Complementarity Problems in Restructured Natural Gas Markets

Steven Gabriel[1] and Yves Smeers[2]

[1] Assistant Professor, Department of Civil and Environmental Engineering, Applied Mathematics and Scientific Computation Program, University of Maryland, College Park, Maryland 20742 U.S.A.
sgabriel@umd.edu;www.eng.umd.edu/∼sgabriel

[2] Tractebel Professor of Energy Economics, Department of Mathematical Engineering and Center for Operations Research and Econometrics, Université catholique de Louvain, Louvain-la-Neuve, Belgium
smeers@core.ucl.ac.be;www.core.ucl.ac.be

Summary. The restructuring of the gas industry did not so far generate the same modeling activity as in electricity. While the literature of activity in electricity market models is now abundant, it is still rather scant on the gas side. This paper surveys some of the existing models and attempts to take advantage of the wealth of knowledge available in electricity in order to develop relevant models of restructured gas markets. The presentation is in three parts. The first one gives a blueprint of the market architectures inherited from the European and North American gas legislation. It also introduces a prototype optimization model and its interpretation in terms of perfect competition between agents operating on the restructured market. The second part extends the model to the case where marketers have market power. The third part considers more complex issues related to regulation of access to the network and existence of market power with different types of agents. Equilibrium models are commonly formulated as complementarity problems and the same mathematical programming framework is adopted here. Many models are single stage; there are generally easy to formulate and well known computationally. But many phenomena require two stage models that are much more intricate and on which much less is known. The paper is thus also aimed at pinpointing possible avenues for mathematical programming research.

1 Introduction

Natural gas markets in Europe and North America have recently witnessed significant changes brought about by government regulation and other market forces. An example of a regulatory measure is the U.S. Federal Energy Regulatory Commission (FERC) order 636, requiring open access service to qualified shippers (www.ferc.gov). In essence, this order transformed gas pipelines from buyers, transporters, and sellers of gas to open access transporters paving the way for new entities such as marketers to become more significant players that might exert market power. In the European Union, similar legal measures for dividing the gas sellers and network operators have also been considered [13] as part of the restructuring and deregulation of the natural gas markets.

The EU currently imports 45 % of its natural gas [32] and this share is expected to rise given limited resources in the EU [5]). Four countries, Russia, Norway, Algeria, and the Netherlands accounted for some 87.7% of all EU gas imports in 2001 ([8] from Energie Bulletin 4145 p.5). Given the declining resources of the United Kingdom, the Netherlands will be the only major internal supplier in the coming years [5]. The potential for market power among the few producers is apparent and natural gas supply security has been addressed in the so-called "Green Paper" [12] and the European Commission (EC) directive 2004/67/EC. The increase in natural gas demand is driven in part by environmental concerns such as the Kyoto Protocol [42] and the fact that natural gas has a lower carbon content than oil or coal [33]. Other reasons for increased importance of natural gas such as the long-term supply situation, or cost-effectiveness are important factors as well.

From a modeling perspective, the traditional system optimization approach for the restructured natural gas markets in Europe and North America will not be the best choice. First, and in contrast with electricity, the gas market, whether in North America or in Europe has never been an integrated system amenable to a full optimization problem given the potentially divergent interests of the main players. Second, given the realities of the new marketplace, such models will fail to capture the important (potential) oligopolistic behavior of market players (e.g., producers in Europe, marketers in Europe or North America).

In general, the introduction of competition in the network industries (e.g. electricity, telecommunication, natural gas) stems from the following idea: one should keep (or allow to be kept), a single company for those activities considered as a natural monopoly, i.e., not competitive by default. One should allow entry in other activities to permit competition.

One way to model this mixture of regulated and non-regulated behavior, with the latter being either perfect or imperfect competition, is to depict all the market players solving separate optimization problems. The Karush-

Kuhn-Tucker (KKT) conditions [2] for these optimization problems taken together with market-clearing conditions constitute a market equilibrium problem typically expressed as a nonlinear complementarity problem (NCP) [30]. Complementarity problems or the related variational inequality problems (VI) have been studied in a variety of engineering and economic settings for a number of years (Facchinei and Pang, 2003). However, only recently have there been NCP/VI (hereafter called "complementarity") models of the natural gas market with full market detail. Some previous examples of imperfect competition models for natural gas markets in Europe concentrating on specific market segments include the early works of [31] and [11] who considered Nash-Cournot producers and a Stackelberg production market, respectively. These works concentrating on the production side were extended for example, in [29] and [6], who considered stochastic aspects and a duopoly of producers, respectively. These models all departed from traditional system optimization approaches such as maximizing total surplus [40] but lacked sufficient market detail on all the players as might be found in large-scale, detailed system optimization market models such as: the Natural Gas Transmission and Distribution Model of the U.S. Department's National Energy Modeling System and its predecessors ([1]; [38], [38]; [15] [18]), the Gas Systems Analysis Model for the North American market ([20], [19]), to name just a few.

Two recent models, have combined both sufficient market detail with the complementarity approach for the new markets in Europe and North America. The first model, GASTALE ([3]; [14]) based in part on the work by [25], [26]. considers Nash-Cournot producers with conjectured supply functions for the European market. In addition it also includes perfectly competitive transportation and storage sectors combined with multiple consumption sectors and seasons. Gabriel et al. ([21],[22],[23]) have developed a model of the North American natural gas market in which marketers compete non-cooperatively against each other as Nash-Cournot players with the transporation, production, storage, and peak gas sectors taken to be perfectly competitive. Also, multiple seasons and consumption sectors are modeled.

Given the recent restructuring in natural gas markets and their importance to the energy sector, an analysis of appropriate modeling formulations is needed. This is the main goal of this paper. In Section 2 we briefly describe the functions of the various market players and provide a simple illustrative example to clarify. We also recall some mathematical programming paradigms that are used in the rest of the paper. In Section 3 we describe as a starting point, perfectly competitive behavior for these players and analyze the resulting KKT conditions for each of the players' optimization problems. Section 4 contrasts this behavior with imperfect competition among some of the players, analyzing the key differences. Both the perfect competition model of Section 3 and the imperfect competition models of Section 4 are relatively easy mathematical programming problems. Section 5 introduces more difficult considerations. It introduces transmission problems that it treats both in an average cost and Ramsey-Boiteux context. The first model guarantees neither the existence nor the uniqueness of an equilibrium. The second model is a non-convex optimization problem. More complex situations of imperfect competition are treated in Section 6, where one envisions situations where different classes of agents operating in the gas market may have market power. This leads to two-stage equilibrium problems that may not have pure strategy solutions. Many of these models have not been treated yet in the literature. The paper is thus a survey of work to be done as well. This is the message developed in the conclusion.

2 Natural Gas Market Players

The supply chain for natural gas begins with producers that extract gas from either onshore or offshore reservoirs. The producers can be assumed to potentially exert market power (as is the case in Europe) or behave in a manner consistent with perfect competition (as in North America). The next step is to transport the gas from production sites to either storage facilities, the citygate, or directly to the consumption sectors (e.g., residential, commercial, industrial, and power generation). Pipeline companies own and operate these transportation routes and are subject to regulated rates (e.g., by FERC in the U.S. and by National Regulatory authorities (NRA) in the EU). Storage operators take advantage of seasonal arbitrage by buying and injecting gas into storage in the low demand season (non-winter) and then selling it to consumers in the high demand season (winter). Storage operators can be taken to be regulated or oligopolistic depending on the local regulations. The EU Directive 2003/55/EC has much weaker regulatory requirements on storage than on transport. It only imposes access to storage on negotiated terms but does not impose any price regulation. Owners of storage facilities are thus only subject to general competition law and possibly to any additional regulatory obligation imposed by the Member States where their facilities are located. Marketers (also known as shippers) are responsible for contracting with pipeline companies to procure the gas and sell it to end-users. The marketers are generally less subject to national regulation and can reasonably be modeled as players with market power given their important position and the new deregulated marketplace for natural gas. Specifically, in the EU, Directive 2003/55/EC does not impose any regulation to marketers which are thus only subject to general competition law and possibly to the regulation that individual Member States may find necessary. Additonally, one can also consider peak demand players who supply extra gas in times of high demand. This supply may be in the form of liquefied natural gas (LNG) or propane/air mixtures. Perfect or imperfect competition could be appropriate for these players as well.

