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Bundling by Competitors and the Sharing of Profits

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Abstract

We discuss the welfare effects of bundling two products offered by two symmetric firms. We first show that, in terms of welfare, a monopoly does better than a duopoly in which each firm sell its good and that a monopoly selling the bundle does better than if it sells the bundle and the two goods separately. We also show that the choice of the mechanism for sharing the profits, obtained from the sales of the bundle, might have dramatic positive or negative effects – even when the various optional mechanisms yield equal splits. In particular, the use of the Shapley value yields the highest total and consumer surpluses and the lowest producer surplus, while the weighted Shapley value totally reverses the outcome and yields profits which are very close (over 99%) to the full monopoly profits. Hence, as in the case of bundling by a monopolist, when competitors bundle they assist each other in deterring entry. However, in addition when competitors bundle, they can implicitly cooperate via the setting of the profit sharing rule and increase their profits at the expense of customers. This issue calls for some further attention by regulators .

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1. Introduction

Bundling is a pervasive policy used by firms. Beginning with Stigler (1968), most of the literature considers bundling by a monopolist as a tool for price discrimination (Adams and Yellen, 1976, Matutes and Regibeau, 1992, McAfee, McMillan and Whinston, 1989, Schmalensee, 1984, Spence, 1980), especially when the valuations of the two bundled commodities are negatively correlated, but also when both products in the bundle have independent valuations (McAfee, McMillan and Whinston, 1989). The gains decrease with positive correlation and vanish when this correlation is perfect. More recently, Nalebuff (1999, 2004) showed that the main gains for the bundling producer come from entry deterrence. In particular, he proves that the pure price discrimination effect may increase the profit of a bundling monopolist by some 9 percent, but if he can make entry deterrence effective, his profits can more than double.

In this paper, we look at pure welfare effects of competitive bundling by competitors and abstain from the entry deterrence issue which was the focus of Nalebuff. We consider bundles of goods, produced by two competing firms, and allow for a mixed policy where, in addition to offering the bundle, each participating firm continues to offer its own good.

Such practices are common in the cable television industry,[3] where broadcast networks (such as ABC, CBS, or NBC) or cable television networks (MTV, CNN) bundle channels (HBO, SHO etc.) into services, where usually, each channel can also be bought *à la carte*. See Crawford (2000, 2002), and Coppejans and Crawford (1999) for an analysis of this case. In 2003, Comcast, the largest US cable operator, decided to offer discount packages to customers subscribing simultaneously to its Internet access and to cable television services. The Comcast bundle forced its Internet customers to also sign up for its cable services while allowing its cable customers not to sign up to its Internet

[3] Though in some cases mixed bundling is not an option.

services.[4] Bundles composed of broadband and Internet access services are being offered as joint ventures between ISP's and cable companies. Telecommunication companies bundle local and long distance call services[5], provided by different carriers. See Linhart et al. (1995) for an analysis.

The practice of bundling by competitors is also observed more and more frequently in the world of tourism, transportation, culture and entertainment. In particular, museum passes, which give visitors (tourists or residents) unlimited access to a list of participating museums, during a limited period of one to several days (perhaps a whole year for residents), have become very common in many cities and countries,[6] or even across nations.[7] Since this happens mainly in the not-for-profit sector, no anti-competitive concerns have been brought up so far.

Bundling by competitors raises several interesting problems. The first issue is price setting. Throughout most of the literature, prices are set via joint profit maximization by the single firm, offering both the individual goods and the bundle. In our case, individual prices are set by each firm, while the price of the bundle is set jointly by the participating firms or by an agent on whom they agree. A plausible solution is to have a jointly owned subsidiary (called the *bundle identity*), which introduces the bundle and conducts its pricing, marketing and sales. Naturally, this subsidiary will maximize its own profit and will share it somehow among the participating partners. This leads to a two stage pure strategy non cooperative game in which the bundling entity sets the price of the bundle in the first stage, while each partner sets the price for his own product (also offered as a part

[4] See Randolph May, The storm over broadband bundling, http://news.com.com/2010-1071-997226.html. (last accessed in December 2003).
[5] Or cellular and wire line services.
[6] In 2000, such passes exist in Amsterdam, Barcelona, Bologna, Bonn, Budapest, Copenhagen, Helsinki, Lisbon, London, Luxemburg, Montreal, Paris, Philadelphia, Salzburg, Stockholm and Vienna. The Netherlands (Nederlandse Museumjaarkaart), the UK (Great British Heritage Pass), Flanders (OKV-Museumkaart), and Switzerland (Passeport Musées Suisses), and probably many other countries, have country-wide museum passes. A search, employing Google's Web search engine, over the phrase "museum joint pass" yielded (as of February 2004) 162,000 hits, compared to a few hundred two years earlier. Of course, many of these are multiple or irrelevant hits.
[7] The Pass-Musées du Rhin Supérieur gives access to museums in the Upper-Rhine region, which includes some parts of Eastern France, Western Germany and Switzerland.

of the bundle) in the second stage, taking into account his share in the profits of the bundle entity.

This brings us to the second problem, related to the sharing of the profits generated by the bundle. So far, this was not considered an issue in the literature which assumes that the bundle is usually offered by a single firm, a case in which the sharing of profits is of little interest. However, if the bundled goods are produced by competing firms or by different divisions of a firm managed as profit centers, then it becomes important to devise a sharing rule, if only for accounting purposes. Linhart et al. (1995) and Ginsburgh and Zang (2003) have independently shown that the Shapley value is a convenient allocation rule[8] applicable for some of these cases, namely when each bundled good is usable only once, when the buyer can also choose not to consume all the goods in the bundle[9] and when her actual consumption can be recorded (for accounting purposes). All these properties are satisfied by museum passes and long distance call bundles. The problem is that due to the combinatorial nature of the Shapley value, computation is practically impossible for the general case. Linhart et al. (1995) and Ginsburgh and Zang (2003) show that, in the particular cases they consider, this computation becomes straightforward: The profit generated from each unit of the bundle is equally divided according to its actual usage. For the museum pass problem in particular, the Shapley value allocation is obtained by dividing the income from each pass equally among the museums that were actually visited by this particular pass holder. The rule is easy to implement and the sharing easy to compute. Moreover, as shown by Ginsburgh and Zang (2004), it provides the right incentives for the participating partners who have no interest to deviate, and does not lead to counterintuitive and inconsistent results (as do many other allocation rules used for museum pass programs).[10]

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[8] See also Mirman, Tauman and Zang (1985), for a survey concerning the sharing of joint costs.

[9] Note that for many types of bundled products, pricing will be such that the consumer will not purchase the bundle unless she intends to consume all its components. This is not the case for services, such as TV-channels (only some can be watched for part of the time) or museums (only some of the museums included in the pass may be visited).

