Forward Markets May not Decrease Market Power when Capacities are Endogenous

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Abstract

This paper analyzes the properties of three capacity games in an oligopolistic market with Cournot players. In the first game, capacity and the operation of that capacity is determined simultaneously. This is the classic open-loop Cournot game. In the second game capacity is decided in the first stage and the operation of that capacity is determined in the second stage. The first stage decision of each player is contingent on the solution of the second-stage game. This is a two-stage, closed-loop game. We show that when the solution exists, it is the same as the solution in the first game. However, it does not always exist. The third game has three stages with a futures position taken between the capacity stage and the operations stage and is also a closed-loop game. As with the second game, the equilibrium is the same as the open-loop game when it exists. However, the conditions for existence are more restrictive once a futures market is added. When both games have an equilibrium, the solution values are identical. The results are very different from games with no capacity stage as studied by Allaz and Vila (1993), which have been used to argue that futures markets can ameliorate market power.

1 Introduction

One of the important questions in the theory of oligopolistic markets is the role of futures markets in mitigating market power. The literature on this subject typically examines the effect of futures markets on production levels in oligopolistic markets without explicit capacity decisions. By adding a capacity decision for each player before the futures decision, we increase the realism of the game for capital-intensive industries in a commodity business and derive results that are different from those in the literature.

Understanding the effects of futures markets has taken on new importance given the problems in the California electricity market. Here the use of futures markets by the regulated electricity purchasers was restricted to 20% of expected sales and the problems in that market in 2000 were partially blamed on the lack of an active futures market that could have locked in lower rates on much of the capacity.

The original work on the potential of futures markets to mitigate market power is by Allaz (1992) and Allaz and Vila (1993). They wrote two of the early papers on this and derived the remarkable result that with Cournot players oligopolistic producers increase production just from the existence of a futures market. This result has intuitive appeal: the futures position fixes the price for a portion of the production and reduces the quantity that is subject to lower prices from increased production. This increases the marginal revenue in the spot market for any production level, thereby increasing the equilibrium quantity. In fact, they show that as the number of periods increases, the equilibrium in a duopoly converges to the competitive equilibrium.

Their work has led to a growing literature with articles confirming or negating the result. None of these articles have addressed the effect of capacity decisions on the extent to which futures decisions can alter production decisions. In the next section we survey the literature illustrating both sides of the debate on the effect of futures markets.

Next we examine the open-loop Cournot game where capacity and production decisions are made simultaneously. We then develop closed-loop games without and with a futures market. In closed-loop games the capacity decision of each player is made knowing how this decision affects the production decision of the other player, while taking the capacity decision of the other player as given. Our first closed-loop game determines capacity in the first stage, followed by the operation of the capacity in the second stage. This is different from the standard open-loop game where the capacity and production decisions are made simultaneously and each player sees the other player's capacity and production decisions fixed. Our main results in this section are that an equilibrium might not exist, but if it exists, it is the same as the open-loop equilibrium where the capacity and production decisions are made simultaneously.

The last game has a futures stage between the capacity stage and the production stage. This is a three-stage closed-loop game. Here the capacity decisions are made knowing their effects on futures decisions, which are made knowing their effects on the spot game. The futures market plays a complex role. An equilibrium might not exist, but if it exists, it is the same as the open-loop equilibrium. The difference from the previous game is that there exist parameters for which the game without futures has an equilibrium but the game with futures does not. The underlying mechanisms are different from the Allaz and Vila model. In their model, increasing the futures position of a player decreases production by the other player. However, in the game with a capacity stage, both players operate at capacity and the marginal value of the capacity is the cost of capacity. Thus, increasing the futures position of a player only decreases the marginal value of the other player's capacity, not its production. We show that this property reproduces the single-stage Cournot solution. We illustrate the results with an example.

We conclude with a discussion of the realism of this result. For example, this model is deterministic. Adding uncertainty, or the equivalent load duration curve can have an important impact.

2 Literature review

Some of the literature on the effects of futures markets on spot markets is generic to all markets while the rest of the literature can be grouped along two different dimensions. The first dimension is Bertrand versus Cournot models with supply-function equilibria falling in between. The second is electricity markets versus traditional commodity markets.

2.1 Cournot electricity markets

Powell (1993) notes that the generators in the British market have a monopoly on the ability to offer contracts for differences, which are essentially futures contracts. This allows them to charge a premium for these contracts when the regional electricity companies are risk averse. He argues for a standard futures contract to mitigate this power. Green (1999) shows that the effect of a futures market depends on the conjectural variation player i assumes about player $-i$. He shows that a Cournot player does not enter into futures contracts and a Bertrand player does contract for all of its production at marginal cost. His results differ from Allaz and Vila because he solves the open-loop game where the futures and production decisions are made simultaneously, thus the conjectural variation is 0 in the Cournot case. Because Allaz and Vila solve a two-stage, closed-loop game, the conjectural variation is greater than zero while still being Cournot.

Gans, Price, and Woods, (1998) reaffirm the Allaz and Vila results. However, they note that contracts can be used to restrict entry, leading to higher prices in the long run. Their paper provides an example that clearly illustrates the phenomenon. Newbery (1998) shows the reverse can be true if generators use contracts to block new entrants. This is in the vein of the literature that starts with the paper by Spence (1977) where an incumbent builds in one stage and the entrant invests in the next.

Harvey and Hogan (2002) start from Allaz and Vila's recognition that both players are worse off after they take their futures position and note that the two-stage game has the payoffs of the prisoner's dilemma game with the game repeated indefinitely. It is well known that in practice, when the prisoner's dilemma game repeated for a large number of periods, the players cooperate in the early periods. They argue that the players learn to cooperate without colluding by avoiding the futures market. A counter argument is that if the players are risk averse, they enter the futures market for managing risk and then have the same second-stage spot game.

Using data from the beginning of the restructured markets in Australia, Wolak (2000) finds that a higher level of contracts induces increased production. He also notes that at high enough levels, contracts can lead to production levels with negative prices.

Joskow and Tirole (2002), look at the effects of the allocation of transmission rights on the electric grid. They conclude that if producers in importing regions or consumers in exporting regions own financial rights, market power is aggravated. The converse is that if producers in exporting regions and consumers in importing regions hold rights, market power is mitigated. Kamat and Oren (2002) examine the effects of zonal pricing with and without transmission constraints. They reproduce the Allaz and Vila results when the transmission constraints are not binding and show that binding constraints mitigate the effect of forward markets.

2.2 Bertrand electricity markets

Haskel and Powell (1992) show that in a contract market that is based on price offers, the market clears with price equal to marginal cost. Thus, a futures market cannot lead to increased production.

2.3 Other commodities and experiments

Le Coq and Orzen (2002) do laboratory experiments with students to test the extent to which futures markets affect spot markets. They find that a futures market leads to increased production, but not to the extent that theory would predict. Adding a futures market is not as effective as increasing the number of players because the students behaved more competitively than theory would predict.

Goering and Pippenger (2002) show that for durable commodities the optimal strategy for a monopolist is to buy in forward markets even at a premium to the spot price. This commits the monopolist to not flood the market with the durable good it produces, an example of which is metals. The commitment not to flood the market makes it possible for customers to buy more, knowing the value of their purchases will not be eroded. The monopolist has to pay a premium because the seller has the risk of being squeezed.

Mahenc and Salanie (2002) show that in a Bertrand market with partially differentiated products the optimal strategy is to take a long position in the product market. This raises the spot price and increases the profits for both players. Since prices are strategic complements, both players have an incentive to buy rather than sell futures, and, unlike Allaz and Vila, there is no prisoner's dilemma game. They note that this behavior was observed in coffee markets in 1977. They also show that the less differentiated the good, the higher the spot price

2.4 Capacity expansion

Wu, Kleindorfer, and Sun (2002) have a capacity-expansion model in electric power with options. They did not characterize the existence of the solution or its properties with and without the options market.

Murphy and Smeers (2003) look at the capacity-expansion game as a two-stage game without a futures market in the context of electricity markets. They are able to show that the equilibrium is unique if it exists and that it does not always exist. They show that the two-stage, closed-loop formulation leads to greater capacity than an open-loop, single-period formulation. This is because each player sees the other's production decisions change in response to its increase in capacity. In some ways this is an alternative approach to imputing conjectural variation while retaining the Cournot formulation.

The next section contains a simplified version of the model in Murphy and Smeers in that we use a deterministic demand level without a load duration curve. The model has a capacity game followed by a production game.

3 Model definition and an example

3.1 Model definition

3.1.1 Cost structure

Assume that generation units can be entirely characterized by their investment and variable operations cost. For a given utilization rate (see Stoft (2002) for a discussion), these costs can be expressed in $/Mwh. We let

ν_i be the per unit production cost,

k_i be the per unit capacity cost

3.1.2 Demand curve

We consider a single time period and let

$$p = \alpha - q \tag{1}$$

be the demand curve in that period.

3.1.3 Variables

The most complex model considered in this paper, the three-stage closed-loop game, assumes that agents build some capacity in a first stage, trade on the forward market in a second stage and on the spot market in the third stage. Because the model is fully deterministic there is no need to distinguish between forward and futures contracts and we use these terms interchangeably. We let

x_i be the capacity invested by player i

y_i be the forward position, and

z_i be the spot generation.

This decomposition implies that y is traded in the futures market and $z - y$ in the spot market. This is the decomposition assumed in Allaz-Vila. It has different possible interpretations in electricity markets. In a standard market design interpretation, y would be traded in the day ahead market and $z - y$ in real time. In a pure bilateral system, y would correspond to OTC contracts and $z - y$ would be the trade in real time. Assuming again the most complex three-stage model, profit accruing at different stages of the decision process can be computed as follows.

