# How well can one measure market power in restructured electricity systems ?[*]

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**Abstract**

The integration of national electricity systems into a single internal European electricity market is not progressing well with the result that the level of competition in the sector remains unsatisfactory. This had led to proposals to apply ex ante remedies that directly bear on the structure of national incumbents. These measures involve quantitative recommendations such as virtual auctioning of capacity or divestitures that increase the number of competing firms. The evaluation of these measures partly relies on computable oligopoly models of the restructured electricity sector. This paper analyses the recent literature of these models and concludes that they are not currently capable of providing the degree of legal and regulatory certainty that the importance of these ex ante remedies requires. The state of the art in these models is such that their results reflect more a set of non-testable assumptions than observed facts or unambiguous theory. More academic work is necessary before these models can be applied in a legal or regulatory context. The conclusion is that this work on the structure of national electricity market distracts from the fundamental objective to introduce competition in the power sector by integrating the national markets into a single electricity market.

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# 1 Introduction

The successive "Benchmarking reports on the Implementation of the Gas and Electricity Internal Market" of the European Commission (EC-2002, EC-2003a, EC2005) conclude to the lack of integration of the electricity markets of the Members States and the poor development of competition in this sector. Further action may thus be expected. It can come from two sides. One possibility is to see further harmonisation actions taken pursuant Article 95 of the Treaty. This can take the form of new harmonisation laws or be limited to a serious implementation of the recent Directives and Regulation (EC-2003b, EC-2003c, EC-2003d). This would improve the architecture (the design) of the European electricity market. An alternative is to rely on competition law. This has been repeatedly announced by the Commissioners in charge of competition and mentioned in several reports on competition policy (Monti (2002), (2003a) and (2003b)), (EC-2003e, EC-2004). Both the Commissioner and the Commission invoke the existence of the market architecture resulting from the new harmonisation laws referred to above and explain that the conditions for the development of a workable competition in the industry are now met. They then conclude that the application of competition law in the gas and electricity industries is now justified. They take stock of the new competition policy that induces national regulatory and competition authorities to work in close cooperation with European competition authorities. This new policy paves the way to the extension of competition type remedies from standard ex post actions taken after a violation of competition law has been observed and proved to their ex ante application by national regulators to prevent potential abuses (Klotz and Nyssens (2004)). This evolution towards ex ante actions echoes the views of economists such as Newbery (2003) and Wolak (2004a), who also advocate completing the traditional ex post approach of competition law by ex ante regulatory measures.

The dichotomies between ex ante or ex post measures on the one hand, and actions for improving the architecture or the structure of the market on the other, constitute the background of this paper. The exercise of market power in the restructured electricity system currently draws considerable attention from economists on both sides of the Atlantic. The typical argument is that an incumbent company has a dominant position in its historical market and that it either uses it or will eventually use it. This entails considerable damage for the consumers and justifies ex ante measures to mitigate this potential exercise of market power. A traditional approach is to recommend structural remedies, for instance by forcing divestiture of some capacity. This happened in the US in California and New England, but not in PJM. It also took place in England and Wales during the Pool period. It is now sometimes advocated in continental Europe (Newbery et al. (2003)). We argue that this recommendation is ill devised in the European continental market and this essentially for two reasons. One is that the European internal electricity market is construed on the principle that competition would develop as a result of the integration of the different national markets. Market integration, "the most fundamental object of the Community" in the words of the Court of Justice (Consten and Grundig v. Commission (1996)), underlies the whole European process and also applies to electricity. Integrating the national electricity markets into a single market would drastically reduce the dominance of even the largest generators. Market integration can be achieved by ex ante actions improving transmission and balancing in the European electricity system. The integration process is taking place very slowly, if at all, because the adopted harmonization measures are not adequate. This argument is developed in Smeers (2005). The second reason is that we are insufficiently equipped to devise well-tuned structural (concentration) remedies that are proportional to the objective pursued. The reason is that our current knowledge of market power in electricity is not sufficient to act with the regulatory or legal certainty that these important decisions require. This is the theme of this paper. Combining the two arguments we conclude that acting on the structure (concentration) of the national electricity markets in order to mitigate the market power of incumbents in their historical market, as is sometimes proposed, is, first, at variance with the fundamental objective of market integration set by the Treaties and, second, possibly counter productive in terms of efficiency.

Market shares and indices such as the Hirschman Herfindal index (HHI) are traditionally used to assess dominant positions in concentrated markets (see Twomey et al. (2004) for a survey of methodologies for assessing market power in electricity). This approach does not apply well to restructured electricity markets where it has been found that companies with very small market share can sometimes exert market power. This happens in periods of high demand when generation capacity is tight and the plants of even the smallest companies are required in order to satisfy demand. This unusual feature of the electricity market is a direct consequence of the inelasticity of short-term demand (see Stoft (2002), chapter 1-1). Other tools than market share or concentration indices are thus necessary in order to assess market power. A still relatively recent but growing trend is to use market simulation models, whether based on activity analysis or econometric representation of the generation system. These models combine some market equilibrium paradigm with a dispatch model of the generators. An other type of market simulation models is based on the Supply Function Equilibrium (SFE) introduced by Klemperer and Meyer (1989) and popularised by Green and Newbery (1992) for the electricity industry. We concentrate on the first type of model that we refer to for convenience as stacking/equilibrium model. Our goal in this paper is to assess the extent to which these models are suitable to assess market power, whether for ex post or ex ante analysis.

This objective should not be misinterpreted: we do not question that firms can and may try to exert market power. This even took place when economic intuition suggested that this would not happen as in the initial duopoly of the Pool that was meant to behave  la Bertrand. But the argument, as this first experiment suggests, is that our current understanding of these questions does not allow one to properly measure the exercise of market power in ex post analysis and hence, a fortiori, how market power would be exercised in the future. The result is that some proposed ex ante remedies might be too rough or disproportionate with the objective. In contrast with this imperfect knowledge, we know very well how to integrate the electricity markets of the Member States. We also know that the result of this integration would dramatically mitigate the market power of even the largest European electricity companies. But this integration is stalled. Our claim is thus that working to reduce concentration of the national electricity markets on the basis of a very imperfect knowledge instead of striving to construct an integrating architecture that we know a lot about, is counterproductive.

The paper is organised as follows. Section 2 presents a very brief survey of the literature and introduces some studies that will be referred to in the rest of the presentation. Most market power analyses are based on stacking/equilibrium models of the sole energy market. We discuss them in Section 3 and suggest that they suffer from at least two flaws. The difficulty of measuring short run variable cost is one of them; it has been documented in the literature. We also argue that stacking/equilibrium models concentrate on short run variable costs which may be considerably lower than long run marginal cost in periods of tight demand, that is, when one "observes" most of the exercise of market power. We also indicate that attempts to overcome this problem and get into long-run marginal costs suffer from seemingly irreducible methodological difficulties. The last subsection of Section 4 introduces forward markets. Forward contracting is well known to reduce market power in the short run; the standard argument is that this should be favoured. This simple statement raises considerable practical difficulties though. First, the lack of data makes it difficult if not impossible in Europe to separate the effect of forward contracting and other effects of the unit commitment type in any measurement of market power. Second, forward contracting is an endogenous process and modelling it generates technical difficulties that we are currently far from being able to solve. Last, long term contracts of dominant agents tend to foreclose the market and are therefore looked at negatively by European competition authorities. Section 4 takes on models that combine energy and transmission. It first shows that "single stage" equilibrium models, of the type commonly used in market power assessments lead to asymmetric assumptions of market power in the energy andtransmission markets. This asymmetry was already encountered in the discussion of long run marginal cost in Section 3 and appears as a recurrent shortcoming of these models. It does not result from the choice of the market analyst but is somehow imposed by the modelling technique. We then proceed to show that eliminating this asymmetry meets considerable mathematical and economic difficulties. First, one needs to resort to “two stage” equilibrium models that are considerably more difficult to solve. Second and assuming that these mathematical difficulties are resolved, we still face models that require subtle economic assumptions that cannot be validated on the basis of observation. In short, removing the unavoidable asymmetric assumptions of “single stage” equilibrium models requires introducing arbitrary economic assumptions that make the resulting model non trustable. The last section turns to recent developments proposed to eliminate the mathematical difficulties of the two stage models. We indicate that this is only achieved at the price of reinforcing the need for arbitrary economic assumptions with the result that the model reflects the impact of assumptions made by the modeler more than hypothesis calibrated on the basis of a solid theory or market observations.

The conclusions of the paper have already be announced: our state of knowledge is not sufficient to draw practical recommendations on remedies like the extent of divestiture that would make the European electricity market competitive. This does not mean that we cannot proceed forward introducing competition in electricity. We know very well how to integrate national markets; we also know that market integration is, since the very beginning, a fundamental mandate in the European construction. We should thus use the instruments that we master well and that are in line with the objectives of the Treaties and refrain from playing games with tools that are currently fraught with both methodological and empirical difficulties. Needless to say pursuing the integration objective requires more effort on harmonisation and hence on Article 95 type measures. More than anything else, it requires moving from soft to hard harmonisation law.

Throughout the discussion, the paper resorts to electrical engineering notions that are summarised in Appendix A. More generally, the paper uses notions of Electricity Economics. In order to simplify the presentation, we always refer to Stoft (2002) for discussions of these notions. The discussion is conducted on a two-zone, two-firms example that is presented in Appendix A.4.

# 2 Stacking/equilibrium models in market power assessments

Stacking/equilibrium models have been used in one or another form in several studies of market power both in the US and in Europe. The following only reports on the small sample of studies referred to in the rest of the paper. Many US studies rely on pure dispatch models with no consideration of transmission or reliability. The sole use of a stacking model and an exogenous consumption suggests that it is possible to dispense with the econometric estimation of a demand model. It still requires a careful assessment of the consumption addressed to the centralised generation system, that is, after taking import/export and decentralised generation into account. The estimation of these latter requires the econometric estimation of this competitive fringe, with the result that one cannot really avoid econometric work in the analysis of market power. Joskow and Kahn (2002) and Borenstein, Bushnell and Wolak (2002) (hereafter referred to as BBW-2002) offer prominent examples of this approach in their studies of the Californian market. BBW-2002 introduce reliability in their assessment of the marginal cost of the generating system. Theses studies concentrate on the Californian (e.g. BBW-2002), New England (e.g. Bushnell and Saravia (2002)) and PJM markets (e.g. Mansur (2003)). More recently, Bushnell, Mansur and Saravia (2005) (hereafter referred to as BMS-2005) analyze the same markets, also using stacking/equilibrium models, but without the reliability considerations introduced in BBW-2002 and applied in Bushnell and Saravia (2002) and Mansur (2003). In none of these papers is there any modeling of transmission. Stacking/equilibrium models with transmission considerations were developed by Hobbs and his co-authors in several papers that will be mentioned as we proceed (see Hobbs and Helman (2004) for a general presentation).

Various European institutions also examined market power on the basis of stacking/equilibrium models. A project conducted by the University of Cambridge, IIT (Spain) and ECN (The Netherlands) examined the impact of different economic assumptions on the assessment of the exercise of market power (Neuhoff et al (2005)). These models are of the dispatch/equilibrium type. They encompass a representation of the electrical grid but no consideration of reliability. Models involving similar economic concepts are also used in ECN in The Netherlands (Hobbs et al. (2004a and 2004b)), Spain (Garcia-Alcade (2002)) and the United Kingdom (Green (2004)). Models of this type were also developed in research programmes of the European Union.

# 3 Energy only models

Real electricity markets involve many submarkets (Stoft (2002), chapter 1-8 and Wilson (2002)) but models of restructured electricity systems only consider a few of them. We distinguish, for the sake of this paper, the submarkets of the commodity (energy) and of some essential services like transmission and capacity. Needless to say our remarks apply to models that would include other submarkets like balancing and spinning reserve. The inclusion of these submarkets require more technical complications though, which may explain their relative absence from the literature (except for a few references such as Sidiqui (2003), Kamat and Oren (2004) or Smeers (2003)). Energy is traded in several successive markets namely in long-run bilateral markets, short-term organised or OTC futures markets, day-ahead organised markets, and possibly several organised daily markets. Electricity is also exchanged in real time in order to maintain the equality between consumption and generation. The real time system constitutes a key organised market (Wilson (2002)) in the pools of the East Coast in the US and in FERC standard market design proposal. They are often administrative constructions in Europe (e.g. see ETSO (2003) for a description of European balancing systems). Not all submarkets exist in all cases.

Most electricity market models used for market power assessment only consider a single energy market. They may embed exogenous forward positions and limit themselves to periods where there is no congestion in order not to model transmission. We begin with these models that concentrate on a single energy market but first briefly review their use in market power assessment studies.

Consider an electricity market where one has access to price observations. In most cases, these will be hourly prices observed on a power exchange, a pool (see Stoft (2002), chapter 1-8 for a discussion of the difference between these two systems) or the real time market. One can distinguish two approaches to the assessment of market power. A first approach uses a stacking model (dispatch or unit commitment, see Appendix A) to compute the marginal cost of the generation system in different periods of time. It then compares these marginal costs to the prices observed in these periods. Joskow and Kahn (2002) and BBW-2002 are prominent examples of this approach. More formally, let $P_t^0$ be the observed price in some hour $t$. One computes (by simulation, using a Stacking model) the marginal cost $C_t'$ in the hour. One then concludes on exercise of market power on the the basis of $P_t^0 - C_t'$.

An alternative approach is to resort to so-called counter factual assumptions as in BMS-2005. In this approach one considers two extreme competition assumptions namely perfect and Cournot competitions and one simulates the market in each of these assumptions using a stacking/equilibrium model. This approach requires the following operations. Let $P_t^{pc}$ be the simulated price of a perfect competition equilibrium in hour $t$. Let $P_t^{cc}$ be the simulated price of a Cournot equilibrium in that same hour. One observes $P_t^0$ and locates it in the interval $[P_t^{pc}, P_t^{cc}]$ and concludes on the exercise of market power. The common approach in these studies is to use an optimal dispatch as the underlying stacking model. Note that supply function equilibria used in other studies of market power lie between $P^{pc}$ and $P^{cc}$. The second approach immediately leads to proposals of ex ante remedies that we claim are not justified. By changing the number of firms (e.g. as a result of divestitures), these models allegedly allow one to test the impact of remedies on market power Arellano (2003) is a typical example of this approach. We shall argue in the following that the existing models are much too dependent on arbitrary assumptions to undertake that analysis with any degree of confidence.

### 3.1 A simple stacking/equilibrium models

Market power studies often do away with congestion. We follow a similar approach in these studies: we consider the two-zone example presented in Appendix A.2 and first assume that all line capacities are infinite in order to eliminate congestion. Recall that the example only assumes two firms; $g_i$ and $G_i$ respectively denote the (endogenous) generation level and (exogenous) generation capacity at node $i$. $s_{fj}$ is the supply of firm $f$ to node $j$. Note that $s_{fj}$ is generally unambiguous in Cournot but that only $s_{1j} + s_{2j}$ is uniquely defined in perfect competition. $P_j(s_j)$ is the demand function at node $j$ and $C_i(g_i)$ the variable generation cost at node $i$. An equilibrium model, whether of the perfect competition or Cournot type assumes that each firm maximises its profit, that is,

- Firm 1

$$\begin{array}{lll} \max_{s_{1j}} \sum_{j=3,5,6} P_j(s_{1j} + s_{2j})s_{1j} - \sum_{i=1,2} C_i(g_i) & & \\ \quad \text{s.t.} \quad 0 \leq g_i \leq G_i & (\nu_i) & \\ \quad \sum_{j=3,5,6} s_{1j} = \sum_{i=1,2} g_i & (\eta_1) & \end{array} \tag{1}$$

- Firm 2

$$\begin{array}{lll} \max_{s_{2j}} \sum_{j=3,5,6} P_j(s_{1j} + s_{2j})s_{2j} - C_4(g_4) & & \\ \quad \text{s.t.} \quad 0 \leq g_4 \leq G_4 & (\nu_4) & \\ \quad \sum_{j=3,5,6} s_{2j} = g_4 & (\eta_2) & \end{array} \tag{2}$$

where the $\nu$ and $\eta$ are the dual variables of the respective constraints.