[10] Some of the sharing rules may increase the share in the joint profit allocated to museums that were not visited, or induce museums to increase their own entry price in order to reap a larger share of the joint profit.

In order to compare the results of competitive mixed bundling with the better-known case of monopoly considered by Nalebuff, we repeat it here, but also add consumer surplus comparisons which are not considered by Nalebuff. We consider the case of two producers each producing a single good or service at zero marginal production cost, and where the reservation prices of the potential customers are uniformly distributed and independent. This implies that the two producers are symmetric. Depending on the case considered, they may cooperate in introducing the bundle. Using analytical and, given the complexity of the problem, computational tools when necessary, we first show, in Section 2, that if the two firms choose not to offer the individual goods but only the bundle, both consumer and producer (and consequently overall) surplus will increase. Furthermore, we show that if both firms offer their individual goods and the bundle but act as a monopoly, then welfare will increase compared to the case where they only offer their individual goods. This increase, however, comes on account of the customers whose surplus decreases. In Section 3 we consider the effects of the sharing rule when firms offer their individual goods but the bundle is managed by the bundle entity. We analyze two allocation rules: the Shapley value and the weighted Shapley value (Shapley, 1953), where the individual goods prices are taken as weights. In our symmetric two-firm settings both rules end up sharing the profits equally. However, the resulting welfare effects are dramatically different when the two firms take the *mechanisms* of the allocation rules into their optimizing behavior. Indeed, under Shapley value sharing consumer and overall surplus are the largest and producer surplus the smallest. This effect is reversed under weighted Shapley value sharing which generates the smallest consumer surplus and overall welfare. The producer surplus attained here is only 0.7% short of the highest possibly surplus attained by the full monopoly. Thus, if it is up for the producers to choose, then the worst option will be obtained!

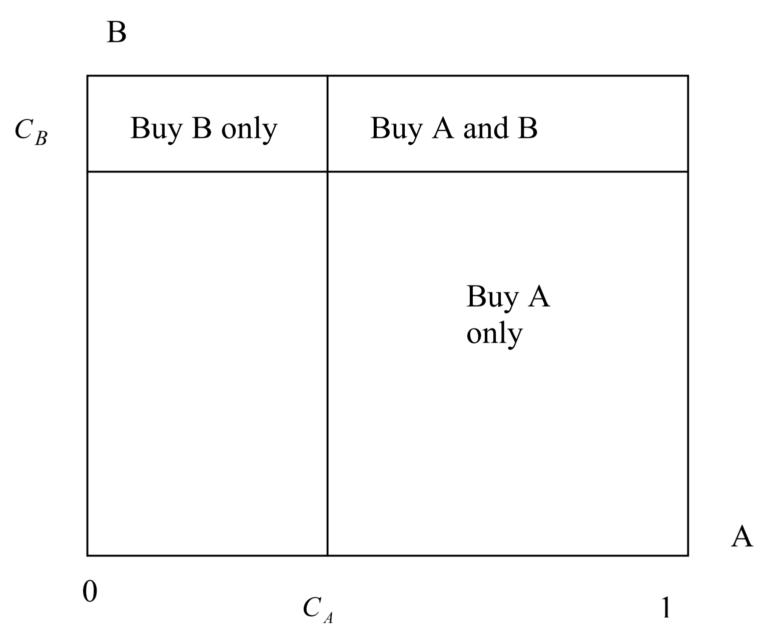
The implication of this is obvious. As in the monopoly case, bundling by competitors serves to discourage entry. In addition to that, by selecting the “right” profit sharing mechanism, the bundling competitors can implicitly cooperate to affect product prices and increase their profits. This might deserve the special attention of the regulators.

2. The General Setup

Following Nalebuff, we assume that there are two firms producing two goods A and B at zero marginal cost. The willingness to pay of customers is uniformly distributed on the (0,1)×(0,1) square (market of size 1 for each good) and there are no budget considerations, implying that the valuations for the two goods are independent. We assume that entry in not possible and hence each firm is a monopoly in its own product market. Each consumer buys one unit of either A, or B or one unit of the bundle at prices C_A, C_B and *C*, respectively.

The situation before the introduction of the bundle is described in Figure 1. Here, customers whose reservation prices are above the prices of the goods namely, those whose reservation prices are to the right of (above) the C_A (C_B) line, will purchase good A (B) at the given price C_A (C_B)

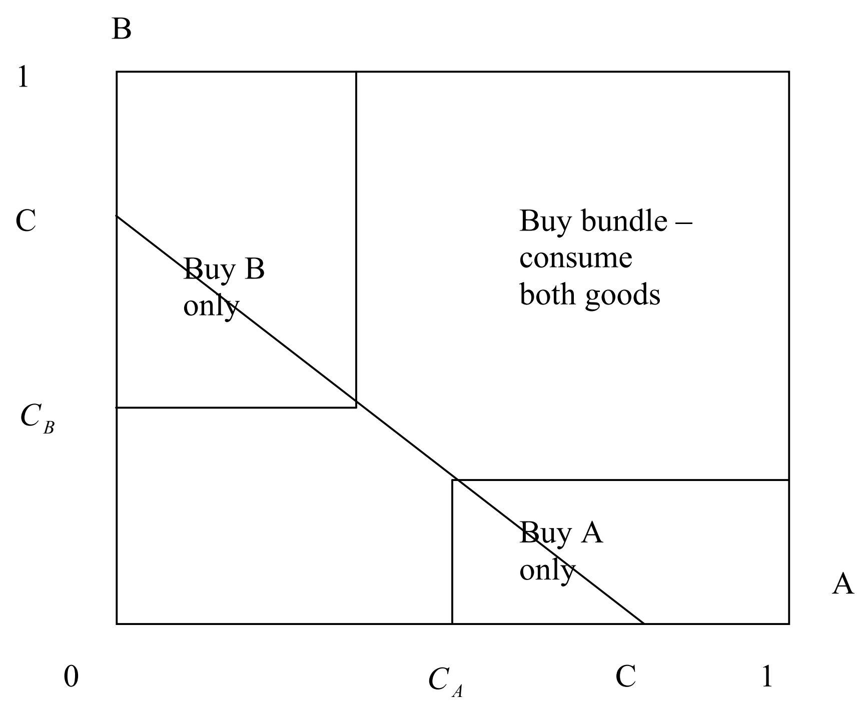
Figure 1. Segmentation of the market without bundle



It follows that the proportion of customers purchasing A is $1 \times (1 - C_A)$, and the resulting revenue or profit is $C_A(1 - C_A)$. Similar expressions are obtained for customers purchasing B. Hence, profits are maximized at $C_A = C_B = 0.5$, and the profit of each firm is 0.25, implying a total profit is 0.5.[11] In Section A1 of Appendix A, we show that the consumer surplus generated is 0.25 and hence the total surplus amounts to 0.75.