Let $-i$ index the player that is not i. The profit accruing to agent i in the spot market, or third stage, is equal to

$$[\alpha - (z_i + z_{-i})](z_i - y_i) - \nu_i z_i \tag{2}$$

By arbitrage the spot and forward prices are equal. The sale in the forward market therefore induces a revenue of $[\alpha - (z_i + z_{-i})]y_i$. There is no cost in trading forward and the forward revenue is equal to the forward profit. Thus the cumulative second- and third-stage profit in the second stage is the operating profit and equal to

$$[\alpha - (z_i + z_{-i})]z_i - \nu_i z_i. \tag{3}$$

Player i incurs a cost $-k_i x_i$ for building capacity in the first stage. This is also its first-stage profit since there is no revenue in the first stage. The cumulative three-stage profit aggregated in the first stage is thus equal to

$$[\alpha - (z_i + z_{-i})]z_i - \nu_i z_i - k_i x_i. \tag{4}$$

3.2 An example

The following example is used throughout the paper to illustrate the results.

3.2.1 Demand

To parameterize the linear demand model, assume a demand of 30 000 Mwh in some hour when the price is \$30/Mwh. We write

$$30 = \alpha - \beta\, 30\,000$$

We consider both low and high values of the long term price elasticity.

In the case with low demand elasticity assume the *long-term* elasticity is .1 $\left(|\frac{p}{q}\frac{dq}{dp}| = .1\right)$ at the point (30,30 000). The values for α and β can be calculated as follows

$$\frac{30}{30\,000\,\beta} = .1 \quad \text{or} \quad \beta = \frac{30}{3\,000} = .01$$

From $30 = \alpha - (.01)(30\,000)$ we get $\alpha = 330$. We now want to convert this expression into $p' = \alpha' - q'$. Suppose we measure quantities in units of 10 Mwh. The reference point becomes (300,3 000) (price is $ 300 for 10 Mwh and demand is 3 000 10 Mwh). β becomes 1 as $\frac{300}{3\,000} = .1$ We rewrite the demand system as

$$\begin{aligned} & p' = \alpha' - q' \\ \text{with} \quad & \alpha' = 3300 \end{aligned}$$

With a higher-long term elasticity of .9, and using the same argument, we measure quantities in units of 30Mwh and rewrite the system $p' = \alpha' - \beta' q'$. The reference point becomes 900, 1000, and the value of the elasticity at that point implies $\beta' = 1$. From $p' = \alpha' - q'$ we get $\alpha' = 1900$.

3.2.2 Technology

The following investment and fuel cost figures are taken from Stoft (2002).

	Investment cost ($/Mwh)	Fuel cost ($/Mwh)
Coal	14.10	11.77
Advanced combustion cycle	7.36	20.78

Expressing everything in 10Mwh for dealing with the small price elasticity and rounding up we select investment and fuel costs equal to

$$\begin{array}{rcccl} K_1 & = & K_{\text{coal}} & = & \$150/10Mwh \\ \nu_1 & = & \nu_{\text{coal}} & = & \$100/10Mwh \\ \\ K_2 & = & K_{\text{gas}} & = & \$75/10Mwh \\ \nu_2 & = & \nu_{\text{gas}} & = & \$200/10Mwh \end{array}$$

Expressing everything in 30Mwh for dealing with the large elasticity and rounding up in the same way, we select the alternative assumption

$$\begin{array}{rcccl} K_1 & = & K_{\text{coal}} & = & \$450/30Mwh \\ \nu_1 & = & \nu_{\text{coal}} & = & \$300/30Mwh \\ \\ K_2 & = & K_{\text{gas}} & = & \$225/30Mwh \\ \nu_2 & = & \nu_{\text{gas}} & = & \$600/30Mwh \end{array}$$

4 The single-stage game

The open-loop game without a futures market is the simplest of the three games considered in this paper. It is the one where agent i simultaneously decides both its investment and sales. The most natural interpretation of this game is one where both agents build capacity and immediately sell all the output of that capacity forward. There is no spot market.

With the standard Cournot assumption, the Nash equilibrium (x_i^*, x_{-i}^*) is obtained in the game when x_i^* solves

$$\max_{x_i \geq 0} \left[\alpha - (x_i + x_{-i}^*)\right] x_i - (\nu_i + k_i)x_i, \qquad i = 1, 2.$$

4.1 Equilibrium conditions

The solution to the game exists and is unique. In order to streamline the comparison of the three games (single, two, and three stages), we concentrate on the case where the single stage game has a single strictly positive equilibrium. Solving the optimization problem of each individual player, one obtains

$$\begin{aligned} &\alpha - 2x_i - x_{-i} - (\nu_i + k_i) = 0 \\ &\alpha - x_i - 2x_{-i} - (\nu_{-i} + k_{-i}) = 0. \end{aligned} \tag{5}$$

This can be solved to give

$$x_i = \frac{1}{3}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big] \qquad i = 1, 2. \tag{6}$$

The price of electricity is equal to

$$\alpha - x_i - x_{-i} = \frac{1}{3}\big[\alpha + (\nu_i + k_i) + (\nu_{-i} + k_{-i})\big]. \tag{7}$$

The unit profit of player i is

$$\frac{1}{3}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big] \tag{8}$$

and the total profit

$$\frac{1}{9}\big[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\big]^2. \tag{9}$$

This solution has x_i strictly positive iff

$$\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i}) > 0 \qquad i = 1, 2. \tag{10}$$

The following is a trivial observation in this game.

Proposition 1 $z_i = x_i$, $i = 1, 2$ *in the open-loop game.*

4.2 Illustration

We here verify that our test example has a strictly positive equilibrium.

Case 1. Small price elasticity

The small long-term elasticity implies $\alpha' = 3300$. We have (recall that units are in 10 Mwh)

$$\begin{aligned} x_1 &= \tfrac{1}{3}(3300 - 2 \star 250 + 275) \\ &= \tfrac{1}{3}\, 3075 = 1025 \end{aligned}$$

$$\begin{aligned} x_2 &= \tfrac{1}{3}(3300 - 2 \star 275 + 250) \\ &= \tfrac{1}{3}\, 3000 = 1000. \end{aligned}$$

Converting to the more usual Mw and Mwh units, installed capacities are respectively 10250 and 10000 Mw for players 1 and 2 respectively. The total production is equal to 20250 Mwh and the price is 3300-2025 = $1275/10 Mwh or $127.5/Mwh. This equipment produces in base since there is a single time period and the price of electricity is $127.5/Mwh.

Case 2. Large long term elasticity

Taking the large long-term price elasticity $(\alpha' = 1900)$ and recalling that $p' = \alpha' - q'$ corresponds to units of 30 Mwh, we get

$$\begin{aligned} x_1 &= \tfrac{1}{3}\big[1900 - 2 \star 750 + 825\big] = 408 \text{ or } 12250 Mw \\ x_2 &= \tfrac{1}{3}\big[1900 - 2 \star 825 + 750\big] = 333 \text{ or } 1000 Mw \end{aligned}$$

Converting again in the standard Mw and Mwh units, investments are respectively 12250 and 1000 Mw for players 1 and 2 and the electricity price is $38.6/Mwh.

5 A two-stage investment/spot model without a forward market

We now consider the case of a two-stage game, namely one where players invest in merchant plants and trade on the spot market. There is no forward market in this model. The Spanish market is an example where there is neither a bilateral nor a futures market. The equilibrium of this model is analyzed by working backward.

5.1 Equilibrium conditions

Let x_i be the capacities inherited from the investment stage. The equilibrium conditions of the spot market are obtained when each agent solves the following optimization problem, which is (2), with $y_i = 0$,

$$\max_{0 \leq z_i \leq x_i} [\alpha - (z_i + z_{-i})]z_i - \nu_i z_i. \tag{11}$$

Here again, the existence and uniqueness in the equilibrium of the spot market are easily established. They are obtained as the solution of the following complementarity system.

$$\begin{aligned} &\alpha - 2z_i - z_{-i} - \nu_i + \omega_i = \lambda_i \qquad i = 1, 2 \\ &x_i - z_i \geq 0 \qquad \lambda_i \geq 0 \qquad (x_i - z_i)\lambda_i = 0 \\ &z_i \geq 0 \qquad \omega_i \geq 0 \qquad z_i \omega_i = 0 \end{aligned} \tag{12}$$

Let $z(x)$ be the solution of these equilibrium conditions as a function of the capacities x inherited from the first investment stage. It is easy to see that $z(x)$ is single valued and continuous in x. Note that $z(x)$ is not continuously differentiable in x.

In order to simplify the presentation, we limit the discussion to the case where the equilibrium satisfies $0 < z_i \leq x_i$, that is, the two producers are active at the equilibrium. Making this simplification, the equilibrium of the spot market satisfies one of the three following possible conditions.

$$\begin{aligned} &\text{(i)} \qquad 0 < z_i < x_i; \qquad i = 1, 2 \\ &\text{(ii)} \qquad 0 < z_i < x_i; \qquad 0 < z_{-i} = x_{-i} \\ &\text{(iii)} \qquad 0 < z_i = x_i; \qquad 0 < z_{-i} = x_{-i} \end{aligned} \tag{13}$$

We now find the equilibrium in the investment game that accounts for the behavior of the players in the spot market. This is commonly referred to as a subgame-perfect equilibrium or closed-loop equilibrium (Fudenberg and Tirole (2000)). Remaining in the simple Cournot framework, we state

Definition 1 *A closed-loop equilibrium of the two-stage game $x^*, z^*(x)$ satisfies the following conditions*

(i) $z^*(x)$ is a Nash equilibrium of the spot market (second-stage game) for every feasible x

(ii) x^* is a Nash equilibrium of the capacity market game (first-stage game) where the payoffs of the agents are

$$\Pi_i(x_i; x_{-i}) = \Pi_i[x_i, z_i^*(x); x_{-i}, z_{-i}^*(x)], \qquad i = 1, 2. \tag{14}$$

If there exists a closed-loop equilibrium $x^*, z^*(x)$, then there exists a feasible neighborhood $N(x^*)$ of x^* (intersection of a ball centered on x^* and the feasible set $x \geq 0$) such that

- $z^*(x)$ is a Nash equilibrium in the spot market for all points x, $x \in N(x^*)$
- x^* is a Nash equilibrium of the capacity market with payoffs $\Pi_i(x_i, x_{-i}); i = 1, 2$, defined as in (14) in that feasible neighborhood $N(x^*)$.