It is now common to express equilibrium models through complementarity formulations. We also adopt this approach. Complementarity formulations rely on so called complementarity conditions of the form

$$x_i \geq 0 \qquad F_i(x) \geq 0 \qquad x_i F_i(x) = 0 \qquad i \in I.$$

These are commonly written under the following form which is adopted throughout in the paper

$$0 \leq x \perp F(x) \geq 0.$$

Define $\dfrac{\partial P_j}{\partial s_j} = P'_j$ and $\dfrac{\partial C_i}{\partial g_i} = C'_i$. Perfect competition assumes price taking firms. This is obtained by stating

$$\frac{\partial}{\partial s_{fj}} P_j(s_{fj} + s_{-fj})s_{fj} = P_j.$$

The equilibrium conditions are then written as

$$0 \leq C'_i + \nu_i - \eta_f \perp g_i \geq 0; \qquad f = 1, i = 1,2;\ f = 2,\ i = 4 \tag{3}$$

$$0 \leq \eta_f - P_j \perp s_{fj} \geq 0; \qquad f = 1,2;\ j = 3,5,6 \tag{4}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; \qquad i = 1,2,4. \tag{5}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{6}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{7}$$

Relation (3) states that a plant $i$ of firm $f$ receives the marginal cost $\eta_f$ of firm $f$ if it generates and that it does not generate if its marginal cost is higher than $\eta_f$. Relation (4) states that firm $f$ delivers to node $j$ if its marginal cost $\eta_f$ is equal to the price $P_j$ in $j$; it does not deliver if its marginal cost $\eta_f$ is higher than $P_j$. Marginal costs $\eta_f$ are equal if both firms deliver to some common node. Relation (5) introduces a scarcity rent on each plant $i$. It sets this scarcity rent at zero if the plant does not operate at capacity. Relations (6) and (7) express free disposal.

The Cournot version assumes that firm sales have an impact on prices and that firms are aware of that impact. The equilibrium conditions are obtained by assuming

$$\frac{\partial}{\partial s_{fj}} P_j(s_{fj} + s_{-fj})s_{1j} = P_j + P'_j s_{fj}$$

which gives the equilibrium conditions

$$0 \leq C'_i + \nu_i - \eta_f \perp g_i \geq 0; f = 1,\ i = 1,2;\ f = 2; i = 4 \tag{8}$$

$$0 \leq \eta_f - P_j - P'_j s_{fj} \perp s_{fj} \geq 0; f = 1,2;\ j = 3,5,6 \tag{9}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; i = 1,2,4. \tag{10}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{11}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{12}$$

Relations (8), (10), (11) and (12) have the same interpretation as in the perfect competition case. Relation (9) replaces the price $P_j$ in $j$ by the marginal revenue $P_j + P'_j s_{fj}$ at that node.

Suppose in order to simplify the discussion that both firms supply the two markets. Let $C'_{fm}$ be the marginal variable cost of the marginal plant of firm $f$ and $\nu_{fm}$ the scarcity margin (possibly equal to zero) of this marginal plant. The exercise of market power is measured by focussing on the equilibrium conditions of the marginal plant of each firm. The perfect competition price satisfy the equilibrium conditions

$$\begin{array}{rclr} P_j^{pc} & = & C'_{fm} + \nu_{fm} & j = 3,5,6 \\ P_j^{pc} & = & C'_4 + \nu_4 & j = 3,5,6. \end{array} \tag{13}$$

The Cournot price $P_j^{cc}$ satisfies

$$\begin{array}{lr} P_j^{cc} + P_j^{'cc} s_{1j} = C'_{fm} + \nu_{fm} & j = 3,5,6; \\ P_j^{cc} + P_j^{'cc} s_{2j} = C'_4 + \nu_4 & j = 3,5,6. \end{array} \tag{14}$$

BMS-2005 assesses the market power at some node $j$ (BMS-2005 only assumes a single node) by locating the observed price $P_j^0$ at that node in the interval $[P_j^{pc}, P_j^{cc}]$. The closer $P_j^0$ to $P_j^{pc}$, the less market power is exerted.

Both the simple comparison of $P_j^0$ with $P_j^{pc}$ and its location in the interval $[P_j^{pc}, P_j^{cc}]$ suffer from several drawbacks. One is due to the difficulty of measuring the marginal costs $C'_i$. This is documented in the literature and is briefly recalled here. The reader is referred to the literature for further details. The second difficulty is methodological; it is elaborated in more detail in the following.

### 3.2 The measurement of the marginal cost

Consider the perfect equilibrium condition where the observed price is compared to the marginal cost.

$$P_j^0 - C'_{fm} - \nu_{fm} = 0$$

signals that firm $f$ does not exert market power. The approach requires to observe $P_j^0$ and uses a stacking model to compute $C'_i$ and $\nu_i$. Studies usually rely on optimal dispatch model for that computation. Two streams of literature have contested our ability to measure the marginal cost on the sole basis of an optimal dispatch model. Harvey and Hogan (2002) and Rajaraman and Alvarado (2003) elaborate on this idea by comparing marginal costs computed by optimal dispatch and unit commitment models. Mansur (2004) and Wolak (2004) come to a similar conclusion, using completely different methods of the econometric type. We briefly review these studies.

The measure of the marginal cost by an optimal dispatch model is simple and transparent but it can be erroneous. Specifically, Harvey and Hogan (2002) argue that the evaluation of the marginal cost found by optimal dispatch models suffers, among other things, from neglecting unit commitment idiosyncrasies. One could obviously argue that some unit commitment constraints and variables make the sole notion of marginal cost illusory. The tradition is to neglect this theoretical argument and to measure the marginal variable cost as the highest variable cost of a running plant not at capacity (see Stoft (2002), section 3-8.2 for a discussion of the practical difficulties of this approach). This reasonable approximation in many cases is erroneous in principle. Harvey and Hogan (2002) argue that it can also lead to serious errors when the solution of a unit commitment problem departs too much from the one of the dispatch model. This happens for instance when it is economical to operate plants of high variable cost and low start-up costs in preference to plants of lower variable cost and higher start-up cost or long minimal running periods. The converse can also happen with certain plants being operated in order to avoid the cost of shutting them down and starting them up again after a few hours. Note also, in contrast with what intuition could suggest, that the solution is not to take the highest fuel cost of the plant in operations. As discussed in O'Neill et al. (2005), Hogan and Ring (2003) and Bjørndal and Jrnsten (2004), none of these fuel costs is a marginal cost in the usual sense. In short the marginal cost computed by dispatch models cannot be trusted. Appendix B reports on some of the features identified in Harvey and Hogan (2002).

Several authors have since confirmed these findings. Rajaraman and Alvarado (2003) elaborated on Harvey and Hogan's arguments again, using optimisation models. Some economists followed suit on the basis of econometric cost estimations. They noted that the unit commitment idiosyncrasies make the production set of a generator non convex and hence prevent the standard derivation of convex cost functions from this production set. Mansur (2004) and Wolak (2003 and 2004b) endeavoured to econometrically estimate cost functions that account for unit commitment idiosyncrasies. Specifically and referring to stacking models, Mansur (2004) assumes perfect competition and specifies a "behavioural" cost function whereby the production in some hour depends on observable data in current, preceding and subsequent hours. These dependencies should not exist if the pure dispatch model adequately represented the behaviour of generators. Mansur finds that the coefficients that express these dependencies are statistically significant and that the model that accounts for unit commitment idiosyncrasies estimated under assumptions of perfect competition gives a better representation of observations.

Wolak (2003 and 2004b) also estimated "behavioural" cost functions but in the context of supply function equilibrium. He considers the Australian and Californian markets for which bids of generators are available. He assumes that these bids are optimal strategies of the generators, given some underlying multiperiod cost function that he wants to eliminate. Cost functions are multiperiod because the unit commitment idiosyncracies couple generation decisions accross periods. He finds that these effects are statistically significant.

In short, quite different approaches suggest that the measurement of marginal cost through a dispatch model commonly found in assessments of market power may not give a proper measurement of thecost effectively faced by generators. The principle of the argument is obvious but its quantification may be tricky. The comparison of the results of unit commitment and optimal dispatch models in Harvey and Hogan (2002) indicate that the effect can be important. This conclusion is confirmed by econometric estimations (Mansur (2004) and Wolak (2003 and 2004b).

## 3.3 Long and short run marginal cost

Suppose for the rest of the discussion that the problem of finding adequate $C_i'$ has been solved satisfactorily. We now concentrate on the methodology that claims to assess market behaviour with respect to counterfactual assumptions, namely perfect competition and Cournot competition such as in BMS-2005 in the US or Neuhoff et al. (2005) in Europe.

Most assessments of market power compare prices to short run variable (essentially fuel) cost of the marginal plant, computed by dispatch models. BBW-2002, Bushnell and Saravia (2002) and Mansur (2003) invoke reliability considerations in order to arrive at a more accurate evaluation of the expected fuel cost of the marginal plant. To the best of our knowledge, no study of market power elaborates on the relation between reliability criteria and long run marginal cost (see Stoft (2002), Part 2 for a discussion of that relation). The absence of any reference to long run marginal costs is worrying. One should question claims that identify the exercise of market power in prices that are not sufficient to recover long run marginal costs.

Long and short run marginal costs are different notions and the choice of one or the other matters. Short and long run marginal costs are equal when capacities are optimal, a property that is automatically achieved in market operating under perfect competition and perfect foresight. Short run marginal costs are higher than long run marginal costs when there is a shortage of capacity and lower when there is an excess. Note also that optimal capacity ex ante is almost always imperfectly adapted ex post when there is uncertainty. In any case, benchmarking the observed price to the marginal variable cost should give an overestimation of the exercise of market power. The equality between short and long run marginal costs can only be stated by invoking scarcity rents on tight capacities (the $\nu_i$ in our formulation). In other words, the sole examination of short run variable costs (here fuel cost) is not necessarily a good estimate of the long run marginal cost. It is thus necessary, in order to conduct the assessment of market power, to go beyond the simple consideration of short run variable (fuel) cost and to consider scarcity rents. This raises an interesting question. The celebrated equality between short run and long run marginal cost holds for perfect competition in perfect foresight and provides a reference counter factual assumption. In contrast, there is no such reference paradigm for imperfect competition. In other words, the methodology that relies on two extreme counterfactual assumptions becomes ambiguous when one takes investments on board. Given the current need for investments in the European power sector, the lack of a reference imperfect competition paradigm is a serious shortcoming when it comes to assess market power.

Because there is no unambigous paradigm of imperfect competition, one needs to resort to ad hoc methods. We discuss two of them; one introduces reliability costs as a surrogate for investment costs. The other selects one (the simplest one) of the many possible interpretations of imperfect competition in an investment world as the benchmark paradigm. None of these approaches properly solves the problem but they at least provide an intuitive approach to assess the excessive character of prices.

## 3.4 Reliability

### 3.4.1 Background

Reliability pervades power engineering. It was a recurrent concern in the regulatory period but largely disappeared in the restructuring literature except for some market power assessment studies such as BBW-2002. Joskow and Tirole (2004) recently brought reliability back to the attention of the research community. We rely on the reliability concepts presented in Part 2 of Stoft (2002). Because electricity demand is not price responsive in the short run, curtailments may be unavoidable when forced outages or sudden demand surges make available capacity insufficient. Electricity is then priced at the value of unserved energy or Lost Load (VOLL) determined by the Regulator. This value, together with the (observed) frequency of curtailment, determine at least in principle the incentive to build new peak capacity. The equilibrium is reached when, for a given VOLL, the frequency of interruptions leads to investments in peak capacities that maintain the current frequency of interruption. This approach was formalised in the former pool of England and Wales where the price of energy charged to the final consumer included a contribution from the probability of curtailment (the Loss of Load Probability or LOLP) and generators were remunerated for making plants available in order to improve reliability. Needless to say, the system can also be in disequilibrium. There may be excess capacity, in which case the frequency of curtailment will be low and the incentive to invest reduced or inexistent. There may also be a shortage of capacity leading to a high curtailment frequency and strong incentives to invest. Changes of demand due to climatic or economic evolutions can also push the system out of equilibrium.

An alternative approach is for the Regulator to set the price at a regulated value of the spinning reserve when this latter is running short. This reasoning again assumes a regulated electricity price in case of scarcity even if there is no curtailment. This is similar to the preceding reasoning except that the regulated price does not have any particular economic interpretation. We consider the case of curtailed energy and VOLL pricing that we discuss in an intuitive way. For the sake of simplification, we also do not consider payments to generators that contribute to reliability, as this would detract from the discussion of the market power embedded in energy prices. Needless to say, this is an important simplification in the assessment of market power as the experience of the former Pool in England and Wales revealed that companies could game reliability mechanisms. The simplification is justified in this paper because these aspects are not present in model based studies of market power. Last, we simplify the model (and make it partially incorrect) by not considering the impact of forced outages and uncertain demand on short term variable (fuel) cost (the standard "probabilistic costing" of power engineering). A rigorous treatment that does not make these simplifications and includes the modelling of payments to generators is given in Ehrenmann and Smeers (2005).

### 3.4.2 Perfect and Cournot competition with reliability

The loss of load probability does not have good mathematical/risk properties (appendix A4). Seen as a function of some load indicator (here noted $s$), it is the derivative with respect to this indicator of the expected unserved energy defined as

$$EUE = E[\max(\mathbf{s} - \mathbf{G}; 0)].$$

Note that in contrast with some common wisdom LOLP, EUE and their derivatives are quite easy to compute in a single area problem.

Assume a regulated VOLL and define a reliability criterion in monetary terms as $R^m(s, G) = \text{VOLL} \star EUE$. Let $m$ be the highest variable cost plant of firm $f$ and assume it is not running at capacity (its scarcity margin is null). An intuitive transposition of the pricing rule of the former England Wales Pool is to price electricity in perfect competition as

$$P_j = C'_{fm} + \frac{\partial R^m}{\partial s} \text{ instead of } P_j = C'_{fm}.$$

This pricing scheme obviously modifies the assessment of market power to the extent that a new cost term is added to the short-term variable cost. Referring to Cournot competition, the pricing rule

$$P_j + P'_j s_{fj} = C'_{fm}$$

is replaced by

$$P_j + P'_j s_{fj} = C'_{fm} + \frac{\partial R^m}{\partial s}.$$

This leads to a different measure of market power. Given the two counterfactual pricing mechanisms

$$\begin{array}{llll} & P_j^{pc} = C'_{fm} + \dfrac{\partial R^m}{\partial s} & & \text{(competitive case)} \\ \text{and} & P_j^{cc} = C'_{fm} + \dfrac{\partial R^m}{\partial s} & - P'_j s_{fj} & \text{(Cournot case).} \end{array}$$

We have market power if

$$P_j^0 - \left( C'_{fm} + \frac{\partial R^m}{\partial s} \right) > 0.$$

The extent of this market power is measured by where $P_j^0$ lies in the interval $[P_j^{pc}, P_j^{cc}]$. Again the closer $P_j^0$ is to $P_j^{pc}$, the smaller the exercise of market power.

