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Multiple Lending and Constrained Efficiency in the Credit Market

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Abstract

In this paper we present a model of credit market with several homogeneous lenders competing to finance an investment project. Contracts are non-exclusive, hence the borrower can accept whatever subset of the offered loans. We use the model to discuss efficiency issues in competitive economies with asymmetric information and non-exclusive agreements. We characterize the equilibria of this common agency game with moral hazard and show that they all belong to the constrained Pareto frontier.

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1 Introduction

This paper is devoted to the analysis of credit markets where lenders strategically compete over the contract offers they make to borrowers. The main aim of the work is to examine the welfare properties of the equilibria of the market interaction. In doing so, we emphasize the role of the contractual externalities that naturally arise in such a framework. At the stage of contracting with a lender, the decision of a single borrower will crucially depend on the contract offers she is simultaneously receiving from all the other lenders.

We set-up a scenario where the offered loan contracts are *non-exclusive*, i.e. a borrower is allowed to accept more than one contracts at a time. Exclusivity clauses are not explicitly imposed in several financial relationships.¹ Many U.S small firms have access to multiple credit sources (Petersen and Rajan 1995) and credit card markets are also clearly non-exclusive situations (see Bizer and DeMarzo (1992), Parlour and Rajan (2001)).

Recently, many important researches developed examples of interactions with non-exclusive contracting with the aim of clarifying the relationship between incentives and competition: the main general results and implications are discussed by Segal and Whinston (2003). In our view, one of the relevant findings of the literature can be summarized as follows: the contractual externalities emerging in this type of interactions can be responsible for existence of *constrained* inefficient equilibria. In other words, a Social Planner who is subject to incentive constraints and feasibility can achieve outcomes that Pareto dominate the equilibrium outcomes of players' interactions.

The present essay proposes an investigation of the welfare properties of the equilibria of a credit market when considering strategic competition among external financiers. Dealing with lender-borrower relationships affected by asymmetric information problems, the inefficiencies arising from multiple contracting would provide some welfare foundation for policy measures.² We study a simple, static and partial equilibrium model of the credit market. We analyze credit relationships by modelling the competition between a finite number of lenders who offer credit lines to a single borrower whose decisions cannot be contracted upon. If agency costs are high enough, competition among financiers delivers non-competitive results, in the forms of credit ra-

¹For a general discussion see Detragiache, Garella, and Guiso (2000).

²The present work should be regarded as part of a research project on welfare foundations for policy intervention, in particular following the literature on the credit channel of monetary policy, which focuses on how monetary variables affect the real economy limiting borrowers' access to credit. A crucial element of this transmission mechanism is the inverse relationship between borrowers' net worth and the external finance premium. A tighter policy induces a rise in the external finance premium through the adverse effect on borrowers balance sheets, that modifies the amount of collateral available for lenders. A restrictive monetary policy raising open-market interest rates may therefore cause a reduction in the amount of lending offered by every financier. The theoretical contributions to this approach share the reference to a principal-agent analysis of credit relationships. As a consequence, the analysis of the transmission mechanism of monetary policy involves only second-best efficient outcomes. Explicitly accounting for credit market equilibria that fail to be Pareto constrained optima (i.e. equilibria that do not belong to the *second best* frontier) constitutes a preliminary step in bringing welfare issues inside the credit view of monetary policy. For basic reference, see Bisin and Gottardi (1999), Holmstrom and Tirole (1997), Repullo and Suarez (2000).

tioning and of positive extra-profits at equilibrium. In terms of welfare, though all the equilibria of this common agency game are constrained-efficient.

Constrained inefficient equilibria have recently been shown to arise in insurance scenarios, the two mains examples are Kahn and Mookherjee (1998), and Bisin and Guaitoli (2004). These papers differ from ours in several respects. Kahn and Mookherjee (1998) consider a model of insurance where the agent (the insured) proposes contract to insurance companies *sequentially*. Both the timing and the bargaining power are different in our set-up, where the lenders simultaneously offer a contract to the agent. Bisin and Guaitoli (2004) allows principals to use menus, i.e. rather than to make a take-it or leave-it offer, each principal is allowed to make several offers and the agent is free to choose one of them.³ Moreover, their equilibrium is not symmetric: not all principals offer the same menu of contracts.

Nevertheless, a crucial difference between our results and theirs comes from the assumptions on agent's preferences, in particular in case of shirking/default. In our model, if the agent takes low effort, she gets with probability one a payoff which is linear in total investment and parameterized to the shirking/default parameter. Introducing such a specification in Kahn and Mookherjee (1998) or in Bisin and Guaitoli (2004) models would destroy the constrained inefficient equilibrium. However, compared to Bisin and Guaitoli (2004), we do not know which would have been the properties of the equilibrium outcome had they consider take-it or leave-it offers rather than menus, or had they considered symmetric equilibria.

In our model, we show that considering a borrower with linear preferences is a sufficient condition for every positive profit equilibrium to be constrained Pareto efficient. Importantly, the same argument applies to the insurance literature. Whenever the assumption of risk-averse agents is removed, then the positive profits equilibria in Bisin and Guaitoli (2004), Kahn and Mookherjee (1998) correspond to second-best allocations.

The discussion is organized in the following way: Section 2 introduces the reference framework provided by Parlour and Rajan (2001), which we set-up in a more standard moral hazard scenario. Then, Section 3 presents the equilibria of this credit market as parameterized by the relevance of the moral hazard problem. Section 4 characterizes the constrained Pareto frontier for this game and provides the welfare analysis of the market equilibria. Section 5 concludes.

2 The model

Credit relationships are represented in a very simple way. The borrower is penniless, though she has access to the technology for the production of the only existing good. The production process is subject to random realizations: if the amount *I* is invested, with probability *p* the production successfully yields G(I) > 0, while with probability (1 - p) the outcome will be 0. The production function G(I) is assumed to be continuous, increasing and strictly concave in *I*:

³This changes the space of relevant mechanisms in the contract design problem underlined, in that the set of equilibria sustained by menus contains the set of equilibria sustained using simple (point) contracts. See Peters (2003). The two sets of equilibria are not comparable, in general.

with positive third derivative, i.e. G'''(I) > 0. Furthermore, the Inada conditions hold $lim_{I\to 0}G'(I) = \infty$ and $lim_{I\to\infty}G'(I) = 0$.