It should be noted that these players each can be modeled as solving an optimization problem in which an abstraction of their operations is assumed. For example, production rates are constrained by the number of available rigs, pressure in the reservoirs, and so on. A full consideration of all engineering aspects for these and the other players would no doubt lead to intractable, non-convex problems, thus making the computation of a complementarity-based equilibrium very difficult at best. For these reasons, an abstraction of their operations is generally taken. Also, it is important to note that in some cases, one parent company may have control over several levels of the natural gas supply chain described above. However, regulations are in effect to try to balance out the field between independent players and ones which are part of a larger company operating on several levels of the supply chain [24]. In the European Union, this control of concentration is left to general competition law.

2.1 An Illustrative Example

To clarify how the natural gas market can be modeled, consider the following example, simplified from Gabriel et al. ([21]) and depicted in Figure 1.

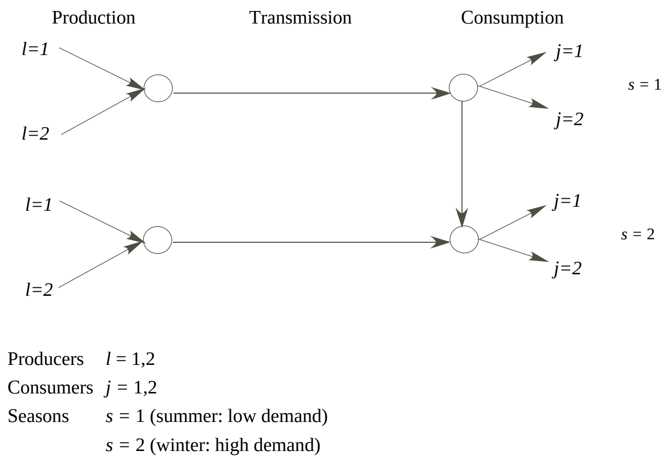

Fig. 1. A simplified example

There are two producers separated from the market by a pipeline (we neglect the distribution system). Storage facilities are located at the end of the pipeline close to the market. There are two market segments, residential and industrial. The problem refers to a single-year horizon decomposed into two seasons. Demand is low in the first season and high in the second. Gas is stored in the low season and extracted in the high season. In order to simplify notation, we assume that both seasons have the same number of days. We also neglect all losses whether from transmission or storage operations. Lastly, depending on the case at hand, it is useful to consider domestic pipelines (entirely located in a country), crossborder pipelines (e.g. crossing an European border) or long distance transportation pipelines (e.g. bringing Russian gas to European borders through Ukraine). Each of these raises new questions which are not treated here. Instead we assume a single pipeline for clarity.

2.1.1. Production

The description of the production of natural gas is reduced to a function giving the cost of extracting the quantity of gas,$(q_{\ell 1}, q_{\ell 2})$ for seasons 1 and 2, respectively, for producer ℓ.

$$\begin{aligned} \text{Cost} = {} & \textstyle\sum_s EC_{\ell s}(q_{\ell s}) \\ & q_{\ell 1} \geq 0, q_{\ell 2} \geq 0 \\ & \text{(EC for Extraction Cost)} \end{aligned} \tag{1}$$

All engineering complexity associated with extracting the gas from the reservoir is thus bypassed. Stylized descriptions of this type are frequently adopted in economics where these functions are used to construct analytical models. In contrast, computable models rely on formulations that allow for more detailed engineering descriptions of the gas production process ([20], [19]). Even though we use a stylized representation of the cost function, such as found in economic models, we keep in mind that one should be able to replace it by a process model of gas production at least as long as one remains within descriptions commonly amenable to optimization models (e.g. [4]).

2.1.2. Transportation

Pipeline transportation is also represented in a very simplified form that neglects all technological characteristics arising from the pressure and flow relationship or representation of compressors; see [10] for details. We simply assume that the pipeline has a maximum capacity $\overline{f}_s$ based on the flow f_s. The owner of the pipeline incurs both short and long-run transportation costs. The costs for the pipeline owner is represented as follows

$$\begin{aligned} \text{Cost} = {} & \textstyle\sum_s TC_s(f_s) \\ & \overline{f}_s - f_s \geq 0 \qquad f_s \geq 0 \qquad s = 1, 2 \\ & \text{(TC for Transportation Cost)} \end{aligned} \tag{2}$$

2.1.3. Storage

Storage is also modeled in the simplest possible form for ease of presentation. Because we neglect losses, the amount recovered in the high demand season is equal to the amount injected in the low demand season. Injection and withdrawal operations respectively cause injection and withdrawal costs and there is also a maximum injection rate. Because there are only two periods, this maximal injection rate also limits the amount that can be stored. Additional constraints on volumetric rates could also be included but are left out from this simple example. The maximal injection rate thus also plays the role of a storage volumetric constraint related to working gas in the reservoir. The associated costs are as follows

$$\begin{aligned} \text{Cost} = {} & IC(i) + WC(w) \\ & i - w \geq 0 \\ & \bar{i} - i \geq 0, w \geq 0 \\ & (IC\text{: injection cost; } WC\text{: withdrawal cost}) \end{aligned} \tag{3}$$

where i, w are the injection and withdrawal amounts, respectively, with $\bar{i}$ the injection capacity.

2.1.4. Demand

In general, the demand for each sector will be a function of the price in that sector, which itself is a (decision) variable to be endogenously determined. For illustrative purposes, we assume a fixed demand in each season. We let

$$d_{js} \text{ be the demand of consumer } j \text{ in season } s. \tag{4}$$

2.1.5. From transaction costs to marketer's costs

The supply of natural gas involves procuring gas from the producers, securing transportation and storage services and selling the gas to the final consumers. These activities imply transaction costs for an integrated company such as the one described by a single optimization model. Because these activities will take on a different interpretation later, it is convenient to single them out in preparation for the rest of the paper. We therefore define the following variables that will later be bundled into a marketer activity. Specifically the marketing department of the integrated company

$$\begin{array}{ll} \text{procures gas from producer } \ell \text{ in season } s & mq_{\ell s} \\ \text{procures transmission services in season } s & mf_s \\ \text{procures storage services (injection and withdrawal)} & mi \text{ and } mw \\ \text{sells gas to customer } j \text{ in season } s & md_{js} \end{array} \tag{5}$$

with the first letter m denoting that it is a marketing variable. For the sake of simplification we shall only refer to the transactions costs due to the selling of the gas (variable md_{js}) and not consider the other marketing variables in the following. We let mn_{js} be the unit cost of selling gas to consumer j in season s.

2.1.6. A system optimization model

As a point of comparison, it is natural to first state the overall natural gas problem in standard production management terms or as a system optimization problem. Specifically, there are costs to produce, transport and store the gas before delivering it to the final consumers. There are also transaction costs of coordinating these activities. As indicated above, we limit our description to the sole transaction costs incurred because of the marketing of gas (sales activity). The most efficient approach in optimization terms is to minimize the sum of all these costs given as

$$\begin{array}{ll} \min \sum_\ell \sum_s EC_{\ell s}(q_{\ell s}) + \sum_s TC_s(f_s) + IC(i) & \\ \qquad + WC(w) + \sum_j \sum_s mn_{js} \cdot md_{js} & \\ \text{s.t.} \quad \sum_i q_{\ell s} - f_s \geq 0 \qquad s = 1,2 & \\ \qquad f_1 - i \geq \sum_j md_{j1} & \\ \qquad f_2 + w \geq \sum_j md_{j2} & \\ \qquad i - w \geq 0 & \\ \qquad \overline{f} - f \geq 0 & \\ \qquad \overline{i} - i \geq 0 & \\ \qquad md_{js} \geq d_{js} \qquad s = 1,2; j = 1,2 & \\ \qquad q_{\ell s}, f_s, i, w \geq 0 & \end{array} \tag{6}$$

Problem (6) is a very simplified representation of a natural gas market operated by a single integrated company. The primary usefulness is to serve as a basis of comparison for more complicated models to be presented below. Indeed, our goal is to progressively transform this small problem with the view of encompassing some of the concerns typically faced by market analysts, regulators, and economists. We assume throughout the paper that all cost functions are convex and differentiable. This approximation is commonly made in economic models. Differentiability can be relaxed at the cost of more complex formulations that we prefer to avoid in this paper. Adding an assumption of quadratic function would also make our complementarity problems linear complementarity problems (LCP).