$x^*, z^*(x)$ is a local equilibrium if $x^*, z^*(x)$ is an equilibrium in a feasible neighborhood around x^*. This is restated as

Definition 2 *A local closed-loop equilibrium of the two-stage game is a closed-loop equilibrium of the game where x is restricted to a non-empty full dimensional subset of the capacity space.*

We now extend Proposition 1 to the two-stage game. As a first step, we show that the cases (i) and (ii) of (13) cannot hold at equilibrium. This is done in Lemmas 1 and 2.

Lemma 1 *Assume there is a closed-loop equilibrium of the two-stage game. Then case (i) of (13) cannot hold at this equilibrium.*

Proof. Suppose

$$0 < z_i^* < x_i^* \qquad i = 1, 2.$$

The system (12) reduces to

$$\alpha - 2z_i^* - z_{-i}^* - \nu_i = 0 \qquad i = 1, 2 \text{ or } z_i^* = \frac{1}{3}[\alpha - (2\nu_i - \nu_{-i})]$$

There exists a ball centered on x^* such that for all x in that ball, $z^*(x) = z^*$ is the best response. Therefore $(x^*, z^*(x^*))$ is a local equilibrium of the capacity game. For this equilibrium the profit of i before paying for capacity is

$$\frac{1}{9}\big[\alpha - (2\nu_i - \nu_{-i})\big]^2.$$

After paying for capacity, the profit is

$$\frac{1}{9}\big[\alpha - (2\nu_i - \nu_{-i})\big]^2 - k_i x_i^*. \tag{15}$$

However, (15) cannot be a local maximum of the payoff of player i with respect to x_i because we can reduce x_i to improve the payoff.

Lemma 2 *Suppose there exists an equilibrium of the two stage game. Then, condition (ii) of (13) cannot hold at this equilibrium.*

Proof. Assume

$$0 < z_i^* < x_i^* \text{ and } z_{-i}^* = x_{-i}^*.$$

The system (12) reduces to

$$\begin{aligned} z_i^* &= \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i) \\ z_{-i}^* &= x_{-i}^* \end{aligned}$$

Set

$$\begin{aligned} z_i(x) &= \tfrac{1}{2}(\alpha - x_{-i} - \nu_i) \\ z_{-i}(x) &= x_{-i}. \end{aligned}$$

It is trivial, to verify that there exists a ball centered on x^* such that for all x in that ball $z(x)$ is the best response. Using the same argument as in Lemma 1, having $x_i^* > z_i^*$ implies that one can always decrease x_i by a small amount and achieve a higher profit. Therefore, this is not a local equilibrium and hence not an equilibrium.

Proposition 2 *A closed-loop equilibrium of the two-stage game, if it exists, satisfies* $z_i^* = x_i^*$, $i = 1, 2$.

Proof. This immediately derives from Lemmas 1 and 2, and the assumption of existence of the closed-loop equilibrium.

Proposition 2 immediately allows us to infer the equivalence of the open and closed-loop equilibrium when the latter exists.

Theorem 1 *The closed-loop equilibrium of the two-stage game, if it exists, is identical to the open-loop equilibrium of the single-stage game.*

Proof. Let x_i^c and z_i^c, $i = 1, 2$ be the closed-loop solution of the two-stage game. By Proposition 2, this equilibrium, if it exists, satisfies $z_i^c = x_i^c$, $i = 1, 2$. This implies

$$\alpha - 2x_i^c - x_{-i}^c - \nu_i = \lambda_i^c \geq 0, \quad i = 1, 2.$$

Consider a decrease of x_i from x_i^c while keeping x_{-i} equal to x_{-i}^c. Note that as x_i^c decreases, λ_i^c increases. Thus, $z_i = x_i$, $i = 1, 2$ satisfies the spot equilibrium conditions (12). This implies that the first-stage objective function of i is equal to

$$\Pi_i(x_i; x_{-i}^c) \equiv (\alpha - x_i - x_{-i}^c - \nu_i)x_i - k_i x_i$$

when x_i is decreased with $x_{-i} = x_{-i}^c$. Note that $\Pi_i(x_i, x_{-i}^c)$ is concave in x_i. Because x^c is a closed-loop equilibrium, Π_i achieves a maximum at x_i^c given $x_{-i} = x_{-i}^c$. One has

$$\alpha - 2x_i^c - x_{-i}^c - \nu_i - k_i \geq 0$$

and hence

$$\lambda_i^c \geq k_i > 0.$$

Because $\lambda_i^c > 0$, there exists a neighborhood of x^c such that for x in that neighborhood, $z_i = x_i$, $i = 1, 2$ satisfies the system (12) and hence remains the spot equilibrium in that neighborhood. Adapting the above reasoning to variations of x_i

in excess of x_i^c one finds $\lambda_i^c = k_i$. Therefore, the closed-loop equilibrium of the two-stage game x^c, if it exists, satisfies the same conditions (4), as the open-loop equilibrium and hence is identical to it.

We now turn to the question of the existence of the equilibrium in the two-stage game. It is well known from game theory (see Fudenberg and Tirole (2000)) that existence is not guaranteed in general. An easily verifiable condition on investment cost allows one to guarantee the existence of this equilibrium for our particular problem.

Theorem 2 *Suppose we limit the capacity space x to points such that $z(x) > 0$; then there exists a closed-loop equilibrium if $k_i \leq 2k_{-i}$, $i = 1, 2$.*

Proof. Because of Theorem 1, we know that a closed-loop equilibrium of the two-stage game, if it exists, is identical to the open-loop equilibrium of the single-stage game. We, therefore, identify sufficient conditions for the open-loop equilibrium to also be a closed-loop equilibrium. The open-loop equilibrium x^* satisfies $\alpha - 2x_i^* - x_{-i}^* - \nu_i = \lambda_i^* = k_i$.

a) Let $x_i < x_i^*$. Then one can check that the second-stage equilibrium $z^*(x)$ remains $z^*(x) = x$. Because of the optimality properties of the equilibrium of the single-stage game, there cannot be a higher profit for player i when $x_i < x_i^*$.

(b) Let $x_i > x_i^*$. As x_i is increased, three possibilities can occur

(i) λ_i becomes zero before λ_{-i} becomes zero

(ii) λ_{-i} becomes zero before λ_i becomes zero.

(iii) λ_i becomes zero exactly when λ_{-i} becomes zero

We successively consider the three cases in (b).

(i) $\lambda_i = 0$ before $\lambda_{-i} = 0$. Let $\tilde{x}_i$ be the value of x_i for which λ_i reaches 0. One has

$$\begin{aligned} &\alpha - 2\tilde{x}_i - x_{-i}^* - \nu_i = 0 \quad &\text{or} \quad &\tilde{x}_i = \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i) \\ &\alpha - \tilde{x}_i - 2x_{-i}^* - \nu_{-i} > 0 \quad &\text{or} \quad &\alpha - \tfrac{1}{2}\alpha + \tfrac{1}{2}x_{-i}^* - 2x_{-i}^* + \tfrac{1}{2}\nu_i - \nu_{-i} > 0. \end{aligned}$$

That is,

$$\frac{1}{2}\alpha - \frac{3}{2}x^*_{-i} + \frac{1}{2}(\nu_i - 2\nu_{-i}) > 0$$

or equivalently

$$\alpha + (\nu_i - 2\nu_{-i}) > 3x^*_{-i}.$$

Replacing x^*_{-i} by its equilibrium value found in the single stage game (relation (6)), we obtain that

$$\begin{aligned} & \alpha + (\nu_i - 2\nu_{-i}) > \alpha - 2(\nu_{-i} + k_{-i}) + (\nu_i + k_i) \\ \text{or} \quad & 2k_{-i} > k_i. \end{aligned}$$

Therefore case (i) occurs iff $2k_{-i} > k_i$. Suppose this inequality holds. One has $\alpha - 2z_i - x^*_{-i} - \nu_i = 0$ with $z_i < x_i$ for $x_i > x^*_i$. This implies that λ_{-i} never reaches 0, which in turn implies that $z_{-i} = x^*_{-i}$ and the profit Π_i is constant for $x_i > x^*_i$. Therefore, choosing $x_i > \tilde{x}_i$ cannot improve the profit of player i. Summing up, assuming $2k_{-i} > k_i$, $i = 1, 2$, neither player can increase its profit by increasing x_i with respect to the open-loop solution. This open-loop equilibrium is thus also a closed-loop equilibrium.

(ii) $\lambda_{-i} = 0$ before λ_i. Using the same steps as in (i), $k_i > 2k_{-i}$ which violates the assumption that $2k_{-i} \geq k_i$.

(iii) $x_i = \widetilde{x}_i$ simultaneously makes λ_i and λ_{-i} equal to 0. One thus has $k_i = 2k_{-i}$ and

$$\alpha - 2\widetilde{x}_i - x^*_{-i} - \nu_i = 0 \text{ and } \alpha - \widetilde{x}_i - 2x^*_{-i} - \nu_{-i} = 0. \tag{16}$$

Let $x'_i = \widetilde{x}_i + \varepsilon, \varepsilon > 0$. $z_{-i} = x^*_{-i}$ and $z_i = \widetilde{x}_i < x'_i$ are the equilibrium in the spot market by (16). Thus,

$$\Pi_i(x'_i, x^*_{-i}) = \Pi_i(\widetilde{x}_i, x^*_{-i}) - k_i\varepsilon < \Pi_i(\widetilde{x}_i, x^*_{-i}).$$

By the optimality of x^*_i, in the range $x^*_i \leq x_i \leq \widetilde{x}_i$ and the concavity of the profit function in this range

$$\Pi_i(x_i^*, x_{-i}^*) > \Pi_i(x_i, x_{-i}^*) \geq \Pi_i(\widetilde{x}_i, x_{-i}^*) > \Pi_i(x_i', x_{-i}^*)$$

which shows that it does not pay to increase x_i beyond x_i^*.