There are $N \ge 2$ lenders (indexed by $i \in N = \{1, 2, ..., N\}$) who compete over the loan contracts they simultaneously offer to a single borrower.

Having received all the contracts' proposals, the borrower decides which of them to sign, taking into account that she can accept any subset of them.⁴ She must also take a non-contractible action (effort), that affects the set-up of the production activity and the successful repayment of the loans obtained. The effort can take only the two values p^H and p^L , with $p^H > p^L$. If the high effort p^H is chosen, production takes place and the borrower gets G(I) - R with probability p^H and 0 otherwise. If, on the contrary, the low effort p^L is taken she earns the private benefit BI with probability 1 and loans are not repaid. Without loss of generality, we set $p^L = 0$ and $p^H = p$.

Let us describe the normal form of the game we are considering. Lenders strategically compete over their contractual offers to the single entrepreneur-borrower. The strategy of lender i is given by the choice of the contract C_i . The contract offer of lender i is defined by a repayment line R_i and a loan amount I_i , i.e.

$$C_i = (R_i, I_i) \in C_i \subseteq \mathbb{R}^2_+.$$

Given the space of feasible contract offers for each lender *i*, we define the aggregate space of contracts in the loan sector as $C = \times_i C_i$. The borrower's strategy is therefore given by the map

$$s_b: \mathcal{C} \to \{0,1\}^N \{p,0\}.$$

With a small abuse of notation, we also define the generic element of the set $\{0,1\}^N$ as the array $a_b = (a_b^1, a_b^2, ..., a_b^N)$ where $a_b^i = \{0,1\}$ is the borrower's decision of rejecting or accepting lender *i*'s offer. The choice of the array a_b defines the set of accepted contracts \mathcal{A} :

$$\mathcal{A} = \left\{ i \in N : a_b^i = 1 \right\},\,$$

that is, borrower's decisions are identified by the choice of the effort level and by the definition of the relevant set \mathcal{A} . Her strategy set will be denoted as \mathcal{S}_b , i.e. $s_b \in \mathcal{S}_b$.

We now consider payoffs. The borrower's payoff is defined by:

$$\pi_b = \begin{cases} p[G(I) - R] & \text{if } p \text{ is chosen,} \\ BI & \text{if no effort is taken,} \end{cases}$$

⁴This defines a scenario of *delegated* common agency (Martimort and Stole 2003)

where R and I denote the aggregate repayment and investment respectively, i.e.

$$R = \sum_{i \in \mathcal{A}} R_i$$
 and $I = \sum_{i \in \mathcal{A}} I_i$.

Lender *i*'s payoff is given by: for every $i \in \{1, 2, ..., N\}$

$$\pi_i = \begin{cases} pR_i - (1+r)I_i & \text{if the borrower chooses } p, \\ -(1+r)I_i & \text{if the borrower takes no effort,} \end{cases}$$

whenever his contract is accepted and zero otherwise: $r \in \mathbb{R}_+$ is the lender's cost of collecting deposits.⁵ Observe that lender *i*'s payoff will not directly depend on lender *j*'s strategies. Existence of contractual externalities among lenders is originated by the borrower's behavior only: at the stage of contracting with lender *i*, the action chosen by the borrower depends on the contractual offer she is receiving from lender *j*.⁶ We can therefore model credit market interactions as a sequential game, with a first stage where several lenders are playing a simultaneous move game and a second stage where the borrower decides on acceptance/rejection of each offer and finally exerts effort. In formal terms, loan relationships are represented by the following common agency game Γ :

$$\Gamma = \{(\pi_i)_{i \in \mathbb{N}}, \pi_b, \mathcal{C}, \mathcal{S}_b\}.$$

3 Credit market equilibria

This section discusses the properties of the subgame perfect Nash equilibria of the game Γ . We parallel the discussion given by Parlour and Rajan (2001) on the equilibria of the credit market with the aim of emphasizing their welfare properties. For this reason, we do not provide a detailed analysis of the set of equilibrium allocations.⁷ With the present note, we want to characterize the constrained efficient Pareto frontier of this economy and show that competition among lenders sustains here only second-best (constrained) efficient allocations.

We start by introducing the following assumption.

Assumption 1 We consider B < 1.

Observe that given Assumption 1, the choice of low action determines a social loss of BI - I(1+r), where I is the aggregate level of investment. Hence there cannot be any equilibrium in which this low action is implemented. We analyze the cases where p is implemented, i.e. in

⁵Lenders here do not have infinite endowment. They rely on the deposit market to finance entrepreneurial activity.

⁶This is usually referred to as the absence of *direct externalities* among principals. Most common agency models have been developed in such a simplified scenario. Examples of recent researches where direct externalities among principals are considered include Bernheim and Whinston (1998) and Martimort and Stole (2003).

⁷For the complete characterization, refer to Parlour and Rajan (2001).

every equilibrium the borrower will be given incentives not to undertake the low action. The relevant Incentive Compatibility constraint will therefore be:⁸

$$p\left[G\left(\sum_{i\in\mathcal{A}}I_i\right) - \sum_{i\in\mathcal{A}}R_i\right] \ge B\sum_{i=1}^N I_i.$$
(1)

Observe that if the low action is chosen, then the borrower has always the incentive to accept the whole array of offered contracts. This greatly simplifies the incentive analysis.