These pricing principles are readily embedded into the perfect and Cournot equilibrium conditions models. Specifically the (simplified) perfect competition equilibrium conditions can be stated as

$$0 \le C'_i + \nu_i + \frac{\partial R^m}{\partial s} - \eta_f \perp g_i \ge 0; f = 1,\ i = 1,2;\ f = 2,\ i = 4 \quad (15)$$

$$0 \le \eta_f - P_j \perp s_{fj} \ge 0; f = 1,2;\ j = 3,5,6 \quad (16)$$

$$0 \le G_i - g_i \perp \nu_i \ge 0; i = 1,2,4. \quad (17)$$

$$0 \le \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \ge 0 \quad (18)$$

$$0 \le g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \ge 0 \quad (19)$$

These conditions are interpreted as follows. At equilibrium a generator that sells to some consumers pays (and charges) the cost of the marginal decrease of reliability that this sale entails. This interpretation requires that the Regulator prices reliability at VOLL and charges generators (or consumers) for it. Recall, as indicated above, that we do not model here the payment that generators receive by making some capacity available in order to improve reliability.

The above complementarity formulation can be readily extended to Cournot competition and is stated as

$$0 \le C'_i + \nu_i + \frac{\partial R^m}{\partial s} - \eta_f \perp g_i \ge 0; f = 1,\ i = 1,2;\ f = 2,\ i = 4 \quad (20)$$

$$0 \le \eta_f - P_j - P'_j s_{fj} \perp s_{fj} \ge 0 f = 1,2;\ j = 3,5,6 \quad (21)$$

$$0 \le G_i - g_i \perp \nu_i \ge 0; i = 1,2,4. \quad (22)$$

$$0 \le \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \ge 0 \quad (23)$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{24}$$

The interpretation of the reliability term is identical. Because the regulator fixes the price of reliability and we exclude the possibility to withhold capacity in order to increase reliability payments, the generators are price takers on the reliability market.

Referring to the first market power assessment method presented in the beginning of this section, market power would then be assessed by comparing the observed prices to a sum of the short run variable cost and a reliability term. In the second market power assessment method, the interval used to assess the observed price would be shifted by the reliability terms. Whatever the approach, the measurement of the exercise of market power would decrease. The higher the demand, the higher the $\frac{\partial R^m}{\partial s}$ term and hence the lower the measurement of market power compared to traditional assessments.

### 3.4.3 Reliability in market power assessment

Most assessments of market power disregard reliability considerations. BBW-2002, Bushnell and Saravia (2002) and Mansur (2003) are exceptions: ideas similar to the above considerations appear in these papers albeit based on a different computational approach. The standard marginal cost of model ((8), (9), (10)) is replaced by an expected marginal cost computed by a Monte Carlo method. The authors consider a sample of 100 random draws of plant availability. The marginal cost curve is constructed for each draw and the marginal cost of satisfying the prevailing net demand computed for each obtained scenario. When net demand exceeds capacity, the marginal cost is set at some regulated level, taken as the price cap on the balancing market in California, that is \$250/Mwh in BBW-2002. It is fixed at the price cap (\$1000/Mwh) on the real time market in PJM and ISONE (only real time markets existed in these systems at the time of the study). The expectation of the computed marginal costs is then compared to the observed price to assess the extent of the exercise of market power. It is worth mentioning that subsequent work by some of these authors (BMS-2005) disregards these reliability considerations and claim that the additional precision to the measurement of market power brought about by this computation does not justify the added complexity!

Prices like \$250/Mwh or even \$1000/Mwh are on the low side of the estimated VOLL that ranges between \$10 000/Mwh and \$100 000/Mwh (Stoft (2002, chapter 2-1)). Also, the size of 100 draws in Monte Carlo computations a priori looks quite small in Monte Carlo terms to capture the impact of extreme and hence rare contingencies that lead to curtailments. Both elements may lead to an underestimation of the cost of reliability in the reported studies. The use of the expected cost function (probabilistic costing) alluded to in Appendix A.4 allows one to bypass the possible shortcomings of the Monte Carlo approach.

### 3.4.4 A capacity market

The above approaches assume that the price of reliability, whether computed through a LOLP or an EUE, is regulated. An alternative approach is to introduce a physical reliability objective to be attained by the generators and let the market price it. The difference between the two methods can be related to the one between a tax on emissions and a market for emission permits. The former assumes that the price is given and let the market decide the reduction of emissions. The latter supposes a given reduction objective and let the market determine the price to achieve it. The above formulations of the competitive and Cournot equilibrium are readily amenable to these alternative approaches. Let $R(s, G)$ be the physical measure of reliability (e.g. the expected unserved energy) and

$$\widetilde{R}(s, G) \equiv R(s, G) - \overline{R} \geq 0 \qquad (\mu) \tag{25}$$

the physical reliability objective. Let $\mu$ be the dual variable of that objective. The perfect and Cournot competition models are respectively expressed as follows.

*Perfect competition equilibrium conditions*

$$0 \leq C_i' + \nu_i + \mu \frac{\partial R}{\partial s} - \eta_f \perp g_i \geq 0; f = 1,\ i = 1, 2;\ f = 2, i = 4 \tag{26}$$

$$0 \leq \eta_f - P_j \perp s_{fj} \geq 0; f = 1, 2;\; j = 3, 5, 6 \tag{27}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; i = 1, 2, 4 \tag{28}$$

$$0 \leq \widetilde{R}(s_3 + s_5 + s_6; G) \perp \mu \geq 0. \tag{29}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{30}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{31}$$

*Cournot competition equilibrium conditions*

$$0 \leq C_i' + \nu_i + \mu \frac{\partial R}{\partial s} - \eta_f \perp g_i \geq 0; f = 1,\; i = 1, 2;\; f = 2, i = 4 \tag{32}$$

$$0 \leq \eta_f - P_j - P_j' s_{fj} \perp s_{fj} \geq 0; f = 1, 2;\; j = 3, 5, 6 \tag{33}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; ti = 1, 2, 4 \tag{34}$$

$$0 \leq \widetilde{R}(s_3 + s_5 + s_6; G) \perp \mu \geq 0. \tag{35}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{36}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{37}$$

Both equilibrium conditions involve an endogenous price of reliability $\mu$ that after multiplication by $\frac{\partial R}{\partial s}$ adds to the standard short run variable cost. This may be interpreted as a surrogate of the long term marginal cost incurred by the generator. A particularly interesting case is the one where the reliability criterion boilds down to a reserve margin objective. Take a very simple version of $\widetilde{R}$

$$\widetilde{R}(s_3 + s_5 + s_6; G) \equiv G - (1 + \alpha)(s_3 + s_5 + s_6) \tag{38}$$

where $\alpha$ is a reserve margin.

These models can be interpreted as embedding both energy and a capacity markets. (See Stoft (2002) chapter 2-8 for a definition of the capacity market and Creti and Fabra (2004) for an economic analysis.) This duality of markets is a welcome addition. It comes at a price though. The perfect competition model represents generators as price takers both in the energy and capacity submarkets. In contrast, generators of the Cournot model exert market power in the energy submarket but are price takers in the capacity submarkets.

Relations (33) and (35) of the stacking/Cournot model therefore reveal an asymmetry of competitive assumption that will be a recurrent phenomenon in the rest of this paper. Specifically, generators behave  la Cournot on the energy submarket but competitively on the other submarkets (here the capacity submarket) where they take the price as given. Such asymmetric assumptions may be realisic in some cases. This is so in the European electricity and emissions permits markets where incumbents occupy a dominant position in their historical electricity market but are price takers in the global emission permit market. There is no reason however to believe that this asymmetry holds in general. In particular, there is no reason to believe that it holds for the capacity and energy submarkets where incumbents probably have the same dominant position. The drawback of the above mathematical formulation is that the asymmetric formulation is a natural outcome of the formulation of the Cournot assumption on the energy market and that getting rid of it requires a full overhaul of the model as we shall discuss later. The deep reason of this asymmetry is that the standard Cournot formulation requires an explicit expression of the inverted demand curve of the goods and services where market power is exerted. This is in principle available for energy but not for reliability. The asymmetric model therefore results from the asymmetry of available data. The absence of data on demand for reliability was not intended in the early days of restructuring when one expected demand for products with differentiated reliability properties to emerge (see the collection of papers in Oren and Smith (1993)). Reality did not follow suit. Except for the large consumers, that already had interruptible contracts in the pre-restructuring days, reliability differentiated demand remains a dream today. We shall return to this argument of asymmetry several times in the rest of the paper.

#### 3.4.5 Conclusion on reliability

Reliability is directly related to investments in power generation system: a decrease of reliability signals the need for investments. Reliability can thus also be used as a surrogate for modelling long-term marginal costs. There is no technical difficulty to embed reliability criteria in the optimal dispatch model underlying the perfect and Cournot competition models used for assessing market power. One approach is to assume a regulated price of reliability; this gives a marginal cost of reliability that adds to the standard variable fuel cost. This approach relates to the architecture of the former England and Wales pool. It requires the Regulator to price reliability. An alternative assumption is to suppose that the Regulator introduces a capacity market. In both cases, a contribution due to reliability is added to the variable fuel cost with the result that the measure of market power is decreased. In contrast we are in uncharted waters when the Regulator does not intervene at all in reliability and there is no way for consumers to express their willingness to pay for reliability and be charged for it. Except for the interruptible contracts of some large consumers, it is currently impossible to differentiate the delivered electricity in terms of reliability and hence to have it directly priced by the market. The absence of regulatory intervention on reliability or capacity issue amounts to a missing market (see Ehrenmann and Smeers (2005) for a modeling approach of these different issues).

Reliability or capacity pricing certainly leads to a better assessment of the long run marginal cost of generators by introducing an additional component to their fuel cost. But the approach suffers from a methodological drawback in the Cournot assumption. The resulting model supposes an asymmetric competitive position of the generators in the energy and capacity markets. This asymmetry can be justified in some cases but is unrealistic in general. This question will appear in a recurrent way in the future. It is a fundamental weakness of the Cournot counterfactual assumption.

## 3.5 An investment model

### 3.5.1 Background

Capacity expansion models have been extensively researched in the pre-restructuring period. Except for attempts to apply the theory of real options to plant valuation, much less attention has been devoted to the investment question in the post restructuring literature. Capacity expansion models can be used to illustrate the well known equality of long and short run marginal costs in optimised production systems. An additional well-known result is that perfectly competitive markets operating in perfect foresight optimize the production system. Long run and short run marginal costs are thus equal under the joint assumption of perfect competition and certainty. This immediately suggests to extend the old capacity expansions models to accommodate the perfect competition counterfactual assumption. This can indeed be done by even developing these models into large general equilibrium models (e.g. MARKAL-MACRO in Loulou et al. (2004)). The situation is different for the Cournot assumption. As already observed, there is no universal paradigm of imperfect competition in an expanding market. The problem is a multistage game that can take on different interpretations depending on the assumptions made on the type of competition in the different investment and operations stages of the game and on the rationality of the agents for forecasting the outcome of successive games. Moreover, these games may not have pure strategy equilibrium, which obviously raises questions about their usefulness for policy analysis. Still it is worthwhile to explore whether some long-term capacity expansion models reflecting an imperfect competition paradigm cannot be used to extend the Cournot counter factual assumption to the case where investment is needed in the power system. We here suggest a simple Cournot type model that offers a minimal deviation with respect to the perfect competition model of capacity expansion.

### 3.5.2 A capacity expansion Cournot model

Consider first the simple case where the power sector is operated in a single time segment. The interpretation of the proposed extension of the Cournot model is that generators only build new plants if they can sell their output forward over the long run. This can be interpreted in terms of the former long-term power purchase agreements. This interpretation remains relevant as some new PPA are now concluded in the US.

The extension of the short term Cournot model of Section 3.1 to the investment case is obtained as follows. The variables $s$ and $g$ retain their former interpretation. We let $G_i$ be again generation capacity at node $i$, that we consider endogenous in this model. We let $K_i$ denote the per/Mwh investment cost. Firm 1 solves

$$\begin{array}{lll} \max \sum_{j=3,5,6} P_j(s_{1j}+s_{2j})s_{1j} - \sum_{i=1,2}[C_i(g_i)+K_iG_i] & & \\ \quad \text{s.t.} \quad 0 \leq g_i \leq G_i & (\nu_i) & \\ \qquad\qquad \sum_{j=3,5,6} s_{1j} = \sum_{i=1,2} g_i & (\eta_1) & \end{array} \tag{39}$$

taking the $s_{2j}$ given. A similar model can be written for Firm 2 that takes $s_{1j}$ as given.

The equilibrium conditions can then be written as follows

$$0 \leq C_i' + \nu_i - \eta_f \perp g_i \geq 0; f=1,\ i=1,2;\ f=2,\ i=4 \tag{40}$$

$$0 \leq \eta_f - P_j - P_j' s_{fj} \perp s_{fj} \geq 0; f=1,2;\ j=3,5,6 \tag{41}$$

$$0 \leq K_i - \nu_i \perp G_i \geq 0; i=1,2,4 \tag{42}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; \tag{43}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{44}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{45}$$

where relation (42) states that one only invests in a new plant if the scarcity rent is equal to the (per Mwh) investment cost (see Stoft (2002), chapter 1-3) for a discussion of investment costs expressed in energy units). Assuming that one invests in technology $i$ (and hence that one also operates plant $i$ to capacity) and sells to market $j$ the equilibrium conditions (40) and (41) become

$$P_j + P_j' s_{ij} = C_i' + K_i. \tag{46}$$

This condition leads again to a different view of market power. The standard counterfactual Cournot condition

$$P_j + P_j' s_{1j} = C_{fm}' \tag{47}$$

is now replaced by

$$P_j + P_j' s_{1j} = C_i' + K_i \qquad (G_i > 0) \tag{48}$$

There is thus market power if $P_j - C_i' - K_i > 0$.

The reality is that the investment problem is slightly more complicated because firms invest to satisfy demand in several time segments. Relation (48) is thus not a true equilibrium condition as the model needs to be expanded to tackle this more complex set up. This extension can be done easily. Specifically, let $h$ designate a time segment and assume all time segments to be of the same length. $K_i$ now designates the investment cost referred to the total number of hours covered by these segments. Firm 1's solves

$$\begin{array}{lll} \max \sum^h \sum_{j=3,5,6} P_j^h(s_{1j}^h + s_{2j}^h)s_{1j}^h - \sum_{i=1,2} C_i^h(g_i^h) - K_i G_i & & \\ \quad \text{s.t.} \quad 0 \leq g_i^h \leq G_i & (\nu_i^h) & \\ \quad \sum_{j=3,5,6} s_j^h = g_1^h + g_2^h & (\eta_1^h) & \end{array} \tag{49}$$

assuming the $s_{2j}^h$ given. A similar model is written for Firm 2.