This type of approach has been extensively used in the discussion of the restructuring of the electricity industry. Many arguments have been developped on the basis of electric power models, comparatively as simple as problem (6) and were eventually transformed into full size computable models for looking at policy and strategic questions. We adopt the same philosophy: starting from a simple optimization model, we progressively introduce economic questions that reflect some of the aspects of the restructuring of the natural gas sector. While there has been considerable modeling activity along these lines in the electricity sector, this has not taken place yet in the gas sector.

The approach is also interesting from an optimization point of view. Some of the models emerging from the process are standard complementarity problems which are now well understood. Other models are optimization problems subject to equilibrium constraints. These problems are much more recent even though their literature is already abundant. Also, other models are equilibrium problems subject to equilibrium constraints, a particular case of Generalized Nash Equilibrium problems. These are quite recent models that turn out to be quite difficult to analyze and computationally challenging to solve. At this stage, such models have received little attention in the literature. Last, but possibly not least, the simplified mathematical programming problems formulated here, can easily be made more challenging by adding all the technological complexities neglected in this presentation.

Classes of mathematical programming problems

Before proceeding with building up a more complicated model of (6), we recall the KKT conditions, complementarity problems, and other mathematical programs that are relevant. A detailed discussion of the properties of these various mathematical programs as well as applications thereof can be found in (Facchinei and Pang, 2004). We use throughout the notation $0 \leq a \perp b \geq 0$ which expresses the set of relations

$$a \geq 0 \qquad b \geq 0 \qquad ab = 0.$$

1. Karush-Kuhn-Tucker Conditions for a Convex Optimization Problem
   Consider a standard nonlinear programming problem of the form

$$\begin{array}{ll} \min & f(x) \\ s.t. & g_i(x) \leq 0 \;\; i = 1, \ldots, m \\ & h_j(x) = 0 \;\; j = 1, \ldots, p \end{array}$$

   where $f, g_i : R^n \to R$, are convex functions and $h_j : R^n \to R$ are affine functions. The KKT conditions are then sufficient for optimality ([16]). These conditions are to find a decision vector $x \in R^n$, an inequality Lagrange multiplier vector $u \in R^m$, and an equality Lagrange multiplier vector $v \in R^p$ such that

$$\begin{array}{rr} \nabla f(x) + \sum_i u_i \nabla g_i(x) + \sum_j v_j \nabla h_j(x) = \; 0 & \\ g_i(x) \leq 0, u_i \geqslant 0, g_i(x) u_i = 0 & \forall i \\ h_j(x) = 0, \; v_j \text{ unconstrained} & \forall j \end{array}$$

   These KKT conditions are a special case of a nonlinear complementarity problem with both equations and inequalities, called a mixed complementarity problem (MCP) and given as follows.

2. Mixed Complementarity Problems
   Find $x \in R^{n_1}, y \in R^{n_2}$ such that

$$\begin{aligned} 0 &\leq F(x, y) \perp x \geq 0 \\ 0 &= G(x, y) \end{aligned}$$

where $F : R^{n_1} \times R^{n_2} \rightarrow R^{n_1}$, $G : R^{n_1} \times R^{n_2} \rightarrow R^{n_2}$ and in general. (These problems are monotone in the context of this paper.)

More generally, one may want to optimize a certain function $\Pi(x, y, z)$ of three sets of variables $x \in R^{n_1}, y \in R^{n_2}, z \in R^{n_3}$. The z vector represents the "first stage" variables whereas x and y represent the "second stage" variables. A typical constraint set consists of two sets of restrictions. First, there are regular constraints on the upper level variables of the form $z \in S$. Secondly, the second stage variables must satisfy some mixed complementarity problem for fixed values of the first stage variables z. This problem is given as follows.

3. Mathematical Programming Problem Subject to Equilibrium Constraints (MPEC)

$$\begin{aligned} &\max_{x,y,z} \Pi(x, y, z) \\ &\text{s.t. } 0 \leq F(x, y; z) \perp x \geq 0 \\ &\quad 0 = G(x, y; z) \\ &\quad z \in S \end{aligned}$$

which is in general a non-convex problem and computationally challenging. A well-known example of an MPEC is the bilevel programming problem in which the lower level constraints are the optimality conditions for a second-stage problem.

MPEC problems can be generalized to equilibrium problems with equilibrium constraints (EPEC). A specific example of an EPEC is as follows: Let K agents have first stage decision variables $z_k, k = 1, \cdots, K$. Each of these agents seeks to maximize an objective function $\Pi^k(x, y, z_k, z^*_{-k})$ where z^*_{-k} represents the optimal but fixed values for the other players. This objective function is optimized subject to the constraint that $z_k \in S_k$ and equilibrium constraints such as specified in the MPEC problem. The full problem is thus to find $z^*_k, k = 1, \cdots, K, x, y$ as follows.

4. Equilibrium Problems subject to Equilibrium Constraints

$$\begin{aligned} z^*_k \text{ solves } &\max_{x,y,z_k} \Pi^k(x, y, z_k, z^*_{-k}) \\ \text{s.t. } &\ 0 \leq F(x, y; z_k, z^*_{-k}) \perp x \geq 0 \\ &\ 0 = G(x, y; z_k, z^*_{-k}) \\ &\ z_k \in S_k \end{aligned}$$

This problem, like the MPEC, is computationally difficult given that it is in general a non-convex problem and existence of a solution (here a pure strategy equilibrium) is not guaranteed even under standard compactness assumptions on the feasible region. The solution of an EPEC problem, if it exists, is a subgame perfect equilibrium ([17]).

3 A Perfect Competition Model

3.1 Demand functions

Market models commonly assume that demand reacts to prices. Short-term (real time) demand of electricity is the exception where demand is commonly assumed to be insensitive to price. This reaction is represented by a demand function, which, for concreteness, we assume to be affine and downward sloping. We let

$$d_{js}(p_{js}) \text{ and } p_{js}(d_{js}) \tag{7}$$

be, respectively, the demand and inverse demand functions of consumer j, in season s. Using the inverse demand function, one introduces the willingness to pay function given as

$$WP_{js}(d_{js}) = \int_0^{d_{js}} p_{js}(\xi)d\xi. \tag{8}$$

We assume, in order to simplify the discussion, that the prices will automatically turn out positive.

3.2 Basic assumptions

Perfect competition assumes that all agents are price-takers. This means that agents optimize their profit or utility subject to prices that they take as given. This assumption does not imply that these prices are exogenous to the system, but simply that these agents see them as such. An expanded version of problem (6) more amenable to an interpretation in terms of an equilibrium is given in problem (9) in which all variables are taken to be nonnegative. In preparation for its interpretation in terms of an equilibrium model, this version also assumes that the marketing/sales activity of the integrated company has been split in several marketing/sales activities k each under the responsability of a different independent marketer k, with its own activity variable and cost. Problem (9) differs from problem (6) in two respects. First it explicitly introduces the demand functions (7) via the willingness to pay function (8). Second it reformulates the constraints by introducing new variables that are easier to interpret in terms of unbundled gas activities. Specifically this latter difference between the two formulations allows for an explicit representation of all the transactions of the marketers and the introduction of a possibly different unit marketing cost mn^k of each marketer k in the objective function. It also separates the production, transportation and storage activities.

$$\begin{aligned} \max \textstyle\sum_j \sum_s WP_{js}(d_{js}) - \sum_\ell \sum_s EC_{\ell s}(q_{\ell s}) - \sum_s TC_s(f_s) \\ - IC(i) - WC(w) - \textstyle\sum_k mn^k(\sum_j \sum_s md^k_{js}) \end{aligned} \tag{9.1}$$

s.t.

$$q_{\ell s} - \sum_k mq_{\ell s}^k \geq 0 \ (wp_{\ell s}) \text{ wellhead price} \tag{9.2}$$
$$\sum_\ell mq_{\ell s}^k - mf_s^k \geq 0 \ (bp_s^k) \text{ border price} \tag{9.3}$$
$$mf_1^k - mi^k - \sum_j md_{j1}^k \geq 0 \ (cg_1^k) \text{ citygate price} \tag{9.4}$$
$$mf_2^k + mw^k - \sum_j md_{j2}^k \geq 0 \ (cg_2^k) \text{ citygate price} \tag{9.5}$$
$$\sum_k md_{js}^k - d_{js} \geq 0 \ (p_{js}) \text{ price paid by consumer} \tag{9.6}$$
$$f_s - \sum_k mf_s^k \geq 0 \ (\tau_s) \text{ transmission price} \tag{9.7}$$
$$mi^k - mw^k \geq 0 \ (\mu^k) \text{ value of gas in storage for marketer } k \tag{9.8}$$
$$i - \sum_k mi^k \geq 0 \ (ip) \text{ injection charge} \tag{9.9}$$
$$w - \sum_k mw^k \geq 0 \ (wp) \text{ withdrawal charge} \tag{9.10}$$
$$\overline{f}_s - f_s \geq 0 \ (\rho_s) \text{ transmission congestion} \tag{9.11}$$
$$\overline{i} - i \geq 0 \ (\lambda) \text{ storage congestion charge} \tag{9.12}$$