The investment level that maximizes the aggregate surplus $S = \pi_b + \sum_i^N \pi_i$ defines the *first best* level of investment, which will be referred to as I^* :

$$I^* = \underset{I}{\operatorname{argmax}} S \equiv \underset{I}{\operatorname{argmax}} pG(I) - I - rI,$$

where I^* is such that $pG'(I^*) = 1 + r$ and that the corresponding surplus is positive.⁹

If there were no incentive problem (i.e. if the borrower were not taking any hidden action), then every equilibrium would involve the *first-best* amount of lending I^* . When strategic behavior of the borrower is considered under the additional assumption of exclusive contracting, i.e. when we explicitly consider incentive constraints but we further assume that the borrower can only accept one contract at a time, then lenders compete à la Bertrand over contracts; at equilibrium they get zero profits and the borrower appropriates the whole surplus.

If we allow for non-exclusive contracting, then we formally enter into a common agency set-up. Given the high degree of externalities involved in the analysis, positive profits equilibria and low levels of aggregate investment are a typical feature in general. In our model, as will be shown, we can sustain zero-profit equilibria with competition among lenders offering nonexclusive contracts for some parameter values, which make the moral hazard problem very mild.

Definition 1 A (pure strategy) equilibrium of the game Γ is an array $[(\tilde{R}_i, \tilde{I}_i)_{i \in N}, (\tilde{a}_b^i)_{i \in N}, p]$ such that:

- the borrower is optimally choosing the set of accepted contracts A (i.e. she is choosing her optimal array a_b ∈ {0,1}^N) and implementing the high level of effort;
- for every lender i = 1, 2, ..., N, the pair $(\tilde{R}_i, \tilde{I}_i)$ is a solution to the following problem:

$$\max_{R_i,I_i} pR_i - (1+r)I_i$$

s.t.

$$p\left[G\left(\sum_{j\neq i}\tilde{I}_{j}\tilde{a}_{b}^{j}+I_{i}\tilde{a}_{b}^{i}\right)-\left(\sum_{j\neq i}\tilde{R}_{j}\tilde{a}_{b}^{j}+R_{i}\tilde{a}_{b}^{i}\right)\right]\geq B\left(\sum_{j=1}^{N}\tilde{I}_{j}\tilde{a}_{b}^{j}+I_{i}\tilde{a}_{b}^{i}\right).$$
(2)

⁸Notice that this incentive compatibility controls for the incentive on aggregate default, which is the only relevant case due to the monotonicity assumption on the borrower's payoff in this case.

⁹To make the problem meaningful, we assume that such an $I^* \in \mathbb{R}_{++}$ exists. That is, we will restrict the analysis to the (exogenous) lenders' cost of funds r such that $r \in [0, \bar{r})$, where \bar{r} is such that $I^*(\bar{r}) = (G')^{-1} \left(\frac{1+\bar{r}}{p}\right)$.

This inequality is the borrower's Incentive Compatibility constraint and it is formulated in terms of aggregate investment and aggregate revenues. The borrower has no endowment, and her exogenous reservation utility is zero so that her participation decision will be always satisfied. This constraint defines lender *i*'s set of feasible contracts under non-exclusivity.

We can characterize equilibrium allocations in terms of the incentive parameter *B*. More precisely, we introduce the threshold value B_z , which defines the lowest level of incentives compatible with the *first best* level of investment:

$$B_{z} := \frac{pG(I^{*}) - (1+r)I^{*}}{I^{*}}.$$
(3)

If $B = B_z$, then the first-best investment I^* is feasible and the Incentive Compatibility constraint is binding. By equation (3), if I^* is implemented then the borrower gets the entire surplus. Lenders' profits are equal to zero in the aggregate and the corresponding aggregate repayment will be R^* such that $pR^* - (1+r)I^* = 0$.¹⁰

Whenever $B > B_z$ allocations giving zero-profits to lenders can be sustained only with a level of debt lower than I^* .¹¹ We denote this level of aggregate investment $\overline{I}(B) < I^*$. On the contrary, if $B < B_z$ it is then possible to achieve I^* and at the same time to leave some extra-surplus to lenders.

We denote $I_B = \min{\{\overline{I}(B), I^*\}}$ the highest level of investment that is at the same time feasible and such to guarantee to the borrower the full appropriation of the social surplus. Fig. 1 identifies B_z using the total surplus hump-shaped curve, $S = pG(I^*) - (1+r)I^*$, and straight lines starting from the origin with slope equal to B.

If $B > B_z$ then $\overline{I}(B)$ is the maximum incentive compatible level of aggregate investment. If $B < B_z$, the intersection of the corresponding straight line from the origin with the curve *S* will be on the right-hand side of the first-best level of investment, that maximizes total surplus, and hence I^* will always be feasible. When the incentive to undertake a low action is small enough, the impact of asymmetric information is reduced and it is possible to show that only a Bertrand outcome can be sustained at equilibrium. In such a situation every lender i = 1, 2, ..., N is offering the loan amount $I_i = I^*$, i.e. the first best is achieved, and the repayment line $R_i = R^* = \frac{I^*(1+r)}{p}$ that gives him zero extra profits. This is stated in the following:

Proposition 1 Denote $B_c := \frac{pG(I^*) - I^*(1+r)}{2I^*}$. Whenever $B \leq B_c$, then the only outcome that can be supported as a (pure strategy) equilibrium of the game Γ is (R^*, I^*) .

Proof. The proof is given in the Appendix. \blacksquare

The intuition for the result is the following: consider a scenario where N - 2 lenders are not active, while each of the remaining two can offer a contract associated to a debt level of I^* , given that $2B_c = B_z$. If $B = B_c$, then the borrower is indifferent between accepting any of the two contracts and accepting both of them and taking a low level of effort. As long as every single

¹⁰This of course implies that every lender earns zero profit, given that they are symmetric and limited liability holds.

¹¹That is, the first best investment level I^* cannot be implemented.



Figure 1: Graphical representation of B_z

lender *i* offers a contract different from the zero-profit one $\left(R_i = \frac{I^*(1+r)}{p}, I_i = I^*\right)$, a Bertrand argument applies: the two-lenders competition determines undercutting to each other's offers until the marginal cost of funds meets the marginal revenues.