Consider now the case where $k_i > 2k_{-i}$. Define $\widetilde{\widetilde{x}}_i$ as the point where λ_{-i} becomes zero, as in case (b) (ii) of Theorem 2. We first note that $\widetilde{\widetilde{x}}_i$ satisfies $\alpha - \widetilde{\widetilde{x}}_i - 2x_{-i}^* - \nu_{-i} = 0$ or after substitution of the value of x_{-i}^*, $\widetilde{\widetilde{x}}_i = x_i^* + k_{-i}$. Following the reasoning of Theorem 2, the open-loop equilibrium can fail to be a closed loop equilibrium only if i has an incentive to invest x_i beyond the point $\widetilde{\widetilde{x}}_i$ where $\lambda_i > 0$ and $\lambda_{-i} = 0$. We explore this situation. Because $\lambda_{-i} = 0$, $z_{-i} < x_{-i}$ and the second-stage equilibrium implies for $x_i > \widetilde{\widetilde{x}}_i$ as long as $\lambda_i > 0$.

$$\begin{aligned} \alpha - 2x_i - z_{-i} - \nu_i &= \lambda_i > 0 \\ \alpha - x_i - 2z_{-i} - \nu_{-i} &= 0 \end{aligned}$$

with $z_{-i}(x_i) = \frac{1}{2}(\alpha - x_i - \nu_{-i}) < x_{-i}^*$. Replacing z in $\Pi_i(x, z)$ by this expression while keeping $z_i = x_i$ in expression (14), the profit becomes

$$\begin{aligned} \Pi_i(x_i; x_{-i}^*)] &= \left[\alpha - x_i - \tfrac{1}{2}(\alpha - x_i - \nu_{-i}) - \nu_i\right] x_i - k_i x_i \\ &= \tfrac{1}{2}\left[\alpha - x_i + (\nu_{-i} - 2\nu_i)\right] x_i - k_i x_i. \end{aligned} \tag{17}$$

An optimum of $\Pi_i(x_i; x_{-i}^*)$ for $x_i > \widetilde{\widetilde{x}}_i$ can occur only if the derivative of $\Pi_i(x_i, x_{-i}^*)$ at $\widetilde{\widetilde{x}}_i$ is positive. Assume it is positive. What we ultimately need is that the optimal objective function value of player i for $x_i > \widetilde{\widetilde{x}}_i$ is larger than the optimum at the open-loop equilibrium. We thus compute the optimal $x_i \geq \widetilde{\widetilde{x}}_i$ that equates the derivative of $\Pi_i(x; x_{-i}^*)$ to zero and verify that this optimal x_i falls in the region where (17) is a valid expression of the profit of i. Let $\widetilde{\widetilde{\widetilde{x}}}_i$ be this value; it is equal to

$$\widetilde{\widetilde{\widetilde{x}}}_i = \frac{1}{2}\left[\alpha + \nu_{-i} - 2\nu_i - 2k_i\right] = \frac{1}{2}\left(\alpha + \nu_{-i}\right) - (\nu_i + k_i).$$

Replacing x_i by $\widetilde{\widetilde{\widetilde{x}}}_i$ in (17) we obtain

$$\Pi_i(\widetilde{\widetilde{\widetilde{x}}}_i; x_{-i}^*) = \frac{1}{8}[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)]^2. \tag{18}$$

In order for $\tilde{\tilde{\tilde{x}}}_i$ to be an optimal response of player i to $x_{-i} = x^*_{-i}$, we need to find the condition that guarantees that

(a) (18) is indeed the correct expression of $\Pi_i(\tilde{\tilde{\tilde{x}}}_i, x^*_{-i})$, that is, $\tilde{\tilde{\tilde{x}}}_i \geq \tilde{\tilde{x}}_i$ and $z_i = x_i$ when $x_i = \tilde{\tilde{\tilde{x}}}_i$.

(b) $\frac{1}{8}\big[(\alpha + \nu_{-i}) - 2(\nu_{-i} + k_i)\big]^2 > \frac{1}{9}\big[(\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}\big]^2$.

We take up these two questions in the following lemma.

Lemma 3 *Suppose*

$$k_i > \frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i},$$

then

$$\Pi_i(\tilde{\tilde{\tilde{x}}}_i, x^*_{-i}) = \frac{1}{8}\big[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)\big]^2$$

is the profit of player i.

Proof. We first find the condition for $\tilde{\tilde{\tilde{x}}}_i \geq \tilde{\tilde{x}}_i = x^*_i + k_{-i}$. We need

$$\frac{1}{2}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] - \frac{1}{3}\big[\alpha + \nu_{-i} + k_{-i} - 2(k_i + \nu_i)\big] > k_{-i}$$

or

$$\frac{1}{4}\big[\alpha + \nu_{-i} - 2(k_i + \nu_i)\big] > 2k_{-i}.$$

Consider now the conditions that guarantee $z_i = x_i$ for $x_i = \tilde{\tilde{\tilde{x}}}_i$. One has $z_i = x_i$ if

$$\alpha - 2x_i - z_{-i}(x_i) - \nu_i \geq 0$$

or after replacement of $z_{-i}(x_i)$

$$\frac{\alpha}{2} - 3x_i + \frac{\nu_{-i}}{2} - \nu_i \geq 0.$$

The maximal value of x_i, x_i^m, that satisfies this condition is

$$x_i^m = \frac{\alpha + \nu_{-i} - 2\nu_i}{3}.$$

One thus needs that $\tilde{\tilde{\tilde{x}}}_i \leq x_i^m$ or

$$\frac{3}{2}(\alpha + \nu_{-i}) - 3(\nu_i + k_i) < (\alpha + \nu_{-i}) - 2\nu_i$$

that can be rewritten as

$$\frac{1}{2}(\alpha + \nu_{-i}) - \nu_{-i} - k_i < 2k_i$$

or

$$\frac{1}{4}\left[(\alpha + \nu_{-i}) - 2(k_i + \nu_i)\right] < k_i$$

which completes the lemma.

We now turn to condition (b) for $\tilde{\tilde{\tilde{x}}}_i$ to be an optimal response of player i to $x_{-i} = x_{-i}^*$. Recall from (9) that the profit at x_i^*, x_{-i}^* is equal to

$$\frac{1}{9}\left[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\right]^2.$$

The equilibrium exists if the following inequality is true and might not exist when the inequality is reversed.

$$\frac{1}{9}\left[\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}\right]^2 > \frac{1}{8}\left[\alpha + \nu_{-i} - 2(k_i + \nu_i)\right]^2.$$

Taking the square root of both sides we get

$$\frac{3}{2\sqrt{2}}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] - \alpha + 2(\nu_i + k_i) - \nu_{-i} < k_{-i}$$

which reduces to

$$\left(\frac{3}{\sqrt{2}} - 2\right)[\alpha + \nu_{-i} - 2(k_i + \nu_i)] < 2k_{-i}. \tag{19}$$

It is now possible to infer a condition for x_i^* not to be the optimal response of player i to $x_{-i} = x_{-i}^*$.

Lemma 4 *Let*

$$k_i > \frac{1}{4}\left[\alpha + \nu_{-i} - 2(k_i + \nu_i)\right] > \left(\frac{3}{\sqrt{2}} - 2\right)\left[\alpha + \nu_{-i} - 2(k_i + \nu_i)\right] > 2k_{-i} \tag{20}$$

then $\tilde{\tilde{\tilde{x}}}_i$ is the optimal response to $x_{-i} = x_{-i}^*$, not x_i^*.

Proof. the result follows from the combination of Lemma 3 and relation (19).

Consider the case where the condition of Lemma 3 that guarantees $z_i = x_i$ when $x_i = \tilde{\tilde{\tilde{x}}}_i$ is violated. It might still be possible to find a response of player i that is better than x_i^*. We now explore this case.

Say $\tilde{\tilde{\tilde{x}}}_i \geq x_i^m$, we now have to calculate the profits at x_i^m and compare with the profits in the open-loop solution. Here

$$z_i = x_i^m = \frac{1}{3}(\alpha + \nu_{-i} - 2\nu_i)$$

and

$$\begin{aligned}\Pi_i(x_i^m, x_{-i}^*) &= (\alpha - \tfrac{2}{3}\alpha + \tfrac{\nu_i}{3} + \tfrac{\nu_{-i}}{3} - \nu_i - k_i)\left(\tfrac{\alpha+\nu_{-i}-2\nu_i}{3}\right) \\ &= \tfrac{1}{9}(\alpha - 2\nu_i + \nu_{-i} - 3k_i)(\alpha + \nu_{-i} - 2\nu_i)\end{aligned}$$

Thus, if $\tilde{\tilde{\tilde{x}}}_i > x_i^m$, the equilibrium exists if

$$(\alpha - 2\nu_i + \nu_{-i} - 3k_i)(\alpha + \nu_{-i} - 2\nu_i) < [\alpha - 2(\nu_i + k_i) + \nu_{-i} + k_{-i}]^2.$$

This condition can be restated after some manipulation as

$$(k_i - 2k_{-i})[\alpha - 2(\nu_i + k_i) + \nu_{-i}] < 2k_i^2 + k_{-i}^2. \qquad (21)$$

The following theorem summarizes the necessary and sufficient conditions for the equilibrium to exist.

Theorem 3 *Suppose we limit the capacity space x to points such that $z(x) > 0$. Suppose also that $k_i > 2k_{-i}$ for some i.*
A closed loop equilibrium exists if (19) is violated and (21) holds true.
A closed loop equilibrium does not exist if (20) holds or

$$\frac{1}{4}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] > k_i$$

and (21) is reversed.

An important special case is when both players have identical costs. The properties of this game follow directly from this corollary.