We now turn to the setup following the introduction of the bundle. Here, customers purchase the individual products or the bundle. This is illustrated in Figure 2, which shows how the $(0,1) \times (0,1)$ square will be partitioned.

Figure 2. Segmentation of the market with mixed bundling



[11] This includes the profit from those who bought both products, that is $(C_A + C_B)(1 - C_A)(1 - C_B) = 0.25$.

1. Customers whose reservation price for A is not smaller than C_A and whose reservation price for B is not greater than $C - C_A$ will purchase A only.
2. Customers whose reservation price for B is not smaller than C_B and whose reservation price for A is not greater than $C - C_B$ will purchase B only.
3. Customers whose reservation price for the bundle (the sum of the individual products reservation prices) is not smaller than C, and their reservation prices for A and B are not smaller than $C - C_B$ and $C - C_A$ respectively, will purchase the bundle.

Note, from Figure 2, that the case where only the bundle is being offered (Nalebuff, 1999, 2004) obtains when $C_A = C_B = C$. Using the figure, the number of buyers of A and B is $(1-C_A)(C-C_A)$ and $(1-C_B)(C-C_B)$, respectively, while $\frac{1}{2}\times\left((2-C)^2-(1-C_A)^2-(1-C_B)^2\right)$ will buy the bundle. It follows that the profits generated by the bundle are given by

(1) $$\Pi(C_A, C_B, C) = C\times\frac{1}{2}\times\left((2-C)^2-(1-C_A)^2-(1-C_B)^2\right),$$

while the profits generated from the sales of the individual products are:

(2) $$\Pi_A(C_A, C_B, C) = C_A(1-C_A)(C-C_A),$$

(3) $$\Pi_B(C_A, C_B, C) = C_B(1-C_B)(C-C_B).$$

Naturally, the prices should satisfy the following intuitive relations:

$C \le C_A + C_B$, $C_A \le C$, $C_B \le C$.

Before turning to the effect of sharing rules on the profits generated by the bundle, we discuss a few basic cases.

The case where only the bundle is being offered

This case is discussed extensively in Nalebuff. It boils down to assuming that the two firms decide to cooperate in a cartel and offer only the bundled product. Their profit is obtained from (1), substituting $C_A = C_B = C$, so that $\Pi(C) = C(2 - C^2/2)$. The profit maximizing price is $C = \sqrt{2/3} = 0.816$, yielding a profit of $\Pi = 0.544$, and, by (A.15) in Section A3 of Appendix A, a consumer surplus of 0.26.

Maximum industry profits (the full monopoly case)

This is the case where the two firms offer both the bundle and individual products and act as a cartel which maximizes the sum of the individual and bundling profits, given by (1) - (3). Its objective function is:

$$\begin{aligned}\Pi_T(C_A, C_B, C) = {} & C_A(1 - C_A)(C - C_A) + C_B(1 - C_B)(C - C_B) \\ & + \tfrac{1}{2} \times C \times \left((2 - C)^2 - (1 - C_A)^2 - (1 - C_B)^2\right),\end{aligned} \tag{4}$$

and the optimization problem solved is:

$$\begin{aligned}& \underset{C_A, C_B, C}{Max}\ \Pi_T(C_A, C_B, C) \\ & s.t. \quad 0 \le C_A \le 1, \\ & \qquad\ \ 0 \le C_B \le 1, \\ & \qquad\ \ 0 \le C \le 2, \\ & \qquad\ \ C_A + C_B \ge C, \\ & \qquad\ \ C_A \le C, \\ & \qquad\ \ C_B \le C.\end{aligned} \tag{5}$$

The numerical solution to the above problem yields
$C_A = C_B = 0.667,\ C = 0.862,\ \Pi_A = \Pi_B = 0.043, \Pi = 0.462$, with the maximal possible total industry profit of 0.549. Following the analysis of Section A.2 in Appendix A, the consumer surplus is 0.229, and hence total surplus amounts to 0.778.

The first best solution

This solution is attained when total (producer + consumer) surplus is maximized. From Section A2 in Appendix A, the consumers' surplus is obtained by adding (A4), (A9) and (A14), and assuming symmetry:

(6)
$$S_C(C_A, C) = (C - C_A)(1 - C_A)^2 + \tfrac{1}{3}(1 - C_A)^3 - \tfrac{8}{3} - 4C + 2(2 + C) + \tfrac{1}{3}(1 + C - C_A)^3 + 2C(1 + C - C_A) - \tfrac{1}{2}(2 + C)(1 + C - C_A)^2 .$$

while the profit $\prod_T$ is given by (4). The corresponding optimization problem becomes:

(7)
$$\begin{aligned} &\underset{C_A, C}{Max}\ \prod_T(C_A, C_A, C) + S_C(C_A, C)) \\ &s.t. \quad 0 \le C_A \le 1, \\ &\qquad\ 0 \le C \le 2, \\ &\qquad\ 2C_A \ge C, \\ &\qquad\ C_A \le C , \end{aligned}$$

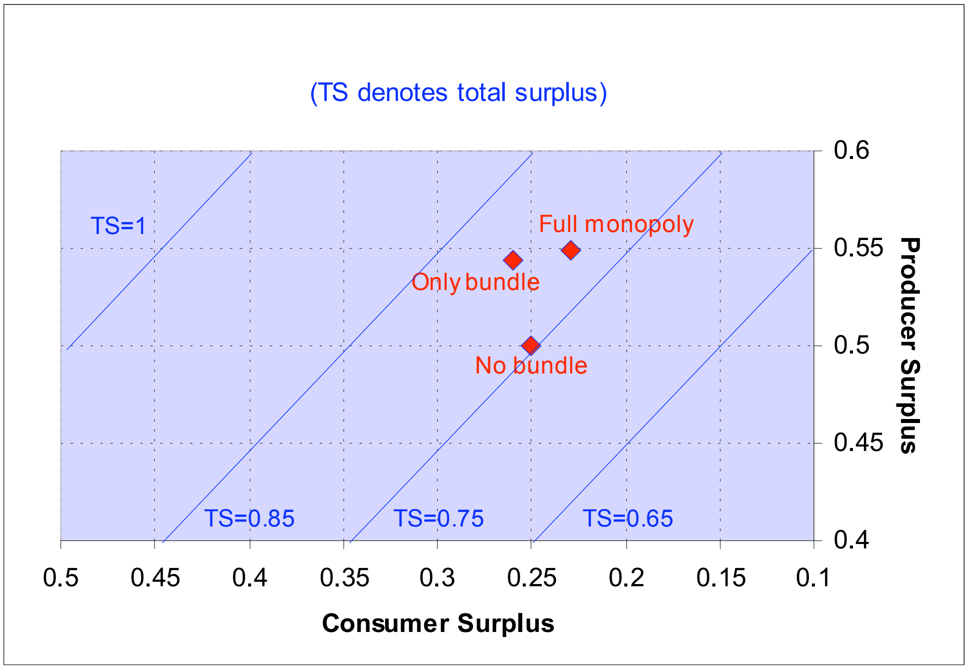
The numerical solution to this problem yields zero prices and profits, and a consumer surplus of 1, which is also the total surplus.

