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ENDOGENOUS R&D SYMMETRY IN LINEAR DUOPOLY WITH ONE-WAY SPILLOVERS

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Abstract

A duopoly model of cost reducing R&D-Cournot competition is extended to study the endogenous timing of R&D strategic investment. Under the assumption that R&D spillovers only flow from the R&D leader to the follower, sequential and simultaneous play at the R&D stage are compared, in order to assess the role of technological externalities in stimulating or attenuating endogenous firm asymmetry. The only timing structure of the R&D stage sustainable as subgame-perfect Nash equilibrium involves simultaneous play and zero spillovers.

Keywords: Endogenous symmetry; Endogenous timing; Stackelberg equilibrium.

JEL classification: C72; D43; L11; L13

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1. Introduction

Many studies of strategic R&D adopt multi stage games in which firms' prior investments lower the cost of production in the product market. The model by d'Aspremont and Jacquemin (1988) is a leading example for the two-stage, two-firm case. In the first stage firms reduce their initial unit costs by investing simultaneously in R&D. R&D generates an external effect (spillover), a fraction of each firm's autonomous cost reduction flowing without payment to the rival. In the second stage, firms engage in Cournot competition in the product market, given their effective unit cost reductions. Many other studies on this field of research build on this standard framework.[1]

Issues in these studies are usually restricted to symmetric equilibria. Following Henriques (1990), Amir and Wooders (1998) show that the symmetric non cooperative equilibrium of the d'Aspremont and Jacquemin model is sometimes unstable, however. In this case two other asymmetric and stable equilibria must also exist. These latter might represent the appropriate benchmark of analysis.

The main concern in this paper is endogenous asymmetry of firms competing in the research activity. The study here claims that differences in firms' R&D levels, besides resulting from the instability of the model, might also reflect the endogenous emergence of strategic roles for firms. For instance, firm asymmetry might reflect R&D leadership, and represent the outcome of a sequential game where -say- the larger firm moves first in the R&D stage and acts as a Stackelberg leader. The issue of *ex post* asymmetry is hence addressed in terms of endogenous assignment of both a timing structure (simultaneous or sequential) and a players' role configuration (leader,

[1] The literature on strategic R&D has been pioneered by Ruff (1969). Other seminal contributions are Dasgupta and Stiglitz (1980) and Spence (1984). Among more recent studies, see Kamien *et al.* (1992), Suzumura (1992), and Hinloopen (2003).

follower) to a given R&D game.

Studying the strategic positions of firms as an equilibrium phenomenon is motivated by the inadequacy of the Stackelberg equilibrium as solution concept. As is shown by Gal-Or (1985), first or second mover advantages in a Stackelberg game occur when actions are strategic substitutes or complements, as in quantity competition with substitute goods and in price competition with differentiated goods, respectively. So providing a sequential game with an exogenous timing structure may not be justified when competitors are assumed to be *ex ante* identical.

Many studies have tried to overcome this flaw.[2] Hamilton and Slutsky (1990) propose a foundation of the Stackelberg equilibrium concept in terms of endogenous timing of firms' actions. Given a basic duopolistic game, they construct an extended framework by adding a precompetitive stage in which leadership is assigned. Namely, at the precompetitive stage, firms simultaneously commit to move early or late in the subsequent basic game. The equilibrium of the resultant game induces a pair of actions in the basic game as well as the order of moves according to which the basic game itself is played.

The present study compares sequential and simultaneous R&D by introducing two games of strategic R&D investments, each involving Cournot competition in the product market. The former is a two stage game with simultaneous play at the R&D stage. The second one is a three stage game with sequential play and perfect information in the R&D phase. The equilibrium concept introduced by Hamilton and Slutsky is used to address the issue of endogenous timing of R&D decisions.

Endogenous timing of R&D decisions has been already studied by Amir *et al.*

[2] Many contributions reinterpret the Stackelberg solution as a special case among a large family of equilibria of a certain class of simultaneous games. See, for instance, Saloner (1987) and Maggi (1996).

(2000). For a version of the d'Aspremont and Jacquemin model that allows for differentiated products and firm specific spillovers, they identify a partition of the parameters space in terms of the equilibrium timing structure that obtains. When the ratios of own spillover rate over demand cross-slope is high (low) for both firms, then only sequential (simultaneous) R&D is observed, with both strategic role configurations. When only one firm's ratio is sufficiently large, sequential play in the R&D stage prevails, with that firm acting as the first mover.

This study departs from the existing literature because of the central assumption here that spillovers only flow from the R&D leader to the follower. Firms' research activity is supposed to generate external effects only when R&D investments are sequential. In this case spillovers are unidirectional and emanate from the first to the second mover. Conversely, when firms invest simultaneously, spillovers are assumed to be zero.

This specification is meant to reflect the different attitudes towards learning and imitation that leaders and followers may display. It gives the role of leader a primary position in the dissemination of technological progress. It also fits well the temporal connotation that leadership assumes in traditional oligopoly theory. Spillovers here take place in the inventive output through imitation. The observation of rivals' results is a precondition for imitation. In this sense, benefits from spillovers should be precluded to a first mover. And assuming that the product market opens as soon as R&D investments are undertaken, implies no mutual observation of inventive results in the case of simultaneous R&D, thereby inducing zero spillovers.

Empirical evidence of advantages accruing to latecomers is provided, among others, by Tellis and Golder (1996).[3] Technological leapfrog of early movers driven by

[3]Numerous examples include the cases of Procter and Gamble in the disposable diaper market

knowledge externalities occurred, for instance, in the global semiconductor industry, where Samsung, Hyundai, and LG exploited spillovers to catch up with their American rivals.[4]

As specific as it is, the spillover mechanism adopted here might be associated to a large class of research activities, independently of both their dimension and degree of *technical* substitutability. It applies more closely to development stages of process innovations, in which case firms' absorptive capacities[5] play a minor role whereas imitation is easier and faster compared to the case of pure research. Kamien *et al.* argue that the R&D process associated with a symmetric spillover structure (each firm benefits from knowledge leakages flowing from the rivals) has a multidimensional heuristic nature. Firms follow different research approaches each involving trial and error. Conversely, symmetric spillovers could not be generated by a one-dimensional R&D process, that is, in the case in which research activity is undertaken accordingly to a unique approach which is common to all firms. To the extent that there exists a single research path followed by all the competing firms, spillovers can only flow from the more R&D intensive firm to the rivals. Such is the case of the strategic investment game with simultaneous moves and one-way spillovers studied by Amir and Wooders (1999), which gives rise to asymmetric equilibria only.[6] It is worth emphasizing that the R&D process underlying the one-way spillover structure in this paper is not necessarily one-dimensional. Spillovers here take place over time through observation of the rival's results and flow from the first to the second mover rather than from

(in which Chux was the first to move), Sony and JVC in the videorecorder market (pioneered by Ampex), and Coca Cola's Tab and Diet Pepsi in the diet cola mass market (where the first to enter was Royal Crown). Using historical analysis on fifty product categories, the authors find that the rate of failure of early leader brands is 47 percent.

[4]See Cho, Kim and Rhee (1998).

[5]For a definition of absorptive capacity, see Cohen and Levinthal (1989). Also see Kamien and Zang (2000).

[6]This result is due to the discontinuity of firms' reaction functions in R&D space along the diagonal which, in turn, reflects the mentioned assumption about externalities.

the more to the less R&D intensive firm. This implies zero spillovers with simultaneous play at the R&D stage, even in the presence of asymmetric investments and/or *technical* complementarity[7] of research projects. On the other hand, sequential play induces one-way spillovers, even when research projects are *technically* substitutes and/or multidimensional.