If the incentive to take the low action falls between B_c and B_z , then zero profits equilibria may arise only if N is large enough. The intuition is the following: consider a scenario where $B < B_z$ and N - 1 lenders are offering the contract (R_i, I_i) , where $I_i = \frac{I^*}{N-1}$ and $R_i = \frac{I^*(1+r)}{(N-1)p}$ is the repayment level that guarantees zero-profits to the i - th lender when offering the loan amount $\frac{I^*}{N-1}$. Then, the borrower will accept all of them and implement the high level of effort. There is therefore room for the *n*-th lender to offer the zero profits contract (R_n^*, I_n) ; if this offer is accepted, then I^* can in principle be implemented. The closer is B to B_z , the higher the number of lenders N - 1 that is necessary to guarantee that the offer of the *n*-th lender will be feasible. More formally, we have the following proposition:

Proposition 2 If $B \in (B_c, B_z)$, then there exists a critical number of lenders N_B such that for all $N > N_B$ the aggregate allocation (R^*, I^*) is an equilibrium outcome.

Proof. The proof is discussed in the Appendix.

3.1 Equilibria with positive profits

If we consider the case $B > B_z$, then positive profits equilibria are a general feature of the analysis. These equilibria are such that every lender is active in the market, though the aggregate investment level turns out to be strictly lower than I_B . A form of credit rationing is therefore implied by competition over financial contracts. Whenever $B > B_z$, we are in the increasing part of the social surplus function S = pG(I) - I - rI represented in Fig. 1. As a consequence, a single lender *i* offering a zero-profit contract can profitably deviate if all the others are playing a zero-profit strategy: a Bertrand outcome cannot be sustained at equilibrium.

In particular, we are able to show the existence of a (symmetric) positive profit equilibrium where all lenders are active: each of them offers the same amount of credit \tilde{I} and fixes the repayment \tilde{R} . Existence of this equilibrium is established in the following proposition:

Proposition 3 If $B \in [B_z, B_m)$, then there is a critical number of lenders N_B such that for every $N \ge N_B$, there exist a positive profit equilibrium. The equilibrium outcome $(N\tilde{R}, N\tilde{I})$ can be characterized through the following set of equations:

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = p\left[G\left((N-1)\tilde{I}\right) - (N-1)\tilde{R}\right],\tag{4}$$

$$p\left[G\left(N\tilde{I}\right) - N\tilde{R}\right] = BN\tilde{I},\tag{5}$$

$$(N-1)\tilde{I} > I_m. \tag{6}$$

Proof. The proof is given in the Appendix.

At equilibrium, all the *N* existing lenders are active and the borrower is indifferent between accepting N - 1 or *N* contracts (Eq. 4) while exerting high effort. This *no-side-contracting* condition is crucial to establish existence of equilibria with positive profits in several works on moral hazard in insurance economies because it prevents additional purchases of insurance.¹² Furthermore, when the borrower accepts *N* contracts, her Incentive Compatibility constraint will bind (Eq. 5). Finally, the aggregate level of credit issued by N - 1 lenders is strictly greater than I_m that corresponds to the investment chosen by one monopolistic lender (Eq. 6).

Equilibria with positive profit may also emerge when the incentive to take low action is relatively small. In such a case, the first-best level of investment I^* will be achieved but the distribution of the total surplus will be rather favorable to the lenders. This equilibria can be shown to exist for every $B \in (B_c, B_l]$ where $B_l := \frac{pG(I_m) - I_m(1+r)}{I^* + I_m}$ and is smaller than B_z . They are sustained by latent contracts, i.e. contracts which are not bought at equilibrium and are used to deter entry. Existence of such equilibria is stated in the following:

¹²see Bisin and Guaitoli (2004) and Kahn and Mookherjee (1998)

Proposition 4 For every $B \in (B_c, B_l]$, there exists a pure strategy equilibrium where only one contract (say, contract i) is bought. The contract guarantees a positive profit to the lender. Furthermore, there is a second lender (say, lender j) who offers a zero profit contract that is not accepted.

Proof. The proof is given in the Appendix.

The equilibrium of Proposition 4 is sustained by latent contracts, i.e. contracts not traded at equilibrium and act as a device to deter potential entrants. The analysis of these sort of equilibria has been introduced in Arnott and Stigliz (1993) and developed by Bisin and Guaitoli (2004).

The main concern of this note is to characterize the welfare properties of credit market equilibria when multiple lenders compete over loan contracts. The next section will therefore provide a welfare analysis of the equilibrium outcomes associated to the game Γ .

4 Welfare analysis

We will provide here a description of the economy's feasible set, that is the set of players' payoffs corresponding to the allocations implementable by a Social Planner. We introduce the notion of Social Planner and the related concept of *constrained* efficiency in the same way as it is done in the literature on incentives in competitive markets (see for instance Bisin and Guaitoli (2004)). The social planner will choose the aggregate investment level *I* and the aggregate repayment *R* to maximize his preferences over the aggregate feasible set that is usually referred to as the *utility possibility set*.¹³

We will henceforth denote π_L the payoff earned by lenders in the aggregate credit sector and π_b the corresponding borrower's payoff. Let us start considering the first-best situation, where the relevant constraints faced by the planner are those imposed by technology and resources (together with limited liability requirements). The corresponding utility possibility set is:

$$\mathcal{F}(\pi_L, \pi_b) = \left\{ (\pi_L, \pi_b) \in \mathbb{R}^2_+ : \pi_L + \pi_b \le pG(I^*) - I^*(1+r) \right\}.$$
(7)

The frontier of the set \mathcal{F} is referred to as the first-best Pareto frontier. All the arrays (π_L, π_b) belonging to this Pareto frontier are such that there does not exist a pair $(\pi'_L, \pi'_b) \in \mathcal{F}$ with $\pi'_L \ge \pi_L$ and $\pi'_b > \pi_b$ or $\pi'_L > \pi_L$ and $\pi'_b \ge \pi_b$. In our set-up, the first-best Pareto frontier is defined by the function $\pi^*_L(\pi^*_b)$.