Dual variables are written to the right of each constraint together with their interpretation. The dual variables of constraints (9.2) to (9.5) are respectively, the wellhead prices (wp), border prices (bp), and citygate prices in summer and in winter (cg). The other dual variables can also be usefully interpreted. Specifically p_{js} is the price paid by consumer j in season s; τ_s is the transmission charge in season s, ip and wp respectively the injection and withdrawal charges into and from storage; ρ_s is the congestion charge of the pipeline, λ the congestion charge of storage facilities, and μ^k is the implicit price of gas in storage for marketer k. Note that except for the introduction of the possibly different transaction costs of the marketing activity and the addition of different marketing variables, this model is equivalent to problem (6). As we argue next, because of the new variables, it is amenable to an interpretation in terms of the behavior of the agents in the market. Note also that balance inequalities are written under the "free disposal assumption" i.e., they hold as equalities when the commodity/service price is positive.

3.3 *KKT* Conditions, Complementarity Formulations and Agent Behavior

We now proceed to establish the KKT conditions of problem (9) and interpret them in terms of agent behavior in perfect competition. This interpretation paves the way to the introduction and formulation of different assumptions of imperfect competition.

3.3.1. Producers' behavior

The relation

$$0 \leq \frac{\partial EC_{\ell s}(q_\ell)}{\partial q_{\ell s}} - wp_{\ell s} \perp q_{\ell s} \geq 0. \tag{10}$$

expresses that each producer maximizes its profit at the prevailing price in the season. If producer ℓ is active in season s ($q_{\ell s} > 0$), then the wellhead price is equal to the marginal cost.

3.3.2. Pipeline operator behavior

The conditions

$$\begin{aligned} 0 \leq \frac{\partial TC_s(f_s)}{\partial f_s} - \tau_s + \rho_s \perp f_s \geq 0 \\ 0 \leq \overline{f}_s - f_s \perp \rho_s \geq 0 \end{aligned} \tag{11}$$

state that the pipeline operator maximizes the profit accruing from the use of the pipeline at the prevailing price. If the pipeline is used ($f_s > 0$), this price is equal to the sum of a marginal transportation cost and a congestion cost ($\tau_s = \frac{\partial TC_s(f_s)}{\partial f_s} + \rho_s$). The congestion cost ρ_s is only different from zero when the pipeline is full ($f_s = \overline{f}_s$).

3.3.3. Storage operator behavior

The conditions describing the behavior of the storage operator can be stated as follows.

$$\begin{aligned} 0 \leq \frac{\partial IC(i)}{\partial i} - ip + \lambda \perp i \geq 0 \\ 0 \leq \frac{\partial WC(w)}{\partial w} - wp \perp w \geq 0 \\ 0 \leq (\bar{i} - i) \perp \lambda \geq 0 \end{aligned} \tag{12}$$

These define storage operation charges and can be interpreted as follows. There is a charge λ on injection facilities only when there are congested, i.e. $\lambda > 0$ implies $i = \bar{i}$. If the injection facilities are used ($i > 0$) the injection charge is equal to the sum of the marginal injection cost and the congestion charge: $ip = \frac{\partial IC(i)}{\partial i} + \lambda$. The withdrawal charge is equal to the marginal withdrawal cost when $w > 0$: $wp = \frac{\partial WC}{\partial w}$.

3.3.4. Consumer behavior

The condition

$$0 \leq -\frac{\partial WP_{js}}{\partial d_{js}} + p_{js} \perp d_{js} \geq 0 \tag{13}$$

expresses that the marginal willingness to pay for gas is equal to the price when there is consumption, that is

$$d_{js} > 0 \Rightarrow \frac{\partial WP_{js}}{\partial d_{js}} = p_{js}.$$

3.3.5. Marketers' behavior

The appearance of marketers is a key element of the restructuring of the gas industry. Marketers emerge from the optimization models as agents that take on former coordination activities that involved procuring the commodity and transportation and storage services as well as marketing the gas. They compete against each other, and as a result put competitive pressure on other agents that are not in a monopoly position (e.g. producers in the EU). Each of the marketer's tasks is described in complementarity form as follows.

3.3.6. Procuring the gas

$$0 \leq wp_{\ell s} - bp_s^k \perp mq_{\ell s}^k \geq 0. \tag{14}$$

When $mq_{\ell s}^k > 0$, the border price charged to marketer k is equal to the wellhead price.

3.3.7. Shipping the gas

$$0 \leq bp_s^k - cg_s^k + \tau_s \perp mf_s^k \geq 0. \tag{15}$$

When $mf_s^k > 0$, the citygate price of marketer k is equal to the sum of the border price charged to marketer k and the transmission price.

3.3.8. Procuring storage services

$$\begin{aligned} &0 \leq ip + cg_1^k - \mu^k \perp mi^k \geq 0 \\ &0 \leq \mu^k + wp - cg_2^k \perp mw^k \geq 0, \\ &mi^k = mw^k. \end{aligned} \tag{16}$$

Note that relation (9.8), $mi^k \geq mw^k$, must hold with equality. Indeed, suppose $mi^k > mw^k \geq 0$, then $\mu^k = 0$ by (9.8). This also implies $ip = cg_1^k = 0$ (by the first complementarity condition of (16)). $i \geq \sum_{k'} mi^{k'} > mi^k > 0$, (12) would then imply $\frac{\partial IC}{\partial i} = 0$, ... which means that the cost of the whole supply chain vanishes to zero in season 1. We exclude this case for economic reasonableness.

The difference of citygate prices between seasons 2 and 1 ($cg_2^k - cg_1^k$) for marketer k is equal to what it has to pay for storage services ($ip + wp$) when it uses these services ($mi^k = mw^k > 0$). This is an intertemporal arbitrage condition.

3.3.9. Marketing the gas

$$0 \leq cg_s^k + mn^k - p_{js}(d_{js}) \perp md_{js}^k \geq 0. \tag{17}$$

When $md_{js}^k > 0$, the price offered to consumer j in season s is equal to the sum of the citygate price of marketer k and the marketing cost mn^k.

4 Imperfect Competition: Market Power of the Marketers

4.1 Background and Definition of the agents

The above discussion is rather straightforward both in mathematical and economic terms. It is well known that KKT conditions of convex problems can be expressed as complementarity conditions and that they can be interpreted in economic terms under our assumptions of convexity (see Section 2.1.6). But this economic interpretation is very specific. It only refers to perfect competition, that is to conditions where all agents are price takers. The interest of the KKT conditions in this model stems from the fact that we would like to modify each of these complementarity conditions in order to better represent the reality of the market. Indeed, European producers do not necessarily behave as price-taking agents. Transmission may be regulated both in the US and Europe resulting in their charging their average cost. Marketers may have a dominant position in their home market in Europe or in some large fraction of the market in the US and hence not behave according to the perfect competition paradigm. Similarly storage owners could be regulated or be in a position to exert market power. In short, one would like to construct a model that resembles the above KKT model at least in terms of its structure, but differs from it in specific market aspects. We begin by briefly motivating this approach.

4.1.1. Unbundling of the transportation and merchant activities

It is commonly assumed, but by no means proved in theory or practice, that the transportation infrastructure is a natural monopoly. This implies that one should not expect competition or, at least much competition to develop, in transportation. We take the extreme view (which is true in Spain and France but not in Germany) that there is a single transportation company operating the infrastructure. Transportation, because it is a monopoly should be regulated both in terms of the conditions of access and its pricing. In other words one cannot expect that competition will naturally lead to relation (11). One would thus need to impose some pricing regulation on the transportation activity.

4.1.2. Unbundling of storage and merchant activity

Storage is essential for gas operation. Storage can only be developed at certain sites and the incumbent European companies currently already operate most sites. It would seem natural to also unbundle storage from the marketing operations. This can be done in two ways: one is to make storage competitive, that is either to transfer ownership to other agents or to auction its capacity; an alternative is to regulate the access to storage. For the sake of brevity we shall not elaborate here on the regulation of the storage activity or on the market power that storage owners can exert. For the sake of simplicity we retain the perfect competition assumption model in (16).