Corollary 1 *When $k_i = k_{-i}$ and $\nu_i = \nu_{-i}$, the closed-loop equilibrium exists and is equal to the open-loop equilibrium.*

6 The three-stage game: the capacity game with a forward market

We now turn to the more complex game of a merchant plant that can contract part of its production in the forward market, trading the rest in the spot market. Given the value of having a futures market to reduce market power from the Allaz Vila results, in this section we develop the corresponding results for a capacity game that precedes taking a futures position. The definitions of the (local) closed-loop equilibrium of the two-stage game can be readily extended to the three-stage game after introducing some additional notation. Specifically, we let z be the vector of total production in the spot market, y the amount traded forward and x the installed capacity. The three-stage game can be solved backward as follows. A spot-price equilibrium z is a vector-valued function $z(x, y)$ where z_i solves

$$\max_{0 \leq z_i \leq x_i} \left\{ \Pi_i^s(x, y; z_i, z_{-i}^*) = [\alpha - (z_i + z_{-i}^*)](z_i - y_i) - \nu_i z_i \right\} \qquad \text{for } i = 1, 2.$$

Assuming that this equilibrium solution exists and is unique, one can write, using (3),

$$\Pi_i^f(x; y) = \Pi_i^s[x, y; z(x; y)].$$

A forward equilibrium y is then a vector-valued point-to-set map $y(x)$ (we shall see that $y(x)$ need not be unique) where $y_i(x)$ solves

$$\max_{y_i} \Pi_i^f(x; y_i; y_{-i}^*) \qquad i = 1, 2.$$

Assuming that this equilibrium solution exists, we define, using (4) and the above expressions

$$Z_i(x) = z_i[x; y(x)] \qquad i = 1, 2.$$

We show that even though $y(x)$ is not unique, $Z_i(x)$ is unique. We can thus define

$$\Pi_i(x_i, x_{-i}) = \\ \{\alpha - [Z_i(x_i, x_{-i}) + Z_{-i}(x_i, x_{-i})] - \nu_i\} Z_i(x_i, x_{-i}) - k_i x_i \; i = 1, 2.$$

The capacity equilibrium solution, if it exists, is a vector x^* where x_i^* solves

$$\max_{0 \le x_i} \Pi_i(x_i, x_{-i}^*) \qquad i = 1, 2.$$

We therefore extend Definition 1 as follows.

Definition 3 *A closed loop equilibrium of the three-stage game $x^*, y^*(x), z^*(x, y)$ satisfies the following conditions*

- *(i) $z^*(x, y)$ is a Nash equilibrium of the spot market (third-stage game) for every feasible x, y,*
- *(ii) $y^*(x)$ is a Nash equilibrium of the forward market (second stage game) for every feasible x,*
- *(iii) x^* is a Nash equilibrium of the capacity market (first stage-game).*

We now proceed to examine the different stages of this equilibrium.

6.1 The spot market equilibrium for given forward positions

Let y_i, $i = 1, 2$ be the forward position of the two agents. The equilibrium conditions on the spot market can be written as

$$\begin{array}{llll} \alpha - 2z_i - z_{-i} - \nu_i + y_i + \omega_i = \lambda_i & & & i = 1, 2 \\ (x_i - z_i) \ge 0 & \lambda_i \ge 0 & (x_i - z_i)\lambda_i = 0 & i = 1, 2 \\ z_i \ge 0 & \omega_i \ge 0 & z_i \omega_i = 0 & i = 1, 2. \end{array} \qquad (22)$$

Note that y_i can be either positive or negative, corresponding to selling or buying in the futures market. Assume there exists an equilibrium $x^*, y(x^*), z[x^*; y(x^*)]$.

Then the equilibrium $z^* = z[x^*, y(x^*)]$ of the spot market satisfies one of the following conditions

$$\begin{array}{llll} \text{(i)} & 0 < z_i^* < x_i^* & i = 1, 2 & \\ \text{(ii)} & 0 < z_i^* < x_i^* & 0 < z_{-i}^* = x_{-i}^* & \\ \text{(iii)} & 0 = z_i^* < x_i^* & 0 < z_{-i}^* < x_{-i}^* & \\ \text{(iv)} & 0 = z_i^* < x_i^* & 0 < z_{-i}^* = x_{-i}^* & \\ \text{(v)} & 0 < z_i^* = x_i^* & 0 < z_{-i}^* = x_{-i}^* & \end{array} \tag{23}$$

We again exclude cases (iii) and (iv) where one agent is driven out of the spot market in order to shorten the discussion.

We extend Propositions 1 and 2 to the case of the three-stage game, that is, we prove that if a closed-loop equilibrium exists, it satisfies $z_i = x_i$, $i = 1, 2$ and find conditions for the existence of this equilibrium. The following lemmas are analogous to those proved in Section 5.

Lemma 5 *An equilibrium, if it exists, cannot satisfy case (i) of (23).*

Proof. Let $x^*, y^* = y(x^*), z^* = z[x^*; y(x^*)]$ be the equilibrium and assume that it satisfies condition (i) of (23). The equilibrium conditions are

$$\begin{aligned} &\alpha - 2z_i^* - z_{-i}^* - \nu_i + y_i^* = 0 \\ &\alpha - z_i^* - 2z_{-i}^* - \nu_{-i} + y_{-i}^* = 0 \\ &0 < z_i^* < x_i^* \qquad i = 1, 2 \end{aligned}$$

Replacing $\nu_i + k_i$ by $\nu_i - y_i^*$ in the expression of the solution of the single-stage (open-loop) equilibrium (6),

$$z_i^* = z_i^*(y^*) = \frac{1}{3}[\alpha - 2(\nu_i - y_i^*) + (\nu_{-i} - y_{-i}^*)]. \tag{24}$$

There exists a neighborhood $N(y^*)$ of y^* such that (24) satisfies $0 < z_i(x^*, y) < x_i^*$ and hence remains an equilibrium of the spot market. Inserting these expressions in

$\Pi_i^s[x^*, y, z]$,

$$\Pi_i^f[x^*, y] = \frac{1}{9}[\alpha - 2(\nu_i - y_i) + (\nu_{-i} - y_{-i})]^2. \qquad (25)$$

Using the first order equilibrium condition,

$$y_i^* = \frac{1}{5}[\alpha - (3\nu_i - 2\nu_{-i})]$$

and $z_i^* = \frac{2}{5}[\alpha - (3\nu_i - 2\nu_{-i})]$. Thus, there exists a neighborhood $N(x^*)$ of x^* such that

$y^*(x) = y^*$ and $Z^*(x) = z^*[x, y^*(x)] = z^*$ are the best responses to any x in $N(x^*)$.

For any x in $N(x^*)$

$$\Pi_i(x) = \frac{2}{25}\left[\alpha - (3\nu_i - 2\nu_{-i})\right]^2 - k_i x_i.$$

Because $x_i^* > z_i^*$ and y_i^* does not depend on x, $\Pi_i(x)$ increases by slightly decreasing x from x_i^* which contradicts the assumption that x, $y^*(x)$, $Z^*(x)$ is an equilibrium.

We now rule out case (ii).

Lemma 6 *An equilibrium, if it exists cannot satisfy case (ii) of (23).*

Proof. Let $x^*, y(x^*), z[x^*, y(x^*)]$ be an equilibrium satisfying case (ii). One has

$$\begin{array}{ll} \alpha - 2z_i^* - x_{-i}^* - \nu_i + y_i^* = 0 & 0 < z_i^* < x_i^* \\ \alpha - z_i^* - 2x_{-i}^* - \nu_{-i} + y_{-i}^* = \lambda_{-i}^* & 0 < z_{-i}^* = x_{-i}^*. \end{array}$$

If it is an equilibrium, it is also a local equilibrium. Keeping x fixed at x^* and letting y move around $y(x^*)$, we find the following solution to the system

$$\begin{aligned} z_i &= \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) \\ \lambda_{-i} &= \alpha - 2x_{-i}^* - \nu_{-i} + y_{-i} - \tfrac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) \\ &= \tfrac{\alpha}{2} - \tfrac{3}{2}x_{-i}^* - \tfrac{1}{2}(2\nu_{-i} - \nu_i) + \tfrac{1}{2}(2y_{-i} - y_i). \end{aligned}$$

We consider the impact of a modification of y_i on the payoff of player i in the forward market given y_{-i}^* fixed. The spot price is equal to

$$\alpha - z_i - x_{-i}^* = \alpha - \frac{1}{2}(\alpha - x_{-i}^* - \nu_i + y_i) - x_{-i}^* = \frac{1}{2}(\alpha - x_{-i}^* + \nu_i - y_i).$$

The corresponding profit accruing to player i in the forward market, that is, after taking into account forward positions, is equal to

$$\begin{aligned}(\alpha - z_i - x^*_{-i} - \nu_i)z_i &= \tfrac{1}{2}(\alpha - x^*_{-i} - \nu_i - y_i)\tfrac{1}{2}(\alpha - x^*_{-i} - \nu_i + y_i) \\ &= \tfrac{1}{4}[(\alpha - x^*_{-i} - \nu_i)^2 - y_i^2].\end{aligned}$$

By assumption, y^*_i maximizes player i's payoff for given y^*_{-i} and x^*. This implies that y^*_i must be zero. Player i's medium term payoff on the forward market is thus $\frac{1}{4}[(\alpha - x^*_{-i} - \nu_i)]^2$. This implies that the profit achieved on the capacity market is $\frac{1}{4}[(\alpha - x^*_{-i} - \nu_i)]^2 - k_i x^*_i$. Reducing x^*_i by a small amount to $x_i < x^*_i$, $y_i = 0$ remains the optimal strategy on the futures market and z^*_i remains unchanged and strictly less than x_i. This reduction improves player i's payoff in the capacity market which was therefore not optimal. This proves the lemma.

6.2 Characterization of the closed-loop equilibrium

Using these two lemmas, Proposition 3 extends Proposition 2 to the three-stage game.

Proposition 3 *A closed-loop equilibrium of the three-stage game, if it exists, satisfies* $z_i = x_i, i = 1, 2.$

With this result it is clear that the spot-market equilibrium $z_i = x_i$ is unique assuming a capacity market equilibrium.

Our next goal is to generalize Theorem 1 and to again show that if an equilibrium of the three-stage game exists, then it is the open-loop equilibrium. As with the two-stage game, we also find that this equilibrium exists only under the conditions in Theorem 1. We analyze this more complex case by partitioning the space of investment variables into different subsets where we further characterize equilibrium properties.

In the subsequent lemmas we treat cases where $z_i < x_i$. Although this cannot occur at equilibrium, this can be a property of a disequilibrium point that is relevant to showing an equilibrium does not exist. We thus have to establish the nature of the forward and spot-market equilibria for all possible $x_i > 0$.