Observe that the payoffs functions $\pi_L(R,I)$ and $\pi_b(R,I)$ evaluated at the high level of effort are both linear in the aggregate repayment *R*. As a consequence, the first-best Pareto frontier

¹³In particular, given that the N lenders are homogeneous, the *social welfare function* will be a weighted sum of the payoffs of the N lenders and of the borrower.



Figure 2: The first-best Pareto frontier

will be a downward-sloping 45-degree line. By using the variable pR as a transfer, we can draw the first-best Pareto frontier as the one depicted in Fig. 2.

Every point on the first-best Pareto frontier corresponds to the optimal investment level I^* . In particular, point *A* identifies a situation where the whole surplus is distributed to the borrower, $\pi_b^* = pG(I^*) - (1+r)I^*$, so that $pR = (1+r)I^*$, i.e. $\pi_L^* = 0$. On the contrary, if $\pi_b^* = 0$ then from (7) we get $pR = pG(I^*)$, i.e. lenders are receiving everything and the borrower is left at her reservation utility of zero (point *A'*).

Let us now define the second-best allocations, i.e. the set of allocations implementable by a planner who is facing informational constraints. The *constrained utility possibility set* is the set of outcomes (π_L, π_b) such that:

$$\mathcal{F}'(\pi_L, \pi_b) = \left\{ (\pi_L, \pi_b) \in \mathbb{R}^2_+ : \pi_L \le \pi_L^{**}(\pi_b^{**}, B), \ \pi_b \le \pi_b^{**} \text{ for every } \pi_b^{**} \in [0, pG(I^*) - I^*(1+r)] \right\},$$

where for every given π_b^{**} , $\pi_L^{**}(.)$ is such that:

$$\pi_L^{**}(\pi_b^{**}, B) = \max_{R, I} pR - (1+r)I,$$
(8)

s.t.

$$pR - (1+r)I + \pi_b^{**} \le pG(I) - (1+r)I, \tag{9}$$

$$\pi_b^{**} \ge BI. \tag{10}$$

With respect to the first-best problem, we have introduced here the Incentive Compatibility requirement in equation (10). Observe that for a given π_b^{**} , the lender's maximization problem is monotone in *R*, hence equation (9) will bind at the optimum. We can therefore substitute the expression for *pR* obtained in (9), in the objective function. The system (8)-(10) can be rewritten as:

$$\pi_L^{**}(\pi_b^{**}, B) = \max_r p G(I) - \pi_b^{**} - (1+r)I, \tag{11}$$



Figure 3: The First and Second-Best Pareto frontiers for $B < B_z$

Notice that the *constrained utility possibility set* and the second-best Pareto frontier are parameterized by a given incentive structure B. Recall that we defined B_z as the level of the incentive

s.t.

parameter such that:

$$pG(I^*) - (1+r)I^* = B_z I^*,$$

implying that $pR = (1 + r)I^*$, i.e. lenders make zero profits. Hence,

$$\forall B < B_z \quad pG(I^*) - (1+r)I^* < BI^*.$$

That is equation (12) is slack and the first-best is feasible in the second-best problem. In particular, the point $(\pi_b^{**}, \pi_L^{**}) = (pG(I^*) - (1+r)I^*, 0)$ belongs to the second-best Pareto frontier (Fig. 3). Hence given $B < B_z$, there is room to reduce π_b^{**} without making the constraint (12) binding. There will therefore be an interval of entrepreneur's utilities, i.e. $\pi_b^{**} \in [BI^*, pG(I^*) - I^*(1+r)]$, such that the second-best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ coincides with the first best one $\pi_L^*(\pi_b^*)$ (Fig. 3). By further reducing the entrepreneur's payoff we get to $\pi_b^{**} = BI^*$ and $\pi_L^{**} = pG(I^*) - I^*(1+r) - BI^*$. Every further reduction in π_b^{**} will imply a decrease in the investment level.

If we consider the case $B > B_z$, equation (12) will always be binding at the optimum level of investment, hence it is not possible to sustain the first-best investment level I^* . As a consequence, for every $B > B_z$ the second-best frontier $\pi_L^{**}(\pi_b^{**}, B)$ will always lie below the first best one, as it is depicted in Fig. 4.





Figure 4: The First and Second-Best Pareto frontiers for $B > B_z$

Hence, while for the cases of relatively mild incentive problem the second-best frontier has a linear part where the first-best level of investment is implemented, when the moral hazard becomes harsher the frontier contracts inwards.

No matter the value of *B*, the highest possible payoff for the lending sector corresponds to the monopolistic allocation, when the entrepreneur is squeezed to a payoff of $\pi_b^{**} = BI_m$ and the lenders appropriate all the rest.¹⁴ Whenever $\pi_b^{**} < BI_m$ every reduction in π_b^{**} calls for a reduction in π_L^{**} . In the limit the only way to set $\pi_b^{**} = 0$ is to fix an investment level equal to zero, so that there will not be anything left for lenders either. We finally argue that the concavity of G(I) will induce a concavity in the second-best Pareto frontier (Fig. 3 and Fig. 4).

Lemma 1 Take any $B \in [0,1]$ then for every $\pi_b^{**} \leq BI^*$ the frontier $\pi_L^{**}(\pi_b^{**}, B)$ is a concave curve. In particular, $\pi_L^{**}(\pi_b^{**}, B)$ has a maximum in $\pi_b^{**} = BI_m$. For every $\pi_b^{**} < BI_m$, $\pi_L^{**}(\pi_b^{**}, B)$ is monotonically increasing.

Proof. If (12) is not binding, we are back to the linear part of the frontier, which is trivially concave. The interesting case is that of a binding incentive compatibility constraint (12), then $I = \frac{\pi_b^{**}}{B}$. Given π_b^{**} and *B*, then *I* is uniquely determined. As a consequence, we get:

$$\pi_L^{**}(\pi_b^{**},B) = pG\left(rac{\pi_b^{**}}{B}
ight) - rac{\pi_b^{**}}{B}(1+r) - \pi_b^{**},$$

that is a strictly concave function of π_b^{**} . In particular, for $B > B_z$, the second-best Pareto frontier is strictly concave.