A summary of the above cases appears in Table 1 and Figure 3.

Table 1. Summary of results

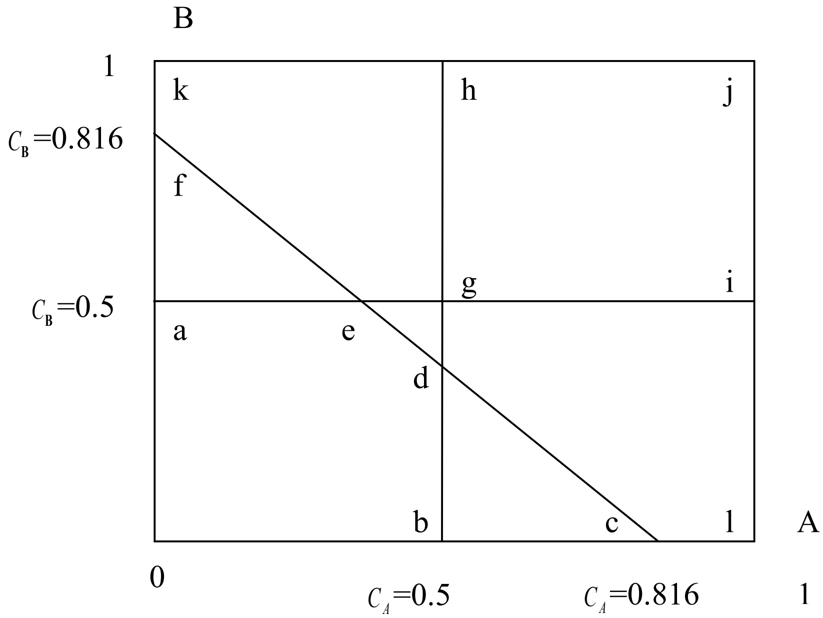
Indicator	**Operating mode**			
	No bundle	**Only bundle**	**Full monopoly**	**First best**
$C_A = C_B =$	0.5		0.667	0.00
C		0.816	0.862	0.00
$\prod_A = \prod_B =$	0.25		0.043	0.00
$\prod =$		0.544	0.462	0.00
Producers' surplus	0.5	0.544	0.549	0.00
Consumers' surplus	0.25	0.26	0.229	1.0
Total surplus	0.75	0.804	0.778	1.0

Figure 3. Surplus chart



The results show that the "bundle only" state generates higher welfare than the competitive "no bundle" state. Furthermore, it improves on both consumer and producer surpluses. To understand this first recall that initially the two firms were monopolies in their product markets. The increase of consumer surplus is demonstrated in Figure 4. It follows from the fact that there are either more customers who have increased their surplus than customers whose surplus went down or that those who improved did better than those who did not. In particular, there are customers who have purchased both products separately (area ghji) and whose surplus went up, as well as some who purchased the bundle, once it became available, but could afford neither A nor B before (area edg). This offsets the welfare loss from those few customers who have purchased one of the two products when available separately, but could not afford the bundle (areas afe and bdc). The remaining customers (areas fkhge and cligd) split, some improving and some worsening their surplus.

Figure 4. The consumer surplus picture



The "full monopoly" case, where firms offer both products and the bundle but act as a cartel, also generates higher welfare than the "no bundle" case. But here welfare is improved through an increase in firms' profits while consumer surplus decreases.

When we compare the "full monopoly" to the "bundle only" case, then it is evident that the price of the bundle increases (now gaining monopoly profits also from customers who seek the individual products only). This, in turn, implies an increase in the firms' profits coupled with a decrease in consumer surplus. The net comparison reveals that "full monopoly" generates less welfare than the "bundle only" case.

We summarize these results in Proposition 1.

Proposition 1[12]

1. The "bundle only" case increases both consumer and producer (and hence overall) surplus compared to the "no bundle" case.
2. The "full monopoly" case is welfare improving compared to the "no bundle" case, but consumers' surplus decreases.
3. "Full monopoly" generates less welfare than the "bundle only" case.

3. Bundling by Competitors. The Sharing Rules

We now discuss the case where the two firms cooperate in offering the bundle while maintaining competition in the individual products. We therefore need to specify how the bundling operation works and how its profits are distributed, that is to specify a sharing mechanism. When the two firms are symmetric, each reasonable sharing mechanism will end up with a 50-50 split. However, since firms behave strategically, they implicitly take into account the marginal effects of the sharing mechanism. This generates different equilibrium welfare outcomes. It follows that, if the choice of the sharing mechanism is not regulated, then firms will agree on the mechanism that generates larger profits for them, without paying attention to a possible decrease in consumer surplus.

We assume that the bundling operation is managed independently by an autonomous profit maximizing entity (the *bundle entity*) owned by the two firms, and operating at zero marginal cost. The profit of the bundle entity is distributed among its owners according to some pre-specified rule to be discussed later. Both firms set their prices to maximize their own profits, which include the profit distributed by the bundle entity. This leads to the following two-stage non-cooperative game.

[12] The producer surplus effects had already been derived by Nalebuff (1999, 2004).

<u>Stage 1</u>: The bundle entity determines C, the bundle price (anticipating the reaction of the individual firms in stage 2).

<u>Stage 2:</u> Given the bundle price, firms set their individual product prices C_A and C_B.

In the sequel, we will consider symmetric sub-game perfect Nash equilibria in pure strategies for the above game.

The stage 1 profit function is (1), where C_A and C_B are determined in stage 2. Hence, the stage 1 problem, solved by the bundle entity, anticipating the stage 2 reaction, is

(8) $$\underset{0 \le C \le C_A + C_B}{Max} \ \Pi[C_A(C), C_B(C), C].$$

The stage 2 profit functions Π_A and Π_B vary according to the specific profit distribution rule that applies. The basic structure of these profit functions is:

(9) $$\Pi_A(C_A, C_B, C) = C_A(1 - C_A)(C - C_A) + \beta_A \Pi(C_A, C_B, C),$$

(10) $$\Pi_B(C_A, C_B, C) = C_B(1 - C_B)(C - C_B) + \beta_B \Pi(C_A, C_B, C),$$

where β_A and β_B are the shares of A and B in the profits of the bundle entity. Note that both can be either constants or functions of C_A, C_B and C, and that, in general, β_A and β_B may or may not sum up to 1. In the two cases discussed below the β 's will sum up to 1.

We now pursue the analysis employing two potential sharing rules, the Shapley value, and the weighted Shapley value (Shapley 1953). We also assume that customers who purchase the bundle consume both goods. This is a natural assumption since these are also available individually and the pricing is such that the consumer does not purchase the bundle unless she intends to consume both goods.