4.1.3. Making marketing competitive

In contrast with storage and transportation, there is no restriction on having several marketers operating in a given territory. Specifically all former gas companies can have a marketing activity. Because they know the producers of gas and the main characteristics of the gas consumers, this implies that they can compete with each other in different geographic segments of the market. Needless to say, the incumbent in some European country is likely to know more about the demand sector of his country than about other countries, at least in a first stage. But this is not sufficient to refrain from entering other markets or from trying to team up with smaller agents operating in other markets. This justifies unbundling the marketing activities and allowing for different marketers in every market.

In short, we thus assume in the following that there is a single pipeline company and a single storage company. The transportation activity is regulated. We do not make any special assumption on storage that remains ruled by (12), that is, at marginal cost pricing. We suppose that there are several marketers that buy and resell gas and procure transportation and storage services, possibly exerting market power.

4.2 Price Discrimination and Arbitrage

Even though there may be several marketers in a single market, it is unlikely that it will immediately become perfectly competitive. This implies that one looks for a Nash equilibrium with respect to some strategic variables. It is common and easy to use quantities as strategic variables (à la Cournot). We shall later use a similar Cournot assumption for representing producers. This will lead to a much more difficult EPEC problem. According to this assumption, each marketer optimizes its profit, assuming the quantitative actions of the others given. In order to illustrate the principle, consider for a moment the simpler problem of marketer k buying gas in season s at citygate prices cg_s^k respectively. These marketers incur marketing costs mn^k. In perfect competition they will sell the gas to segment j at the price p_{js} satisfying

$$p_{js}(d_{js}) = cg_s^k + mn^k, \quad k = 1, 2. \tag{18}$$

Both marketers will sell to segment j if the quantities $cg_s^1 + mn^1$ and $cg_s^2 + mn^2$ are equal. If not, only the marketer with the smallest $cg_s^k + mn^k$ will remain in that market segment.

The situation is different with a Nash-Cournot assumption. We adopt the standard notation to let $-k$ designate marketers other than k. Under this assumption and with this notation, marketer k solves the problem

$$\max_{md_s^k} p_{js}(md_s^k + md_s^{-k})md_s^k - (cg_s^k + mn^k)md_s^k. \tag{19}$$

Assuming a positive sale ($md_s^k > 0$), one sees that the pricing condition (18) is replaced by

$$p_{js}(d_{js}) + md_s^k \frac{\partial p_{js}}{\partial d_{js}} = cg_s^k + mn^k, \quad k = 1, 2. \tag{20}$$

The only difference between the perfect and Nash-Cournot competition is thus the replacement of $p_{js}(d_{js})$ by $p_{js}(d_{js}) + md_s^k \dfrac{\partial p_{js}(d_{js})}{\partial d_{js}}$.

Applying this reasoning to the previously derived KKT conditions, the Nash-Cournot behavior of the marketers can be inserted into the above model by simply replacing relation (17) by

$$0 \leq cg_s^k + mn^k - p_{js}(d_{js}) - md_{js}^k \frac{\partial p_{js}(d_{js})}{\partial d_{js}} \perp md_{js}^k \geq 0. \tag{21}$$

In this relation the gas price collected by the marketer from customer j in time segment s is replaced by the marginal revenue from the same client in that period. The rest of the KKT conditions remain unchanged.

This model is amenable to some variations. One can assume that all marketers behave à la Cournot. Alternatively, one can suppose that the incumbent marketer retains a dominant position and that the entering marketers behave competitively, that is that they are price-takers. One would then have a mix of relations (17) for the entrants and (21) for the incumbent. This could be justified for instance if an entrant believes that it is too small to try to exert market power in this new market. The entrant therefore prefers to leave the task of maintaining a relatively high gas price to the incumbent and simply behaves as a price-taker.

The possibility of having this mix of behaviors introduces alternative possible formulations. One may simply combine the competitive relations describing the Cournot (21) and competitive (17) behaviors. This is the situation where the incumbent naively considers the actions of the entrant as given. Alternatively, one could assume that the incumbent takes the actions of the entrant into account when planning its strategy. It then chooses its marketing action taking into account the reaction of the entrant. This latter interpretation complicates the problem as shown below.

Price discrimination does not occur in perfect competition but is a standard outcome of market power. In order to analyze this phenomenon, consider again the perfect competition model and suppose that marketer k supplies both consumers 1 and 2 in season s $(md_{1s}^k > 0, md_{2s}^k > 0)$. Relation (17) becomes

$$p_{1s} = p_{2s} = mn^k + cg_s^k. \tag{22}$$

One sees that the prices paid by the two customers in season s are identical. Consider now the Cournot model and make the similar assumption that marketer k supplies both consumers in season s. Relation (21) becomes

$$p_{1s}(d_{1s}) + md_{1s}^k \frac{\partial p_{1s}}{\partial d_{1s}}(d_{1s}) - mn^k - cg_s^k = 0 \text{ and}$$
$$p_{2s}(d_{2s}) + md_{2s}^k \frac{\partial p_{2s}}{\partial d_{2s}}(d_{2s}) - mn^k - cg_s^k = 0. \tag{23}$$

This time one cannot conclude that $p_{1s} = p_{2s}$. The prices to the two consumers could be different and therefore price discrimination could occur. An interesting question is whether price discrimination can persist in an open market. This is where new agents, namely arbitrageurs, intervene.

Arbitrageurs are new agents that take advantage of price differences existing in a market. They buy where the price is lower and sell where it is higher if the difference exceeds their transaction costs. Suppose, in order to simplify the problem, different consumer prices as results from the Cournot pricing of the marketers, zero transportation costs between the customers and negligible transaction costs are present. The following modeling of arbitrageurs has been introduced by [37] for the electricity sector and is presented here for natural gas. Suppose an arbitrageur that buys a quantity a from a first consumer paying a lower price and sells this amount to a second consumer with a higher price. The arbitrageur can make a profit and will expand this trading until the prices of both consumers are equal. This can be formalized by imposing that an arbitrageur solves the following problem

$$\max_{as}[p_{1s}(d_{1s} + a_s^*) - p_{2s}(d_{2s} - a_s^*)]a_s \qquad (a_s \text{ unconstrained}) \tag{24}$$

where a_s represents the amount that is arbitraged. It is important to note that the a_s^* in $p_{1s}(d_{1s} + a_s^*)$ and $p_{2s}(d_{2s} - a_s^*)$ is not a decision variable to the arbitrageur (based on the perfect competition assumption). The arbitrageur is supposed to be a price taker. He/she trades as long as $p_{1s} \neq p_{2s}$ but does not take the impact of his/her trade on the price into account. This is the usual assumption of a competitive agent: it implies that the market settles at a value a_s for which

$$p_{1s}(d_{1s} + a_s^*) - p_{2s}(d_{2s} - a_s^*) = 0. \tag{25}$$

This effect can be readily inserted in the model (10) to (17) by adding both the variables a_s and the constraints (25), $s = 1, 2$ to the set of complementarity conditions.

