CORE DISCUSSION PAPER 2005/86

ENDOGENOUS FIRM ASYMMETRY AND COOPERATIVE R&D IN LINEAR DUOPOLY WITH SPILLOVERS

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In a linear model of cost reducing R&D/Cournot competition, firm asymmetry is shown to be sustainable as subgame perfect Nash equilibrium with R&D competition only if the productivity of research is sufficiently large relative to the benefits from imitation. In such a case, industry-wide cost reduction and firms asymmetry are increasing and decreasing functions of the spillover rate, respectively. In the absence of spillovers, a symmetric joint lab generates higher consumer surplus and social welfare than a pair of asymmetric competitors. If spillovers are not too small, asymmetric R&D competition is advantageous to consumers, but not to firms.

Keywords: Endogenous asymmetry; Cournot instability; R&D cooperation.

JEL Numbers: C72; L13; O32.

[0] I thank Rabah Amir, Antonio Purpura, and Vincent Vannetelbosch for helpful comments. Financial supports from Università degli Studi di Palermo are gratefully acknowledged. This text presents results of the Belgian Program of Interuniversity Poles of Attraction initiated by the Belgian state, Prime Minister's office, Science Policy Programming. The scientific responsibility is assumed by the author.

1 Introduction

This paper deals with endogenous heterogeneity of firms involved in strategic research activity (R&D). Its main interest is the occurrence of equilibria where *ex ante* identical firms undertake different levels of R&D thereby operating as asymmetric competitors in the product market. The analysis here conforms to a dominant paradigm for strategic R&D consisting of multi-stage oligopoly models where prior R&D investments lower the marginal costs of production at the market stage. Most such models have been recently concerned with performance comparisons between different R&D scenarios, given non cooperative behavior in the product market. The key assumption of R&D spillovers is made most often.

The articles by d'Aspremont and Jacquemin (1988)- henceforth AJ- and by Kamien *et al.* (1992) are leading examples for the two-stage case.[1] A common result of their analysis is that both industry-wide cost reduction and total output are larger under R&D cooperation than under R&D competition, whenever the spillover rate is sufficiently large. In this case R&D cooperation increases consumer surplus and hence social welfare independently of any direct public intervention.

Performance comparisons in these studies have been usually restricted to the symmetric equilibrium of the non cooperative case. Henriques (1990) argues that comparing cooperative and non cooperative solutions is meaningful only when the latter are stable under Cournot best reply dynamics, however.[2] Considering only the case of zero spillovers, Amir and Wooders (1998) show that when the AJ model is not stable, two other asymmetric and locally stable equilibria in R&D levels arise. Then they

[1] The literature on strategic R&D has been pioneered by Ruff (1969). Among more recent studies, see Suzumura (1992), Katsoulakos and Ulph (1998), Amir (2000), and Hinloopen (2000, 2003).

[2] Cournot best reply dynamics is defined by each player best replying to the previous action of the rival. About the instability of the Cournot equilibrium see, among others, Seade (1980) and Dixit (1986).

provide various justifications for taking the asymmetric equilibria as the appropriate standard of analysis, resting on both theoretical support and experimental evidence.[3] In this spirit, Cournot instability is a source of endogenous heterogeneity.[4]

Focusing on the asymmetric equilibria for non cooperative R&D, Amir and Wooders (1998) find that R&D cooperation *under the constraint of identical investments* is sometimes unattractive for firms, in that it generates smaller combined profits than *asymmetric* R&D competition with no spillovers.[5] More generally, benefits to asymmetric competitors are direct implications of well known properties of Cournot oligopolies. As shown by Bergstrom and Varian (1985, 1985a), industry output and hence price and consumer surplus are not affected by any sum preserving reallocation of constant marginal costs among Cournot competitors, provided the equilibrium is interior. So, changes in social surplus will only depend on the way such a reallocation affects industry profit through the aggregate cost of production. This latter, for a given marginal costs sum, is maximized by symmetric allocation of marginal costs among firms. Hence, the private gross benefit of operating the reallocation is positive whenever this latter is feasible. In multi-stage models of cost reducing R&D/Cournot competition, this tendency towards asymmetric cooperative solutions is attenuated by the cost of undertaking asymmetric actions in the R&D phase. An example of such a cost is the efficiency loss due to investing unequally in the first stage under decreasing returns to R&D.[6]

[3] A major reason is that Cournot best reply dynamics never converges to the symmetric, unstable equilibrium.

[4] Also see Matsuyama (2002).