Specifically, we first consider the case where the investment variables satisfy $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$. (This is the case where both players use all of their generation capacity in the spot market). Lemma 7 characterizes the equilibrium in the forward market for that case. We then turn to the situation where one of the above inequalities is violated. This corresponds to the case where one of the players has invested in too much capacity in the sense that its marginal operating profit (marginal revenue – operating cost) on the spot market is negative when both capacities are fully used. Lemmas 8 and 9 show that the other player realizes that there is an overinvestment at the tentative equilibrium; it takes advantage of the situation and uses the forward market to drive the over-built player out of the forward market.

Lemma 7 *Let (x_i, x_{-i}) satisfy*

$$\alpha - 2x_i - x_{-i} - \nu_i > 0 \qquad i = 1, 2$$

then

$$y_i \geq \tilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0, \quad i = 1, 2$$

is a closed-loop equilibrium of the forward market.

Proof. Take x given and let $\widetilde{y}_i = \widetilde{y}_i(x)$, $i = 1, 2$ for this given x. One has

$$\alpha - 2x_i - x_{-i} - \nu_i + \widetilde{y}_i = 0, \quad i = 1, 2$$

and hence $z_i = x_i$ is an equilibrium on the spot market.

We want to prove that any $y_i \geq \widetilde{y}_i$ is the best response of player i to a futures position $y_{-i} \geq \widetilde{y}_{-i}$ of player $-i$. Suppose $y_i > \widetilde{y}_i$, one has

$$\begin{aligned} \alpha - 2x_i - x_{-i} - \nu_i + y_i &= \lambda_i > 0 \\ \alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} &= \lambda_{-i} \geq 0 \end{aligned}$$

and $z_i = x_i$ remains an equilibrium on the spot market. Taking $y_i > \tilde{y}_i$ therefore maintains the profit of player i, whatever $y_{-i} \geq \widetilde{y}_{-i}$ is selected by player $-i$.

Take $y_i < \widetilde{y}_i$, $y_{-i} \geq \widetilde{y}_{-i}$. z_i becomes smaller than x_i and one can write the equilibrium conditions of the spot market as

$$\begin{aligned} \alpha - 2z_i - x_{-i} - \nu_i + y_i &= 0 \\ \alpha - z_i - 2x_{-i} - \nu_i + y_{-i} &= \lambda_{-i} > 0. \end{aligned}$$

This implies

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i)$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\left[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\right].$$

The optimum of the profit of player i is achieved for $y_i = 0$ with a payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This is the global optimum of player i if and only if

$$0 = y_i < \widetilde{y}_i = -(\alpha - 2x_i - x_{-i} - \nu_i) < 0 \quad \text{which is a contradiction.}$$

Therefore, $y_i < \widetilde{y}_i$ cannot be the best response of player i to $y_{-i} \geq \tilde{y}_{-i}$. Thus $\tilde{y}_i(x)$, $i = 1, 2$ is a closed-loop equilibrium of the forward market and any $y_i \geq \tilde{y}_i(x)$, $i = 1, 2$ is also a closed-loop equilibrium of the forward market.

We now examine the case of overbuilding by player i. Consider now what happens when one of the relations

$$\alpha - 2x_i - x_{-i} - \nu_i > 0 \qquad i = 1, 2$$

is violated. Let

$$\alpha - 2x_i - x_{-i} - \nu_i < 0 \quad \text{and} \quad \alpha - x_i - 2x_{-i} - \nu_{-i} > 0.$$

This case corresponds to case (ii) of Theorem 2 in the game without a futures market. The analysis is more complicated with a futures market because we have to analyze the resulting futures positions of the players.

Lemma 8 shows that player $-i$ can always drive player i out of the forward market by selecting y_{-i} large enough.

Lemma 8 *For a given (x_i, x_{-i}), if $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$, then $y_i = 0$ is the optimal response of player i to any $y_{-i} \geq \max \widetilde{y}_{-i}(x)$.*

Proof. We first show that $z_i < x_i$ when $y_i = 0$. Suppose player $-i$ takes a position $\bar{y}_{-i} \geq \widetilde{y}_{-i}(x)$.
We first claim that the equilibrium in the spot market is

$$\alpha - 2z_{-i} - x_{-i} - \nu_i = 0$$
$$\alpha - z_i - 2x_{-i} - \nu_{-i} + \bar{y}_{-i} = \lambda_i > 0.$$

To see this, first note that, because $\alpha - 2x_i - x_{-i} - \nu_i < 0$, there exists some $z_i < x_i$ (that we assume > 0) that solves $\alpha - 2z_i - x_{-i} - \nu_i = 0$. Note that the definition of $\tilde{y}_{-i}(x)$ implies $\alpha - x_i - 2x_{-i} - \nu_{-i} + \widetilde{y}_{-i}(x) = 0$ and hence any $z_i < x_i$ and $y_{-i} > \widetilde{y}_{-i}(x)$ satisfies $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} > 0$, which shows that $z_i < x_i$ and $z_{-i} = x_{-i}$ is the equilibrium.

Consider the reaction of player $-i$ to $y_{-i} > 0$. Because $y_{-i} \geq \widetilde{y}_{-i}(x)$, $\alpha - x_i - 2x_{-i} - \nu_{-i} + y_{-i} > 0$, $\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} > 0$ for all $z_i < x_i$. Therefore, $z_{-i} = x_{-i}$ whenever $y_{-i} \geq \widetilde{y}_{-i}(x)$, whatever the position of player i on the forward market. Consider the following strategies of player i, keeping in mind that $y_{-i} \geq \widetilde{y}_{-i}(x)$ implies $z_{-i} = x_{-i}$, whatever i does on the forward market. Because the shape of the objective function depends on the value of y_i, we treat two cases:

(i) $y_i \geq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$

(ii) $y_i \leq \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$

Note first that player i's payoff in case (i), remains constant at $(\alpha - x_i - x_{-i} - \nu_i)x_i$ for all $y_i \geq \widetilde{y}_i(x)$. Therefore player i cannot improve its payoff by selecting $y_i \geq \widetilde{y}_i(x)$ and the optimum in case (ii) is a global optimum.

Player i's payoff in case (ii) can be computed as follows. Because $y_i \leq \widetilde{y}_i(x)$, $z_i \leq x_i$ and z_i solves

$$\begin{aligned} &\alpha - 2z_i - x_{-i} - \nu_i + y_i = 0 \\ &\alpha - z_i - 2x_{-i} - \nu_{-i} + y_{-i} = \lambda_{-i} > 0. \end{aligned}$$

As in Lemma 7, the optimal response of player i is

$$z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i + y_i) < x_i$$

and

$$\Pi_i^f(x; y_i, y_{-i}) = \frac{1}{4}\big[(\alpha - x_{-i} - \nu_i)^2 - y_i^2\big].$$

The maximum profit is achieved for $y_i = 0$ with the player i payoff equal to $\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2$. This will be the global optimum of player i's payoff if one has both

$$0 = y_i < \widetilde{y}_i(x) = -(\alpha - 2x_i - x_{-i} - \nu_i) > 0$$

and

$$\frac{1}{4}(\alpha - x_{-i} - \nu_i)^2 > (\alpha - x_i - x_{-i} - \nu_i)x_i.$$

The first condition is true by assumption. To verify the second condition, first note that it can be rewritten

$$(\alpha - x_{-i} - \nu_i)^2 - 4(\alpha - x_{-i} - \nu_i)x_i + 4x_i^2 > 0$$

or

$$(\alpha - 2x_i - x_{-i} - \nu_i)^2 > 0$$

which is always satisfied.

The optimal reaction of player i is thus $y_i = 0$ when player $-i$ selects $y_{-i} \geq \bar{y}_{-i}$ and $\alpha - 2x_i - x_{-i} - \nu_i < 0$. Note that this solution is unique by the strict concavity of the objective function in this range. This proves the lemma.

The difference between this result and the result of Allaz and Vila is that player $-i$ operates at capacity and does not change its spot position in response to player i's actions. We now show that the best response of player $-i$ is $y_{-i} \geq \widetilde{y}_{-i}(x)$. Thus, given (x_i, x_{-i}), we have the equilibrium in the futures market (even though this is not an equilibrium in the capacity market).

Lemma 9 *Suppose* $\alpha - 2x_i - x_{-i} - \nu_i < 0$ *and* $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$. *Then* $y_{-i} \geq \widetilde{y}_{-i}(x)$ *is the optimal reaction of player* $-i$ *to* $y_i = 0$.

Proof. With $y_i = 0$, define $\widetilde{z}_i$ such that $\alpha - 2\widetilde{z}_i - x_{-i} - \nu_i = 0$. Because $\alpha - 2x_i - x_{-i} - \nu_i < 0$, $\widetilde{z}_i$ is smaller than x_i. We consider three cases because the shape of the objective function of player $-i$ depends on whether the spot decisions are at capacity. We examine the following strategies of player $-i$ on the forward market.

(i) y_{-i} is selected to guarantee $z_{-i} = x_{-i}$.

(ii) y_{-i} is selected to optimize the payoff in the range where $z_i < x_i$, $z_{-i} < x_{-i}$.

(iii) y_{-i} is selected to optimize the payoff in the range where $z_i = x_i$, $z_{-i} < x_{-i}$.

We successively consider these three cases and compute the resulting payoff for player $-i$.