Price discrimination can also take place between seasons. Suppose that the marketer uses storage services. In perfect competition (17) implies that the difference between the prices charged to a given consumer is equal to the difference between the citygate prices in these seasons (see the discussion of storage operations in section 3.3.5). This difference is itself equal to the sum of the marginal injection and withdrawal costs, to which one also adds a congestion cost in case the storage capacity is full. This is expressed in the following relation

$$\begin{aligned} p_{j1} - cg_1^k - mn^k = 0 \\ p_{j2} - cg_2^k - mn^k = 0 \end{aligned} \tag{26}$$

which imply

$$p_{j1} - p_{j2} = cg_1^k - cg_2^k.$$

Taking the Cournot assumption where the prices charged to a consumer in the two seasons have been replaced by the marginal revenues accruing from these consumers, one obtains

$$\begin{aligned} p_{j1}(d_{j1}) + md_{j1}^k \frac{\partial p_{j1}}{\partial d_{j1}}(d_{j1}) - cg_1^k - mn^k = 0 \\ p_{j2}(d_{j2}) + md_{j2}^k \frac{\partial p_{j2}}{\partial_{j2}}(d_{j2}) - cg_2^k - mn^k = 0. \end{aligned} \tag{27}$$

This does not imply that $p_{j2} - p_{j1} = cg_2^k - cg_1^k$. The price difference between the two seasons is not necessarily equal to the difference between the citygate prices and hence to the sum of the marginal injection and withdrawals charges and a possible congestion cost. In other words, there may be price discrimination. This price discrimination between seasons has been pointed out for the case of reservoir management in electricity in [7]. It also appears here in natural gas. The implication of the market power here is a non-optimal use of the storage compared to the perfect competition case. Arbitrageurs can again intervene to reduce the price discrimination between seasons. An arbitrageur here is an agent who buys a quantity in the first, low-price period and releases it in the higher price period. The arbitrageur does not buy gas from the producers (it would be an other marketer in that case); he/she simply takes a position between the two periods. Needless to say the arbitrageur incurs the storage costs, in this case the sum of the marginal injection and withdrawal costs and the possible congestion charge in case storage facilities are full. The arbitrageur therefore solves the following problem

$$\max \left[p_{j2}(d_{j2} + a^*) - p_{j1}(d_{j1} - a^*) - \left(\frac{\partial IC(i + a^*)}{\partial} + \frac{\partial WC(w + a^*)}{\partial w} + \lambda \right) \right] a \tag{28}$$

where he/she takes $\frac{\partial IC}{\partial i}$, $\frac{\partial WC}{\partial w}$ and λ as given. Solving the problem will imply that the prices between two seasons will satisfy the relation

$$p_{j2}(d_{j2} + a^*) - p_{j1}(d_{j1} - a^*) = \frac{\partial IC(i + a^*)}{\partial i} + \frac{\partial WC(w + a^*)}{\partial w} + \lambda. \tag{29}$$

Again this effect can be readily inserted in the model (10)-(17) by adding both the variables a and relation (29) to the set of complementarity conditions at least if one assumes that one has an analytic expression of both $\frac{\partial IC}{\partial i}$ and $\frac{\partial WC}{\partial w}$. One also needs to replace $\bar{i} \geq i$ be $\bar{i} \geq i + a$. We saw before that the Cournot marketer could anticipate the actions of the spatial arbitrageurs expressed in relation (25) (clairvoyant marketer) or take them as given (myopic marketer). The same distinction can be made here with respect to the behavior of marketers vis à vis the seasonal arbitrageurs (relation (29)). The case of the naïve arbitrageur is straightforward to model: one simply replaces relation (21)-(22) by the pair

$$\begin{aligned} &0 \le cg_s^k + mn^k - p_{is}(d_{js}+a) - md_{js}^k \tfrac{\partial p_{js}}{\partial d_{js}} \perp md_{js}^k \ge 0 \\ &p_{1s}(d_{1s}+a) - p_{2s}(d_{2s}-a) = 0. \end{aligned} \tag{30}$$

In contrast with the naïve Cournot marketer, the clairvoyant marketer foresees the action of the arbitrageur and takes them into account in its sales. Metzler et al. (2003) have shown that both assumptions lead to the same outcome in electricity markets. It is conjectured that the same result holds here. The reader is referred to Metzler et al. (2003) for an in-depth discussion of this question.

5 Regulated Transportation

5.1 Background

It was argued before that there will likely remain a single transportation company in each EU Member State after restructuring has taken place. This transportation company therefore has a dominant position in the transportation market and hence needs to be regulated. Germany is the only proponent of an alternative approach and argued for a long time that transportation of natural gas is a competitive activity. And indeed some competition developed. But Directive 2003/55/EC applies to all Member States and Germany will need to comply with the common approach which is to regulate gas transportation. It remains to be seen how it will meet the regulation requirement. Regulation should facilitate the proper access to transportation infrastructure. The exact meaning of "proper" has been extensively discussed in the literature on access pricing in network industries (mainly in telecommunication). We note that our formulation (11) implements a marginal cost pricing of transportation services and a congestion charge when the capacity of the pipeline is saturated. This congestion cost is charged to all marketers. Marginal cost pricing has been vigorously discussed in the context of access to the electric power network where it gave rise to the famous disputes between proponents of the flowgate and nodal models and to the discussion of zonal/nodal pricing in the United States. It also gives rise to various issues of market power in the transportation of electricity. We shall not discuss these questions here because in contrast with electricity, congestion in natural gas transport does not seem yet to be a major issue. Besides marginal cost pricing we consider two other approaches to transportation pricing, namely average cost and Ramsey-Boiteux pricing. Average cost pricing is the most widely accepted tariff structure in practice even though it has little economic virtue. By contrast, Ramsey Boiteux pricing is a sophisticated way to allocate costs. Its application to utilities was made famous by Boiteux's seminal contribution to electricity pricing. It has been extensively discussed in the context of access to telecommunication infrastructure. Its application to natural gas is due to [9]. We model these approaches without any attempt to summarize the extensive discussions that they generated.

5.2 Average cost pricing

Average cost pricing is the preferred access pricing method in practice. It consists of setting a price that allows the network owner to cover its cost including a proper rate of return on capital. To illustrate the principle, consider the simple situation depicted in Figure 2 with two marketers. One assumes that the charge is set at regular time intervals by the regulator on the basis of the transportation cost and on some historical or prospective view of the flow in the pipeline.

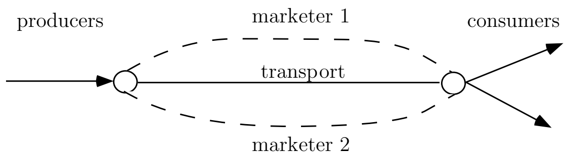

Fig. 2. two marketers and a transporter

Let tc and F be respectively the variable and the fixed cost of the network (see Figure 3). Assume two marketers who respectively ship f_1 and f_2 through the network. A plausible average cost access tariff is given by the unit rate τ_s

$$\tau_s = \frac{F}{f_1 + f_2} + tc \tag{31}$$

One can again think of two possible implementations of this tariff. In a first "naïve" implementation, the marketers do not foresee that increasing the amount of demanded transmission service will decrease the unit rate τ_s. In another interpretation, they anticipate this change. Replacing (11) by (31) and keeping the rest of the KKT conditions unchanged models the naïve interpretation.

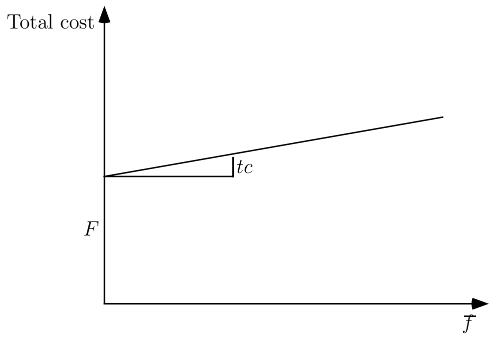

Fig. 3. Cost function of the transportation activity

5.3 Infeasible problems and multiple equilibria

All models covered up to this point can be converted into convex optimization problems, at least under standard conditions on the cost function (convexity), the demand curve (downward sloping) and for the Cournot model, revenue function (concavity). They are thus guaranteed to have a convex set of solutions. In contrast the introduction of average cost pricing prevents this conversion into a convex optimization problem. The complementarity problem becomes nonlinear and ceases to be monotone as a result of the decreasing unit rate τ_s (31) replacing (11). This may make the model infeasible or introduce multiple equilibria. This is illustrated in Figure 4 for the cost function of the transportation activity shown in Figure 3. The example assumes a single consumer, no storage, zero marginal gas production cost and a marketer who needs to pay for transportation priced at average cost. The figure illustrates two situations that correspond to different levels of the fixed charge F. Curve (1) corresponds to the case of relatively low value of F; the average cost curve intersects with the demand curve at two points so that here are two equilibria. When the fixed charges of the pipeline are too high (curve (2)), the transporter cannot find a demand level that pays for the cost of the network. This lack of equilibrium may seem unrealistic if the fixed charges are limited to the sole cost of the network. This phenomenon proved dramatically relevant before the restructuring of the US gas sector in the 1980's when the fixed charges to be recovered by the marketers (at that time the pipelines companies) included the take or pay commitments of long-term contracts.