The Shapley Value Sharing

This sharing was suggested by Ginsburgh and Zang (2003, 2004) as the better allocation rule for distributing the income of museum-pass programs. Since each museum continues to offer individual entry, the problem is similar to the mixed bundling context considered here. For the particular case of two firms and under the assumption that each bundle buyer consumes both A and B, the Shapley value will share the income from the bundle equally between the two firms regardless of their individual actions. Hence $\beta_A = \beta_B = 1/2$, and it follows from (9) and (10) that the stage 2 profit functions are:

(11)
$$\begin{aligned}\Pi_A(C_A, C_B, C) &= C_A(1-C_A)(C-C_A) + \tfrac{1}{2}\Pi(C_A, C_B, C)\\ &= C_A(1-C_A)(C-C_A) + \tfrac{1}{4}\times C\times\left((2-C)^2-(1-C_A)^2-(1-C_B)^2\right),\end{aligned}$$

(12)
$$\begin{aligned}\Pi_B(C_A, C_B, C) &= C_B(1-C_B)(C-C_B) + \tfrac{1}{2}\Pi(C_A, C_B, C)\\ &= C_B(1-C_B)(C-C_B) + \tfrac{1}{4}\times C\times\left((2-C)^2-(1-C_A)^2-(1-C_B)^2\right),\end{aligned}$$

where Π is given by (1). The stage 2 optimization problem for firm A is:

(13)
$$\begin{aligned}&\underset{C_A}{Max}\ \ \Pi_A(C_A, C_B, C)\\ &s.\,t.\quad 0 \le C_A \le 1,\\ &\qquad\quad C_A \le C.\end{aligned}$$

Firm B's stage 2 problem is similar and there is also a combined constraint $C \le C_A + C_B$, which, given that firms are identical, can be expressed as $C_A \ge C/2$, and $C_B \ge C/2$. Assuming interior solutions, the first order necessary conditions for (13) (and firm's B similar problem) are quadratic equations and the solution, which should be satisfied in the equilibrium of the two-stage game, is:

(14)
$$C_A = C_B = \frac{4+5C-\sqrt{(4+5C)^2-72C}}{12}.$$

Following the above analysis and (8), the stage 1 problem becomes:

$$
(15)\quad
\begin{aligned}
&\underset{C_A, C}{Max}\ \frac{C}{2}(2-C)^2 - C(1-C_A)^2 \\
&s.t.\quad C_A = \frac{4+5C-\sqrt{(4+5C)^2-72C}}{12}, \\
&\qquad 2C_A \ge C, \\
&\qquad C_A \le C, \\
&\qquad 0 \le C_A \le 1, \\
&\qquad 0 \le C \le 2.
\end{aligned}
$$

A plot of the profit function of (15) appears in Figure B.1 of Appendix B, where it is shown to be unimodal (though non-concave) in *C*. A numerical solution to the problem yields: $C_A = C_B = 0.403,\ C = 0.647,\ \Pi_A = \Pi_B = 0.24,\ \Pi = 0.362$, with a total industry profit of 0.479. Using the results of Section A2, Appendix A, one can show that the resulting consumer surplus is 0.4, and hence total surplus amounts to 0.879, which is 17% larger than the surplus of 0.75 obtained before the introduction of the bundle.

The Weighted Shapley Value Sharing

The Weighted Shapley Value shares the joint income of the bundle entity proportionally to the individual prices of the two goods. This implies $\beta_A = C_A/(C_A + C_B)$ and $\beta_B = C_B/(C_A + C_B)$. By (9) and (10), the second stage profits are

$$
(16)\quad
\begin{aligned}
\Pi_A(C_A, C_B, C) &= C_A(1-C_A)(C-C_A) + \frac{C_A}{C_A + C_B} \times \Pi(C_A, C_B, C) \\
&= C_A(1-C_A)(C-C_A) + \frac{C_A}{C_A + C_B} \times \frac{C}{2}\left((2-C)^2 - (1-C_A)^2 - (1-C_B)^2\right),
\end{aligned}
$$

$$
(17)\quad
\begin{aligned}
\Pi_B(C_A, C_B, C) &= C_B(1-C_B)(C-C_B) + \frac{C_B}{C_A + C_B} \times \Pi(C_A, C_B, C) \\
&= C_B(1-C_B)(C-C_B) + \frac{C_B}{C_A + C_B} \times \frac{C}{2}\left((2-C)^2 - (1-C_A)^2 - (1-C_B)^2\right).
\end{aligned}
$$

The second stage optimization problem for firm A is (13), where Π_A is now given by (16), leading to the following first order necessary condition for firm A:

$$(18)\qquad \begin{aligned}&(1-C_A)(C-C_A)-C_A(C-C_A)-C_A(1-C_A)\\&+\frac{C_B}{(C_A+C_B)^2}\times\frac{C}{2}\times\left((2-C)^2-(1-C_A)^2-(1-C_B)^2\right)\\&+\frac{C_A}{C_A+C_B}\times\frac{C}{2}\times 2\times(1-C_A)=0.\end{aligned}$$

Since A and B are symmetric, (18) can be written as

$$(19)\qquad \begin{aligned}&(1-C_A)(C-C_A)-C_A(C-C_A)-C_A(1-C_A)\\&+\frac{C}{8C_A}\times\left((2-C)^2-2\times(1-C_A)^2\right)+\frac{C}{2}\times(1-C_A)=0.\end{aligned}$$

Differentiating (18) once again, with respect to C_A and taking symmetry into account, yields the following second order condition for a maximum

$$(20)\qquad \begin{aligned}&-2(C-C_A)-2(1-C_A)+2C_A-\frac{C}{8C_A^2}\times\left((2-C)^2-2(1-C_A)^2\right)\\&+\frac{C}{2C_A}\times(1-2C_A)\le 0.\end{aligned}$$

Both (19) and (20) should be satisfied in the equilibrium of the two-stage game.
In view of the above, the following first stage problem was solved numerically

$$(21)\qquad \begin{aligned}&\underset{C_A,C}{Max}\ \frac{C}{2}\times(2-C)^2-C\times(1-C_A)^2\\&s.t.\quad (19),\ (20),\\&\qquad 2C_A\ge C,\\&\qquad C_A\le C,\\&\qquad 0\le C_A\le 1,\\&\qquad 0\le C\le 2.\end{aligned}$$