As a consequence, the main result of the present study is different from Amir and Wooders, with the only timing structure sustainable in subgame-perfect equilibrium involving simultaneous play in R&D choices, zero spillovers, and firm symmetry. As will be clarified later, for the framework discussed here, a symmetric equilibrium always exists in the case of simultaneous play at the R&D stage. When the model is globally stable under Cournot best reply dynamics, the symmetric equilibrium is unique. So simultaneous play represents the benchmark for firm symmetry.

Unlike the case discussed by Hamilton and Slutsky, and Amir *et al.*, R&D leadership in this study is not trivially preferred to simultaneous play. Since spillovers are unilateral and take place over time, firms' payoff functions are different under simultaneous and sequential play. Hence, the corresponding Cournot-Nash and Stackelberg equilibria cannot be compared in the usual way, as they refer to two *different games*, with different payoff functions. Intuitively, R&D leadership is attractive for small spillovers only, whereas the opposite applies to followership. For sequential R&D to be sustainable in equilibrium, an interval of the spillover parameter must exist, within which both the leader and the follower are better off than in the case of simultaneously

[7] Following Katsoulacos and Ulph (1998), research investments are said pure substitutes from the *technical* point of view when firms undertake identical innovative activities, just duplicating each other's research. Conversely, they are pure complements when the results achieved by each firm add to that of its rivals. The former case is implied by one-dimensional R&D processes, while the latter encompasses multidimensional R&D processes and represents the usual benchmark for most recent studies of strategic R&D, including d'Aspremont and Jacquemin.

play. In the remainder of the paper it is shown that this particular interval is empty. Only simultaneous investment in R&D can be observed in equilibrium, independently of the magnitude of the spillover rate.

The remainder of the paper is organized as follows. Section two describes the basic framework. Section three introduces the issue of endogenous timing, states the main assumptions, and characterizes the equilibrium of the game. Section four concludes. The appendix reports proofs of all the propositions and discusses an extension of the model to the case of multiple equilibria under simultaneous play.

2. The basic game

This section introduces two games, denoted by G_{sim} and G_{seq}, each assuming Cournot competition in the product market. For the sake of the analysis, G_{sim} and G_{seq} are thought of as two different versions of a basic game of cost reducing R&D-product market competition, representing the benchmarks for the timing structure they respectively involve. G_{sim} is a two stage game with simultaneous moves in the R&D stage and coincides with the original model by d'Aspremont and Jacquemin, with the major exception of a zero spillover rate. G_{seq} is a three stage game with sequential moves and perfect information in the R&D stage, with the first mover acting as a Stackelberg leader and giving unidirectional spillovers.

2.1. G_{sim}: simultaneous moves in the R&D stage and zero spillover rate

Consider a two-stage duopoly game in which firms 1 and 2, before engaging in Cournot competition, can exploit a cost reducing opportunity by investing resources in R&D. In the first stage, R&D levels are chosen simultaneously and non cooperatively; subsequent Cournot competition is subject to firms' first stage R&D levels.

Firms are *ex ante* identical. Namely, they use the same linear production technology, with unit cost $c > 0$, share the same R&D opportunities, and face the same inverse demand function $P(q_1 + q_2) = a - (q_1 + q_2)$, where q_i denotes the final output to firm $i = 1, 2$.

The innovative process is taken as deterministic.[8] It generates autonomous cost reductions under decreasing returns to R&D investments. Roughly speaking, when defining the optimal R&D level, firms trade off the fixed cost of the innovation against a marginal cost reduction in the product market.

With simultaneous play at the R&D stage, R&D outputs are perfectly appropriable, *i.e.* the spillover rate is equal to zero. Hence, given a pair of R&D investments (x_1, x_2), the cost reductions accruing to firms just depends on own private expenditures. Namely, the cost reduction for firm 1 is equal to $\min\left\{\lambda\sqrt{x_1}, c\right\}$, $\lambda > 0$. $A_1 \equiv \left\{x_1 : 0 \leq x_1 \leq \left(\frac{c}{\lambda}\right)^2\right\}$ identifies firms 1's undominated R&D action set (similarly for firm 2).[9]

A strategy for firm i is a pair $S^i_{sim} \equiv (x_i, q_i)$, with $x_i \in A_i$, and $q_i : A_i \times A_j \to R_+$; $i, j = 1, 2; i \neq j$. Attention is restricted to subgame-perfect Nash equilibria (SPNE); hence, each firm considers its own overall profit conditionally on the second stage Cournot equilibrium.[10]

[8] For a stochastic version of the d'Aspremont and Jacquemin model see Hauhenscild (2003).

[9] The specification of the R&D technology used here is equivalent to that introduced by D'Aspremont and Jacquemin, according to which, in the first stage of the game, firm 1 chooses a cost reduction level y_1, facing an R&D cost of $\frac{\gamma}{2}y_1^2$, $\gamma > 0$. Given the pair of actions (y_1, y_2), the effective cost reduction of firm 1 is equal to $y_1 + \beta y_2$, where β is the spillover parameter. Therefore, the R&D production function $y_1 = \lambda\sqrt{x_1}$ represents the inverse mapping of the R&D cost function used by D'Aspremont and Jacquemin, with $x_1 = \left(\frac{1}{\lambda}y\right)^2$ and $\lambda = \sqrt{\frac{2}{\gamma}}$.

[10] Production levels are strategic substitutes. Moreover, under Assumption 1, stated below, the Cournot Nash equilibrium of the product market subgame is unique.

The Cournot Nash output and profit for firm 1 are

$$q_1^C(x_1, x_2) = \frac{a-c+2\lambda\sqrt{x_1}-\lambda\sqrt{x_2}}{3}, \text{ and} \tag{1}$$

$$\Pi_1^C(x_1,\ x_2) = \frac{(a-c+2\lambda\sqrt{x_1}-\lambda\sqrt{x_2})^2}{9} - x_1 \tag{2}$$

respectively (and similarly for firm 2). In section 3.2 below the assumption is made that $a > 2c$, under which the validity of (1) and (2) is ensured.

2.2. G$_{seq,i}$: three-stage game with sequential moves in the R&D stage

Consider the same framework as for the two-stage game, but think of the R&D phase as having two stages, *i.e.,* displaying sequential play with perfect information, with firm i playing as first mover and firm j as second mover, $i = 1, 2,\ i \neq j$.

In the first stage of $G_{seq,i}$, firm i sets its R&D expenditure as a Stackelberg leader. In the second stage, firm j, upon observing i's R&D level, reacts. Then the product market instantaneously opens and Cournot competition takes place.

With sequential play at the R&D stage, only the second mover can learn from, or imitate, its rival. So R&D spillovers flow from the first to the second mover only. For any pair of R&D expenditures $(x_{i,}\ x_j)$, the effective cost reductions for firm i and j are

$$cr_i(x_i) = \min\{\lambda\sqrt{x_i}, c\}, \text{ and} \tag{3}$$

$$cr_j(x_i, x_j) = \min\left\{\lambda\left(\sqrt{x_j} + \theta\sqrt{x_i}\right), c\right\} \tag{4}$$

respectively, where θ denotes the spillover parameter, $\theta \in [0, 1]$.

The R&D undominated action sets of firm i and firm j are

$$A_i = \left\{ x_i : 0 \leq x_i \leq \left(\frac{c}{\lambda}\right)^2 \right\}, \text{ and } A_j \left\{ x_j : 0 \leq x_j \leq \left(\frac{c}{\lambda} - \theta\sqrt{x_i}\right)^2 \right\} \tag{5}$$

respectively. A strategy for firm i is a pair $S^i_{seq,i} \equiv (x_i, q_i)$, with $x_i \in A_i$, and $q_i : A_i \times A_j \to R_+$. A strategy for firm j is a pair $S^j_{seq,i} \equiv (r_j, q_j)$, with $r_j : A_i \to A_j$, and $q_j : A_i \times A_j \to R_+$. Strategies $S^j_{seq,j}$, and $S^i_{seq,j}$ for game $G_{seq,j}$ are defined analogously.