The Cournot equilibrium conditions are readily obtained as

$$0 \leq C_i'(g_i^h) + \nu_i^h - \eta_f^h \perp g_i^h \geq 0; f = 1,\ i = 1,2;\ f = 2,\ i = 4 \tag{50}$$

$$0 \leq \eta_f^h - P_j^h - P_j'^h s_{fj}^h \perp s_{fj}^h \geq 0; f = 1,2;\ j = 3,5,6 \tag{51}$$

$$0 \leq K_i - \sum_h \nu_i^h \perp G_i \geq 0; i = 1,2,4 \tag{52}$$

$$0 \leq G_i - g_i^h \perp \nu_i^h \geq 0 \tag{53}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{54}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{55}$$

This modeling is more complex than the reliability based assessment of long term marginal cost that only involves an easily computed term $\frac{\partial R}{\partial s}$ for each $h$. In contrast the investment approach requires a linkage of the complementarity conditions of each $h$ through the coupling consraint

$$\sum \nu_i^h = K_i.$$

### 3.5.3 Conclusion on capacity expansion models

The above model provides an extension of the counter factual Cournot assumption to the investment problem. It modifies the standard evaluation of market power in two senses. First, the impact of the short run variable cost must be adapted to account for the change of operating mode due to the imperfectly competitive energy market. A second modification is the need to account for the capacity cost. The model is more complicated to compute than the usual short run Cournot equilibrium problem but remains quite amenable to existing software. Its drawback is of a methodological nature: this model is just one of the many possible models that one can construct to adapt the counter factual Cournot assumption to an investment context. It is indeed necessary to supplement the usual Cournot hypothesis with additional assumptions in order to move from an operation only to an investment and operations model. These assumptions bear on the sequencing of the investment, future and spot market and the rationality of the player exerting market power in the face of this sequence of markets. Murphy and Smeers (2004a and 2004b) illustrate some possible alternatives and show that different assumptions can lead to quite different results. In particular some of these assumptions may hamper the existence of a pure strategy equilibrium, a feature that is obviously embarrassing for policy analysis in general and market power evaluation in particular. The reason of this shortcoming is deep: we lack a good unambiguous theory of how market power is exerted in general and particularly when investments are at stake.

## 3.6 Forward markets

### 3.6.1 Background

Energy is traded in several successive markets in restructured electricity systems. These may be decentralised forward markets, centralised or OTC futures markets, day-ahead, intraday and real time markets. The above models only consider a single energy market that is generally interpreted as a day-ahead or real time market. The impact of forward contracting on the exercise of market power has been recognised in the work of Newbery (1998), Green (1999) and

Wolak (2000). It has since been repeatedly emphasised in various publications. The lack of forward contracts has been mentioned as one of the many causes that led to the Californian debacle. The benefits accruing from long term contracts for mitigating market power have also been pointed out in studies of PJM and ISONE and investigations of the former E&W pool. Possibly in contrast with the above, one should note that the European Commission has always objected in its concentration cases to the conclusion of very long-term contracts by firm enjoying a dominant position on the market on the ground that the would foreclose the market. We shall not get into this question here but simply note that long term contracts, if they limit the exercise of market power of agents in the short run (the argument found in the electricity restructuring literature) may also limit the entry of new agents (the argument of European competion authorities) in the long run.

Forward contracting is an important part of the generators problem. We here indicate some of the questions raised by the extension of the perfect and Cournot counter factual assumptions to the inclusion of forward contacting. The discussion is conducted on the standard short run problem with no investment consideration.

### 3.6.2 The perfect competition assumption

Forward contracting does not imply any modification of the perfect competition energy models at least as long as one does not introduce uncertainty. The forward price is equal to the spot price, which is itself equal to the marginal cost. The perfect competition counter factual assumption therefore remains unchanged.

### 3.6.3 The Cournot competition assumption

The situation is different for the Cournot assumption. It is straightforward to formally accommodate exogenous forward contracting in the Cournot model. Let $s^c_{fj}$ be the forward contract of firm $f$ to consumer $j$ and $P^f_j$ be the contract price. $G_i$ is fixed again and all symbols have their former interpretation. Firm

1's problem is rewritten as follows

$$
\begin{array}{ll}
\max_{s_{1j}} \sum_{j=3,5,6} P_j^f s_{1j}^c + P_j(s_{1j}+s_{2j})(s_{1j}-s_{1j}^c) - \sum_{i=1,2} C_i(g_i) & \\
\quad \text{s.t.} \quad 0 \le g_i \le G_i & (\nu_i) \\
\quad \quad \sum_{j=3,5,6} s_{1j} = \sum_{i=1,2} g_i & (\eta_1) \\
\quad \quad s_{1j} \ge 0 &
\end{array}
\tag{56}
$$

assuming $s_{2j}$ and $s_{ij}^c$.

The equilibrium conditions can be written as

$$0 \le C_i' + \nu_i - \eta_f \perp g_i \ge 0; f = 1; i = 1,2; f = 2; i = 4; \tag{57}$$

$$0 \le \eta_f - P_j - P_j'(s_{fj} - s_{fj}^c) \perp s_{fj} \ge 0; f = 1,2; j = 3,5,6 \tag{58}$$

$$0 \le G_i - g_i \perp \nu_i \ge 0; i = 1,2,4. \tag{59}$$

$$0 \le \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \ge 0 \tag{60}$$

$$0 \le g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \ge 0 \tag{61}$$

Let $m_f$ be firm's $f$ marginal plant, and assume some $s_{fj} > 0$ and $\nu_{mf} = 0$. These conditions lead to the simpler Cournot equilibrium condition

$$P_j + P_j'(s_{1j} - s_{1j}^c) = C_{mf}'. \tag{62}$$

This relation leads to quite different situations where the observed price can be higher or lower than the marginal cost, depending on the level of contracting. We can indeed have

$$
\begin{array}{lll}
s_{1j}^c > s_{1j} & \text{if} & P_j < C_{fm}' \\
s_{1j}^c = s_{1j} & \text{if} & P_j = C_{fm}' \\
s_{1j}^c < s_{1j} & \text{if} & P_j > C_{fm}'
\end{array}
\tag{63}
$$

Prices are higher than marginal costs if firm $f$ is undercontracted (short on sales, long on capacity) and smaller than marginal cost otherwise (long on sales, short on capacity). Both situations reflect the exercise of market power.

This extension of the Cournot model, as simple as it may look, raises various difficulties though. We successively consider data, modelling and numerical issues.

**Data issues.** While the stacking/Cournot model is easily modified to account for forward contracts, its use for measuring market power requires information on the extent of contracting. The reality is that one knows almost nothing in Europe on forward contracting. Specifically the main publication, Platt's, reports price quotation but no volume information. It is thus impossible to identify the extent of contracting and thus the modified Cournot model in a useful way. Specifically relations like $P_j < C'_{fm}$ can be due to long term contracts or unit commitment constraints without one being able to differentiate between the two explanations on the basis of existing data. The result is that any interpretation of the difference $P_j^0 - C'_{fm}$ is blurred by the existence of different possible explanations.

**Modelling issues.** Forward contracts raise more than data issues. Forward contracting is an endogenous process and its modelling requires transforming the single stage model into a two-stage game problem. This raises several difficult questions. Forward contracts are financial contracts and it is tempting to resort to the standard perfect arbitrage postulate of finance theory in order to model the relation between forward and spot prices at equilibrium. This is the approach adopted in Allaz and Vila (1993)'s seminal paper. The perfect arbitrage assumption takes on a particularly simple form in the deterministic world commonly assumed in studies of market power. It implies that the prices of the forward and spot markets are equal, as any discrepancy between these two prices would signal unexploited profit opportunities. The perfect arbitrage in an imperfectly competitive market is more demanding in terms of rational expectation as agents must be able to foresee the outcome of the imperfectly competitive market.

Some authors have questioned the validity of the perfect arbitrage assumption in electricity. Kamat and Oren (2004) argue on the basis of a stylised model that, far from obeying the perfect arbitrage assumption, the sequencing of electricity markets can be used for further price discrimination and thus to enhance market power. Their discussion concentrates on the day ahead and balancing markets, which admittedly may be quite different from the sequence of the forward and day-ahead markets. In contrast with these claims, Joskow reports that arbitrage between the day ahead and real time markets is functioning well in the restructured markets of the East Coast. Other studies have examined the arbitrage on transmission contracts in NYISO and concluded that it is imperfect (Bartholomew et al. (2003)) or may depend on the architecture of the market such as the introduction of virtual bidding (Saravia (2003)). The underling cause of these departures from the standard perfect arbitrage assumption is that the financial electricity markets are not very liquid, a feature that may be due to the fact that they remain quite different from the ideal financial or standard commodity markets.

The observation that arbitrage might be imperfect is important as it questions the fundamental perfect arbitrage assumption. It is also embarrassing because one does not have a ready substitute for this assumption. Dropping it therefore introduces a fundamental ambiguity in the modelling process. One knows well how to model perfect arbitrage, or perfect price discrimination, but one does not know what to do otherwise. Specifically the idea that arbitrage between the forward and spot markets may not be perfect leaves it completely uncertain how to model the formation of expectation in the forward market with respect to the outcome of the spot market. Very much like the assumption of rational expectation, it is very difficult to substitute the assumption of perfect arbitrage.

**Computational issues.** Neglect the above worries for a moment and accept the perfect arbitrage assumption. The modelling of endogenous forward contacts transforms the single stage Cournot model into a two-stage model. There is a growing literature that develops on the basis of the seminal contribution of Allaz and Vila (1993). Most of this literature is based on models that do not embed inequality constraints of the type present in the dispatch or unit commitment problems. This simplification is convenient but has an impact. Dropping these inequality constraints allows one to find an explicit solution of the second stage game (in this case the spot market) that the modeller can move forward to the first stage game (in this case the forward market). Green

(1999) and (2003) and Newbery (1998) rely on that property. The same is true for perfect competition models such as Bessembinder and Lemmon (2002) or Siddiqui (2003). The first stage game then retains all the convexity properties commonly used to prove existence and uniqueness of the first stage equilibrium (in this case the forward market). This approach fails for modelling power systems as soon as one accounts for the numerous machines constraints present in dispatch and unit commitment models. The second stage problem (the spot market) is then modelled as a linear complementarity problem in the easiest case, that is when using an optimal dispatch model. Moving its solution forward to the first stage game (the forward market) destroys all convexity properties. The result is that the forward market may not have pure strategy equilibrium. The situation is worse if one adopts a unit commitment model for representing the second stage problem. It is indeed impossible to state equilibrium conditions for this second stage and thus to construct a first stage model. Whether the impossibility to prove or find a pure strategy equilibrium in the forward market is important or not in practice is an empirical question. But some remarks may hint at a possible answer. Liquidity is notoriously difficult to develop in the power markets. This may signals that agents remain wary of instruments that would appear at first sight of utmost usefulness to mitigate the considerable risks embedded in electricity prices. A possible explanation of that a priori akward behaviour is that the possible inexistence of an equilibrium of the forward market observed in theory finds its way into some erratic behaviour of these markets in practice. In any case, very much like for the capacity expansion model discussed before, the absence of pure strategy equilibrium drastically reduces the usefulness of the obtained model for policy evaluation in general and for assessing the extent to which the appetite for forward contracting of a company decreases its market power in particular.

### 3.6.4 Conclusion on forward contacting

Forward contracts reduce the incentive to exert market power on the short run market. Exogenous forward contracts can be embedded in Cournot models if data is available. The inclusion of these contracts may not always be very convincing in practice. Forward contract and unit commitment idiosyncrasies can indeed have similar effects on the relation between prices and short run variable costs with the result that it is difficult to disentangle one from the other in ex post studies of market power. The situation is more intricate in an ex ante analysis. Forward contracting is indeed an endogenous process and its modelling is fraught with difficulties. The natural approach is to model it as a two-stage game. This raises two difficulties. On the modelling side forward contacting requires a perfect arbitrage assumption that may not hold as well in electricity as in other markets. On the solution side, the two stage game may not have pure strategy equilibrium.

# 4 Energy and Transmission Models

## 4.1 Background

Like reliability, the grid is an externality that needs to be internalised in order to make the restructured electricity market efficient. This internalisation requires a market design or architecture. The US experience shows that the choice of this architecture may already generate considerable discussions even when one limits oneself to the perfect competition counter factual assumption. BMS-2005 states that the counter factual assumptions allow one "to abstract away from the detailed market rules and regulations in each market and examine the range of equilibrium price outcomes that would be predicted from considering market structure alone". This suggests that one can study the impact of the structure of the market without bothering about its architecture. This claim is widely accepted and is also stated in several occasions in Stoft (2002). It has not been proved though. We bypass the discussion of the choice of a market architecture in transmission and assume nodal pricing because of its conceptual simplicity and its growing approval in practice. We do not simply abide to the claim that perfect and Cournot model can abstract from market architecture and concentrate on energy models that also account for a transmission submarket.

Let $w_i$ be the injection/withdrawal charge for transmission at node $i$ and

$\eta_f$ be the marginal variable cost of firm $f$, $f = 1, 2$. Neglecting scarcity rents for the sake of simplification and recall that lines (1–6) and (2–5) are the only ones constrained in the six node problem of Appendix A, the optimality conditions of the transmission constrained optimal dispatch problem derived in Appendix A can be restated as the following perfect competition equilibrium conditions

$$0 \leq C_i' - \eta_f + w_i \perp g_i \geq 0; f = 1,\ i = 1, 2;\ f = 2,\ i = 4 \tag{64}$$

$$0 \leq \eta_f - P_j - w_j \perp s_{fj} \geq 0;\ f = 1, 2;\ j = 3, 5, 6 \tag{65}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{66}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{67}$$

$$0 \leq \overline{F}_\ell - \sum_{i=1,2,4} PDF_{i(\ell)} g_i + \sum_{j=3,5,6} PDF_{j(\ell)} s_j \perp \lambda_\ell \geq 0 \tag{68}$$

$$\ell = (1-6), (2-5)$$

where

$$w_i = \lambda_{(1-6)} PDF_{i(1-6)} + \lambda_{(2-5)} PDF_{i(2-5)} \tag{69}$$

This complementarity model can be interpreted as consisting as two sets of equilibrium conditions respectively on the energy and transmission submarkets coupled via the $w_i$. Relations (64) to (67) state the equilibrium on the energy markets already encountered in preceding models, after modifications to account for the transmission price between the node and some hub. Relations (68) to (69) are new. They express the equilibrium on the submarket of transmission services. The $w_i$ link the two submarkets.

## 4.2 The Cournot counterfactual assumption

It is straightforward to modify the above expressions to accommodate Cournot competition in the energy market. One obtains the relations

$$0 \leq C_i' - \eta_f + w_i \perp g_i \geq 0 \qquad f = 1, i = 1, 2; f = 2, i = 4 \tag{70}$$

$$0 \leq \eta_f - P_j - P_j' s_{fj} - w_j \perp s_{fj} \geq 0 \quad f = 1, 2; j = 3, 5, 6 \tag{71}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{72}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{73}$$

$$0 \leq \overline{F}_\ell - \sum_{i=1,2,4} PDF_{i(\ell)} g_i + \sum_{j=3,5,6} PDF_{j(\ell)} s_j \perp \lambda_\ell \geq 0 \tag{74}$$
$$\ell = (1-6), (2-5)$$

where

$$w_i = \lambda_{(1-6)} PDF_{i(1-6)} + \lambda_{(2-5)} PDF_{i(2-5)}. \tag{75}$$

As in the model (32) to (37) involving reliability, these relations reflect different competitive assumptions in the energy and transmission submarkets. Generators have pricing power in energy but are price takers in the transmission submarket. The history of the former Pool in England and Wales provides evidence of the exercise of market power on the reliability market. It is generally admitted that generators can also exert market power in transmission (Berry et al. (1999), Borenstein et al. (1999), Cardell et al. (1997)) even in a nodal pricing architecture. We are again facing the embarrassing asymmetry of assumptions that appears as soon as one introduces another submarket in a Cournot energy model. Here again we shall need to resort to a two stage game model to remove this asymmetry. We discuss this question in Section 4.5, after introducing spatial arbitrage.