Defining the constrained Pareto frontier of the economy gives us more intuitions about the welfare implications of competition over loan contracts. The existence of positive profits equilibria and some form of rationing in credit markets where an arbitrarily large number of homogeneous lenders is competing, turn out to be the by-product of the competitive process itself under asymmetric information. In such circumstances, a single planner who faces the same informational constraints as the lenders cannot implement credit markets allocations that Pareto dominate the equilibrium outcomes of the strategic interactions between *N* lenders and a single borrower.

The equilibria with positive profits and latent contracts described in Proposition fall in the region where the incentive levels $B < B_z$: there it is always possible to sustain the first best level of investment I^* together with $\pi_b^{**} > BI^*$. Hence, the latent contracts are just a device for a different sharing of the surplus. The equilibrium level of investment would be the same that a social planner would choose when solving (11) - (12) with a slack incentive compatibility constraint. This equilibrium allocation would correspond to a point on the linear part of the second best Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$ where it coincides with the first best one.

With respect to the efficiency properties of the equilibria described in Proposition 3 we state the following:

¹⁴Notice that every monopolistic investment depends on the value of the incentive parameter, hence it should be written $I_m(B)$.

Proposition 5 Take a $B > B_z$ and consider the positive profits equilibrium defined in Proposition 3. Then, if we denote as $\tilde{\pi}_b$ and $\tilde{\pi}_L$ the payoffs earned by the single borrower and by all the lenders, respectively, we have that the pair $(\tilde{\pi}_b, \tilde{\pi}_L)$ belongs to the constrained Pareto frontier $\pi_L^{**}(\pi_b^{**}, B)$.

Proof. We first introduce a useful definition. Assume that the borrower earns $\tilde{\pi}_b$ in the positiveprofits equilibrium, we denote $\tilde{\pi}_L(\tilde{\pi}_b)$ the lenders' payoff induced by $\tilde{\pi}_b$ at equilibrium. Let us now take $\pi_b^{**} = \tilde{\pi}_b$ and construct the equilibrium relationship $\tilde{\pi}_L(\tilde{\pi}_b)$. In the positiveprofits equilibrium defined by (4) - (6) each lender offers the same contract (\tilde{I}, \tilde{R}) and in the aggregate the borrower buys all contracts and exerts high effort. The borrower is indifferent between accepting N or N - 1 contracts. Let us call I^A the amount of credit issued and pR^A the expected revenues of the lenders. Given that the Incentive Compatibility constraint is binding in this equilibrium, we then have $\pi_b^{**} = \tilde{\pi}_b = BN\tilde{I}$, that implies:

$$I^A = \frac{\tilde{\pi}_b}{B} = \frac{\pi_b^{**}}{B}.$$
(13)

where we denoted $I^A := N\tilde{I}$.

Given the borrower payoff and the number of active lenders N, the aggregate investment level I^A that supports $\tilde{\pi}_b$ at equilibrium is uniquely determined. In particular, the Incentive Compatibility constraint of the equilibrium defines the same level of aggregate investment of the second-best problem. This investment level I^A determines the aggregate surplus of the economy as:

$$S^{A} = pG(I^{A}) - (1+r)I^{A}, (14)$$

and the lenders' payoff once deduced the borrower's utility π_{b}^{**} :

$$\pi_L^{**}(\pi_b^{**}) = S^A - \pi_b^{**} = p G \left(I^A \right) - (1+r) I^A - B I^A.$$
(15)

Notice that the payoff the credit sector earns is strictly positive:

$$\pi_L^{**}(\pi_b^{**}) = pR^A - (1+r)I^A > 0.$$
(16)

In particular, the system of equations (13) - (15) identifies a pair (π_L^{**}, π_b^{**}) belonging to the frontier of the *constrained utility possibility set* $\mathcal{F}'(\pi_b, \pi_L)$.

5 Conclusion

We constructed a common agency framework for the credit market, where under the assumption of risk neutral preferences for the agent when choosing low action, every positive-profit equilibrium turns out to be constrained Pareto efficient. Despite the externalities originated by strategic competition over financial contracts, borrower's preferences are such that the Incentive Compatibility constraint is always binding. As a consequence, inefficient outcomes cannot be sustained at equilibrium. Interesting extensions of this framework to discuss the effects of competition under non-exclusive contracting both at individual and aggregate level would call for enriching the contractual scheme to make it more sensitive to the incentive problems.

A Appendix

Proof of Proposition 1 in the text.

We first prove that the first best investment level I^* is an equilibrium outcome whenever $B \le B_c$. We consider the following array of offered contracts:

$$\{(R_i, I_i) = (R_j, I_j) = (R^*, I^*) \text{ for } i \neq j; \ (R_k, I_k) = (0, 0) \ \forall \ k \neq i, j\}.$$
(17)

That is, there are two lenders, say lender *i* and lender *j* who offer the first best allocation, while all other lenders are offering the null contract (0,0). The borrower is indifferent between accepting the i - th and the j - th contract; given that $B \le B_c$, accepting all contracts and choosing low action is never a best reply.

In such a scenario, no lender has a profitable deviation given that the first best outcome is implemented and the borrower's profit is maximized.

Now, let us show that R^* , I^* is also the unique equilibrium outcome. In other words, we show that no positive profit equilibrium can exist for $B \le B_c$. Observe that every positive profit equilibrium must imply a binding Incentive Compatibility constraint, otherwise some lender whose contract is accepted can raise his repayment and make the constraint binding. That is, we should have:

$$p\left[G\left(\sum_{i\in\mathcal{A}}R_i\right)-\sum_{i\in\mathcal{A}}I_i\right]=B\sum_{i=1}^NI_i.$$
(18)

We have to consider two cases:

•
$$\sum_{i=1}^{N} I_i \leq I^*$$
.