Attention is restricted to SPNE. Hence, each firm sets its own R&D expenditure conditionally on the Cournot equilibrium of the market subgame. The Nash output and profit for firm i are

$$q_i(x_i, x_j) = \frac{a - c - \lambda\left(\sqrt{x_j} + (\theta - 2)\sqrt{x_i}\right)}{3}, \text{ and} \tag{6}$$

$$\Pi_i(x_i, x_j) = \left(\frac{a - c - \lambda\left(\sqrt{x_j} + (\theta - 2)\sqrt{x_i}\right)}{3}\right)^2 - x_i \tag{7}$$

respectively. For firm j, they are equal to[11]

$$q_j(x_i, x_j) = \frac{a - c + \lambda\left(2\sqrt{x_j} + \sqrt{x_i}(2\theta - 1)\right)}{3}, \text{ and} \tag{8}$$

$$\Pi_j(x_i, x_j) = \left(\frac{a - c + \lambda\left(2\sqrt{x_j} + \sqrt{x_i}(2\theta - 1)\right)}{3}\right)^2 - x_j. \tag{9}$$

3. Endogenous timing

In this section the question of endogenous timing of R&D investments is addressed according to the Hamilton and Slutsky approach. Starting from the basic game in-

[11] Analogously to what reported for game G_{sim}, Assumption 1 is Section 3.2 below ($a > 2c$) ensures the validity of (6), (7), (8) and (9).

troduced in section 2, an extended game is constructed by adding a preplay stage in which timing decisions are taken, with firms' preplay actions consisting of timing announcements for R&D investments. The SPNE of the extended game induces a pair of actions in the basic game as well as the timing structure according to which the R&D phase of the basic game is played.

3.1. The extended game and the preplay stage

Take the basic game to be as described in section 2 above. At the preplay stage, firms simultaneously announce whether they move early or late in the R&D stage of the subsequent basic game. The action set of each player is denoted $T \equiv \{E, L\}$, where E stands for *early*, and L for *late*.

The extended game is obtained by addition of the preplay stage to the basic game. Once timing announcements are made, they become common knowledge and the specific version of the basic game they induce is played. If the two firms select identical actions (*i.e.* (E, E) or (L, L)), then the R&D stage of the basic game exhibits simultaneous play, that is G_{sim} is played. If they choose different actions (*i.e.* (E, L) or (L, E)), then the R&D stage of the basic game is played sequentially, that is G_{seq_i} is played, with i being identified by the specific sequential structure that obtains.

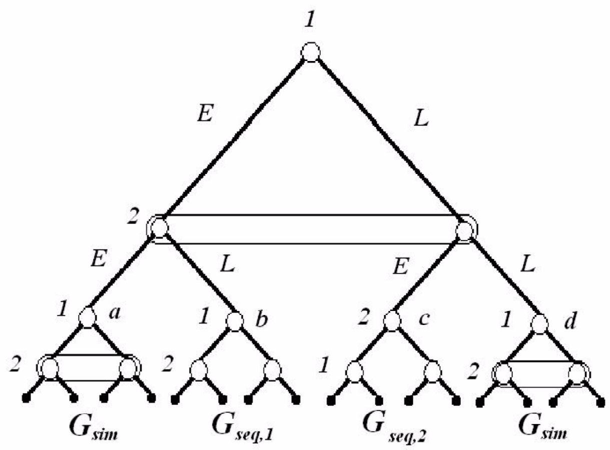

Figure 1: The extended game

Figure 1 describes the extensive form of the extended game.[12] At each node $i = a, b, c, d$, a different version of the basic game is played according to the particular timing structure identified by the path to i.

A strategy for firm i in the extended game is a pair (t_i, ϕ_i), with $t_i \in T$, and $\phi_i : T^2 \to \{S^i_{sim}, S^i_{seq,i}, S^i_{seq,j}\}$, where S^i_{sim}, $S^i_{seq,i}$, and $S^i_{seq,j}$ are defined as in subsections 2.1 and 2.2 above. Attention is restricted to SPNE. Hence, for a given timing decision of the rival, when making a timing announcement in the preplay stage, either firm considers its own payoff as defined in the SPNE of the specific version of the basic game that would obtain.

3.2. Assumptions

Since the payoffs of G_{sim} represent the limit of those of G_{seq} as the spillover rate tends to zero, the following assumptions are stated relative to the case of sequential

[12] An oval shape surrounding two nodes defines an information set. To simplify the description of the game tree, figure 1 considers only the R&D stage of each version of the basic game, the product market stage being invariant and involving Cournot competition.

play in the R&D stage.

Assumption 1: $\frac{a}{c} > 2$

Assumption 1 ensures that, for any version of the basic game, the Nash equilibrium of the market subgame is unique, with both firms active on the market. Assumption 1 restricts attention to interior solutions of the market subgame thereby emphasizing the role of R&D with respect to endogenous asymmetry.

Assumption 2: If $\theta \leq 0.5$, then $\frac{a}{c} < \frac{4.5(1-\theta)}{\lambda^2}$, and if $\theta > 0.5$, then $\frac{a}{c} < \frac{9}{\lambda^2(2-\theta)}$

Assumption 2 rules out the case in which, under the SPNE of G_{sim}, both firms obtain a full cost reduction. Namely, it excludes the case in which the intersection of firm's R&D reaction curves in game G_{seq} identifies the pair of R&D levels $\left((\frac{c}{\lambda})^2, (\frac{c}{\lambda}(1-\theta))^2\right)$.[13] Observe that if Assumption 2 did not hold, then a sequential order of moves in the R&D stage would induce a Pareto dominating outcome.[14]

Assumption 3: $\lambda < \sqrt{1.5}$

Assumption 3 ensures that the R&D stage of G_{sim} has a unique symmetric equilibrium which is globally stable under best reply dynamics. The assumption at hand

[13] Recall that firms' reaction functions in G_{sim} and G_{seq} are different. Figure 3.1 describes the linearized R&D reaction functions for game G^1_{seq}. If $\theta < 0.5$ both of them slope down, and Assumption 2 rules out the case in which the reaction function of the follower (firm 2) just consists of the curve $\frac{c}{\lambda} - \theta\sqrt{x_1}$, along which it achieves a full cost reduction for any expenditure level of the leader. If $\theta > 0.5$, part of the reaction function of the follower slopes up; in this case, Assumption 2 prevents firm 1 from reacting to an expenditure level of $\frac{c}{\lambda} - \theta\sqrt{x_1}$ by playing $\left(\frac{c}{\lambda}\right)^2$, thereby obtaining a full cost reduction.

[14] In this case, both firms would obtain a full cost reduction under the SPNE of G_{seq} as well. The first mover could not do better than choosing the same R&D level as that identified by the intersection of the two R&D reaction functions of G_{seq}, *i.e.* its maximal R&D expenditure, since otherwise it would reach a lower profit-indifference level for any R&D decision of the rival. It follows that the leader of G_{seq} would be better off than a player of G_{sim}, with both of them obtaining a full cost reduction without receiving spillovers. By contrast, the follower of G_{seq} would be better off than a player of G_{sim} since, for a given production cost of the rival, the spillovers flowing from the leader would allow it to undertake a smaller R&D expenditure for any given cost reduction level. In this case, the extended game has three SPNE equilibria. One equilibrium involves simultaneous play in the R&D stage, with both firms moving late. The other two equilibria involve sequential play with both order of moves. These latter clearly dominate (weakly) the former.

is equivalent to requiring that the product of the slopes of the two linearized R&D reaction functions in G_{sim} is smaller than one. Imposing an analogous restriction on the R&D reaction functions of G_{seq} amounts to assuming[15]

$$\theta > \theta_s \equiv \frac{6\lambda - \lambda^3 - \sqrt{(\lambda^6 - 36\lambda^2 + 81)}}{3\lambda}. \tag{10}$$

The two conditions are not equivalent, since for any $\lambda \in \left(\sqrt{1.5}, 1.5\right)$,[16] there exist admissible values of θ satisfying (10). Namely, (10) does not imply Assumption 3, so that assuming $\theta > \theta_s$ would not guarantee the stability of the symmetric SPNE of the R&D game of G_{sim}. Nevertheless, the RHS of (10) is negative whenever Assumption 3 holds. Namely, Assumption 3 implies (10), so that $\theta < \theta_s$ obviously implies $\lambda > \sqrt{1.5}$. The effect of relaxing Assumption 3 is discussed in the appendix, where the hypothesis is made that $\theta < \theta_s$. When the symmetric equilibrium with simultaneous play is unstable, other stable equilibria obtain, implying asymmetric investment in R&D. By contrast, under Assumption 3, simultaneous R&D represents the benchmark for firm symmetry.