### 4.3 Cournot model and price discrimination

Cournot generators exert market power by, among other things, price discriminating between their customers. Model (70) to (75) could be expanded to reveal two types of price discrimination. One is standard in Cournot models: customers in a given node pay different prices if their demand curves for electricity are different. This discrimination is unlikely to be relevant in stacking/Cournot models since gross demand is generally modeled as inelastic and price dependence of the net demand results from the inclusion of import/export and a competitive fringe, which are node but not customer specific. We therefore only assume a single net demand curve in each node and hence do away with this price discrimination. The other price discrimination is more unusual, but also more interesting. There is a price discrimination between the energy and transmission prices in the sense that the price paid for transmission between two nodes is not equal to the difference of the energy prices between these two nodes. Hobbs (2001) first noted this shortcoming and suggested introducing arbitrageurs that trade electricity between nodes.

The proposal makes considerable sense. Restructured electricity systems developed both numerous markets and a trading activity to arbitrage between these markets. But the observation of both the US and European electricity sectors suggests that this activity is subject to vagaries. The analysis of the boom and bust of the trading activity in electricity is a subject of its own; it suffices to say here that arbitraging between different locations may not always be easy. Newbery et al. (2003) argue that this is the case in the Benelux market because there is currently no power exchange in Belgium. Imperfect arbitrage may also be due to the poor liquidity of these exchanges. It is indeed sometimes sugested by practitioners that the existing European power exchanges should be merged into a single European PX in order to reach a sufficient level of liquidity. Other studies already mentioned (Bartholomew et al. (2003) and Saravia (2003)) also suggest that spatial arbitraging is not always perfect even in markets endowed with a quite sophisticated architecture. As in the discussion of forward contracting, the question arises as to whether spatial arbitrage is effectively taking place. As with forward markets, perfect arbitrage or perfect discriminations are ideal modeling assumptions. The problem is that they are probably unrealistic and one does not know what intermediate assumption to make. We elaborate on this question using the above energy/transmission models.

## 4.4 Cournot model and arbitrage

Perfect arbitrageurs eliminate any discrepancy between the transmission price between two nodes and the difference between energy prices at these nodes. Arbitrage cannot involve the strategic players as they freeze their injections into the grid. Consider therefore arbitrage between consumptions nodes and let $a_j$ be the energy injected/withdrawn by arbitrageurs at node $j$. Let $s_j = s_{1j} + s_{2j}$. The behaviour of an arbitrageur can be modelled as

$$\begin{array}{r} \max \sum_{j=3,5,6} [P_j(s_j + a_j) - w_j] a_j \\ \text{s.t.} \quad \sum_{j=3,5,6} a_j = 0. \end{array} \tag{76}$$

The solution of this problem is unbounded except if

$$P_j - w_j = P_{j'} - w_{j'} \qquad j, j' = 3, 5, 6. \tag{77}$$

Relations (77) are thus the conditions imposed by the arbitrage activity at the equilibrium. It remains to insert these conditions in the stacking/Cournot model.

Hobbs (2001) and Metzler et al. (2003) studied that question. They consider two cases that differ by the attitude of the strategic generators with respect to the arbitrageurs. In a first assumption the strategic generators take the actions of the arbitrageurs on the energy market as fixed. This is the usual Nash assumption where each agent seeks to optimize its actions taking the actions of the other agents as fixed. This approach only demands to augment the Cournot equilibrium conditions (70) and (71) with the arbitrage variables $a_j$ and the arbitrage equilibrium conditions (77). Neglecting again scarcity rents for the sake of simplificity of the presentation, one obtains the expanded conditions.

$$0 \leq C_i'(g_i) - \eta_f + w_i \perp g_i \geq 0; \qquad f = 1,\ i = 1, 2;\ f = 2,\ i = 4 \tag{78}$$

$$0 \leq \eta_f - P_j(s_j + a_j) - P_j'(s_j + a_j) s_{fj} - w_j \perp s_{fj} \geq 0; \tag{79}$$

$$f = 1, 2;\ j = 3, 5, 6$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{80}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{81}$$

$$\tag{82}$$

$$P_j(s_j + a_j) - P_{j'}(s_{j'} + a_j') = w_j - w_{j'}, \quad j, j' = 3, 5, 6. \tag{83}$$

$$0 \leq \overline{F}_\ell - \sum_{i=1,2,4} PDF_{i\ell} g_i + \sum_{j=3,5,6} PDF_{j\ell} s_j \perp \lambda_\ell \geq 0 \tag{84}$$

$$\ell = (1-6), (2-5) \tag{85}$$

where

$$w_i = \lambda_{(1-6)} PDF_{i(1-6)} + \lambda_{(2-5)} PDF_{i(2-5)}. \tag{86}$$

Because of (83), any discrepancy between transmission prices and difference of energy prices at equilibrium is eliminated.

In a second assumption the strategic generators anticipate the actions of the traders and take them into account in their optimisation. This is a Stackelberg type behaviour. In order to model it, define $a(s; w)$ as the $a_j$, $j = 3, 5, 6$ that are solutions of (83) expressed as function of $s, w$. Because the $P_j$ are differentiable functions, the $a$ are also differentiable. The Stackelberg model is stated as

$$0 \leq C_i'(g_i) - \eta_f + w_i \perp g_i \geq 0; \qquad f = 1,\ i = 1, 2;\ f = 2,\ i = 4 \tag{87}$$

$$0 \leq \eta_f - P_j[s_j + a_j(s; w)]; \qquad f = 1, 2;\ j = 3, 5, 6 \tag{88}$$

$$- P_j'[s_j + a_j(s, w)](1 + \frac{\partial a_j}{\partial s_{fj}}) s_{fj} - w_j \perp s_{fj} \geq 0.$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{89}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{90}$$

Meltzer, Hobbs and Pang (2003) show that both assumptions give the same result. This is comforting. It is indeed impossible to a priori identify which of these behavioural assumptions is more realistic. Both are equally plausible and any difference between their implications would create an ambiguity in the measuring of market power. The question is whether there exist other arbitrage operations whose impact depends on assumptions on the behaviour of strategic generators with respect to the arbitrageur.

## 4.5 Cournot model and the System Operator (SO)

The System Operator is a special arbitrageur. In a perfect competition model, the system operator (SO) trades electricity between all nodes, taking into account the capabilities of the network. This means that, in contrast with the activities of the traders that are in principle only based on price differences, the arbitrage operations of the SO are also limited by physical constraints. These arbitraging operations are perfectly taken care of in the perfect competition model (64) to (69). In this model the SO implicitly trades energy between all nodes, taking the transmission constraints into account. The arbitraging possibilities of the SO are more limited in the Cournot model where the injections of the strategic generators are fixed and the SO cannot therefore arbitrage between them. But the consumers are price takers and the system operator can arbitrage between them so as to make the transmission prices between any two consumer nodes equal to the difference of energy prices between these nodes. In other words, the SO is an arbitrageur that buys and sells between all consumers nodes so as to satisfy the constraints of the network while maximizing welfare when injections from Cournot generators are given. This is an arbitrage activity, that is constrained by line capacities. We model both the perfect competition and Cournot cases.

Let $y_n$ be the purchase/sale of the SO ($y_n$ is unconstrained) at some node $n$ ($i$ or $j$). Suppose that the SO does not exert market power; we first consider the perfect competition case where the SO arbitrages between all nodes. The result of this arbitrage can be represented as

$$0 \leq \overline{F}_\ell - \sum_{1,2,4} PDF_{i\ell}(g_i + y_i) + \sum_{j=3,5,6} PDF_{j\ell}(s_j + y_j) \tag{91}$$

$$\perp \lambda_\ell \geq 0 \quad \ell = (1-6), (2-5)$$

$$\sum_{i=1,2,4} y_i + \sum_{j=3,5,6} y_j = 0 \tag{92}$$

$$w_i = \lambda_{(1-6)} PDF_{i(1-6)} + \lambda_{(2-5)} PDF_{i(2-5)} \qquad i = 1, \cdots, 6 \tag{93}$$

$$P_n - P_{n'} = w_n - w_{n'}, \qquad n = 1, \cdots, 6 \tag{94}$$

where the argument of $P_n$ is $g_n + y_n$ or $s_n + y_n$ depending on whether $n$ is an injection or withdrawal node. The equilibrium conditions of the perfect competition model can be stated as

$$0 \leq C_i'(g_i + y_i) - \eta_f + w_i \perp g_i \geq 0 \tag{95}$$

$$0 \leq \eta_f - P_j(s_j + y_j) - w_j \perp s_{fj} \geq 0. \tag{96}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{97}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{98}$$

to which one adds the relations (91) to (94).

Turning now to the Cournot competition (and noting that $y_i = 0, i = 1, 2, 4$ in this case) one can again transpose the representation of the interactions between strategic generators and arbitrageurs to model the behaviour of the strategic generators with respect to the SO. The first assumption is that the strategic generators take the actions of the SO as given and conversely. This is again the Nash assumption where each agent optimises its actions taking the actions of the others as given. This is a single stage equilibrium problem which is formulated as follows

$$0 \leq C_i'(g_i) - \eta_f + w_i \perp g_i \geq 0 \tag{99}$$

$$0 \leq \eta_f - P_j(s_j + y_j) - P_j'(s_j + y_j)s_{fj} - w_j \perp s_{fj} \geq 0 \tag{100}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{101}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{102}$$

$$\begin{aligned} 0 \leq \overline{F}_\ell - \sum_{1,2,4} PDF_{i\ell} g_i + \sum_{j=3,5,6} PDF_{j\ell}(s_j + y_j) \\ \perp \lambda_\ell \geq 0 \qquad \ell = (1-6), (2-5) \end{aligned} \tag{103}$$

$$\sum_{j=3,5,6} y_j = 0 \tag{104}$$

$$w_i = \lambda_{(1-6)} PDF_{i(1-6)} + \lambda_{(2-5)} PDF_{i(2-5)} \qquad i = 1, \cdots, 6 \tag{105}$$

$$P_j(s_j + y_j) - P_{j'}(s_{j'} + y_{j'}) = w_j - w_{j'} \qquad j = 3, 5, 6 \tag{106}$$

The alternative is to suppose that the strategic generators take the reaction of the System Operator into account when optimising their action in the energy market. To model this assumption, note $y(s)$ and $w(s)$ to be the solution of the complementarity relations (103) to (106). Assume for notational simplicity (but incorrectly) that one can define the partial derivatives $\frac{\partial y_j}{\partial s_j}$. The behaviour of the strategic player is then described as

$$0 \leq C_i'(g_i) - \eta_f + w_i(s) \perp g_i \geq 0 \tag{107}$$

$$\begin{aligned} 0 \leq\ & \eta_f - P_j(s_j + y_j(s_j)) \\ & - P_j'(s_j + y_j(s_j))(1 + \frac{\partial y_j}{\partial s_j}) s_{fj} - w_j(s) \perp s_{fj} \geq 0. \end{aligned} \tag{108}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{109}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{110}$$

This is a two stage equilibrium model or an equilibrium problem subject to equilibrium constraints (EPEC).

The dichotomy of economic assumptions is identical to the one encountered for the treatment of the arbitrageurs. But there are major differences in the outcome. The constraints (83) that represent the outcome of the actions of the arbitrageur are linear relations while the relations (103) to (106) that represent the result of the SO's action are complementarity relations. This mathematical difference has drastic consequences. Meltzer et al. (2003) result on arbitrageurs does not hold any more for the SO. Choosing one or the other assumption on the behaviour of the generators changes the result. Because there is little empirical reason for selecting between these assumptions, the Cournot counter-factual assumption becomes ambiguous.

## 4.6 Single state vs. two stage models

The representation of the arbitrageur and of the SO in the two-stage model introduces an interesting new feature with respect to the standard Cournot model. When modeling the arbitrageur or the SO, a reasonable assumption is that generators behave strategically with respect to both the consumers (the energy market) and the other agents (the arbitrageurs or the SO). The two-stage model allows one to represent that assumption. The result is that the asymmetry found in the simple extension of the Cournot model to the representation of the reliability ((32) to (35) and the transmission market ((70) to (75) is eliminated in a two-stage model. This latter explicitly assumes that the strategic players are able to influence the actions of the agents operating in the other markets (the arbitrageurs or the system operator), that they reckon that possibility, and that they take advantage of it. Technically this is achieved by representing the other market, arbitrage or transmission as equilibrium constraints imposed on the actions of the strategic generators and not simply as a set of additional complementarity conditions to be tackled at the same level. The same approach would allow one to represent strategic behaviour on the reliability market: it would suffice in order to do so to consider the reliability function $R(s;G) - \overline{R} \geq 0$ as constraints on the action of the strategic generators and not simply as an additional relation to be satisfied (see Ehrenmann and Smeers 2005).

The two-stage model therefore does away with the asymmetric behaviour of the strategic generators with respect to different submarkets (energy, reliability, transmission). This gain comes at a price. We already suggested before that it is difficult to identify a priori if strategic generators effectively anticipate the actions of the arbitrageurs. The same is true when it comes to anticipating the actions of the SO. Perfect information and full rationality demand that strategic players do anticipate the reactions of the SO and take advantage of this anticipation. But these may also be heroic assumptions with little empirical support. Two papers explore this question.

Ehrenmann and Neuhoff (2003) take advantage of the flexibility offered by the modelling of single and two stage games to explore two proposals of the organisation of cross border trade of electricity in the European Community. The first proposal, referred to as coordinated auction (ETSO (2001)), supposes a separation of the electricity market into successive transmission and energy markets. The transmission market clears before the energy market. Traders first bid into the transmission market in order to acquire transmission rights. Equipped with these transmission rights, they then bid into the energy market. The second proposal considers integrated transmission and energy markets where traders only bid for energy and the TSO simultaneously allocate energy and transmission rights. The intuition is that the integrated approach is superior; some analysis conducted on stylised model (Neuhoff (2003)) confirms that it is indeed the case. The question is whether the results obtained on stylised analyses can be confirmed by experiments conducted on more realistic situations.

Ehrenmann and Neuhoff (2003) expand the formalism adopted in the stylised model into a computable model. The work is remarkable in several aspects. From a modelling point of view, it no longer sees the choice between the single and two-stage representations of energy and transmission submarkets simply as an assumption but tries to relate it to proposed institutional arrangements. From an empirical point of view, the work illustrates the key role played by rational expectation in these models. While the assumption may be crucial in various economics models, it sometimes appears unrealistically demanding in practice. From a practical point of view and directly relevant to the object of this paper, the work also illustrates that the choice of one or the other modelling paradigm (single or two stage model) can have a significant impact on the results, that is on simulated price levels. This is a key finding for market power studies: the counter factual Cournot assumption becomes ambiguous as soon as one introduces other submarkets and there is no clear cut assumption to make on the interactions between these submarkets.