If the total amount of offered loan is lower than I^* , then a single lender, say lender *i*, can profitably offer a debt level $I_i=I^*$. Recalling that whenever $B \le B_c$ we have $p[G(I^*) - I^*(1+r)] \ge BI^*$, then there is room for the i - th lender to offer the loan amount I^* together with a positive repayment R_i . This behavior constitutes a profitable deviation for the lender, since he is able to appropriate of the payoff originally shared amongst the active lenders.

•
$$\sum_{i=1}^{N} I_i > I^*$$
.

In this case there will be for sure lenders whose contracts are not bought at equilibrium. To show that no positive profit equilibria can be sustained in this case we first assume that $\sum_{i \in \mathcal{A}} I_i < I^*$. In such a case, let us consider any lender *i* whose contract (I_i, R_i) is not accepted. By offering the loan amount $I'_i \in (0, I_i)$ he can make the borrower's payoff from accepting all contracts and playing low action strictly lower; then, there exist a repayment R'_i such to give incentives to lender *i* to profitably deviate and to the borrower to accept the contract (R'_i, I'_i) on top of those contained in the set \mathcal{A} . Analogously, if $\sum_{i \in \mathcal{A}} I_i = I^*$, then it is possible to show that every lender *i*, with $i \in \mathcal{A}$, can profitably reduce the amount of loan he is offering without inducing the borrower to modify the optimal choice of \mathcal{A} .

Proof of Proposition 2 in the text.

Consider the case of $B \in (B_c, B_z)$ and a given number of lenders *N*. If every lender offers the contract $(R', I') = \left(\frac{R^*}{N-1}, \frac{I^*}{N-1}\right)$, it is incentive compatible for the borrower to accept N-1 contracts and exert high effort (p). We want to show that these prescriptions (strategies) for each lender and for the borrower constitute an equilibrium of the game Γ .

Notice that in the case we described, the borrower obtains the first best aggregate level of investment buying N-1 contracts, attaining her maximum expected payoff and each single lender gets zero profits. Let us evaluate if there exist profitable deviations.

Given what her opponents offer, lender *i* can never propose a loan that the borrower will accept and guarantees herself positive profits. When all $j \neq i$ lenders offer (R', I'), whatever lender *i* proposes, the borrower can always buy the remaining N-1 contracts and achieve her maximum payoff. Hence, it is a best response for lender *i* to offer (R', I') when all other lenders offer (R', I').

To guarantee that the borrower has no profitable deviations, we have to eliminate incentives to shirk

$$pG\left(\frac{N-1}{N-1}I^*\right) - (N-1)\frac{I^*(1+r)}{N-1} \ge BN\frac{I^*}{N-1},\tag{19}$$

that is, we want that the utility she gets from buying all *N* contracts and exerting low action be lower than the first best payoff:

$$pG(I^*) - I^*(1+r) \ge BN \frac{I^*}{N-1}.$$
(20)

This translates onto an incentive parameter that satisfies the following:

$$B \le \frac{pG(I^*) - I^*(1+r)}{I^* \frac{N}{N-1}}.$$
(21)

Let us define $B_N = \frac{pG(I^*) - I^*}{I^* \frac{N}{N-1}}$. As *N* increases, $B_N \to B_z$. Hence for every $B \in (B_c, B_z)$ there exists a N_B such that for every $N > N_B$, $B \le B_N$ and the borrower has no incentive to deviate from buying N - 1 contracts and choosing *p*. There does not exist any contract for any lender *i* that gives her positive profits and is accepted by the borrower. Hence, (R', I') for each lender and the borrower accepting N - 1 contracts and exerting high effort constitute an equilibrium.

Proof of Proposition 3 in the text.

The proof is organized in two steps. First, we show that there is an aggregate contract $(N\tilde{R}, N\tilde{I})$ which is a solution of the system (4)-(5) and satisfies (6). In a next step we show that the strategy profile $(R_i, I_i) = (\tilde{R}, \tilde{I})$ for every lender i = 1, 2, ...N together with the borrower decision of accepting all contracts and choosing the high level effort is a subgame perfect equilibrium of the game Γ .

Considering (4) and (5) together we get:

$$p\left[G\left((N-1)\tilde{I}\right) - \left(1 - \frac{1}{N}\right)G\left(N\tilde{I}\right)\right] - B\tilde{I} = 0.$$
(22)

We define $f(I|N,B) = p[G((N-1)I) - (1 - \frac{1}{N})G(NI)] - BI$.

Observe that we are considering aggregate investment level that should belong to the interval $[I_B, I_m]$, given the system (4)-(6). Now, we also denote $I_B^o = \frac{I_B}{N}$; as a consequence, we have:

$$f(I_B^o) = pG\left(\frac{(N-1)}{N}I_B\right) - pG(I_B) - \frac{1+r}{N}I_B.$$
(23)

Given that the function G(.) is concave and recalling that $G'(I_B) > 1 + r$, we have that

$$pG(I_B) - \frac{(1+r)}{N}I_B > p\left[G\left(\frac{(N-1)I_B}{N}\right)\right],\tag{24}$$

so that $f(I_B^o) < 0$.

Using a similar argument, and recalling the definition of B_m we can check that for every $B < B_m$ there exist an N_B large enough such that $\forall N \ge N_B$ we get:

$$f\left(\frac{I_m}{N-1}\right) > 0. \tag{25}$$

Given the continuity of the function f(.), for every $N \ge N_B$ there exists a value $\tilde{I}(B,N)$ such that $f(\tilde{I}) = 0$; given \tilde{I} , the value of R satisfying (4)-(5) can be defined in a direct way.

Now, we have to show that at equilibrium every lender will offer the contract (\tilde{R}, \tilde{I}) and that the borrower will always have an incentive to accept all contracts and to implement the high action.