The numerical solution is

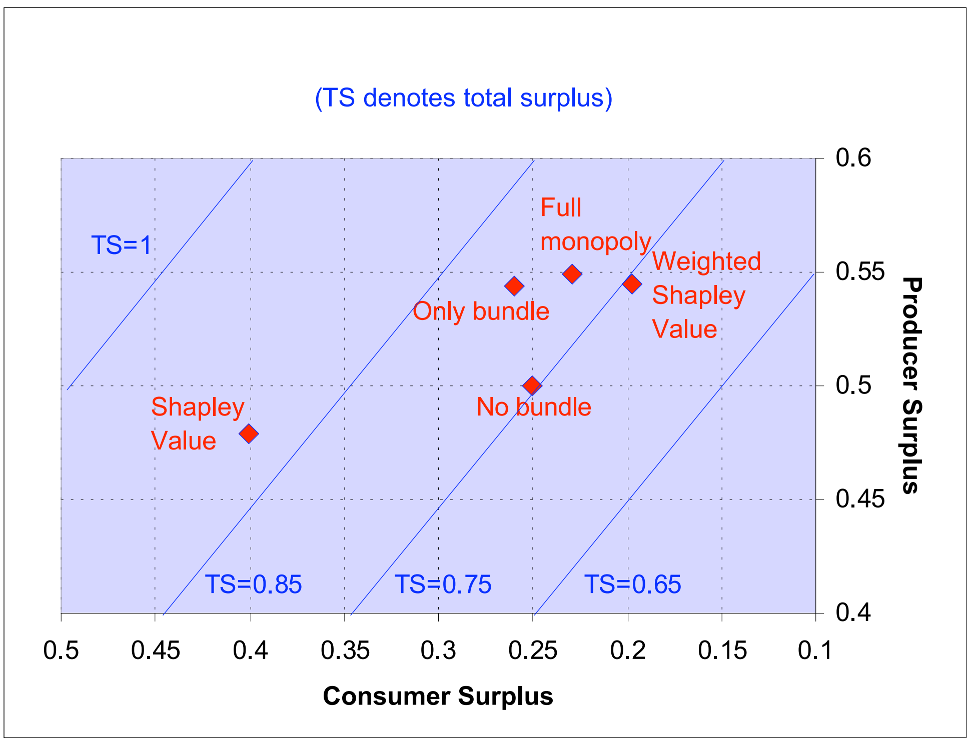
$C_A=C_B=0.725,\ C=0.921,\ \prod_A=\prod_B=0.272, \prod=0.467$, with a total industry profit of 0.545. Consumer surplus is 0.197, and total surplus amounts to 0.742, which is smaller than the surplus obtained before the introduction of the bundle.

An updated summary of the results appears in Table 2 and Figure 5.

Table 2. Summary of main numerical results

Indicator	Operating mode					
	No bundle	Only bundle	Full monopoly	First Best	Shapley Value	Weighted Shapley Value
$C_A = C_B =$	0.5		0.667	0.00	0.403	0.725
C		0.816	0.862	0.00	0.647	0.921
$\prod_A = \prod_B =$	0.25		0.043	0.00	0.24	0.272
$\prod =$		0.544	0.462	0.00	0.361	0.467
Producers' surplus	0.5	0.544	0.549	0.00	0.479	0.545
Consumers' surplus	0.25	0.26	0.229	1.0	0.4	0.197
Total surplus	0.75	0.804	0.778	1.0	0.879	0.742

Figure 5. Surplus chart



Note that the Shapley value (or the fixed 50-50 sharing) yields the highest consumer and total surplus (next to the first best). However, from the producers' point of view using the weighted Shapley value for sharing is more likely since it yields higher industry profits.

We state our findings in Proposition 2.

Proposition 2

Excluding the first best case:

1. The Shapley value sharing yields the *largest* consumer surplus and welfare and the *smallest* producer surplus.
2. The weighted Shapley value sharing yields the *smallest* consumer surplus and welfare and the *largest* producer surplus next to the full monopoly solution.
3. Both allocation mechanisms end up splitting the profit of the bundle entity equally between the two firms. The nature of the mechanism affects outcomes, though both splits yield equal shares, and could therefore become an issue for regulation.

A problem arises if the producers are free to choose the sharing mechanism since they will prefer the weighted Shapley value allocation which is inferior, in terms of consumer and total surplus. Indeed, by choosing the weighted Shapley value the producers can guarantee themselves a profit of 0.545 which is only a 0.7% smaller than the maximum possible industry profit of 0.549 attained by the full monopoly. Producers might argue that both sharing rules eventually lead to a 50-50 split. However, since the formulas utilized by the two rules are different, outcomes are also different.

4. Conclusions

Notwithstanding the entry deterrence issue, raised by Nalebuff, our results show that it is welfare improving for a monopoly to introduce a bundle next to its two regular goods. This is counterintuitive on two accounts. First, a monopoly selling two goods and the bundle is better than a duopoly in which each firm sells one good. Secondly, a monopoly which sells the bundle only, generates larger total welfare (and smaller profits) than the one which sells both the bundle and the two goods separately.

Our second result is concerned with the case in which the bundle is sold by a profit maximizing entity that is owned by the two firms which continue selling competitively both goods. Here counterintuitive results are obtained depending upon the method used to share the joint profit. Though both mechanisms suggested here end up splitting the bundle profit equally, one mechanism increases welfare and consumer surplus while the other is welfare and consumer surplus decreasing, when compared to the other options discussed. Regulation, focusing on the sharing mechanism, is thus necessary here, even in the case of an oligopoly.

It is likely that the gist of our results will generalize to situations where there are more than two symmetric firms, selling one good each, though computations could become cumbersome. More work is needed to deal with cases in which (some) firms sell and bundle more than one good, and the case in which there is asymmetry.

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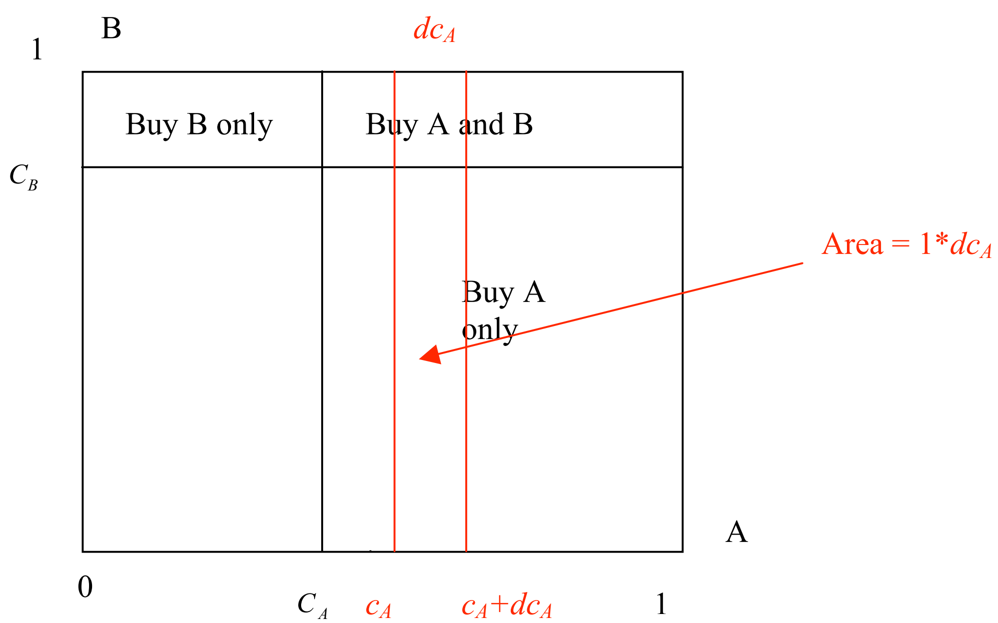
Appendix A. Computation of Consumers' Surplus