The impact of equally plausible but non identifiable assumptions is confirmed in Neuhoff et al. (2005). The work compares models developed by different research teams by applying them on a stylised representation of the North-western Europe electricity market. All teams model the perfect competition and Cournot counter factual assumptions. The models encompass the energy and transmission submarkets. The results of the work are directly relevant to our analysis. All models give the same results for the perfect competition assumption. This illustrates, not only that there are no error in the models but, more to the point of this paper, that there is no ambiguity in the definition of the perfect competition counter factual assumption. In contrast the results of the different models are at variance, sometimes significantly, in the Cournot counter factual assumption. These divergences can be traced to the representation of the interactions between the energy and transmission submarkets. Specifically bounded rationality of the strategic generators plays a key role. Needless to say, it is extremely difficult, if not totally impossible, to make a plausible assumption on the degree of rationality of the strategic generators.

As already mentioned before, besides difficult modelling choices, the recourse to two-stage models presents a technical but irreducible difficulty. From a mathematical point of view, these two stage models are non-convex problems that may not have any pure strategy equilibrium or conversely that may have several equilibrium. They have a mixed strategy equilibrium but this is of little help in practice. First we are at this stage unable to compute mixed strategies; second they are difficult to interpret in practical terms, and in particular in terms of exercise of market power. It should be recalled here that one encounters the same difficulty in two-stage forward and spot Cournot model of the sole energy market. While the standard Alaz Vila (1993) result always has pure strategy equilibrium, the introduction of the inequality constraints of the dispatch model for representing the spot market destroys this nice property. One cannot extend an Alaz-Vila type result for a stacking/Cournot model with an endogenous forward market.

## 4.7 Conclusion on energy and transmission models

Two stage models allow one to do away with the asymmetric behavioural assumptions that arise in simple expansions of the Cournot energy models to other submarkets such as transmission or reliability. Two stage models are also important for modelling the arbitrage activity of traders or transmission system operators. This additional flexibility is in principle welcome; but it also opens a range of possibilities that theory is currently unable to help us select from. One knows well how to model perfect arbitrage and perfect discrimination but the reality probably lies in between and one does not know where. It is possible, in principle, to model perfect rationality in a two-stage model even though this assumption appears extremely demanding in these models and one does not know what alternative assumption to select. Furthermore, two-stage models may lack pure strategy equilibrium, which always raises doubts about their validity. In any case, the domain of uncertainty that this extended modelling approach allows makes the sole notion of a Cournot counterfactual assumption and its use for measurement market power ambiguous.

# 5 Back to single stage models

## 5.1 Background

Two-stage models make it possible to construct multi submarket equilibrium models that exhibit symmetric behaviours of the strategic players in the different submarkets. Specifically if one assumes that generators behave  la Cournot with respect to the final consumers, one would also expect that they are able to influence the capacity market, the actions of arbitrageurs and those of the transmission system operator. A similar concern arises with respect to environmental markets such as local emission permits (e.g. green certificates in the EU). Supposing for a moment that one can specify reasonable assumptions on the behaviour of the strategic players in these submarkets, the question arises whether we are able to construct and solve the two-stage models that the insertion of these different submarkets imply. The answer is positive in principle but negative in practice. Each submarket equilibrium requires a representation through complementarity inequalities. This leads to very complex equilibrium problems subject to equilibrium constraints (EPEC). The mathematical programming paradigm extends already complex MPEC model (Luo et al. (1996)) and one just begins to study it. Hobbs and his co-authors developed the so-called Conjectured Supply Function Equilibrum approach that bypasses this difficulty and offers a possible way out.

## 5.2 The conjectured supply function approach

The conjectured supply function approach was introduced in Day et al. (2002). The underlying modelling idea is that strategic players are able to conjecture the reaction of the market to their action. We first illustrate the idea on the sole energy market. Let $P^*, s^*$ be equilibrium prices and supplies in the energy market. The authors assume that firm $f$ conjectures that a change of the price at node $i$ from $P_i^*$ to $P_i$ will entail a change of the supply of all the other producers (noted $-f$) from the equilibrium value $s^*_{-fi}$ given by

$$s_{-fi} - s^*_{-fi} = SFC_{-f}(P_i - P_i^*). \tag{111}$$

where $\mathrm{SFC}_{-f}$ is a parameter conjectured by firm $f$. Introducing this formalism in our two-firms example, Firm 1's problem becomes

$$\begin{array}{l} \max_{s_{1j}} \sum_{j=3,5,6} P_j(s_{1j} + s_{2j})s_{1j} - \sum_{i=1,2} C_i(g_i); \\ 0 \leq g_i \leq G_i \\ \sum_{j=3,5,6} s_{1j} = \sum_{=1,2} g_i \\ s_{2j} = s^*_{2j} + SFC_{2j}[P_i(s_{1j} + s_{2j} - P_i^*]. \end{array} \tag{112}$$

A similar problem being stated for Firm 2. Note that in contrast with the standard Cournot model, the actions of the other firms are no longer fixed in Firm 1's problem. Very much like the more standard conjectural variations, relation (111) introduces a dependence of $s_{2j}$ with respect to the action of player 1. The corresponding expanding market power model is derived by replacing $P_j' s_{1j}$ in the equilibrium conditions of the Cournot model by an expression that properly accounts for this dependence. In order to see this, consider the derivative

$$\frac{\partial}{\partial s_{1j}} P_j(s_{1j} + s_{2j})s_{1j}$$

where

$$s_{2j} = s^*_{2j} + SFC_{2j}[P_j(s_{1j} + s_{2j}) - P_i^*].$$

It is easy to see from the expression (111) of the conjectured supply function that

$$\frac{\partial s_{2j}}{\partial s_{1j}} = SFC_{2j}\left[P_j'\left(1 + \frac{ds_{2j}}{ds_{1j}}\right)\right]$$

or

$$\frac{\partial s_{2j}}{\partial s_{1j}} = \frac{SFC_{2j}P'}{1 - SFC_{2j}P'}.$$

We can thus write

$$\frac{\partial}{\partial s_{1j}} P_j(s_{1j} + s_{2j})s_{1j} = P_j + \frac{P'}{1 - SFC_{2j}P'} s_{1j}. \tag{113}$$

The new equilibrium conditions are thus obtained by replacing $P's_{1j}$ in equilibrium conditions of the standard Cournot model by $\frac{P'}{1 - SFC_{2j}P'} s_{1j}$. This expression reduced to the standard Cournot model when $SFC_{-2j} = 0$.

Using this derivation, one can write the equilibrium conditions of the energy only model as

$$0 \leq C_i' + \nu_i - \eta_f \perp g_i \geq 0; f = 1,\ i = 1, 2;\ f = 2,\ i = 4 \tag{114}$$

$$0 \leq \eta_f - P_j - \frac{P_j'}{1 - SFC_{2j}P_j'} s_{fj} \geq 0; f = 1, 2;\ j = 3, 5, 6 \tag{115}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; i = 1, 2, 4. \tag{116}$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{117}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{118}$$

It is easy to show that this assumption is similar (in fact identical after some transformations) to one obtained with the more standard conjectural variations that assumes that the rest of the market $(-f)$ reacts to a change of supply of firm $f$ according to

$$\frac{ds_{-fi}}{ds_{fi}} = \gamma_f$$

Specifically, the range of equilibrium of the energy market that can be spanned by varying the coefficient $SFC_{-f}$ is the same as the range of equilibrium obtainable by changing the coefficients $\gamma_f$. It would thus seem that little is gained by the approach compared to the more traditional conjectural variations. As we shall see, the real advantage of the conjectured supply function is that it can be extended to submarkets for which no inverted demand curve is available.

### 5.3 Conjectured supply function of other markets

Pang et al. (2004) extend the idea of conjectured supply functions from the sole energy market to consider other submarkets such as transmission and emission limits. The principle of this extension can be traced to the remark made in Section 3.4.4 about the origin of the asymmetric behavioural assumptions of strategic generators in the energy and other submarkets. We argued in that section that the existence of the inverted demand function makes it possible to directly model market power in the energy market. In contrast, the demand for the other services can only be derived from a model of these services such as a reliability function or a representation of the network. It is only by including equilibrium conditions of these submarkets as constraints of the Cournot energy model that one is able to model the response of these submarkets to the actions of the strategic generators. This is at the origin of the two-stage models discussed before. The extension of conjectured supply function bypasses the need to model these submarkets. It assumes that agents are able to conjecture the reaction of the other submarkets to their decision on the energy market. We illustrate this principle on the transmission market that was treated in Section 4.5.

Suppose that firm $f$ believes that an increase of its injection/withdrawal at some node (e.g. $g_1$ for an injection at node 1 or $s_3$ for a withdrawal at node 3) induces a change of the equilibrium transmission price $w_i^*$. Assume further that this change is given by relations such as

$$\begin{aligned} -w_{f1} + \{w_1^* - WTC_{f1}(g_1 - g_1^*)\} = 0 \\ -w_{f3} + \{w_3^* + WTC_{f3}(s_3 - s_3^*)\} = 0. \end{aligned} \tag{119}$$

Inserting these conjectured responses into Firm 1's model, one obtains

$$\begin{array}{ll} \max_{s_{1j}} \sum_{j=3,5,6}[P_j(s_{1j}+s_{2j})+w_j]s_{1j} - \sum_{i=1,2}[C_i(g_i)+w_i]g_i; & \\ 0 \leq g_i \leq G_i & (\nu_i) \\ \sum_{j=3,5,6} s_{1j} = \sum_{=1,2} g_i & (\eta_1) \\ s_{2j} = s_{2j}^* + SFC_{2j}[P_i(s_{1j}+s_{2j}) - P_i^*] & j=3,5,6 \\ w_j = w_j^* + WTC_{1j}(s_{1j} - s_{1j}^*), & j=3,5,6 \\ w_i = w_i^* - WTC_{1i}(g_{1i} - g_{1i}^*), & i=1,2. \end{array} \tag{120}$$

with a similar model being written for Firm 2. Introducing the expression of $w_i$ in the equilibrium conditions (114)-(115), one obtains

$$\begin{array}{l} 0 \leq C'_i + \nu_i - \eta_f + w_i + WTC_{fi} g_{f1} \perp g_i \geq 0; \\ f = 1,\ i = 1, 2;\ f = 2,\ i = 4 \end{array} \tag{121}$$

$$0 \leq \eta_f - P_j - \frac{P'_j}{1 - SFC_{2j} P'_j} s_{fj} - w_j - WTC_{fj} s_{fj} \perp s_{fj} \geq 0; \tag{122}$$

$$f = 1, 2,\ j = 3, 5, 6$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0; i = 1, 2, 4$$

$$0 \leq \sum_{i=1,2} g_i - \sum_{j=3,5,6} s_{1j} \perp \eta_1 \geq 0 \tag{123}$$

$$0 \leq g_4 - \sum_{j=3,5,6} s_{2j} \perp \eta_2 \geq 0 \tag{124}$$

This model embeds a representation of the strategic behaviour of generators in both the energy and transmission submarkets. It is single stage, which considerably simplifies both the analysis of the conditions of the existence of an equilibrium and its computation. Similar "conjectures" can be made for price of emission allowance (e.g. green certificate) reliability (e.g. capacity market) without significantly complicating the model. It suffices to specify the coefficients of the conjectured responses.

The approach is rich and general. It overcomes most of the difficulties encountered with two-stage models. One can easily expand the number of submarkets that interact with the energy markets, without unduly increasing the technical complexity of the resulting model. Cournot models expanded with conjectured supply functions of different submarkets indeed retain their good convexity properties and are therefore amenable to computation. But this gain also has a price. It requires a definition of the conjectured supply functions and thus of the coefficients SFC and WTC. These can only be obtained from structural econometric modeling. But developments of this area for complex complementarity models is scant (see Garcia-Alcade et al. (2002) for an example on the energy market). Barring the estimations of these coefficients, the expanded Cournot model is simply more ambiguous than ever.

### 5.4 Conclusion on conjectured supply curves

Conjectured supply curves models bypass the intrinsic non convexity difficulties of two-stage models. They lead to single stage models whose convexity properties can be neatly studied. The underlying idea is to assume demand functions (the conjectured supply functions) in these submarkets where there is no readily available demand function. The principle immediately points to the weakness of the approach. These conjectured supply functions assume some parameters that are difficult, if not impossible to calibrate from field data. The lack of an unambiguous theory for choosing between single and two-stage game models is replaced by the need to resort to structural econometric models (estimation of the coefficient of the conjecture responses) that remain largely unexplored at this stage (see Garcia-Alcade et al. (2002) for some examples). Here too, the counterfactual Cournot assumption is ambiguous.

## 6 Conclusion

The exercise of market power in restructured electricity systems is currently the object of intense interest both in Europe and the US. The subject has a distinctive feature in Europe. The restructuring of the electricity sector was initially meant to integrate the national electricity systems into a single European electricity market. This would have mitigated the market power of even the largest incumbents. The integration process is taking place at a desperately slow pace, if at all. The result is that most former regulated utilities now find themselves in a dominant position in their traditional market simply because this latter has not been integrated in a larger market as initially intended. These companies are vulnerable to both the temptation and the accusation of abusing their dominant position. At the same time European competition law has been revamped in two important directions. Its application has been decentralised as a result of the emphasis placed on the subsidiarity principle. Also, following a trend initiated in the telecommunication sector (Monti (2003c)), more attention is progressively given to ex ante measures taken by national regulatory authorities in network based industries.

This has led to an intense interest for the ex post measure and the ex ante simulation of the exercise of market power.

Traditional concentration indices have quickly proven their limit in this exercise and the attention has been drawn to the potential offered by market simulation models. The principle of the approach is to compare observed prices to simulated prices. This requires specifying reference simulation models. The perfect competition assumption with its emphasis on marginal cost pricing quickly emerged as a natural reference paradigm: perfect competition models simulate one extreme, that is the absence of market power. Another paradigm is necessary in order to assess whether the exercise of market power is excessive. Cournot competition has been proposed as alternative counter factual assumptions. Market power assessment studies based on such market simulation models then quickly developed in different places such as California, PJM, ISONE, Wisconsin in the US. Similar studies also exist in Europe for Benelux, the Scandinavian system, Spain and the European Union. This paper examines the underlying methodology of these studies. The Cournot model is the natural contender for devising ex ante remedies to structural problem. It is indeed the only one, in this approach, that allows one to test the impact of these remedies. Our analysis suggests that the Cournot model has not reached the state where it can be safely used for this purpose.

Whether perfect or Cournot competition, the price simulation models rely on some representation of the generation system. Specifically many studies rely on an optimal dispatch model that is used to compute marginal cost. This optimal dispatch model may be linked to a demand model in order to construct a perfect competition models. Cournot competition can easily be simulated by modifying the perfect competition model.

Some authors have argued that the very measure of marginal cost in these studies might be flawed. The reason is that an optimal dispatch model might result in a set of running plants that is quite at variance with reality. This is due to the fact that unit commitment models, not dispatch models, are used to decide plants start up and shut down. The discrepancies come from the fact that unit commitment models embed a lot of constraints that optimal dispatch do not accommodate.