In what follows, the SPNE of games G_{sim} and G_{seq} are characterized. Comparing the respective payoffs leads to the main results of this study, which is summarized in Proposition 3.2 below.

[15] For a discussion about the stability of the Cournot equilibrium see, among others, Seade (1980), and Dixit (1987). In the present setting, stability is guaranteed whenever $\mu_1^1\mu_2^2 > \mu_1^2\mu_2^1$, where $\mu^i \equiv \frac{\partial \Pi_i(x_i, x_j)}{\partial x_i}$, and subscripts denote partial derivatives. Consider game G_{sim}, and linearize the R&D reaction functions by taking the square root of the R&D levels to be the decision variables. The slope of either firm's linearized R&D reaction function is equal to $\frac{2\lambda^2}{4\lambda^2 - 9}$, that is larger than -1 whenever $\lambda < \sqrt{1.5}$. Consider next G_{seq}. For the sake of the stability analysis, the relevant case is $\theta < 0.5$. Indeed, for larger spillovers, $\mu_1^1\mu_2^2 - \mu_1^2\mu_2^1 > 0$, since $\mu_1^2 > 0$, whereas $\mu_2^1 < 0$. Given firms' linearized reaction functions in R&D space, the condition to be imposed amounts to $\left| \frac{2\lambda^2(2\theta - 1)}{9 - 4\lambda^2} \frac{\lambda^2(\theta - 2)}{9 - \lambda^2(2 - \theta)^2} \right| < 1$, that is equivalent to $\theta > \theta_s$.

[16] Assumption 1 and Assumption 2 imply $\lambda < 1.5$.

3.3. SPNE of $\mathbf{G}_{sim}$

Recall that output levels are strategic substitutes and that the market subgame has a unique Cournot equilibrium. In addition, observe that, under Assumptions 1 and 2, the overall profits of both firms, as defined in (2), (7) and (9) above, are concave in the R&D investment. So, the reaction function of firm 1 at the R&D stage is continuous and single valued and is written

$$r_1(x_2) = \arg\max_{x_1 \in A_1} \left\{ \Pi_1^C(x_1,\ x_2) \right\} = \min \left\{ 4\lambda^2 \left(\frac{a-c-\lambda\sqrt{x_j}}{9-4\lambda^2} \right)^2,\ \left(\frac{c}{\lambda} \right)^2 \right\}, \tag{11}$$

where $4\lambda^2 \left(\frac{a-c-\lambda\sqrt{x_j}}{9-4\lambda^2} \right)^2$ is the unique root of $\frac{\partial \Pi_1^C(x_1,\ x_2)}{\partial x_1} = 0$ (and similarly for firm 2).[17] (11) states that firm 1 -say-, given its rival's investment, never undertakes an R&D expenditure exceeding what is sufficient to achieve a full cost reduction, *i.e.* larger than $(\frac{c}{\lambda})^2$. Larger R&D levels are strictly dominated actions. Since R&D is cost reducing, in the absence of spillovers, R&D investments exhibit the same strategic nature as quantities. Namely, the innovative efforts are strategic substitutes and the R&D reaction functions slope down.

Under Assumptions 1-3, the game has a unique, symmetric and globally stable SPNE in which each firm spends $x^C = 4\lambda^2 \left(\frac{a-c}{9-2\lambda^2} \right)^2$, produces $q_i^C = 3\frac{a-c}{9-2\lambda^2}$, and earns $\Pi^C = \frac{(9-4\lambda^2)(a-c)^2}{(9-2\lambda^2)^2}$.

3.4. SPNE of $\mathbf{G}_{seq}$

The previous considerations about the maximization problem of the firms in G_{sim} apply to G_{seq}. Given the symmetry of the game, just consider the case in which firm

[17] Since the marginal profitability of R&D increases without bound as the R&D expenditure approaches zero, $x = 0$ cannot represent a solution to the maximization problem at hand.

1 moves first in the R&D stage (game $G_{seq,1}$).

The R&D reaction function of the second mover is defined as follows:

$$r_2(x_1) = \arg\max_{x_2 \in A_2} \{\Pi_2(x_1, x_2)\} = \min\left\{ x_2^*(x_1)\ , \left(\frac{c}{\lambda} - \theta\sqrt{x_1}\right)^2 \right\}, \qquad (12)$$

where $x_2^*(x_1) \equiv \left(\frac{2\lambda(a-c+\lambda\sqrt{x_1}(2\theta-1))}{9-4\lambda^2}\right)^2$ denotes the unique root of $\frac{\partial \Pi_2(x_1,x_2)}{\partial x_2} = 0$.

Observe that R&D best response functions can be linearized by taking the square root of the R&D level to be the decision variable. According to (12), the linearized best response curve of the follower corresponds to the line $\frac{c}{\lambda} - \theta\sqrt{x_1}$ whenever the $x_2^*(.)$ rule induces a higher expenditure than what is sufficient to realize a full cost reduction. Along this line, the follower obtains a full cost reduction for any investment level of the leader. If $\theta < 0.5$, then the linearized best response of the follower is downward sloping. For higher spillover rates, its $x_2^*(.)$ part slopes up. An analogous argument for the first mover would point out that the R&D reaction function of the leader slopes down independently of the spillover rate. Finally, observe that when $\theta = 0$, the reaction functions of both players are the same as in the simultaneous moves case. Three examples are depicted in Figure 2.

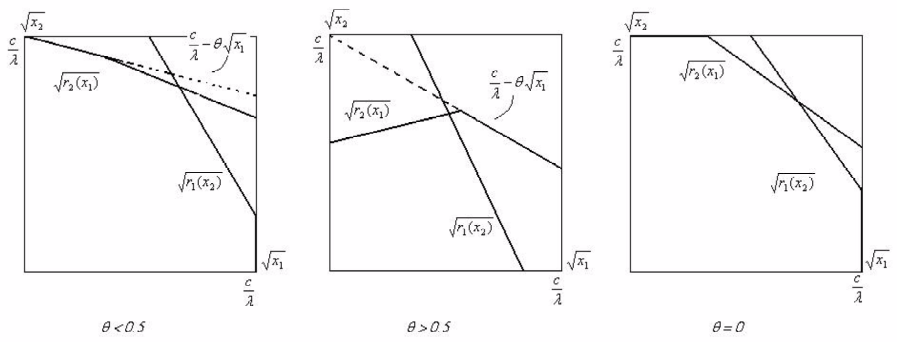

Figure 2: Linearized R&D reaction curves for game $G_{seq,1}$

As a Stackelberg leader, the first mover solves

$$\max_{x_1 \in A_1} \{\Pi_1(x_1, x_2)\} \ s.t. \ x_2 = r_2(x_1). \tag{13}$$

The following proposition describes the SPNE of game $G_{seq,1}$.