(i) Player $-i$ uses the futures market to guarantee the full utilization of its capacity and it takes $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ where $\widetilde{\widetilde{y}}_{-i}(x)$ is defined by

$$\alpha - \widetilde{z}_i - 2x_{-i} - \nu_{-i} + \widetilde{\widetilde{y}}_{-i}(x) = 0.$$

This amounts to selecting

$$y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x).$$

The equilibrium on the spot market associated with $y_i = 0$, $y_{-i} \geq \widetilde{\widetilde{y}}_{-i}(x)$ is $z_i = \widetilde{z}_i$ and $z_{-i} = x_{-i}$. The payoff for player $-i$ is

$$(\alpha - \widetilde{z}_i - x_{-i} - \nu_i)x_{-i} = \frac{1}{2}(\alpha - x_{-i} - \nu_i)x_i.$$

(ii) Let $y_{-i} = \widetilde{\widetilde{y}}_{-i}(x) - \varepsilon_{-i}$ where ε_{-i} is small enough to guarantee that z_i does reach x_i and z_{-i} does not hit zero. z_i and z_{-i} then solve the system

$$\alpha - 2z_i - z_{-i} - \nu_i = 0$$
$$\alpha - z_i - 2z_{-i} - \nu_{-i} + y_{-i} = 0$$

We can solve for z_i and z_{-i} as a function of y_{-i}, as in Lemma 5. Setting $y_i = 0$ in relation (25), the payoff for $-i$ is

$$\Pi^f_{-i}[x; 0, y_{-i}] = \frac{1}{9}\big[\alpha - 2(\nu_{-i} - y_{-i}) + \nu_i\big]^2.$$

The derivative of Π^f_{-i} with respect to y_{-i} is $\frac{4}{9}[\alpha - 2(\nu_{-i} - y_{-i}) + \nu_i]$. At $\widetilde{\widetilde{y}}_{-i}$, when z_{-i} reaches x_{-i}, this derivative is equal to

$$\frac{4}{9}\big[\alpha - (2\nu_{-i} + \nu_i) + 2\widetilde{\widetilde{y}}_{-i}\big] = \frac{4}{9}x_{-i} > 0.$$

Because $\Pi^f_{-i}[x; 0, y_{-i}]$ is concave in y_{-i}, and its derivative at $\widetilde{\widetilde{y}}_{-i}(x)$ is positive, it is still increasing at that point. Thus, the optimum of $\Pi^f_{-i}[x; 0, y_{-i}]$ cannot be $y_{-i} < \widetilde{\widetilde{y}}_{-i}(x)$. This implies that $y_{-i} = \widetilde{\widetilde{y}}_{-i} - \varepsilon_{-i}$ is not the best response by player $-i$.

(iii) The following elaborates on the same concavity argument to prove that decreasing y_{-i} to the level where z_i reaches x_i or z_{-i} reaches 0 cannot maximize $\Pi^f_{-i}[x; 0, y_{-i}]$. There is obviously no gain for player $-i$ to further decrease y_{-i} if z_{-i} hits zero before z_i reaches x_i since its payoff is then exactly zero. Consider the alternative case where z_i hits x_i and z_{-i} is still positive. This occurs for some $\overline{z}_{-i}$ that satisfies

$$\alpha - 2x_i - \overline{z}_{-i} - \nu_i = 0$$
$$\text{or} \qquad \overline{z}_{-i} = \alpha - 2x_i - \nu_i.$$

Consider decreasing y_{-i} further to check the possibility of the resulting price increasing profits. We show that this cannot happen. Let $z_{-i} = \overline{z}_{-i} + \varepsilon$. The corresponding profit of player $-i$ is

$$(\alpha - x_i - \overline{z}_{-i} - \varepsilon - \nu_{-i})(\overline{z}_{-i} + \varepsilon).$$

The derivative of this expression at $\varepsilon = 0$ (for $z_{-i} = \overline{z}_{-i}$) is equal to $3x_i + (2\nu_i - \nu_{-i}) - \alpha$. This expression is positive because it is equal to

$$2(-\alpha + 2x_i + x_{-i} + \nu_i) + (\alpha_i - x_i - 2x_{-i} - \nu_{-i})$$

which is positive by assumption.

The conclusion is that it cannot pay to further decrease y_i beyond the point where $z_i = x_i$. $y_{-i} \geq \widetilde{y}_{-i}(x)$ thus guarantees the maximal profit of player $-i$ when $y_i = 0$.

This completes the proof of the lemma.

We now characterize the solution when player i has excess capacity.

Lemma 10 *Let (x_i, x_{-i}) satisfy $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$ and $\widetilde{y}_{-i}(x) = -(\alpha - x_i - 2x_{-i} - \nu_{-i})$. Then $y_i = 0$, $y_{-i} \geq \widetilde{y}_{-i}(x)$ is a closed-loop equilibrium of the forward market. At that equilibrium $z_i < x_i$.*

Proof. The result is a combination of Lemmas 8 and 9 after noting that $\widetilde{y}_{-i}(x) = -(\alpha - x_i - 2x_{-i} - \nu_{-i}) \geq -(\alpha - \widetilde{z}_i - 2x_{-i} - \nu_{-i}) = \widetilde{\widetilde{y}}_{-i}(x)$.

Lemma 10 has an immediate interpretation. If a player develops its generation capacity up to a point where its marginal revenue is negative when both capacities are operated at their maximums, then the equilibrium on the forward market forces this player to operate below capacity. In short it has effectively invested too much. We now show that this cannot be an equilibrium.

Lemma 11 *There cannot be any equilibrium of the capacity game with a forward market such that $\alpha - 2x_i - x_{-i} - \nu_i < 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} > 0$.*

Proof. Assume such an equilibrium exists. The equilibrium on the forward market is $y_i = 0$ and $y_{-i} \geq \bar{y}_{-i}$ with the corresponding spot equilibrium $z_i = \frac{1}{2}(\alpha - x_{-i} - \nu_i)$, $z_{-i} = x_{-i}$. This spot equilibrium satisfies $z_i < x_i$ and hence cannot be an equilibrium by Proposition 3.

We now explore whether one can have an equilibrium of the capacity game with a forward market such that $\alpha - 2x_i - x_{-i} - \nu_i < 0$, $i = 1, 2$. This corresponds to case (ii) in the game with no forward market. The situation is easily clarified with the following lemma.

Lemma 12 *An equilibrium of the capacity game with a forward market cannot satisfy* $\alpha - 2x_i - x_{-i} - \nu_i < 0$, $i = 1, 2$.

Proof. If such an equilibrium exists, it satisfies $z_i = x_i$, $i = 1, 2$ by Proposition 3. Because the marginal revenue of both player is negative at this point, this cannot be an optimal position for either of them. Therefore, this is not an equilibrium.

On the basis of the above, we conclude that an equilibrium of the capacity game with a forward market, if it exists, satisfies $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$ and $z_i = x_i$, $i = 1, 2$. We infer the following proposition.

Proposition 4 *An equilibrium of the capacity game with a forward market, if it exists, satisfies* $\alpha_i - 2x_i^* - x_{-i}^* - \nu_i \geq 0$, $i = 1, 2$.

Proof. The result is immediately derived from lemmas 11 and 12.

We can then prove the extension of Proposition 2 to the three-stage game.

Proposition 5 *An equilibrium of the capacity game, if it exists, is the open-loop equilibrium.*

Proof. Assume an equilibrium of the three-stage game exists. By Proposition 4, one has $\alpha - 2x_i - x_{-i} - \nu_i \geq 0$, $i = 1, 2$. $\alpha - 2x_i - x_{-i} - \nu_i$ is also the marginal operating profit accruing to player i from its operation on the forward and spot market (both players select y_i such that $z_i = x_i$). The optimality of player i's action in the capacity game implies that the marginal operating profit is equal to k_i. We therefore need $\alpha - 2x_i - x_{-i} - \nu_i - k_i = 0$, $i = 1, 2$. These are the conditions for the open-loop equilibrium. We thus conclude that if an equilibrium of the three-stage game exists, it is the open-loop equilibrium.

This means that the capacity game sets capacities at the same level as in the open-loop game. Thus, the futures market cannot be used to expand production in the spot market. Through the capacity game, the players see the destructive competition that results from the futures game and they block this possibility when setting capacity levels.

6.3 Existence of the closed-loop equilibrium

In the game without a futures market, existence is not guaranteed and depends on the values of the parameters. We now develop the corresponding results for the three-stage game. We take the open-loop capacities at equilibrium

$$x_i^* = \frac{1}{3}\left[\alpha - 2(\nu_i + k_i) + (\nu_{-i} + k_{-i})\right] \quad i = 1, 2$$

and show when they are the capacity equilibrium of the three-stage game. We first note that player i never gains if it reduces its capacity with respect to x_i^*, given x_{-i} remaining at x_{-i}^*. The only way the open-loop equilibrium may fail to be a first-stage equilibrium of the three-stage game is if one player can benefit from increasing its investment with respect to x_i^*, x_{-i} unchanged at x_{-i}^*. In order to explore this possibility, we increase x_i. Let $x_i = x_i^* + \varepsilon_i$ while keeping $x_{-i} = x_{-i}^*$.

Consider first the range of values of x_i that keep $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$, $i = 1, 2$. We know that the equilibrium of the forward market is to select y_i, $i = 1, 2$ so that $z_i = x_i$, $i = 1, 2$. Because of the optimality properties of the open-loop equilibrium, we can conclude that player i has no interest in increasing or decreasing x_i as long as one remains in the region $\alpha - 2x_i - x_{-i} - \nu_i > 0$, $i = 1, 2$.

In order to assess whether x_i^* is really the optimal choice of player i, we need to explore what happens when a player leaves the region $\alpha - 2x_i - x_{-i} - \nu_i > 0$ for either $i = 1, 2$. Consider the two possible cases

(i) $x_i = x_i^* + \varepsilon_i$ with $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} > 0$

(ii) $x_i = x_i^* + \varepsilon_i$ with $\alpha - x_i - 2x_{-i}^* - \nu_{-i} = 0$ and $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$

Case (i) is handled by the following lemma.

Lemma 13 *Suppose $k_i < 2k_{-i}$, then $x_i = x_i^*$ is the best reaction of player i to $x_{-i} = x_{-i}^*$.*

Proof. Set $x_i = x_i^* + \varepsilon_i$. Simple replacement in $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ and $\alpha - x_i - 2x_i^* - \nu_{-i} > 0$ shows that case (i) holds if and only if $2k_{-i} > k_i$. I f so $\alpha - 2x_i - x_{-i}^* - \nu_i = 0$ for $\varepsilon_i = \frac{k_i}{2}$.

We know from the above that it is never optimal for i to select $x_i \leq x_i^* + \frac{k_i}{2}$ strictly larger than x_i^*. Consider now $x_i > x_i^* + \frac{k_i}{2}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} > 0$. From Lemma 10 we know that the associated forward and spot equilibrium satisfies $y_i = 0$, $z_i < x_i$. This cannot be an optimal position for i since it can always be improved by slightly decreasing x_i.