We argue in this paper that most studies focus on the comparison of prices with short run variable cost interpreted as "marginal costs". The relevant marginal cost is the long run marginal cost. It is equal to the short run marginal cost when the generation system is optimally dimensioned. Most studies interpret short run marginal costs as fuel costs. It is well known that prices must go beyond fuel costs in order to justify investments. Comparing prices with short run variable cost will exaggerate the measure of market power and may even lead to "observe" the exercise of market power in situations of tight capacities where prices are not sufficient to justify new investments.

This remark immediately raises the question of what should be done in order to benchmark observed prices with respect to long run marginal costs. The answer is not clear. The perfect competition paradigm can easily be extended to a capacity expansion model. In contrast, economic theory does not provide an unambiguous definition of a dynamic Cournot model; this makes the extension of the Cournot paradigm ambiguous. Given the absence of a clear benchmark, one can only search for a surrogate of long run marginal costs in order to get a more correct measure of market power. One possibility is to expand the simulation models with a representation of reliability. This has been done in a few studies but remains largely absent from most of the literature. Still, as we argue, the necessary technology to insert a representation of reliability in both perfect and Cournot competition is available. The absence of a representation of reliability and the bias that can develop in a measure of the exercise of market power are obvious. It is generally admitted that generators exert market power when capacity is tight with respect to demand. It is also in these situations that the marginal value of reliability is highest and hence that the correction of the measure of market power by a reliability term is most justified.

An alternative is to construct multiperiod models that generalise the static Cournot model. As said before, this approach is ambiguous as there are many ways to generalise the static Cournot models. We propose one that has the advantage of simplicity both in terms of computation and economic interpretation. But there are several others. Needless to say, this ambiguity is reflected in the measure of the exercise of market power.

The introduction of reliability considerations in the perfect competition model does not raise any particular questions. The model is readily interpretable and the computation easy. In contrast, its insertion in the static Cournot models point to an embarrassing phenomenon. While it is straightforward to formulate a Cournot model of the sole energy submarket, introducing another submarket, such as capacity, in that model lead to asymmetric assumptions. Generators are Cournot players on the energy market but price takers on the capacity market. Needless to say this asymmetry is not desirable and one would like to get rid of it. In other words, the Cournot counter factual assumption should not impose different behaviours on different submarkets.

The recourse to counter factual assumptions is meant to abstract for market architecture. The experience of market power shows that generators are able to take advantage of flaws in the architecture. This is particularly relevant for Europe where the intended internal market remain largely segmented by countries because of transmission limitations (Haubrich et al. (2001)). It is thus natural to try to expand the counter-factual models to include a transmission submarket. This question has been extensively studies in the literature and one can draw on this wide body of knowledge. Very much like for capacity market, it is straightforward to extend the perfect competition model to include a nodal pricing architecture of transmission. But the Cournot model raises again questions. The straightforward extension of the simple Cournot model of the energy market again leads to asymmetric behavioural assumptions. Generators behave  la Cournot on the energy submarket and are price taker in transmission. This is not acceptable. Recent research shows how to eliminate this asymmetry. But this has some cost. Making the models more symmetric can only be done by introducing assumptions that are extremely hard if not impossible to verify on the market. In short, we may have a robust representation of the perfect competition paradigm (assuming that marginal costs can be assessed with a dispatch model), but we lack a non-ambiguous Cournot model that features more than the energy market.

The elimination of the asymmetric behavioural assumptions raises an other question: the obtained model may lack pure strategy equilibrium. The recourse to mixed strategies makes simulation models essentially useless for policy purposes. Recent research has proposes some method to circumvent this drawback. But these methods again increase the number of assumptions that one need to introduce in the model, with no current possibility to select from these assumptions on a solid basis.

In short the simulation models that we are currently able to construct are ambiguous as soon as one departs from the perfect competition paradigm. They do not offer the legal and regulatory certainty required by ex ante remedies to structural problems. More academic work is needed. In contrast, we know a lot both from theory and practice on market integration. Effectively integrating the electricity markets of the Member States would definitely reduce the market power of the incumbents. But there is currently almost no progress along this path in Europe.

It is illustrative to close this paper with two quotes from the analysis of the exercise of market power in California. The debacle in California is remarkable in its extent and no system has been studied as much after 2000. Still here is how divergent two leading experts can be on the ex post analysis of market power in California three years after the facts.

"With this clarification, the second conclusion is less well understood and more important. Namely, the record to date has not produced anything that has withstood analysis to support a finding that market manipulation, including the exercise of market power, had a major impact on prices during 2000-2001" (Hogan, September 2003).

Compare this statement with the following. "The firm-level results presented below are consistent with the view that the enormous increase in the amount market power exercised in the California market beginning in June of

2000 documented in BBW was due to a substantial increase in the amount of unilateral market power possessed by each of the five large suppliers in California. (Wolak, June 2003).

If one has reached that state of knowledge in an ex post analysis, what can one say with reasonable certainty on ex ante remedies ?

## References


[1] Allaz, B. and J.L. Vila. Cournot competition, forward markets and efficiency. *Journal of Economic Theory*, 59, 1–16, 1993.

[2] Arellano M. S. (2003). Diagnosing and mitigating market power in Chile's electricity industry. Cambridge MIT Institute Working Paper 27.

[3] Bartholomew E. S, Siddiqui, A.S., Marnay, C. and S.S. Oren (2003). The New York transmission congestion contract market: Is it working Efficiently. *Electricity Journal*, November 2003, 1–11.

[4] Bessembinder, H. and M.L. Lemmon (2002). Equilibrium pricing and optimal hedging in electricity forward markets. *Journal of Finance*, 57, 1347–1382.

[5] Berry, C.A., Hobbs, B.F., Meroney, W.A., O'Neill, R.P. and W.R. Stewart, Jr. (1999). Analyzing strategic bidding behavior in transmission networks. *Utilities Policy*, 8(3), 139–158.

[6] Bjørndal, M. and K. Jrnsten (2004). Equilibrum prices supported by dual price functions in markets with non-convexities. Department of Finance and Management Science, Norwegian School of Economics and Business Administration, Bergen, Norway.

[7] Borenstein, S., Bushnell, J.B. and S. Stoft (2000). The competitive effects of transmission capacity in a deregulated electricity industry. *Rand Journal of Economics*, 31(2), 294–325.

[8] Borenstein, S., Bushnell, J.B. and F.A. Wolak (2002). Measuring market inefficiencies in California's restructured wholesale electricity market. *American Economic Review*, 92(5), 1376-1405.

[9] Bushnell, J., Mansur, E.T. and C. Saravia (2005). Market structure and competition: A cross-market analysis of U.S. electricity deregulation demand. WP-126 CSEM Working Paper, University of California Energy Institute, Berkeley.

[10] Bushnell, J. and C. Saravia (2002). An empirical assessment of the competitiveness of the New England electricity market. WP-101 CSEM Working Paper, University of California Energy Institute, Berkeley.

[11] Cardell, J., Hitt, C.C. and W.W. Hogan (1997). Market power and strategic interaction in electricity network. *Resource and Energy Economics*, 19(1–2), 109-137, 1997.

[12] Creti, A. and N. Fabra (2004). Capacity markets for electricity. WP 124 CSSEM Working Paper, University of California Energy Institute, Berkeley.

[13] Day, C., Hobbs, B.F. and J.-S. Pang (2002). Oligopolistic competition in power networks: A conjectured supply function approach. *IEEE Transactions on Power Systems*, 27(3), 597–607.

[14] EC-2002. Second benchmarking report on the implementation of the internal electricity and gas markets. Commission Staff Working paper SEC (2002), 1038, October 2, 2002.

[15] EC-2003a. Third benchmarking report on the implementation of the internal electricity and gas market. Commission draft Staff paper, March 1, 2004.

[16] EC-2003b. Directive 2003/54/EC of the European Parliament and of the Council of 26 June 2003. Official Journal L 176, July 15, 2003, pp. 0037-0056

[17] EC-2003c. Regulation (EEC) No 1228/2003 of the European Parliament and of the council on Conditions for Access to the Network for cross-border exchanges in electricity, June 26, 2003. Official Journal L 176, July 15, 2003, pp. 0001-0010

[18] EC-2003d. Directive 2003/55/EC of the European Parliament and of the Council of 26 June 2003 concerning common rules for the internal market in natural gas and repealing Directive 98/30/EC. Official Journal L 176, July 15, 2003, pp. 0057-0078

[19] EC-2003e. XXXIInd Report on Competition Policy - 2002. Available from DG Competition web site.

[20] EC-2004. European Union Competition Policy - 2003. Available from DG Competition web site.

[21] Ehrenmann, A. and K. Neuhoff (2003). A comparison of electricity market designs in networks. Cambridge MIT Institute Working Paper 31, available from the "CMI Electricity Project" website.

[22] Ehrenmann and Smeers (2005). Market simulation model with reliability. Mimeo.

[23] ETSO (2001). Co-ordinated auctionning a market-based method for transmission capacity allocation in meshed networks. Final report, April 2001, available from ETSO website.

[24] ETSO (2003). Current state of balance management in Europe. Available from ETSO website, December 2003.

[25] Garcia-Alcade, A., Ventosa, M., Rivier, M., Ramos, A. and G. Relano. Fitting electricity market models: A conjectural variation approach. *Proceedings 14th PSCC 2002*, Spain, Seville.

[26] Green, R.J. (1996). Increasing competition in the British Electricity spot market. *Journal of Industrial Economics*, 44(2), 205-216.

[27] Green, R.J. (1999). The electricity contract market in England and Wales. *Journal of Industrial Economics*, 47(1), 107-124.

[28] Green R. (2002) Retail competition and electricity contracts. Cambridge Working paper in Economics CWPE 0406. University of Cambridge.

[29] Green, R. (2004). Did English generators play Cournot ? Capacity withholding in the electricity pool. Cambridge MIT Institute Working Paper 41.

[30] Green and Newbery (1992). Competition in the British electricity spot market. *Journal of Political Economy*, 100(5), 929-953.

[31] Harvey, S.M. and W.W. Hogan. Market power and market simulations. Mimeo, Center for Business and Government, John F. Kennedy School of Government, Harvard University, Cambridge, 2002.

[32] Haubrich, H.J., Zimmer, C., von Sengbusch, K., Li, F., Frisz, W., Kopp, S., Schulz, W., Musgens, F. and M. Peek (2001). Analysis of electricity network capacities and identification of congestion. Available from DG Energy site.

[33] Hobbs, B.F. (2001). Linear complementarity models of Nash-Cournot competition in bilateral and POOLCO power markets. *IEEE Transaction on Power Systems*. 16(2), 194–202.

[34] Hobbs, B.F. and U. Helman (2004). Complementarity-based equilibrium modeling for electric power markets. In D.W. Bunn (eds.), *Modeling Prices in Competitive Electricity Markets*. J. Wiley Series in Financial Economics, Wiley, United Kingdom

[35] Hobbs, B.F.and F.A.M. Rijkers (2004). Strategic generation with conjectured transmission price response in a mixed tranmsission system. Part I: Formulation. *IEEE Transactions on Power Systems*, 19(2), 701-717.

[36] Hobbs, B.F., Rijkers, F.A.M. and A.F. Wals (2004). Strategic generation with conjectured transmission price response in a mixed tranmsission sys-

tem. Part II: Application. *IEEE Transactions on Power Systems*, 19(2), 872–879.

[37] Hogan, W.W. and B.J. Ring (2003). On minimum-uplift pricing for electricity markets. Working Paper, John F. Kennedy School of Government, Harvard University.

[38] Joskow, P. and E. Kahn (2002). A quantitative analysis of pricing behaviour in California's wholesale electricity market during summer 2000: the final world. Cambridge MIT Institute Working Paper 02.

[39] Joskow, P. (2004). Transmission policy in the United States. Cambridge MIT Institute Working Paper 54.

[40] Joskow, P. and J. Tirole (2004). Reliability and competitive electricity markets. WP-129, CSEM Working Paper, University of California Energy Institute, Berkeley.

[41] Kamat, R. and S. S. Oren (2004). Two settlements systems for electricity markets under network uncertainty and market power. *Journal of Regulatory Economics*, 25(1), 5–37.

[42] Klemperer, P.D. and M.A. Meyer. Supply function equilibria. *Econometrica*, 57, 1243-1277, 1989.

[43] Klotz, R. and H. Nyssens (2004). Energy day: First sectoral high-level meeting within ECN. *Competition Policy Newsletter*, 3, 33–35.

[44] Loulou, R., Goldstein, G. and K. Noble (2004). Documentation for the MARKAL family of models. Energy Technology Systems Analysis Programme. Available from http://www.etsap.org/tools.htm, October 2004.

[45] Luo, Z.A., Pang, J.S. and D. Ralph (1996). *Mathematical Programs with Equilibrium Constraints*, Cambridge University Press, Cambridge (U.K.).

[46] Mansur, E.T. (2003). Vertical integration in restructured electricity markets: Measuring market efficiency and firm conduct. WP-117 CSEM Working Paper, University of California Energy Institute, Berkeley.

[47] Metzler, C., Hobbs, B.F. and J.-S. Pang (2003). Nash-Cournot equilibria in power markets on a linearized DC network with arbitrage: Formulations and properties. *Networks and Spatial Economics*, 3(2), 123–150.

[48] Monti, M. (2002). The single energy market: the relationship between competition policy and regulation. Speech 02/01, House of Deputies, 10th Commission on Productive Activities, Commerce and Tourism, Rome, March 7, 2002. Available from DG Competition Web site.

[49] Monti, M. (2003a). Applying EU competition law to the newly liberaliseid nergy markets. World Forum on Energy Regulation, Rome, October 6, 2003.

[50] Monti, M. (2003b). EU competition policy after May 2004. Fordham Annual Conference on International Antitrust Law and Policy, New York, U.S.A.

[51] Monti, M. (2003c). Competition and regulation in the new framework. Presented in the "Public Workshop on 'Electronic Communications Consultation Mechanisms'", Brussels, July 15, 2003.

[52] Monti, M. (2004). Energy liberalisation: moving towards real markets opening. Brussels, European Commission, DG Competition.

[53] Murphy and Smeers (2004a). Generation capacity expansion in imperfectly competitive restructured electricity markets. To appear in *Operations Research.*

[54] Murphy and Smeers (2004b). Forward markets may not decrease market power when capacities are endogenous. Mimeo.

[55] Neuhoff, K. (2003). Integrating transmission and energy markets mitigates market power. Cambridge MIT Institute Working Paper 17.

[56] Neuhoff, K., Barquin, J., Boots, M.G., Ehrenmann, A., Hobbs, B.F. and F.A.M. Rijkers (2005). Network-constrained models of liberalized electricity markets: the devil is in the details. *Energy Economics*, 27, 495–525.

[57] Newbery, D.M. (1998). Competition contracts and entry in the electricity spot market. *Rand Journal of Economics*, 29, 726–749.

[58] Newbery, D.M. (2003). The relationship between regulation and competition policy for network utilities 1. Paper written for the conference "How should competition policy transform itself ? Designing the new competition policy". Japan, November 20, 2003. Cambridge University.