Proposition 3.1. *Let $w \equiv \frac{a}{c}$. In addition, suppose that Assumptions 1, 2, and 3 hold. Then there exist $\underline{\theta}(w,\lambda)$, $w^{+}(\theta,\lambda)$, and $w^{++}(\theta,\lambda)$ such that, in the SPNE of game $G_{seq,1}$, the leader sets its R&D expenditure as follows*

$$x_1 = \begin{cases} \left(\frac{3(a-c)(3-2\lambda^2)(2\lambda^3(1-2\theta)+\lambda(9-4\lambda^2)(2-\theta))}{9(9-4\lambda^2)^2-(2\lambda^3(1-2\theta)-(9-4\lambda^2)(\theta-2)\lambda)^2}\right)^2 & \text{if } \theta \in [\underline{\theta}(w,\lambda), 1], \text{ and } w < w^{+}(\theta,\lambda) \\ \left(\frac{c}{\lambda}\right)^2 & \text{if } \theta < \underline{\theta}(w,\lambda), \text{ and } w < w^{++}(\theta,\lambda) \\ \left(\frac{2\lambda(a-2c)}{(9-4\lambda^2)}\right)^2 & \text{if either } \theta \in \left[\underline{\theta}(w,\lambda), \frac{\lambda^2}{4.5}\right], \text{ and } w > w^{+}(\theta,\lambda), \\ & \text{or } \theta < \underline{\theta}(w,\lambda), \text{ and } w > w^{++}(\theta,\lambda) \end{cases}, \tag{14}$$

and the follower reacts as follows

$$x_2 = \begin{cases} \left(\frac{2\lambda(a-c)\left(\lambda^2\left(2\lambda^2\theta - 2\lambda^2 - 9\theta + 10 + 3\theta^2\right) - 9\right)}{\lambda^2(3\theta(3\theta + 4\lambda^2 - 12) + (108 - 40\lambda^2 + 4\lambda^4)) - 81} \right)^2 & \textit{if } \theta \in \left[\underline{\theta}(w,\lambda), 1\right], \textit{ and } w < w^+(\theta,\lambda) \\ \left(\frac{2\lambda(a - 2c(1-\theta))}{9 - 4\lambda^2} \right)^2 & \textit{if } \theta < \underline{\theta}(w,\lambda), \textit{ and } w < w^{++}(\theta,\lambda) \\ \left(\frac{c\left(9 - 4\lambda^2\right) - 2\theta\lambda^2(a - 2c)}{\lambda(9 - 4\lambda^2)} \right)^2 & \textit{if either } \theta \in \left[\underline{\theta}(w,\lambda), \frac{\lambda^2}{4.5}\right], \textit{ and } w > w^+(\theta,\lambda), \\ & \textit{or } \theta < \underline{\theta}(w,\lambda), \textit{ and } w > w^{++}(\theta,\lambda) \end{cases} \quad (15)$$

A proof is provided in the appendix.[18]

In line with intuition, the leader faces a maximal R&D expenditure only for small spillover rates ($\theta < \underline{\theta}(w,\lambda)$). More precisely, the leader is willing to achieve a full cost reduction only when R&D expenditures are strategic substitutes.[19] On the other hand, the leader induces a follower's full cost reduction only when demand is high relative to marginal costs, that is, when market size is sufficiently large ($w > w^+(\theta,\lambda)$, $w > w^{++}(\theta,\lambda)$).

3.5. The SPNE of the extended game

In this subsection, the main result of this study is presented. Sequential R&D is sustainable as SPNE of the extended game if and only if the two players agree on the role configuration it involves, that is, if and only if it Pareto dominates simultaneous play in terms of firms' profits. If this condition is not satisfied, then, at least one firm can do better by shifting to the same timing decision as the opponent, inducing simultaneous play in the R&D stage.

[18] $\underline{\theta}(w,\lambda)$, $w^+(\theta,\lambda)$, and $w^{++}(\theta,\lambda)$ are defined in the appendix.
[19] See the proof of Proposition 3.1 in the appendix.

As emphasized, G_{sim} and G_{seq} have different payoff functions. When spillovers are relatively large, possible gains for the leader, in terms of unit cost advantages, declines along with the incentives for preemptive R&D. In line with intuition, lemma 1 in the appendix states that a firm prefers to be the leader of G_{seq} rather that a player of G_{sim} only if the spillover rate is sufficiently small. By contrast, the follower's opportunity cost of private R&D increases with the spillover rate. Lemma 2 in the appendix says that a firm accepts to be the follower of G_{seq} only if the spillover rate is sufficiently large.

For sequential play to be sustainable as SPNE an interval of the spillover parameter must exist, within which both the leader and the follower are better off than they would in the case of simultaneous R&D, *i.e.*, within which both firms' profits are larger in game G_{seq} than in game G_{sim}. As summarized by the following proposition, this particular interval is empty; that is, sequential R&D never obtains.

Proposition 3.2. *Suppose that Assumptions* $1, 2$ *and* 3 *hold. Then, the only timing structure of the basic game sustainable in pure strategies SPNE of the extended game involves simultaneous play in the R&D stage and zero spillovers.*

A proof is provided in the appendix.

When inducing a sequence of moves in the R&D stage, firms' timing decisions also determine whether or not R&D generates external effects. Proposition 3.2 also says that firms, by blocking sequential play in the R&D stage, implicitly agree to set the spillover rate at zero.[20]

[20] By contrast, Amir and Wooders find that firms prefer a certain degree of R&D externalities to the case in which research is perfectly appropriable.

4. Concluding remarks

This paper has examined the question of firm diversity in terms of endogenous emergence of an appropriate timing structure for strategic R&D investments. For a linear model of cost reducing R&D-Cournot market competition, sequential and simultaneous play at the R&D stage are compared. Under the assumption that spillovers take place over time and only flow from the R&D leader to the follower, either sequential play with one-way spillovers or simultaneous play with zero spillovers can *a priori* prevail. Because of such link between the spillover mechanism and the order of play, a firm contemplating leadership faces a trade-off between commitment and appropriability of research. Intuitively, large spillovers dissuade candidate leaders. On the other hand, small spillovers prevent a firm from accepting the role of follower. For this framework, only simultaneous play with zero spillovers is observed at the R&D stage. The result provides an instance of externalities as a barrier to firm heterogeneity.[21]

A natural extension of this study is related to the literature on R&D joint ventures and would perform a comparative analysis of non cooperative and cooperative R&D. The present paper suggests that in the case in which spillovers take place over time, the relevant scenario for R&D competition is identified by the unique equilibrium of the extended game, involving simultaneous moves at the R&D stage and zero spillovers. A performance comparison in terms of effective cost reductions (including external effects) and hence consumer surplus, would require a specification of the spillover mechanism for cooperative R&D, possibly including the case of endogenous spillover rates.

[21] See, among others, Roller and Sinclair-Desgagnè (1996). An opposite interpretation of spillovers is due to Eeckhout and Jovanovic (2002). By inducing followers to free ride, spillover rates create permanent inequality among firms.