Consider now $x_i \geq x_i^* + k_{-i}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} \leq 0$. This is an optimal position for i if there exists a forward and spot equilibrium that gives a higher profit than the single-stage equilibrium. Assume such an equilibrium. It cannot satisfy $z_i = x_i$, $i = 1, 2$ because this would give a negative marginal revenue to player i even before incurring investment costs. It cannot satisfy $z_i < x_i$, $z_{-i} = x_{-i}$ because i could improve its position by decreasing x_i. It must thus satisfy $z_i = x_i$ and $z_{-i} < x_{-i}$. Redoing the reasoning of case (ii) in Lemma 8, this implies $y_{-i} = 0$ and $z_{-i} = \frac{\alpha - x_i - \nu_{-i}}{2}$. Replacing in $\alpha - 2x_i - z_{-i} - \nu_i$ the marginal revenue of i is

$$\frac{1}{2}\big[\alpha - 3x_i - (2\nu_i - \nu_{-i})\big]$$

which is equal to

$$\frac{1}{2}\big[-(\alpha - x_i - 2x_{-i}^* - \nu_{-i}) + 2(\alpha - 2x_i - x_{-i}^* - \nu_i)\big].$$

By definition, this expression is negative at $x_i = x_i^* + k_{-i}$. It can only decrease when x_i increases. The marginal revenue of player i is thus negative before incurring investment costs and this cannot be an optimal position.

Case (ii) is treated in Lemma 14.

Lemma 14 *Let* $k_i > 2k_{-i}$ *and*

$$\left(\frac{3}{\sqrt{2}} - 2\right)\left[\alpha + \nu_{-i} - 2(k_i + \nu_i)\right] - 2k_{-i} \leq 0. \tag{26}$$

Then $x_i = x_i^*$ *is the best reaction of player* i *to* $x_{-i} = x_{-i}^*$.

Proof. Following the reasoning of Lemma 12, we can easily verify that case (ii) occurs if and only if $k_i > 2k_{-i}$. We know that selecting x_i such that $x_i^* < x_i \leq x_i^* + k_{-i}$ would imply $z_i = x_i$, $z_{-i} = x_{-i}^*$ which cannot be an optimal payoff of player i in that range of x_i.

Consider now $x_i > x_i^* + k_{-i}$ such that $\alpha - 2x_i - x_{-i}^* - \nu_i > 0$ and $\alpha - x_i - 2x_i^* - \nu_{-i} < 0$. From Lemma 10 we know that the associated forward and spot equilibrium satisfies $y_{-i} = 0$, $z_{-i} < x_{-i}$ and that y_i is selected such that $z_i = x_i$. These conditions are analyzed in the preliminaries to Lemma 3 where we considered conditions that guarantee that (18) is indeed the current expression of $\Pi_i(\tilde{\tilde{x}}_i, x_{-i}^*)$. Because the equilibrium of the forward market guarantees that $z_i = x_i$ for $x_i \geq \tilde{\tilde{x}}_i = x_i^* + k_{-i}$, we conclude by reproducing the reasoning of Lemma 3 that the condition

$$\frac{1}{4}[\alpha + \nu_{-i} - 2(k_i + \nu_i)] > 2k_{-i}$$

implies that

$$\Pi_i(\tilde{\tilde{x}}_i, x_{-i}^*) = \frac{1}{8}[(\alpha + \nu_{-i}) - 2(\nu_i + k_i)]^2.$$

As in Lemma 4 that (26) in the assumptions guarantee that player i cannot improve its position with respect to the open loop profit by moving into a range where $z_{-i} < x_{-i}$.

Consider now $z_i \geq x_i^* + \frac{k_i}{2} > x_i^* + k_{-i}$. We then have $\alpha - 2x_i - x_{-i}^* - \nu_i < 0$ and $\alpha - x_i - 2x_{-i}^* - \nu_{-i} < 0$. These conditions have been encountered in Lemma 13 where they do not lead to an optimal position for player i.

Lemma 14 points to an interesting differentiation between the two and three stage games. In the game without a futures market, when setting capacity, player i satisfies

$$\alpha - 2x_i - z_{-i} - \nu_i \geq 0$$

as long as $x_i \leq x_i^m$. This condition guarantees that $z_i = x_i$ in the spot market.

With a futures market the condition for $z_i = x_i$ in the spot market becomes

$$\alpha - 2x_i - z_{-i} - \nu_i + y_i \geq 0. \quad (27)$$

Lemma 10 guarantees that the equilibrium of the forward market when $\alpha - 2x_i - x_{-i} - \nu_i > 0$ and $\alpha - x_i - 2x_{-i} - \nu_{-i} < 0$ is achieved for y_i large enough and $y_{-i} = 0$ and that (27) indeed holds. Thus the spot market equilibrium condition does not put an added condition on the existence or non existence of an equilibrium as it did in the game without a futures market.

We can conclude with the following theorem:

Theorem 4 *A closed-loop equilibrium of the three-stage game exists if one of the following conditions holds*

(i) $k_i < 2k_{-i}$, $i = 1, 2$

(ii) For $k_i > 2k_{-i}$ *for some* i*, if (26) holds, then the open-loop equilibrium is also the closed-loop equilibrium of the three-stage game.*

The solution does not exist when the inequality (26) is reversed.

The proof derives from applying Lemmas 13 through 14 to both players.

From this we can see that adding a futures game did not change the equilibrium with respect to the two-stage game. However, the game with a futures market has no equilibrium for a larger set of parameter values than the game without a futures market because the condition on the spot-market equilibrium is no longer needed.

6.4 Illustration

In testing for the existence of an equilibrium for a given set of parameters, the solution has to be checked for conditions $z_1 = x_1$, $z_2 < x_2$, and $\overset{\approx}{\tilde{x}}_1 > x_1^* + k_2$. So that $k_1 > 2k_1$, we modify the capital costs by adding 1 to k_1 in both cases.

Condition	Low-elasticity case	High-elasticity case
$\overset{\approx}{\tilde{x}}_1$	1499	500
λ_1 (no futures)	-598.5	201.5
Condition (26)	214.0	-328.9
$\overset{\approx}{\tilde{x}}_1 - x_1^* - k_2 > 0$	400	133.7

Table 1: Evaluation of conditions for the existence of an equilibrium

From this table we see that in the high-elasticity case $\overset{\approx}{\tilde{x}}_1$ is not high enough for$z_2 < x_2$. Thus, the equilibrium exists with and without a futures market. With the low-elasticity case, $\lambda_1 < 0$ and the equilibrium does not exist when there is no futures market. However, the equilibrium does exist when there is a futures market. Thus, for a reasonable parameter set, adding a futures market can lead to disequilibrium in the market.

In the low-elasticity case, by setting the capacity and operating costs for player 1 to 555 and 100 respectively and 75 and 200 for player 2, the equilibrium does not exist in the case without a futures market.

7 Conclusion

By adding a capacity game we provide the players with the foresight to avoid the prisoners' dilemma game of Allaz and Vila. Thus, we can see that the futures market may not be the panacea for eliminating oligopolistic profits in the long run. The implications are that market monitoring in electricity markets needs to be vigilant despite the creation of futures markets. In addition to not increasing compeptitiveness, a futures market can result in no equilibrium for a larger set of parameter values than without a futures market.

In this paper, we do not have a load duration curve, which is necessary to represent one of the critical features of electricity markets. Adding a load curve also represents the situation of deterministic costs and uncertain demand. Clearly, in the time periods where capacity is not binding, a futures market can increase production as described by Allaz and Vila and the solution is different from the open-loop game. This and other differences will be explored in future work.

References

[1] Allaz, B. 1992. Oligopoly, uncertainty and strategic forward markets and efficiency. *International Journal of Industrial Organization*, 10, 297–308.

[2] Allaz, B. and J.-L Vila. 1993. Cournot competition, forward markets and efficiency. *Journal of Economic Theory*, 59, 1–16.

[3] Fudenberg and Tirole. 2000. *Game Theory*. MIT Press.

[4] Gans, J.S., D. Price, and K. Woods. 1998. Contracts and electricity pool prices. *Australian Journal of Management* 23(1), 83–96.

[5] Goering, G. E. and M. Pippenger. 2002. Durable goods monopoly and forward markets. *International Journal of the Economics of Business*, 9(2), 271–282.

[6] Green, R. 1999. The electricity market in England and Wales. *The Journal of Industrial Economics*, 47, 107–124.

[7] Haskel, J.E. and A. Powell. 1992. Contracts and competition: a note. Working paper, Queen Mary and Westfield College.

[8] Joskow, P. and J. Tirole. 2002. Transmission rights and market power on electric power Networks. *RAND Journal of Economics*, 31(3) 450–487.

[9] Kamat, R. and S.S.Oren. 2004. Two-settlement systems for electricity markets under network uncertainty and market power. *Journal of Regulatory Economics*, 25(1), 5–37.

[10] Le Coq, C. and H. Orzen. 2002. Do forward markets enhance competition? Experimental evidence. SSE/EFI Working Paper, Department of Economics, Sveavagen, Stockholm, Sweden, August.

[11] Mahenc, P. and F. Salanie. 2002. Tacit collusion through forward trading. LEERNA, Université des Sciences Sociales de Toulouse, Working Paper.

[12] Murphy, F.H. and Y. Smeers. 2003. Generation capacity expansion in imperfectly competitive restructured electricity markets. *Operations Research*, to appear.

[13] Newbery, D. 1998. Competition, contracts, and entry in the electricity spot market. *RAND Journal of Economics*, 29(4), 726–749.

[14] Powell, A. 1993. Trading forward in an imperfect market: The case of electricity in Britain. *The Economic Journal*, 103, 444–453.

[15] Spence, A. M. 1977. Entry, capacity, investment and oligopolistic pricing. *Bell Journal*, 8, 534–544.

[16] Stoft S. 2002. *Power System Economics: Designing Markets for Electricity*. John Wiley & Sons.

[17] Wolak, F. 2000. An empirical analysis of the impact of hedge contracts on bidding behavior in a competitive electricity market. *International Economic Journal*, 14(2) 1-39.

[18] Wu, D.J., P. R. Kleindorfer. and Y. Sun. 2002. Optimal electric power expansion in the presence of options. *Proceedings of the 35th Hawaii International Conference on Systems Sciences-2002*.