then $D^n f(x)=D^{n-1}(-u''/u')$ and $u'(x)=e^{-f(x)}$.

Claim 2: If $(-1)^n D^n\, f(x)\le 0$ for $2\le n\le m$, then $(-1)^n\, D^n\, u'(x)\ge 0$ for $2\le n\le m$. This claim follows from the following two identities

$$D^n\,u'(x)=\quad D^{n-1}\,(e^{-f(x)}\,(-Df(x))),$$

$$D^{n-1}\,(e^{-f(x)}(-Df(x)))=\sum_{r=0}^{n-1}\frac{n!}{(n-r-1)!r!}D^r\,e^{-f(x)}\,D^{n-r-1}\,(-Df(x))$$

and induction in n after noting that the claim is true for $n=2$.

Claims 1 and 2 together complete the required proof.

Proof of Corollary 1: If *u* satisfies infinitely repetitive risk aversion, then as seen from the proof of Theorem 1 and Table 1, $(-1)^n D^n u_1\le 0$ for each $n\ge 2$, i.e. u_1' is completely monotone. Claim 1 in the proof of Theorem 1 proves that if $(-1)^n D^n u_1\le 0$ for $n\ge 2$, then $(-1)^n D^{n-1}\ (-u''/u')\le 0$ for $n\ge 2$ which proves that $-u''/u'$ is completely monotone.

Note that the complete monotonicity of the first derivative of the utility function and/or of its absolute risk aversion is a necessary but not sufficient condition for infinitely repetitive risk aversion.[8]

How successful is this refinement strategy? As noted earlier, infinitely repetitive risk aversion is satisfied by the class of functions $u(w) = aw^{\alpha}, a > 0$ and $0 < \alpha \leq 1$. The question is therefore whether this is the only class with this property? This is an interesting, but a difficult question. The complete answer is not known, but a partial answer is as follows:

Note, first that the operator $T : u \rightarrow u / u'$ has one and only one fixed point, namely $u(w) = w.$[9]

Theorem 2: If a vN-M utility function $u \in \boldsymbol{u}$ satisfies infinitely repetitive risk aversion, then the successive iterates $u_{n+1} = Tu_n, n = 0, 1, \cdots$ with $u_0 = u,$ converge pointwise to the fixed point of T, i.e., $\lim_{n\to\infty} u_n(w) = w$ for all $w \geq 0.$

Proposition 3: The successive iterates $u_{n+1} = Tu_n, n = 0, 1, \cdots$ and $u_0 \in \boldsymbol{u}$, converge to the fixed point in a finite number of iterations if and only if u_0 belongs to the family $u(w) = aw^{\alpha}, a > 0,\ 0 < \alpha \leq 1.$

Proof of Theorem 2: Since $u \in \boldsymbol{u}$ satisfies infinitely repetitive risk aversion, $u_n \in \boldsymbol{u}$ for each $n \geq 0,$ i.e., $u_n' > 0$ and $u_n'' \leq 0$ for all $n \geq 0.$

(a) For each $n \geq 1,$ since $u_{n-1}'(w)$ is non-increasing in $w, u_n(w) = u_{n-1}(w) / u_{n-1}'(w)$,

$$= (1/u_{n-1}'(w)) \int_0^w u_{n-1}'(z)dz \geq w (\text{using } u(0) = 0).$$

[8] The utility function $u(w) = w/(1+w),$ for example, does not even satisfy RRA but its first derivative and absolute risk aversion are both completely monotone.
[9] Clearly, if u is a fixed point of T, then $u = Tu = u/u'$ implies $u' = 1.$

(b) Since $u_n = u_{n-1}/u'_{n-1}$ and $u''_{n-1} \le 0$, it is seen from differentiation of u_n that $u'_n(w) \ge 1$ for each $w \ge 0$ and $n \ge 1$.

(c) Since $u_{n+1} = u_n / u'_n$ and as shown in (b) $u'_n \ge 1$, for each n, $u_n(w)$ is non-increasing in w for $n \ge 1$.

(d) In view of (c), let $v(w) = \lim_{n\to\infty} u_n(w)$. Then, in view of (a), $v(w) \ge w$.

Since for each w, $u_n(w)$ and $u_{n+1}(w)$ converge to the same limit $u_{n+1}(w) = u_n(w)/u'_n(w)$, $\lim_{n\to\infty} u'_n(w) = 1$ if $v(w) > 0$, i.e., if $w > 0$. This means that if $w > 0$, then $u'_n(w)$ is bounded, i.e., there exist m and n_0 such that $u'_n(w) \le m$ for $n \ge n_0$. Hence the set $\{u'_n(y) : y \ge w, n \ge 1\}$ is bounded, since $u'_n(w)$ is non-increasing in w. Therefore, by the dominated convergence theorem

$$u_n(y) - u_n(w) = \int_w^y u'_n(z)\, dz$$

converges to $\int_w^y 1\, dz = y - w$. Thus, $v(y) - v(w) = y - w$ for $y \ge w > 0$. However, since $u_n(0) = 0$ for each n, $v(0) = \lim_{n\to\infty} u_n(0) = 0$. This means that $\lim_{n\to\infty} u_n(y) = v(y) = y$ for each $y \ge 0$. This completes the proof.