5.4 Ramsey-Boiteux problem statement

Economists working on access pricing in the telecommunication area have extensively promoted the application of Ramsey-Boiteux pricing for access

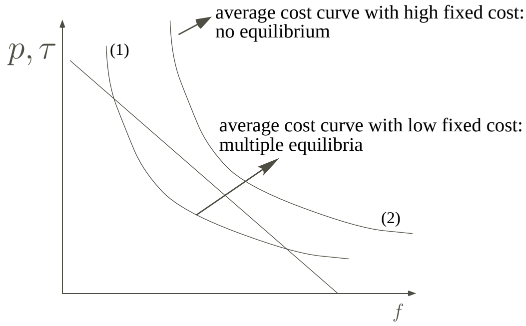

Fig. 4. Non existing and multiple equilibria

to the infrastructure. Cremer et al. (2003) converted this approach to transportation of natural gas. We first introduce the method in a simplified context and then discuss the problem that it raises in the more realistic context (even though extremely simplified) of our example.

Consider the simplified case where there is no storage, a single marketer, one gas producer and two customers as shown in Figure 4.

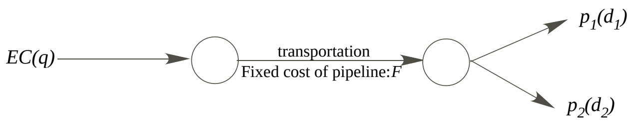

Fig. 5. One producer, one marketer, no storage

Assume the charge to recover through access prices amounts to a single fixed cost of the pipeline ($tc = 0$ in Figure 3). One wants to find access charges for the two customers that maximize economic welfare and allow one to cover the revenue requirement of the pipeline.

Supposing that the whole economy is in perfect competition except for the transportation of natural gas. We note q the production quantity and use τ^j to denote the transport charge to consumer j in the example. τ^j is then given by

$$\tau^j = p_j(d_j) - \frac{\partial EC}{\partial q}(q) \tag{32}$$

where $\frac{\partial EC}{\partial q}$ is the price charged by the producer for its gas in perfect competition. The transport charge is equal to the difference between the price paid by the consumer and the marginal extraction cost of gas. Relation (32) implies that q can be written as a function of d_j and τ^j: let $q(d,\tau)$ be this function. The resulting welfare maximization problem in simplified form is stated as

$$\begin{array}{l} \max \int_0^{d_1} p_1(\xi_1)d\xi_1 + \int_0^{d_2} p_2(\xi_2)d\xi_2 - EC(q) \\ \text{s.t. } \tau^1 d_1 + \tau^2 d_2 \geq F \\ \quad 0 \leq q \leq \overline{f}. \end{array} \tag{33}$$

Note that the KKT conditions of (33) are similar but not identical to (32). (33) is indeed the regulator's problem while (32) represents the equilibrium conditions in a perfectly competitive market.

This formulation assumes that there exists a benevolent regulator that tries to maximize the overall welfare while simultaneously covering the fixed charge of the network. The formulation assumes that the marketer procures the gas at marginal cost which corresponds to a perfectly competitive production market. Alternative assumptions are possible. A perfectly competitive gas production is a quite reasonable in North America, but not in Europe. Whatever the assumption of competition on the production side, Ramsey-Boiteux introduces access charges that are specific to the consumer segment. The consumer which values gas more pays more. This is price discrimination but it is accepted in this context because of the objective pursued, namely an efficient pricing of the infrastructure. In U.S. parlance, the discrimination is not undue. We do not discuss this legal and economic issue here.

Consider the formulation given in (33) and the transmission charges τ^1 and τ^2. The (perfect competition) equilibrium conditions of the rest of the gas market can be written as

$$\begin{array}{l} p_j(d_j) = bp + \tau^j \\ \sum d_j \;\; = q \end{array} \tag{34}$$

where $bp = \partial E/\partial q$ is the border price in the one producer case. This is a square system, which means that the production and demand are entirely determined by bp, τ^1 and τ^2. The Regulator is only responsible for choosing τ^1 and τ^2 while the market will select bp on the basis of q. The Regulator therefore optimizes a criterion that effectively depends on d_j and q by playing on the τ^j. Assume a quadratic cost function EC, then bp is affine. Because we also assumed affine demand functions, the dependence of all variables d on τ^1 and τ^2 is affine. The maximization problem of the Regulator is thus convex. Economists have elaborated at length on the analytic solution of this problem.

The same reasoning could have been made if marketers behaved à la Cournot. The relationship (32) would have been replaced by Cournot equilibrium equations, that is, by replacing the price by the marginal revenue. These resulting expressions would have been affine. The problem of the Regulator would have been different from an economic point of view, but its mathematical structure would have remained unchanged. In both cases, Ramsey pricing is amenable to an analytic solution. Things become much more complex when one turns to a more detailed physical model where the square system of equation (34) is replaced by a complementarity problem.

5.5 Applying Ramsey-Boiteux to the example

The above reasoning can be considered in the more general case of our example. Assume as before that the transport charges (τ_{js}) differentiated by customer and season are known. All the other variables of the market are determined by the equilibrium conditions that describe the behavior of all agents except the transporter. Specifically one defines a restricted equilibrium subproblem RESP(τ) consisting of the following complementarity conditions

- Producer's behavior (10)
- Storage operation behavior (12)
- Consumer behavior (13)
- Marketers behavior (14) to (17)
- All balance inequalities (9.2) to (9.10) holding as equalities.

One notes that the pipeline operator equations (11) that involved the transport charges τ_s are not part of the subproblem. They have been replaced in RESP(τ) by exogenous assumptions on the τ. The result is a well defined restricted equilibrium subproblem RESP(τ) parametrized by the τ_{js}.

RESP(τ) is a complementarity problem, which in this case is equivalent to an optimization problem. It has a convex set of solutions which reduces to a single point when the marginal cost of the producers and the demand functions are affine and non-constant. It is thus possible to define the Ramsey pricing problem using the same philosophy as before: the Regulator selects the τ_{js} in order to maximize a function that depends on the d_{js} and $q_{\ell s}$. While the objective function is concave in these variables, it is no longer concave in the τ_{js}. The relation between the former and the latter is indeed piecewise affine in this case because it is the solution of a linear complementarity problem that is parametrized in τ. This problem is a mathematical programming problem subject to equilibrium constraints (MPEC) as discussed above (see [16]). Note that the formulation can encompass different variants of the restricted equilibrium subproblem. Specifically, there is no difficulty accommodating Cournot marketers instead of perfectly competitive marketers. The variants on arbitrageurs that we discussed in this problem can also be included.

6 Cournot Producers

6.1 A first model [36]

Both the former "gas companies" and the gas producers had market power in the pre-restructuring European market. In contrast gas producers can be seen as largely competitive in the US. The study of market power in the European gas sector through complementarity problems began in Norway and combined both economic analysis and computational methods. Specifically, [36] modeled the European gas market under three assumptions of competition, namely perfect competition, monopoly, and the now standard Cournot assumption. By comparing the results obtained to observation, they concluded that the Cournot model was a realistic representation of the European market of the time. Mathiesen ([34] and [35]) also showed how complementarity problems could be used to solve equilibrium models. We begin our discussion of the market power of the producers by casting this early work in our example that we simplify somewhat further. Consider a hypothetical gas company (that is, a company that bundles merchant, transmission and storage activities) operating in the pre-restructured period. It is regulated at cost and can only charge the sum of the procurement cost and a fixed mark-up that represents its average costs and some previously agreed upon margin. We let $ac^{\ell j}$ be this mark-up when the company procures gas at producer ℓ's location and sells it to market j. Neglect storage operation and assume a single season. Let p_j be the price in market j. A producer ℓ selling to the consumer market j receives a netback $p_j - ac^{\ell j}$ as shown in Figure 6.