[59] Newbery, D.M., van Damme E. and N.H.M. von der Fehr (2003). Benelux market integration: market power concerns. Report of the Market Surveillance Committee. Dutch Competition Authorities. Available from http://www.nmanet.nl

[60] O'Neill, R.P., Sotkiewicz, P.M., Hobbs, B.F., Rothkopf, M.H. and W.R. Steward Jr. (2004). Efficient market-clearing prices in markets with nonconvexities. Forthcoming in *European Journal of Operational Research.*

[61] Oren, S.S. and S.A. Smith (1993). *Service Opportunities for Electric uUtilities: Creating Differentiated Products.* Boston, Kluwer Academic Publishers.

[62] Pang, J.-S., Hobbs, B.F. and C. Day (2004). Electricity market models with conjectured resource price responses. To appear *Mathematical Programming B.*

[63] Rajaraman and Alvarado (2003). (Dis)proving market power. Available from HEPG website.

[64] Saravia, C. (2003). Arbitrageurs on efficiency and market power in the New York Electricity Market. WP-121 CSEM Working Paper, University of California Energy Institute, Berkeley.

[65] Sidiqqui, A. (2003). Managing electricity reliability risks through the forward markets. *Network and Spatial Economics*, 3(2), 225–263.

[66] Smeers, Y. (2003). Market incompleteness in regional electricity transmission. Part II. The forward and real time markets. *Network and Spatial Economics*, 3(2), 175–196.

[67] Smeers, Y. (2005). The European experience suggests that strong harmonisation is necessary in order to integrate restructured electricity markets. Paper prepared for the EPRI Conference "In Search of Alternative Pathways", San Francisco, May 2005.

[68] Stoft, S. (2002). *Power System Economics: Designing Markets for Electricity.* Wiley-IEEE Press.

[69] Twomey, P., Green, R., Neuhoff, K. and D. Newbery (2004). A review of the monitoring of market power. CMI EP71 Working Paper, Cambridge Working Papers in Economics.

[70] Wilson, R. (2002). Architecture of electric power markets. *Econometrica*, 70(4), 1299–1340.

[71] Wolak, F. (2000). An empirical analysis of the imapct of hedge contracts on bidding behaviour in a competitive electricity amrket. *International Economic Journal*, 14(2), 1–39.

[72] Wolak, F. (2003). Identification and estimation of cost functions using observed bid data: An application to electricity. In M. Dewatripont, L.P. Hanse, and S. J. Turnovsky (eds.), *Advances in Econometrics: Theory and Applications, Eight World Congress, Volume II.* Cambridge University press, 133-169.

[73] Wolak, F. (2004a). Lessons from international experience with electricity market monitoring. Center for the Study of Energy Markets Working Paper 134, University of California Energy Institute, Berkeley, 2004.

[74] Wolak, F. (2004b). Quantifying the supply-side benefits from forward conracting in wholesale electricity markets. Available from Frank Wolak website, November 2004.

[75] Wolak, F. (2004c). Managing unilateral market power in electricity. Available from Frank Wolak website, November 2004.

[76] Yao, J., Oren, S. and I. Adler (2004). Computing two settlement Cournot equilibria in electricity Markets. *Proceeding of the 37th Hawaii International Conference on Systems Sciences HICSS 37. Big Island, Hawaii, January 5-8, 2004.*

# Appendix A: Electrical Engineering Background

## A.1 Dispatch and unit commitment models

Market power assessments in electricity most commonly rely on some underlying optimal dispatch model. A more complex model, namely the unit commitment is also sometimes invoked. We refer to them as stacking models. These models were developed by the power industry in order to solve short-term problems (from an hour to a week) whereby one optimizes the operations of the generation system in order to satisfy an exogenously demand. The optimal dispatch model works on an hourly basis and assumes that the set of running plants is given. The model then chooses the operation level of each running plant in order to satisfy the demand at minimal cost. Optimal dispatch plants can also easily be transformed into an equilibrium model. These models can be formulated as follows.

**The dispatch model.** Let $g_i$ be the generation level of generator $i$, $C_i(g_i)$ its cost function and $G_i$ its exogenous capacity; $s_j$ is the (exogenous)demand of consumer$j$. The optimal dispatch problem is formulated as

$$\begin{array}{lll} \min & \sum_i C_i(g_i) & \\ \text{s.t.} & \sum_i g_i \geq \sum_j s_j & (\eta) \\ & 0 \leq g_i \leq G_i & (\nu_i) \end{array} \tag{A.1.1}$$

The demand constraint is here written as an inequality, using the standard free disposal assumption. This inequality holds as equality under standard economic assumptions (increasing and convex cost functions). The conversion of the dispatch problem into an equilibrium model is most conveniently described by first considering the welfare optimisation version of the dispatch problem. Make $s_j$ endogenous and suppose that consumers have an inverted demand function: $P_j(s_j)$. The welfare problem is stated as

$$\begin{array}{lll} \min & \sum_i C_i(g_i) - \sum_j \int_0^{s_j} P_j(\xi) d\xi & \\ & \sum_i g_i \geq \sum_j s_j & (\eta) \\ & 0 \leq g_i \leq G_i & (\nu_i) \end{array} \tag{A.1.2}$$

Models such as this only involve short-term variable costs that mainly consist of fuel expenses. Demand appears as a fixed amount that needs to be satisfied in the optimisation model and as a argument of the inverted demand in the equilibrium model. The complementarity version of the model is stated below and interpreted as follows. Let $\nu_i$ be the scarcity rent of capacity of generator $i$, and $\eta$ the market price. (Note that the scarcity rent $\nu_i$ of capacity $G_i$ should not be confused with a market power premium.) One writes

$$0 \leq C_i'(g_i) + \nu_i - \eta \perp g_i \geq 0 \tag{A.1.3}$$

$$0 \leq G_i - g_i \perp \nu_i \geq 0 \tag{A.1.4}$$

$$0 \leq \eta - P_j(s_j) \perp s_j \geq 0 \tag{A.1.5}$$

$$0 \leq \sum_i g_i - \sum_j s_j \perp \eta \geq 0. \tag{A.1.6}$$

The equilibrium conditions can easily be interpreted: (A.1.3) states the standard equality between prices and marginal cost for positive generation level; (A.1.4) indicates that the scarcity rent is null for plants that are not operating at capacity, (A.1.5) states that prices should be lower than the marginal willingness to pay for the first unit if consumption is to be different from zero. Last relation (A.1.6) states the equality between supply and demand.

The optimal dispatch is a subproblem of the unit commitment problem that we here present in the simplest possible form. Consider a set of successive hourly demands over a day (24 hourly demands) or a week (168 hourly demand). The cost structure of generation plants is modified as follows. Besides the proportional charge, one also introduces a start up cost. The problem of finding how to meet a daily or weekly given demand at minimal cost therefore requires both selecting which plants are running in each hour and their operations level. This problem is formulated as follows.

**The unit commitment model (simplest possible version).** One first decomposes the day (week) into a succession of hours $h = 1, ..., H$. Consumers have a demand $s_j^h$ (inverted demand function $P_j^h(s_j^h)$) in each hour $h$. There is a fixed cost to start up a machine

$$\begin{array}{lrcll} \text{e.g.} & C_i(g_i^h) & = & K_i + c_i g_i^h & \text{if} \quad g_i^h > 0, g_i^{h-1} = 0 \\ & & = & c_i g_i^h & \text{if} \quad g_i^h > 0, g_i^{h-1} > 0. \end{array} \tag{A.1.7}$$

The unit commitment problem is stated as

$$\begin{array}{ll} \min & \sum_h \sum_i C_i(g_i^h) \\ \text{s.t.} & \sum_i g_i^h = \sum_j s_j^h \ \forall \ h \\ & 0 \leq g_i^h \leq G_i. \end{array} \tag{A.1.8}$$

where the demand constraint must be written as an equality because of the more complex, non convex form of the objective function. Indeed, the objective function of this model now comprises both fuel costs and start up costs. These may induce keeping a plant runing (and hence oversatisfying demand) in order to avoid shutting down and restarting a plant. This is not possible in electricity where consumption must always be equal to generation, hence the equality formulation. We briefly mention other complications that are not modeled here for the sake of simplicity but are crucial in practice. Running plants cannot operate at an arbitrary level but need to remain above some minimal generation level. There are constraints on the ramping rate that bound the change of operation level of a plant between two successive periods. Other constraints also impose a minimal downtime between shut-down and start-up of a machine and a minimal running time between start-up and shut-down. We refer in the text to these different features (start up cost, technical minima, ramping rate, minimal downtime and running time constraintsä.) as to the unit commitment idiosyncrasies. The tradition in market power assessment is to mainly work with dispatch models. As we shall see unit commitment problems may also turn out to be relevant.

In contrast with dispatch models, unit commitments models cannot easily be turned into equilibrium models because of the indivisibilities present in the representation of the operations of the plants. (See O'Neill et al. (2004), Hogan and Ring (2003) and Bjørndal and Jrnsten (2004)).

## A.2 The electrical grid

The electrical network imposes several constraints on the operations of the power sector. It is common in restructuring studies to limit oneself to thermal limits on the lines. These are easily expressed by bounding the flows on the lines. Flows are bidirectional, which implies hat the bounds should in principle be imposed on the two possible directions of the flow.

Market integration is mentioned in the text as the natural goal of the restructuring process in the European Union. The following example taken from Chao and Peck (1998) is particularly suitable for discussing market integration issues. The grid has six nodes and eight lines. Generators are located at nodes 1, 2 and 4 and consumers at nodes 3, 5 and 6. Electricity flows on the network according to Kirchhoff laws. These are commonly represented by the so called DC load flow approximation, that can itself be modelled by a linear mapping (the PTDF coefficients) of injection and withdrawals into line flows. These are defined as follows. Assume some reference node in the grid that can be interpreted economically as a hub where all the electricity is traded. An injection at some node $i$ of the network results in flows on all lines that are given by the PTDF. Lines 1-6 and 2-5 are the sole lines assumed to have a thermal limit in Chao and Peck (1998). PTDF on these lines therefore have a particular relevance. We illustrate PTDF on these lines by noting that unitary commercial transaction $s_{13}$ between a generator node 1 and a consumer node 3 results in flows on the line 1-6 computed as

$$PDF_{1(1-6)} - PDF_{3(1-6)}.$$

**Modeling transmission.** Market simultation models that involve a representation of the network often rely on the PTDF representation. The representation of the network is obtained as follows. Select a hub node (e.g. 6) and assume the DC approximation of load flow equations. Compute PTDF of the lines subject to thermal limits with respect to injection and withdrawal at nodes (in the example, two constrained lines (1–6) (2–5) with PTDF, $PTDF_{i(1-6)}$ $PDF_{i(2-5)}$). Let $g_i$, $i = 1, 2, 4$ be the injections and $s_j, j = 2, 5, 6$ be the withdrawals; the constraints on line (1-6) due to the grid can be written

$$(-\overline{F}_{1-6} \leq) \sum_{i=1,2,4} PDF_{i(1-6)} g_i - \sum_{j=3,5,6} PDF_{j(1-6)} s_j \leq \overline{F}_{1-6} \qquad \text{(A.2.1)}$$

with a similar constraint for (2-5).

For the sake of simplicity, we shall only refer to the upper bound on the flow in the rest of the paper.

## A.3 A network based optimal dispatch model

It is commonly admitted (but not proved) that national grids do not create much network bottleneckls but that constraints at the interconnections are serious. These bottlenecks hamper the integration of the national markets with the result that incumbents remain to some extent protected from competition by the limited capacities of the grid at the borders. It thus makes sense to expand the dispatch and unit commitment models to accommodate the network constraints. This is illustrated on the dispatch model

**The optimisation problem**

$$\begin{array}{ll} \min & \sum_i C_i(g_i) \\ \text{s.t.} & \sum_i g_i = \sum_j s_j \\ & (-\overline{F}_\ell \leq) \sum_{i=1,2,4} \mathrm{PDF}_{i\ell}\, g_i - \sum_{j=3,5,6} \mathrm{PDF}_{j\ell}\, s_j \leq \overline{F}_\ell \quad \ell = (1-6), (2-5) \\ & 0 \leq g_i \leq G_i. \end{array} \qquad \text{(A.2.2)}$$

## A.4 Reliability

Reliability concerns pervaded the power industry in regulatory days. They somehow lost their importance in the economic restructuring literature, but recently came back to the forefront in Joskow and Tirole (2004) and have since expanded with concerns for resource adequacy. The former Pool in England and Wales embedded a traditional reliability concept in its pricing system, namely the Loss of Load Probability (LOLP). We briefly present that concept and related notions used in the discussion.

The LOLP is the probability that load is not served. In order to call upon probability notions, we distinguish between deterministic variables used in market simulation models that we note with "normal" caracters and random variables appearing in reliability criteria (in "bold" caracters).

Let $s_j$ be a contract variable appearing in the simultation model and $\mathbf{s}_j$, the realisation of contrat in real time. Capacity is also random which leads to the distinction between $G_i$, the rated capacity and $\mathbf{G}_i$, the available capacity. One then defines the following notions

| | | |
|---|---|---|
| (i) | Loss of Load Probability (LOLP) | $\Pr[\sum_j \mathbf{s}_j \geq \sum_i \mathbf{G}_i]$ |
| (ii) | Expected unserved energy (EUE) | $\mathrm{E}[\max(\sum_j \mathbf{s}_j - \sum_i \mathbf{G}_i; 0)]$ |
| (iii) | Value of Lost Load VOLL | economic cost of unserved energy between \$1000/Mwh and \$ 100 000/Mwh !! |
| (iv) | Expected Cost ($EC(\mathbf{s}, \mathbf{G})$) | Expectation of the dispatch cost with ($\mathbf{s},\mathbf{G}$) random unserved energy priced at VOLL |

As mentioned above, in these expressions the load and capacity variables take on a significance of random variable that expands on their deterministic interpretation in the stacking model. A supply variable $s$ of the stacking model represents a certain profile in a supply contract (e.g. flat if there is a single variable) with some flexibility otherwise. The exercise of the flexibility by the buyer and even the possible deviation from the contractual terms make the supply a truly random variable $\mathbf{s}$. In the same way, the $G$ variable of the stacking model refers to a rated capacity while the available capacity $\mathbf{G}$ is subject to forced outages and hence is a truly random variable.

The LOLP is commonly used in the power industry and easy to compute. If suffers however from inadequate risk properties. The EUE is a related but more suitable criterion. The LOLP is related to the EUE through derivative operator: in some sense the LOLP is the derivative of the EUE.

The value of lost load (VOLL) is another notion that has been present in the profession since the very beginning. It expresses the economic value of the curtailed energy. Even though commonly used, the concept has so far resisted any precise evaluation. Stoft (2002) mentions ranges from \$10 000/Mwh to \$100 000/Mwh.

Last, one can invoke expected dispatch costs. This can be defined as the expectation of the dispatch costs computed with respect to the distribution of plant availability and demand uncertainty, assuming that un-served demand is priced at the VOLL when demand is higher that available capacity. This expectation and some of its derivatives are easy to compute. For the sake of simplification we do not use the expected cost in the paper.

# Appendix B

The following features are mentioned as explanation for the divergence between results from optimal dispatch and unit commitment models.

- sensitivity to data when system is tight
- chronological demand that makes
  short run marginal cost $\neq$ variable cost
  short run marginal cost meaningless (startup cost, minimal run up and shut down constraints, ...)
- aggregation of hours into blocs (reduce marginal cost)
- limited energy plants
- pumped storage and reservoir
- decision under uncertainty
  hydro availability
  stochastic price process
- network constraints
- maintenance and plant availability