APPENDIX

A.1. Proof of Proposition 3.1

Let $w \equiv \frac{a}{c}$. Consider, w.l.o.g., game $G_{seq,1}$. So, the assumption is made that firm 1 is the R&D leader and firm 2 is the R&D follower. Firm1 solves

$$\max_{x_1 \in A_1} \Pi^1 (x_1, x_2) \;\; s.t. \; x_2 = r_2 (x_1), \tag{16}$$

where $A_1 = \left\{ x_1 : 0 \leq x_1 \leq \left(\frac{c}{\lambda}\right)^2 \right\}$ and $r_2 (x_1)$ is defined as in (12). Specifically, $r_2 (x_1) \in \left[0, \left(\frac{c}{\lambda} - \theta\sqrt{x_1}\right)^2\right]$. Suppose first, $x_2 < \left(\frac{c}{\lambda} - \theta\sqrt{x_1}\right)^2$. In this case (16) reduces to

$$\max_{x_1 \in A_1} \Pi^1 (x_1, x_2^* (x_1)) = \max_{x_1 \in A_1} \left\{ \left(\frac{\left(3-2\lambda^2\right)(a-c)+\left(6-2\lambda^2-3\theta\right)\lambda\sqrt{x_1}}{9-4\lambda^2} \right)^2 - x_1 \right\}. \tag{17}$$

Observe that $\frac{\partial \Pi^1 (x_1, x_2^* (x_1))}{\partial x_1} = \frac{\lambda\left(6-2\lambda^2-3\theta\right)\left(\left(3-2\lambda^2\right)(a-c)+\left(6-2\lambda^2-3\theta\right)\lambda\sqrt{x_1}\right)}{\sqrt{x_1}(9-4\lambda^2)^2} - 1$, which increases without bound as x_1 approaches 0. Hence, a nil expenditure level violates a marginal condition of the problem. Furthermore, under Assumptions 1 and 2, the profit function of the leader is concave and the first-order condition is sufficient. Define x_1^* as firm 1's optimal R&D expenditure. Solving (17) yields:

$$x_1^* = \min \left\{ \left(\frac{3(a-c)\left(3-2\lambda^2\right)\left(2\lambda^3(1-2\theta)+\lambda\left(9-4\lambda^2\right)(2-\theta)\right)}{9(9-4\lambda^2)^2-(2\lambda^3(1-2\theta)-(9-4\lambda^2)(\theta-2)\lambda)^2} \right)^2, \left(\frac{c}{\lambda}\right)^2 \right\},$$

where the first term in the brackets is the unique root of $\frac{\partial \Pi^1 (x_1, x_2^* (x_1))}{\partial x_1} = 0$. More specifically, $x_1^* = \frac{3(a-c)\left(3-2\lambda^2\right)\left(2\lambda^3(1-2\theta)+\lambda\left(9-4\lambda^2\right)(2-\theta)\right)}{9(9-4\lambda^2)^2-(2\lambda^3(1-2\theta)-(9-4\lambda^2)(\theta-2)\lambda)^2}$ if and only if

$$\theta > \frac{\left(w\lambda\left(3-2\lambda^2\right)-2\lambda^3+9\lambda-\sqrt{w((9\lambda^2-12\lambda^4+4\lambda^6)(w-2))+\lambda^2(52\lambda^2+4\lambda^4-279)+324}\right)}{6\lambda} \equiv \underline{\theta}(w, \lambda),$$

with $\underline{\theta}(w, \lambda)$ being an increasing function of w. In addition, $\underline{\theta}(w, \lambda) < 0.2$ for any admissible w and λ. So, the leader achieves a full cost reduction only when

R&D investments are strategic substitutes. Finally notice that $\underline{\theta}(w,\lambda) > 0 \iff w > \frac{1}{2}\frac{22\lambda^4-90\lambda^2+81}{\lambda^2(9-9\lambda^2+2\lambda^4)}$, with an admissible region of parameters existing within which $\frac{1}{2}\frac{22\lambda^4-90\lambda^2+81}{\lambda^2(9-9\lambda^2+2\lambda^4)} < \frac{4.5(1-\theta)}{\lambda^2}$ and $\theta < \underline{\theta}(w,\lambda)$. Suppose next $x_2 = (\frac{c}{\lambda} - \theta\sqrt{x_1})^2$. In this case (16) reduces to

$$\max_{x_1\in A_1} \Pi^1\left(x_1, (\tfrac{c}{\lambda} - \theta\sqrt{x_1})^2\right) = \max_{x_1\in A_1}\left\{\left(\frac{a-2c+2\sqrt{x_1}\lambda}{3}\right)^2 - x_1\right\}. \tag{18}$$

Define x_1^{**} as firm 1's optimal R&D expenditure. Solving (18) yields

$$x_1^{**} = \min\left\{\left(\frac{2\lambda(a-2c)}{(9-4\lambda^2)}\right)^2, \left(\tfrac{c}{\lambda}\right)^2\right\}. \tag{19}$$

Calculations show that

$$\Pi^1(x_1^*, x_2^*(x_1^*)) = \begin{cases} \dfrac{(a-c)^2\left(12\lambda^2-4\lambda^4-9\right)}{\lambda^2(108-40\lambda^2+4\lambda^4+12\theta(\lambda^2-3)+9\theta^2)-81} & \text{if } \theta \in \left[\underline{\theta}(w,\lambda), 1\right] \\ \dfrac{c^2\left(\lambda^2\left(81-18\theta+9\theta^2-16\lambda^2\right)-81\right)+a^2\lambda^2\left(9-12\lambda^2+4\lambda^4\right)+6ac\lambda^2\left((1-\theta)\left(3-2\lambda^2\right)\right)}{(9-4\lambda^2)^2\lambda^2} \\ \text{if } \theta < \underline{\theta}(w,\lambda) \end{cases},$$

$$\Pi^1\left(x_1^{**}, (\tfrac{c}{\lambda} - \theta\sqrt{x_1^{**}})^2\right) = \begin{cases} \frac{(a-2c)^2}{9-4\lambda^2} & \text{if } \frac{a}{c}\lambda^2 < 4.5 \\ \frac{a^2\lambda^2-9c^2}{9\lambda^2} & \text{if } \theta > 0.5 \text{ and } \frac{a}{c}\lambda^2 > 4.5 \end{cases}.$$

Observe next that $\max\left\{\Pi^1(x_1^*, x_2^*(x_1^*)), \Pi^1(x_1^{**}, (\frac{c}{\lambda} - \theta\sqrt{x_1^{**}})^2)\right\}$ is equal to:

(i) $\frac{(a-2c)^2}{9-4\lambda^2}$ if

$$\text{either } \theta \in \left[\underline{\theta}(w,\lambda), \frac{\lambda^2}{4.5}\right] \text{ and } w > w^+(\theta,\lambda), \text{ or } \theta < \underline{\theta}(w,\lambda) \text{ and } w > w^{++}(\theta,\lambda);$$

(ii) $\Pi^1(x_1^*, x_2^*(x_1^*))$ if

$$\text{either } \theta > \frac{\lambda^2}{4.5}, \text{ or } \theta \in \left[\underline{\theta}(w,\lambda), 1\right], \text{and } w < w^+(\theta,\lambda),$$
$$\text{or } \theta < \underline{\theta}(w,\lambda) \text{ and } w < w^{++}(\theta,\lambda),$$

where,

$$w^{+}(\theta,\lambda) \equiv \frac{8\lambda^6-18\lambda^2\theta^2-24\lambda^4\theta-72\lambda^2-4\lambda^4+72\theta\lambda^2+81}{\lambda^2(36\theta-9\theta^2-12\theta\lambda^2+36-44\lambda^2+12\lambda^4)}+$$
$$\frac{\sqrt{(16\lambda^8+48\lambda^6\theta-196\lambda^6+36\lambda^4\theta^2+792\lambda^4-252\lambda^4\theta+324\theta\lambda^2-1296\lambda^2-81\lambda^2\theta^2+729)}\left(2\lambda^2-3\right)}{\lambda^2(36\theta-9\theta^2-12\theta\lambda^2+36-44\lambda^2+12\lambda^4)},$$
$$w^{+}(\theta,\lambda) < \frac{4.5(1-\theta)}{\lambda^2} \Longleftrightarrow \theta < \frac{\lambda^2}{4.5},$$
$$w^{++}(\theta,\lambda) \equiv$$
$$\frac{28\lambda^4-54\lambda^2+18\theta\lambda^2-12\lambda^4\theta-2\sqrt{(81\lambda^4-72\lambda^6-486\lambda^4\theta+432\lambda^6\theta+16\lambda^8-96\lambda^8\theta+81\theta^2\lambda^4-36\theta^2\lambda^6)}}{8\lambda^6-16\lambda^4},$$

and $w^{++}(\theta,\lambda) < \frac{4.5(1-\theta)}{\lambda^2}$ if $\theta < \underline{\theta}(w,\lambda)$. The conclusion follows by observing that $\Pi^1(x_1^*, x_2^*(x_1^*)) \gtreqless \Pi^1\left(x_1^{**}, (\frac{c}{\lambda}-\theta\sqrt{x_1^{**}})^2\right) \Longrightarrow x_2^*(x_1^*) \lesseqgtr (\frac{c}{\lambda}-\theta\sqrt{x_1^*})^2$. Calculations yield the best response of the follower. ■