A.1 Before introduction of the bundle

Figure A.1



Here, we let c_A denote the varying reservation price of product A, over which the integration is being carried out. The consumer surplus for each buyer of good A is her reservation c_A price minus the good price C_A, namely, $c_A - C_A$. Hence, the overall consumers' surplus of buyers of good A will be:

(A.1) $$CS_A = \int_{C_A}^{1} (c_A - C_A) dc_A = \left\langle \tfrac{1}{2} c_A^2 - C_A c_A \right|_{C_A}^{1} = \tfrac{1}{2}(1 - C_A)^2 .$$

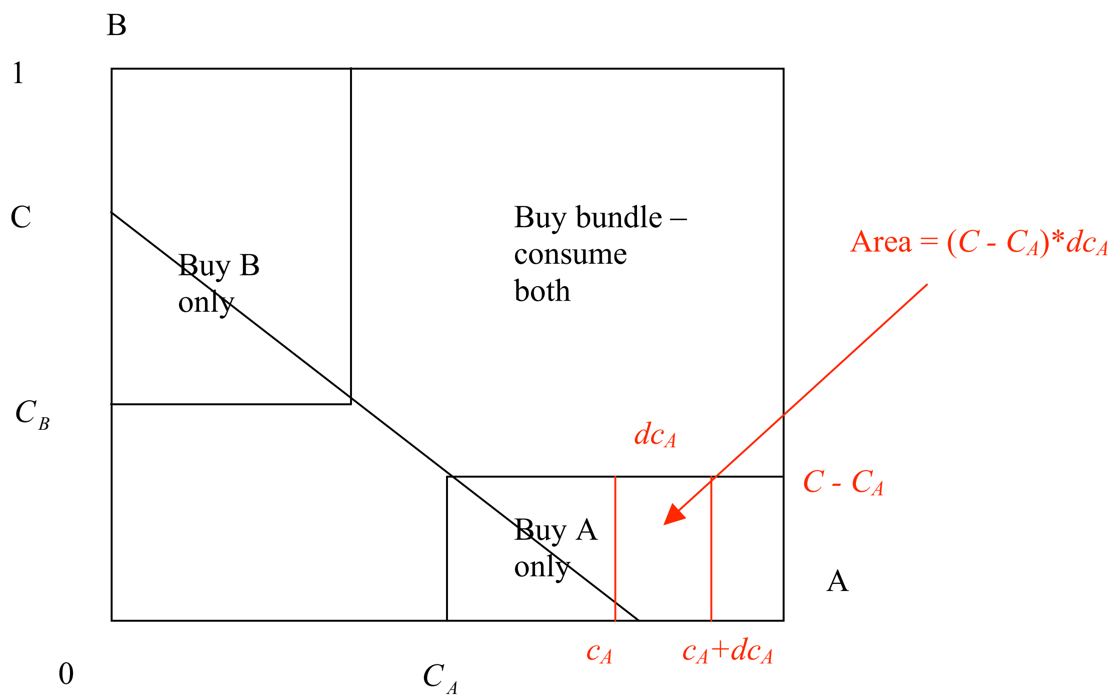
Since $C_A = C_B = 0.25$ was established for this case, it follows that

(A.2) $$Total\ CS = 2 \times CS_A = (1 - C_A)^2 = 0.25 .$$

A.2 Following introduction of the bundle

We first consider those buyers who did not purchase the bundle.

Figure A.2



Here

(A.3) $$CS_A = \int_{C_A}^{1} (c_A - C_A)(C - C_A)dc_A = (C - C_A)\int_{C_A}^{1} (c_A - C_A)dc_A = \tfrac{1}{2}(C - C_A)(1 - C_A)^2 \ .$$

By symmetry, the surplus of consumers who have purchased non-bundled products is

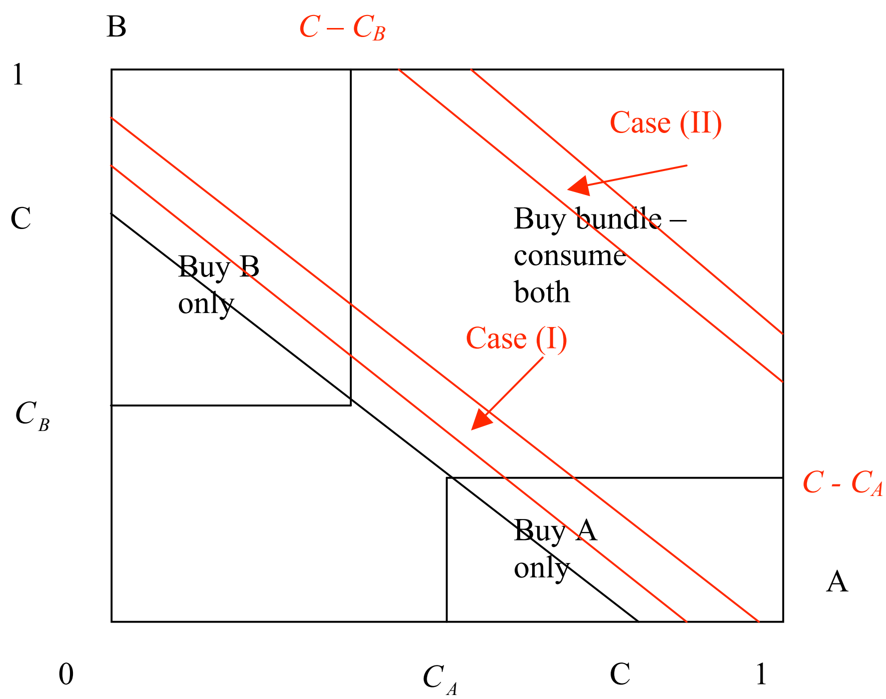
(A.4) $$CS_T = (C - C_A)(1 - C_A)^2 \ .$$

Consider now those customers who have purchased the bundle. Here, we let *c* denote the varying reservation price of the bundle buyers over which the integration is being carried out. The consumer surplus for each bundle buyer is her reservation price *c* minus the bundle price *C*, namely *c* - *C*. Assuming a symmetric outcome $C_A = C_B$, we have to consider two integration areas:

(I) $$C \le c \le 1 + C - C_A$$

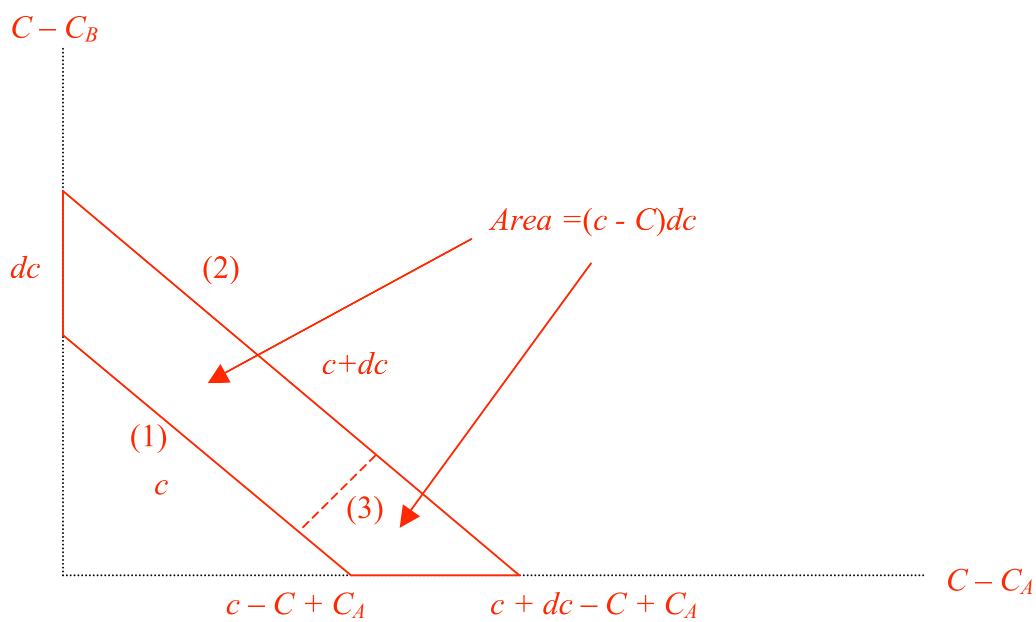
(II) $1 + C - C_A \leq c \leq 2$

Figure A.3



Integration area (I)

Figure A.4



Now

(A.5) $$Length\ of\ (1) = \left[(c - C + C_A - C + C_B)^2 + (c - C + C_B - C + C_A)^2\right]^{1/2} = \sqrt{2} \times (c - C).$$

(A.6) $$Length\ of\ (2) = \sqrt{2} \times (c + dc - C).$$

(A.7) $$Height\ (3) = \left(dc^2 - \tfrac{1}{2}dc^2\right)^{1/2} = \tfrac{1}{\sqrt{2}}dc\ .$$

It follows that the trapezoidal area in Figure A.4 is

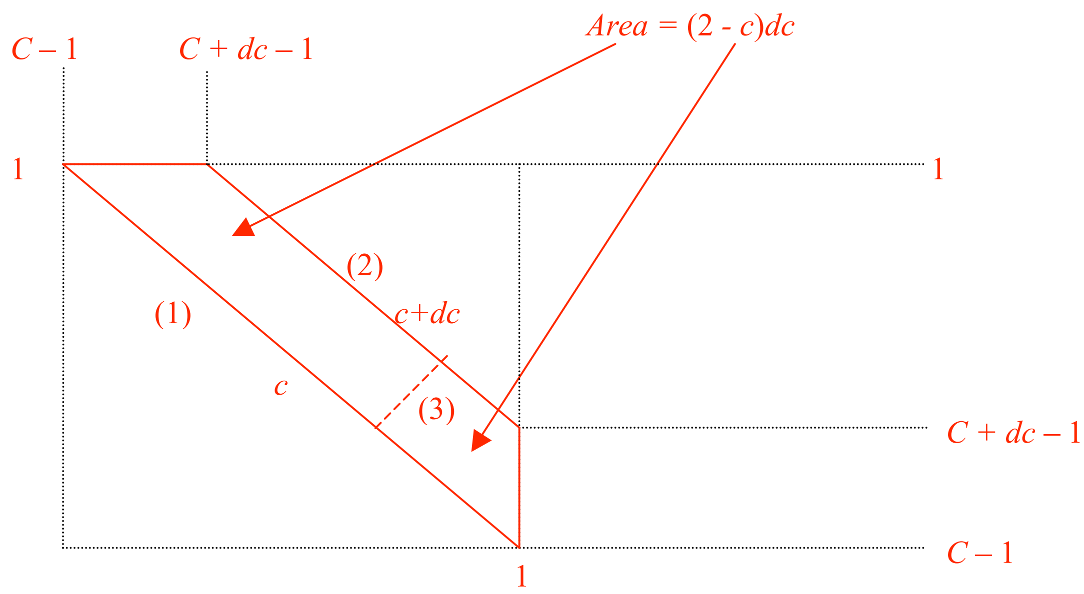
(A.8) $$Area = \tfrac{\sqrt{2}}{2}(2c - 2C + dc)\tfrac{dc}{\sqrt{2}} \cong (c - C)dc\ .$$

and that the consumer surplus is

(A.9) $$CS_{(I)} = \int_{C}^{1+C-C_A} (c - C)^2\, dc = \tfrac{1}{3}(1 - C_A)^3\ .$$

Integration area (II)

Figure A.5



Here:

(A.10) $\quad Length\ of\ (1) = \sqrt{2} \times (2 - c)$.

(A.11) $\quad Length\ of\ (2) = \sqrt{2} \times (2 - c - dc)$.

(A.12) $\quad Height\ (3) = \frac{1}{\sqrt{2}} dc$.

It follows that the trapezoidal area in Figure A.5 is

(A.13) $\quad Area = \frac{\sqrt{2}}{2}(4 - 2c - dc)\frac{dc}{\sqrt{2}} \cong (2 - c)dc$.

and that the consumer surplus is

(A.14)
$$CS_{(II)} = \int_{1+C-C_A}^{2} (c - C)(2 - c)dc = -\tfrac{8}{3} - 4C + 2(2 + C) + \tfrac{1}{3}(1 + C - C_A)^3 +$$
$$+ 2C(1 + C - C_A) - \tfrac{1}{2}(2 + C)(1 + C - C_A)^2 \ .$$

that is, the sum of (A.4), (A.9) and (A.14).

A.3 Following introduction of the bundle. Only the bundle is offered

This analysis follows up from the preceding case by setting $C_A = C_B = C$. These yields

(A.15) $$CS = \frac{(1-C)^3}{3} - \frac{C}{2} + \frac{2}{3}.$$

Appendix B. The stage 1 profit function for the Shapley Value allocation

Figure B.1. Numerical plot of the profit function (15)