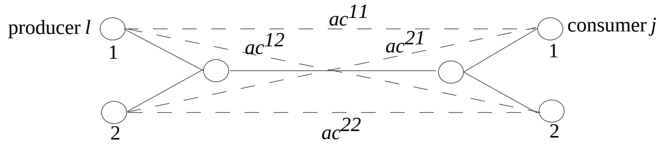

Fig. 6. No storage, fixed gas company margin

The behavior of the Cournot producer 1 can then be described by the following optimization problem

$$\begin{aligned} \max_{md_1^1, md_2^1} \; & p_1(md_1^1 + md_1^{-1})md_1 + p_2(md_2^1 + md_2^{-1})md_2^1 \\ & -ac^{11}md_1^1 - ac^{12}md_2^1 - EC_1(md_1^1 + md_2^1) \\ & \text{s.t.} \quad md_1^1 \geq 0, md_2^1 \geq 0 \end{aligned} \tag{35}$$

where the optimization is carried out on the variables md_1^1 and md_2^1, keeping the sales md_1^{-1} and md_2^{-1} fixed. [36] formulated and solved this problem on a bipartite transportation network with European producers ℓ being the left-hand nodes and European markets being the right hand nodes. The *ac* gave the transportation costs. Different extensions of Mathiesen et al.'s work were made that ultimately led to the GASTALE model mentioned in the introduction ([3]). Golombeck et al. (1995) examined the impact of the introduction of the first European Gas Directive by assuming that it would lead to arbitrage between gas prices inside the border of the European Union. In other words, arbitrageurs would trade gas between the different border points so as to eliminate the price differences that would not be justified by transporting costs. [26] also examined the impact of abolishing export monopolies in the exporting countries. In all these studies, the marketing company was represented by an exogenously given overall cost and margin that we noted *ac*. In contrast, GASTALE introduced market power both at the producer and marketer side. The representation of the latter was simplified with respect to Gabriel et al. [21] in order to make the example more tractable. It does so by implementing an oligopolistic version of the economic notion of double marginalization [27],[28]. See [41] for a discussion for the monopoly case.

6.2 Double marginalization and the GASTALE model

The structure of the European gas market suggests that both the producers and the marketers have market power. The question is whether this duality of market power can be accommodated in computational models. GASTALE extends Golombeck's model to account for this phenomena [3].

Mathiesen et al.'s original model briefly recalled above assumes that the marketers simply add a mark-up to the price that they get from the producers. In other words the margin between the price paid by the consumer and the marginal cost of the producers is shared by the producers and the marketers but the part of the latter is fixed. This is the case when one assumes that all transportation and storage costs are exogenously given and the profit of the marketer is regulated. The price charged by a marketer to a consumer is thus equal to the price at the wellhead plus the sum of the price of transportation and storage including some regulated margins. [3] consider an extension of this view where the marketers behave competitively or à la Cournot. Specifically these authors assume a given number of identical marketers in each market that equally share the demand in that market. All segments are served and hence each marketer sells an equal quantity to each segment. Using this property, Boots et al. can relate the prices charged to the different segments of the final demand to the price charged by the producer to the marketers. Their model is a mix of computational and analytical modeling. The derivation of the demand curve seen by the producers is analytical and relies on the assumption of symmetry of the marketers. The exertion of market power by the producers is computational and directly related to the previous work of [36] and [25]. An interesting objective is to remove the analytical part of this model to make it purely computational. This is necessary if we want to do away with the assumption of symmetric marketers. We shall see that Boots et al.'s approach can in principle be extended by assuming non-identical marketers that behave à la Cournot but at the price of additional computational difficulties. We consider two cases depending on whether the producers behave à la Cournot or à la Bertrand.

6.3 Bertrand producers and competitive or Cournot marketers

Following the standard reasoning of double marginalization we assume that the marketers take the border price bp_s^k as given and that they can buy unlimited quantities at that price. Natural gas is normally considered as an homogeneous product after pretreatment at the well or at the beach (the "border" in *bp*). This suggests representing the competition of the producers à la Bertrand. The producer ℓ that sells to a marketer k at the lowest price gets all the demand in that market. If several producers sell to a marketer they do it at the same price and equally share the demand of that market. This complies with our noting bp_s^k as the price paid by marketer k "at the border" in season s. Given the bp_s^k, one can define a restricted equilibrium subproblem RESP(bp) that represents the behavior of the rest of the market by assembling the complementarity conditions that describe

- The pipeline operator behavior (11)
- The storage operator behavior (12)
- The consumer behavior (13)
- The marketer behavior (14) to (17)
- All balance inequalities (9.2) to (9.10) holding as equalities.

Only the relation (10) describing the behavior of the producers is left out. It is replaced by taking the bp_s^k as parameters. Because of the integrability of the demand functions, RESP(bp) is a complementarity problem that is equivalent to an optimization problem. Introduce the notation

$$mq_s^k = \sum_{\ell} mq_{\ell s}^k$$

to denote the total demand of gas by marketer k in season s. This value can be derived from the solution of RESP(bp). It is unique for each vector bp_s^k if one assumes affine demand functions as we did throughout the paper (affine demand functions are a sufficient but non necessary condition for this result). Then, let $mq_s^k(bp)$ be the demand of gas of marketer k in season s, as a function of the prices bp_s^k found as part of the solution of RESP(bp). Inserting this solution in the profit function of the producers, it is possible to define a new Nash equilibrium problem whereby the producers select the border prices $bp_{\ell s}^k$ at which they sell gas to the marketers in order to maximize their profit. The resulting problem is an equilibrium problem subject to equilibrium constraints but of a type that to the best of our knowledge has not been mentioned let alone studied in the literature. The first-stage competition is of the Bertrand type while the second stage is Cournot. The natural question is whether this model would be relevant in practice. We already indicated that production is competitive in the US so that this model would thus add very little. In contrast there is much talk of the emergence of “gas to gas competition” in the oligopolistic European gas market. A Bertrand competition, where, producers compete in price would thus be worth exploring. The interest of the problem is that, in contrast with all the other models discussed in this paper, Bertrand competition for homogeneous products cannot be modeled through complementarity formulations.

We do not explore this problem any further and turn instead to the more standard formulation where producres have limited possibilities for exerting market power through prices but do so through quantities. This leads to a full two-stage Cournot model.

6.4 Cournot producers and competitive or Cournot marketers

In order to adapt the above formulation to arrive at a problem where both stages are Cournot, consider the case where marketers are not given a certain border price bp_s^k but an import quantity $mq_{\ell s}^k$. In other words producers behave strategically by restricting their sales to marketers. It is easy to see that one can restate the restricted equilibrium subproblem to accommodate this new situation where quantities are the strategic variables. Consider the restricted equilibrium subproblems RESP(mq) consisting of
- The pipeline operator behavior (11)
- The storage operator behavior (12)
- The consumer behavior (13)
- The marketer behavior (14) to (17)
- All balance inequalities (9.2) to (9.10) holding as equalities.

Again the relation (10) describing the behavior of the producer is left out and is replaced by an assignment of $mq_{\ell s}^k$.

This subproblem is again a complementarity problem, which is equivalent to an optimization problem. It has a convex set of solutions which is unique when the marginal cost of the producers are affine and non-constant. Let bp_s^k be the price of the gas found in relation (15). This price, bp_s^k is the marginal value of the gas sold to marketer k in season s, that comes out as a solution of that suproblem. It is thus the price at which marketer k is willing to pay for the gas,when offered the quantities $mq_{\ell s}^k$. We can thus define the mapping $bp_s^k(mq_s)$. This allows one to define a new Nash equilibrium problem for the producers whereby they select the quantities $mq_{\ell s}^k$ that they sell to the marketers. This is stated in

$$\max \sum_{s} \Big[\sum_{k} bp_s^k(mq_{\ell s}^k, mq_{-\ell s}^k) mq_{\ell s}^k - EC_{\ell s}(\sum_{k} mq_{\ell s}^k) \Big] \tag{36}$$

$$mq_{\ell s}^k \geq 0, mq_{-\ell s}^k \text{ fixed.}$$

There is one such intertemporal problem for each producer. The collection of these problems for the different producers and the search of a set of $mq_{\ell s}^k$ that simultaneously solves all of them is an equilibrium problem subject to equilibrium constraints (EPEC). Again, there is no real difficulty accommodating perfectly competitive marketers instead of Cournot marketers or any mix of assumptions that we have seen. The difficulty is indeed to solve such a problem.

7 Conclusions

This paper surveys some work as well as points out work that remains to be done. It considers essential problems brought about by the restructuring of the gas industry in Europe and North America for which one has relatively little knowledge and understanding. We can improve our insight of this market by modeling it on the basis of standard economic assumptions. Models of industrial organization raising questions of direct relevance to the gas market flourish in industrial economics. As it is often the case, their results differ drastically depending on their assumptions. This is confirmed by numerical experiments. As one says "the devil is in the details". The problem is that the devil has considerable potential in the important area of natural gas. It is important to add to the insight provided by economists by also exploring these questions experimentally, in this case computationally. Because of the novelty of the market, there are currently little data in Europe to validate these models. In contrast the restructured US gas market has accumulated several years of experience. This validation process is especially interesting since many of the models arising from industrial economic concepts also turn out to be quite difficult in mathematical programming terms.

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