A.2. Proof of Proposition 3.2

Proposition 3.2 is proved through the following lemmas:

Lemma 1. *Suppose that Assumptions 1, 2, and 3 hold. Then, a firm prefers to be the leader of G_{seq} rather than a player of G_{sim} if and only if the follower of G_{seq} does not obtain a full cost reduction and $\underline{\theta}(w,\lambda) < \theta < \theta_1(\lambda)$, where $\theta_1(\lambda) \equiv \frac{\left(252\lambda^2-324-48\lambda^4+12\sqrt{(729+648\lambda^4-1134\lambda^2+16\lambda^8-164\lambda^6)}\right)}{2(36\lambda^2-81)}$, and $\underline{\theta}(w,\lambda)$ is defined as in the proof of Proposition 3.1 above.*

proof

Let $w \equiv \frac{a}{c}$. Recall that in the SPNE of G_{sim} each firm earns $\Pi^C = \frac{(a-c)^2\left(9-4\lambda^2\right)}{(9-2\lambda^2)^2}$, and that, under Assumptions 1 and 2, $\frac{a}{c} \in \left(2, \frac{4.5(1-\theta)}{\lambda^2}\right)$. Suppose first that, under the SPNE of G_{seq}, the follower obtains a full cost reduction. In this case the equilibrium profit of the leader is equal to $\Pi_l = \frac{(a-2c)^2}{9-4\lambda^2}$. Consider $\Pi^C - \Pi_l = \frac{12a^2\lambda^4-36a^2\lambda^2-16ac\lambda^4+162ac+72c^2\lambda^2-243c^2}{(9-2\lambda^2)^2(9-4\lambda^2)}$. The denominator of the previous expression is positive in the admissible region of the parameters. The numerator is positive if and only if $\frac{\left(32\lambda^4-108\lambda^2\right)}{2(12\lambda^4-36\lambda^2)} < \frac{a}{c} < \frac{\left(108\lambda^2-32\lambda^4\right)}{2(12\lambda^4-36\lambda^2)}$. Since $\frac{\left(32\lambda^4-108\lambda^2\right)}{2(12\lambda^4-36\lambda^2)} < 2$, and

$\frac{\left(108\lambda^2-32\lambda^4\right)}{2(12\lambda^4-36\lambda^2)} > \frac{4.5(1-\theta)}{\lambda^2}$, $\Pi^C > \Pi_l$ for any admissible parameter. Consider next the case in which, in the SPNE of G_{seq}, neither the leader nor the follower obtains a full cost reduction. In this case the equilibrium profit of the leader is equal to $\Pi_l^* = \frac{(a-c)^2\left(12\lambda^2-4\lambda^4-9\right)}{\lambda^2(108-40\lambda^2+4\lambda^4+12\theta(\lambda^2-3)+9\theta^2)-81}$.

Consider $\Pi_l^* - \Pi^C = \frac{(a-c)^2\lambda^2\left(\theta^2\left(36\lambda^2-81\right)+\theta\left(324+48\lambda^4-252\lambda^2\right)-4\lambda^4\right)}{(9\lambda^2\theta^2+12\lambda^4\theta-36\lambda^2\theta+108\lambda^2-40\lambda^4+4\lambda^6-81)(9-2\lambda^2)^2}$. Recall from Proposition 3.1 that, under the hypothesis at hand, $\theta > \underline{\theta}\,(w,\lambda)$. Calculations show that $\Pi_l^* > \Pi^C$ if and only if $\underline{\theta}\,(w,\lambda) < \theta < \theta_1(\lambda)$, where

$$\theta_1(\lambda) \equiv \frac{\left(252\lambda^2-324-48\lambda^4+12\sqrt{(729+648\lambda^4-1134\lambda^2+16\lambda^8-164\lambda^6)}\right)}{2(36\lambda^2-81)} > \underline{\theta}\,(w,\lambda)$$

for some w and λ in the admissible region of the parameters. Finally, consider the case in which, in the SPNE of G_{seq}, the leader obtains a full cost reduction. In this case the equilibrium profit of the leader is equal to

$$\Pi_l^{**} = \frac{c^2\left(\lambda^2\left(81-18\theta+9\theta^2-16\lambda^2\right)-81\right)+a^2\lambda^2\left(9-12\lambda^2+4\lambda^4\right)+6ac\lambda^2\left((1-\theta)\left(3-2\lambda^2\right)\right)}{(9-4\lambda^2)^2\lambda^2}.$$

Recall from Proposition 3.1 that, under the hypothesis at hand, it must be $\frac{a}{c} < w^{++}(\theta,\lambda)$. Computations show that $\Pi_l^{**} - \Pi^C > 0$ if and only if $\theta < \widetilde{\theta}\,(w,\lambda)$, where

$$\widetilde{\theta}\,(w,\lambda) = \frac{4w\lambda^5-24w\lambda^3-6\lambda^3+\sqrt{(81-27\lambda^2-18w\lambda^2+8w\lambda^4+9w^2\lambda^2-4w^2\lambda^4)}\left(4\lambda^2-9\right)+27\lambda+27w\lambda}{\lambda(9-2\lambda^2)}.$$

Since $\widetilde{\theta}\,(w,\lambda) > 0 \Leftrightarrow \frac{a}{c} > w^{++}(\theta,\lambda)$, $\Pi_l^{**} < \Pi^C$ for any admissible parameter. ■

Lemma 2. *Suppose that Assumptions 1, 2, and 3 hold. In addition, suppose that in the SPNE of G_{seq}, neither the leader nor the follower obtains a full cost reduction. Then, a firm prefers to be the follower of G_{seq} rather than a player of G_{sim} if and only if*

$$\theta \geq \theta_2(\lambda) \equiv \frac{\left(24\lambda^2-45-4\lambda^4+\sqrt{(2025-2304\lambda^2+984\lambda^4-192\lambda^6+16\lambda^8)}\right)}{2(6\lambda^2-18)}.$$

proof

Let $w \equiv \frac{a}{c}$. Recall that in the SPNE *of* G_{sim} each firm obtains $\Pi^C = \frac{(a-c)^2(9-4\lambda^2)}{(9-2\lambda^2)^2}$.