Let us start with the borrower's behavior if each lender is playing (\tilde{R}, \tilde{I}) , then the borrower's strategy of accepting *N* contracts and playing *H* is a best reply. Equations (4) and (5) guarantee that when $(N\tilde{R}, N\tilde{I})$ is offered in the aggregate then the borrower cannot deviate by accepting N - 1 contracts and playing *L* anyway: this means that she is not in the decreasing part of her payoff function, so that no deviation involving reductions in the number of accepted contracts will be profitable. In particular, accepting *N* contracts will be a best reply.

Let us consider now the behavior of the *N* lenders. Suppose all (N-1) lenders except lender *i* offer (\tilde{R}, \tilde{I}) and consider lender *i*'s best response. Assume lender *i* offers (R_i, I_i) , his payoff can be measured with respect to the aggregate amount of loans the borrower takes up:

$$\pi_{i} = pR_{i} - (1+r)I_{i}$$

$$= pG(k\tilde{I}+I_{i}) - pk\tilde{R} + (1+r)k\tilde{I}$$

$$-(1+r)I_{i} - \max\left\{pG((N-1)\tilde{I}) - p(N-1)\tilde{R}, B\left((N-1)\tilde{I}+I_{i}\right)\right\}$$
(26)

where π_i is lender *i*'s payoff as a function of (R_i, I_i) and $k = \{0, 1, 2, ..., N-1\}$ is the number of contracts the entrepreneur buys together with the *i*-th. On the right hand side of the equation we represented the surplus at the aggregate amount of investment $k\tilde{I} + I_i$ net of the reimbursements of the *k* lenders offering (\tilde{R}, \tilde{I}) and of the entrepreneur's utility. The entrepreneur can obtain $pG((N-1)\tilde{I}) - p(N-1)\tilde{R}$ accepting the (N-1) contracts and exerting effort *p* or $B((N-1)\tilde{I} + I_i)$ accepting all the contracts offered and choosing low action.

There can be two cases: either $I_i \leq \tilde{I}$ or $I_i > \tilde{I}$. Let us consider first the case when $I_i \leq \tilde{I}$. From the definition of the equilibrium, it is clear that the borrower will always prefer to accept at least (N-1) contracts and exert effort p. In this case, the individual revenue of each of the (N-1) lenders obtained by using (4) and (5) will be:

$$p\tilde{R} = p\left[G\left(N\tilde{I}\right) - G\left((N-1)\tilde{I}\right)\right].$$
(27)

In addition, given $I_i \leq \tilde{I}$ and the concavity of G(.) the entrepreneur will buy all the (N-1) contracts together with the *i*-th, hence lender *i*'s payoff will be:

$$\pi_{i} = pG\left((N-1)\tilde{I} + I_{i}\right) + (1+r)(N-1)\tilde{I} - pG\left((N-1)\tilde{I}\right) - (1+r)I_{i}$$
(28)

which is maximized setting $I_i = \tilde{I}$ and guarantees a payoff of $p\tilde{R} - (1+r)\tilde{I}$.

Consider now the case of $I_i > \tilde{I}$, which induces the low action and a payoff such as:

$$\pi_{i} = pG\left(k\tilde{I} + I_{i}\right) - pkR + (1+r)k\tilde{I} - B(N-1)\tilde{I} - BI_{i} - (1+r)I_{i}$$
(29)

which is increasing in k and takes into account that the contract offered by the lender i could affect the number of contracts the borrower would accept together with exerting high effort.

The first order condition for a maximal π_i with respect to I_i gives:

$$pG'(k\tilde{I}+I_i) - (1+r) - B = 0$$
(30)

which implies $k\tilde{I} + I_i = I_m$.

Hence, $I_i = I_m - k\tilde{I}$ and $(N-1)\tilde{I} > I_m$ imply that the highest number of contracts which can be accepted together with the *i*-th is k = N-2. Hence, the optimal value of *k* is such that: $k = max_{k' \in \{1,2,\dots,N-2\}} \{k' | I_m - k\tilde{I} > 0\}$: it follows I_i cannot be greater than \tilde{I} , which contradicts the initial assumption.

Therefore, the optimal choice of lender *i* can only be $I_i \leq \tilde{I}$ which implies that his best response will be to offer a contract (\tilde{R}, \tilde{I}) .

Hence, the specific contracts (\tilde{R}, \tilde{I}) exist and they are robust to individually profitable deviations when the number of lenders is sufficiently high and $B \in [B_z, B_m)$.

Proof of Proposition 4 in the text.

Given the definition of I_m and I^* and the continuity of G(.) there exists a B_l such that:

$$pG(I_m) - I_m(1+r) = B_l(I_m + I^*).$$
(31)

Now, for every $B \in (B_c, B_l]$ we consider the function $x(I) = pG(I) - I(1+r) - B(I+I^*)$; by continuity there exists an investment level I' such that x(I') = 0.

The equilibrium is defined by one lender, say lender *i*, making positive profits offering the investment $I_i = I^*$ and the repayment R_i s.t. $pR_i = pG(I^*) - I^*(1+r) - (pG(I') - I'(1+r))$. A second lender, say lender *j* offers the zero-profit contract with $I_j = I'$ and $pR'_j = (1+r)I'$. All other lenders $k \neq i, j$ are offering the null contracts (0,0). The borrower is accepting contract *i*, only.

Given the behavior of the other players, lender *i* must offer the borrower at least a payoff of pG(I') - I'(1+r) in order for his contract to be bought. Hence, he has the incentive to set the investment level at I^* so to realize the maximum amount of profits pR_i .

Let us now consider lender *j*: he cannot profitably deviate from the level of investment $I_j = I'$ and be guaranteed that his offer is accepted, without inducing the borrower to select low action.

Given the existence of the latent contract j, no contract offering positive investment level proposed by any of the inactive lenders will be accepted at equilibrium.

Finally, the borrower is indifferent between accepting either contract i or j in isolation and choosing high effort, and buying both contracts and choosing low action. That is, accepting i only is a best reply.

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