Proof of Proposition 3: The proof of "if" part is obvious. We prove the "only if" part. It is easily seen that

$$\frac{u''_n}{u'_n} = \frac{u'_n}{u_n} - \frac{u'_{n+1}}{u_{n+1}} \text{ for } n \ge 0.$$

If the convergence is in finite iterations, then $u_n'' = 0$ for some finite *n*. If $n = 0$, then by integration $u(w) = aw;\ a > 0$. If $n = 1$, then again by integration, and from the fact that $u_n = \frac{u_{n-1}}{u'_{n-1}}$, we obtain $u(w) = aw^{\alpha}$, $a > 0$ and $0 < \alpha < 1$. Similarly, if $n \geq 2$, then

$$u_{n-2} = ae^{(1/\alpha)x^{\frac{1}{\alpha}}} - a,\ a > 0 \text{ and } 0 < \alpha < 1.$$

But this means that u_{n-2} is not concave, which contradicts that $u_{n-2} \in \boldsymbol{u}$. Hence $n \leq 1$, and $u(x) = ax^{\alpha}$, $a > 0$ and $0 < \alpha \leq 1$. This completes the proof.

Theorem 2 and Proposition 3, reduce our question to the following: does infinitely repetitive risk aversion imply that the successive iterates $u_{n+1} = Tu_n$, $n \geq 0$, $u_0 \in \boldsymbol{u}$, converge in a finite number of iterations?

4. Conclusions

We have already referred to some economic applications of repetitive risk aversion in the text. Besides the original application in Aumann and Kurz (1977), another application can be found in Chander and Wilde (1998) and Chander (2000) who show that in an optimal scheme the agent's decision to evade income tax is equivalent to risking his entire income against a possible gain in terms of lower tax payment and then characterize the optimal tax function by imposing repetitive risk aversion. In all these applications, neither decreasing absolute risk aversion nor decreasing relative risk aversion is sufficient for obtaining the required characterizations. Another important application is to the Arrow-Debreu portfolio problem in that repetitive risk aversion is a sufficient condition under which the option to invest in a complete set of Arrow-Debreu securities raises the marginal value of wealth, that is the first derivative of the maximal expected utility or the value function (see Gollier (2001, Propositions 34 and 54). Finally, the canonical class of utility functions

$u(w) = aw^{\alpha}$, $a > 0$ and $0 < \alpha \le 1$, that mirror the infinitely repetitive risk aversion are also the ones most commonly used in financial economics.

We have shown that the concept of RRA can be viewed as a part of the process for refining the set of risk averse utility functions and found an additional justification for mixed risk aversion, which is known to be a sufficient condition for proper risk aversion, in terms of this concept.

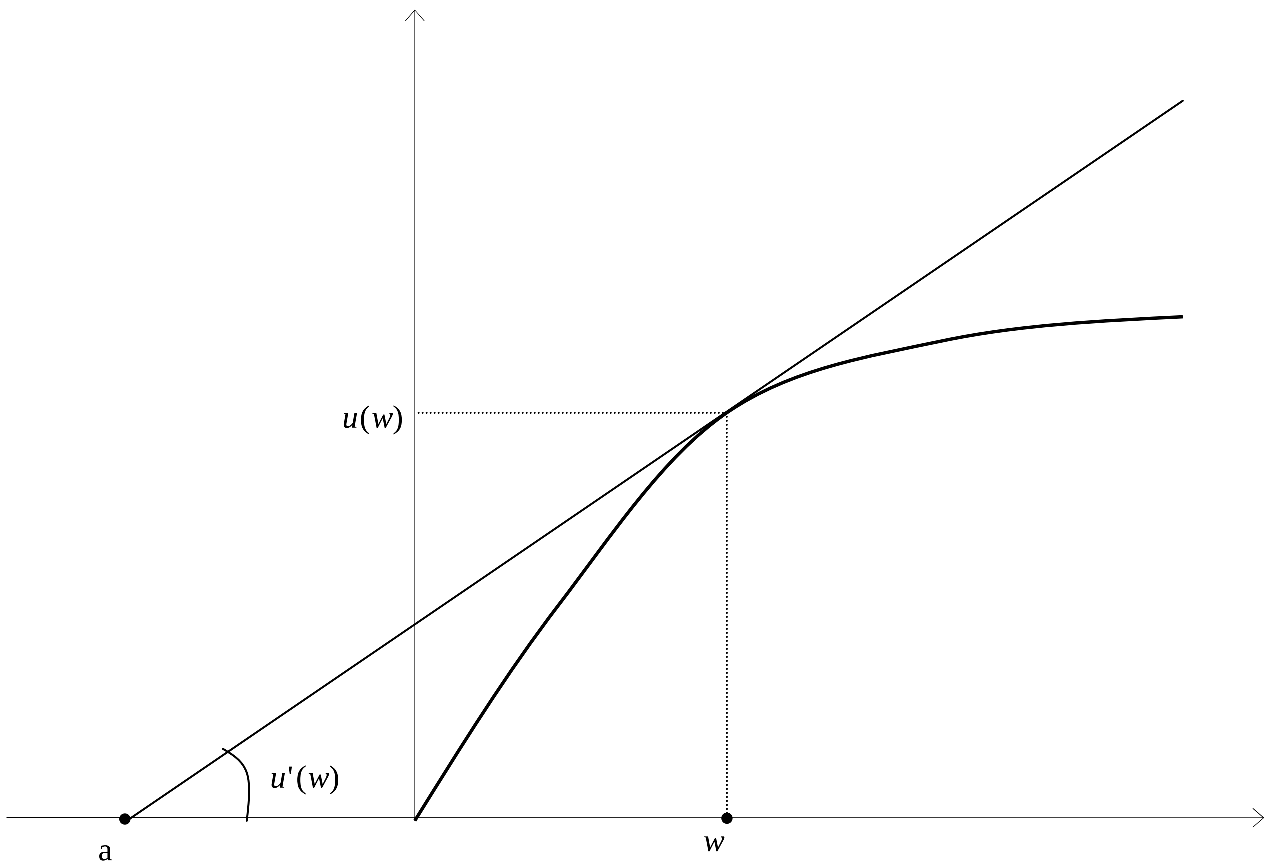

Figure 1

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