Assume that in the SPNE *of* G_{seq} no firm realizes a full cost reduction. In this case, the equilibrium profit of the follower is equal to

$$\Pi_f = \frac{(a-c)^2\left(2\lambda^4(\theta-1)+\lambda^2\left(10+3\theta^2-9\theta\right)-9\right)^2\left(9-4\lambda^2\right)}{\left(9\lambda^2\theta^2+12\lambda^4\theta-36\lambda^2\theta+108\lambda^2-40\lambda^4+4\lambda^6-81\right)^2}.$$

Under the assumptions at hand, $\Pi_f - \Pi^C > 0 \Longleftrightarrow \theta > \theta_2(\lambda)$, where $\theta_2(\lambda) \equiv \frac{\left(24\lambda^2-45-4\lambda^4+\sqrt{(2025-2304\lambda^2+984\lambda^4-192\lambda^6+16\lambda^8)}\right)}{2(6\lambda^2-18)}$. ■

Proof of the proposition

Consider first the case in which either the leader or the follower obtains a full cost reduction in the SPNE of G_{seq}. According to lemma 1 above, the would be leader prefers its own equilibrium payoff under G_{sim} to that a simultaneous player would obtain with G_{seq}. If the candidate follower's equilibrium payoff with G_{seq} is smaller (weakly) than that of a simultaneous player with G_{sim}, then the SPNE of G_{sim} dominates (weakly) that of G_{seq}. In this case the extended game has two SPNE (in pure strategies) both involving simultaneous play at the R&D stage and firms' timing announcements equal to (E, E) and (L, L), respectively. If the follower's equilibrium payoff with G_{seq} is larger than that of a simultaneous player with G_{sim}, then the extended game has a unique SPNE involving simultaneous play at the R&D stage, with both firms moving late. Consider next the case in which, at the SPNE of G_{seq}, neither the leader nor the follower obtains a full cost reduction. According to lemmas 1 and 2, sequential play at the R&D stage is induced in a SPNE of the extended game if and only if the two sets $\{\theta \in [0,1] : \theta < \theta_1(\lambda)\}$ and $\{\theta \in [0,1] : \theta > \theta_2(\lambda)\}$ have non empty intersection. Observe that $\theta_1(\lambda) - \theta_2(\lambda) < 0$, fir any admissible parameter. So, sequential timing cannot be sustained as SPNE of the extended game. Suppose

$\theta < \theta_1(\lambda)$. In this case, the would be follower blocks any timing combination giving rise to sequential moves at the R&D stage, and the extended game has a unique SPNE involving simultaneous play at the R&D stage, with both firms moving early. Suppose next $\theta > \theta_2(\lambda)$. In this case sequential play is blocked by the would be leader, and the extended game has a unique SPNE involving simultaneous play at the R&D stage, with both firms moving late. Finally, suppose $\theta_1(\lambda) < \theta < \theta_2(\lambda)$. In this case, both firms prefer G_{sim} to G_{seq}, and the extended game has two SPNE both involving simultaneous play in the R&D stage, with firms' timing announcements equal to (E, E) and (L, L), respectively. ■

A.3 Relaxing the stability condition

So far, the study of endogenous timing has been restricted to the case in which the SPNE of G_{sim} is unique, symmetric and globally stable under Cournot best reply dynamics, that is, to the region of parameters identified by Assumption 3. Due to this restriction, a relation between the shape of the possible outcomes of the extended game (whether the outcome is symmetric or not) and the order of moves of the related basic game has been established, linking symmetric and asymmetric R&D investments to simultaneous and sequential play respectively.

This section examines the effects of relaxing Assumption 3. For the sake of brevity and ease of reading, no proofs of the following assertions will be presented, the discussion remaining at an intuitive but precise level.

As already mentioned in section 3.2, relaxing Assumption 3 amounts to assuming $\theta < \theta_s$ (with θ_s defined as in (10)) and $\lambda \in \left(\sqrt{1.5}, 1.5\right)$. In the remainder of the analysis, attention will be restricted to this region of the parameters. Consider first G_{sim}. Given the concavity of firms' overall profits with respect to R&D expenditure, each firm's R&D reaction function is continuous and single valued. Within the region of parameters at hand, the symmetric equilibrium of the R&D game is unstable under Cournot best reply dynamics. In this case, two other asymmetric and locally stable equilibria obtain, each involving R&D specialization by the larger firm. Specifically, the two asymmetric equilibria are mirror images of each other, both inducing an R&D expert-beginner configuration with the more R&D intensive firm achieving a full cost reduction.[22]

Let Π be the profit going to each firm under the symmetric equilibrium; define $\overline{\Pi}$ and $\underline{\Pi}$ to be the profits going to the more and the less R&D intensive firm respectively, under the asymmetric equilibrium. It could be checked that, under Assumptions 1 and 2 stated in section 3, $\overline{\Pi} > \Pi > \underline{\Pi}$.

Consider now G_{seq}. Let Π_l and Π_f be the profit going to the leader and the follower, respectively. It is possible to check that, when deciding how much to invest in R&D, the first mover has to compare the overall profits it would obtain under the following alternative candidate solutions. In the first case, the leader chooses its maximal R&D level and obtains a full cost reduction. In the second case, the leader chooses the R&D level maximizing its profits subject to the constraint that the follower achieve a full cost reduction.

Intuitively, the difference between the profits going to the leader under the first and the second solution above is a decreasing function of the spillover rate. Namely, for small spillovers (θ close to 0), in the SPNE of the game, the leader obtains a full cost reduction. On the other hand, for relatively large spillovers (θ close to θ_s), a region of admissible parameters exists within which the SPNE expenditure of the leader induces a full cost reduction of the follower.

[22] See Amir and Wooders (1998).

Finally, consider the extended game. For the case discussed here, the SPNE of G_{seq} is unique, whereas, as already reported, G_{sim} has multiple equilibria. Hence, when solving the extended game by going backward from the final stage to the first, the three equilibria of G_{sim} must be considered separately, as alternative benchmarks for the case of simultaneous play.

Case 1: Consider first the two asymmetric equilibria of G_{sim} as the benchmark for simultaneous play. In line with the discussion above, only two extreme cases are examined. In the first case the spillover rate is small (θ is close to 0). As already mentioned, for such small spillovers, in the SPNE of G_{seq}, the first mover achieves a full cost reduction. It can be checked that, in this case, $\overline{\Pi} > \Pi_l > \underline{\Pi}$ and $\overline{\Pi} > \Pi_f > \underline{\Pi}$. The underlying intuition rests on the fact that, were the spillover to be zero, given a maximal expenditure of the first mover, leader and follower would attain the same profits as would the more and the less R&D intensive firms under the asymmetric equilibrium of G_{sim} respectively. The conclusion follows by noting that the profit of the leader declines, whereas that of the follower increases, in the spillover rate. For case 1 and small spillovers, given the payoffs at hand, it is easy to check that neither a sequential nor a simultaneous timing structure is sustainable as SPNE in pure strategies of the extended game. In addition, the equilibrium in mixed strategies is unique.[23] A possible interpretation is that, with small spillovers, and large productivity of the research activity,[24] all the conceivable combinations of order of moves arise in equilibrium with positive probability.

In the second case, the spillover rate is relatively large, that is, θ is close to θ_s. Let firm 1 be the first mover in game G_{seq}. Focus on the region of parameters

[23] In other words, the extended game is similar to Matching Pennies. See, for instance, Echenique (2004).

[24] That is, $\lambda \in \left(\sqrt{1.5}, 1.5\right)$.

within which, in the SPNE of G_{seq}, firm 2 achieves a full cost reduction. In this region, firm 1 obtains the same profits as the less R&D intensive firm under the asymmetric equilibrium of G_{sim}. On the other hand, firm 2 is better off than the more R&D intensive firm of G_{sim}, since it gets the same cost reduction as the latter while receiving spillovers. More precisely, $\Pi_f > \overline{\Pi} > \Pi_l = \underline{\Pi}$. In this case, there are three SPNE in pure strategies for the extended game. In the first one, the timing structure is simultaneous, with both firms moving late in the R&D stage. In the second and the third one, the timing structure is sequential, with the would be less R&D intensive firm under the selected asymmetric equilibrium of G_{sim} moving first. Note that both the sequential equilibria weakly dominate the simultaneous one.

Case 2: Consider now the symmetric equilibrium of G_{sim} as the relevant benchmark. For spillovers close to zero, it is easy to check that $\Pi_l > \Pi > \Pi_f$. In this case, the extended game has a unique SPNE involving simultaneous play at the R&D stage, with both firms moving early. For θ close to θ_s, $\Pi_f > \overline{\Pi} > \Pi_l = \underline{\Pi}$ clearly implies $\Pi_f > \Pi > \Pi_l$. In this case the only SPNE of the extended game involves simultaneous play at the R&D stage, with both firms moving late.

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