[5] It is worth recalling the general fact that when the equal treatment constraint is not imposed, R&D cooperation always yields higher combined profits than R&D competition. So the case discussed by Amir and Wooders implicitly provides an example where the global solution of the cooperative problem is asymmetric. However, as emphasized in the literature, asymmetric cooperative solutions are easy to conceive but hard to realize. See, among others, Salant and Shaffer (1998). Amir *et al.* (2003) allow for asymmetry in the cooperative solution using a general version of the model of Kamien *et al.* (1992).

[6] This point has been recently emphasized by Salant and Shaffer (1999). Also see Van Long and

In line with the above contributions, this paper addresses the issue of firm diversity in terms of Cournot instability of the symmetric solution in R&D space. The study of asymmetric equilibria is extended to encompass R&D spillovers. A comparison of asymmetric R&D competition and symmetric cooperation through a joint lab is performed to feature the impact of spillovers on both the private benefits and the consumer gains from firm asymmetry. The main results can be summarized as follows.

Increasing spillovers act as a barrier to endogenous asymmetry, in that they raise the opportunity cost of private research while stimulating the dissemination of innovative results through imitation. For the non cooperative scenario, when spillovers are relatively large, firms' incentive for private R&D are attenuated and symmetric R&D profiles prevail. Conversely, when the productivity of research is large relative to the benefits from imitation, *i.e.* the spillover rate is relatively small, the symmetry of the model is broken in favor of an R&D expert-novice configuration with the more R&D intensive firm choosing the maximal R&D level in its action set. This is what here is meant by *R&D specialization* or *full innovation.* As long as interfirm asymmetry is preserved in the equilibrium with R&D competition, both industry-wide cost reduction and consumer surplus increase with the spillover rate. This result is due to the combined effect of persistent specialization in the research activity by the larger firm and increasing externalities. Therefore, firm asymmetry, namely the difference of equilibrium cost reductions and market shares, declines as the spillover rate increases.

The comparative analysis focuses on the parameter region in which the model is unstable. As mentioned, the asymmetric equilibria here represent the benchmark for the non cooperative case. In the absence of spillovers, the joint lab is shown to dominate asymmetric R&D competition in terms of social surplus. When spillovers

Soubeyran (1999).

are introduced, welfare results are ambiguous because of a twofold impact of imitation on industry performance. The paper considers consumer surplus and combined profits separately.

The first effect of spillovers concerns industrywide dissemination of innovative results and is beneficial to consumers. As shown by Amir and Wooders (1998), with zero spillovers, total cost reduction and consumer surplus are higher under the joint lab than with asymmetric R&D competition. Here it is shown that if spillovers are not *too small*, aggregate cost reduction and hence consumer surplus are larger with asymmetric competition than with the joint lab.

The second effect relates to endogenous asymmetry and is detrimental to firms. With zero spillovers, a large level of demand relative to unit costs is sufficient for joint profits to be larger with the joint lab than with asymmetric R&D competition.[7] This paper shows that, by reducing the market share of the more efficient firm, increasing spillovers weakens the conditions under which the joint lab generates higher combined profits.

The remainder of the paper is organized as follows. Section two introduces the model and two organizational scenarios for R&D depicting Nash competition and cooperation via joint lab, respectively. Performance comparisons are provided in section three. Section four concludes. All the propositions are proved in the appendix.

2 The model

2.1 The game and the R&D scenarios

Consider a two-stage duopoly game in which firms 1 and 2, before engaging in Cournot competition, exploit a cost reducing opportunity by investing resources in

[7] See [Amir and Wooders, 1998, Prop.3].

R&D. In the first stage, R&D expenditures are chosen simultaneously. Subsequent Cournot competition is subject to firms' first stage R&D levels.

Firms are *ex ante* identical. Namely, they utilize the same linear production technology, with unit cost $c > 0$, share the same R&D opportunities, and face the same inverse demand function $P(q_1 + q_2) = a - (q_1 + q_2)$, where q_i denotes the final output of firm $i = 1, 2$.

The inventive activity takes place under technological externalities. Namely, for any pair of R&D investments (x_1, x_2), the effective cost reduction for firm i, $i = 1, 2$, depends on its own autonomous cost reduction $\lambda\sqrt{x_i}$, $\lambda > 0$, and, via spillovers, on the rival's one. Let $CR_1(x_1, x_2, \lambda, \theta) \equiv \min\left\{\lambda(\sqrt{x_1} + \theta\sqrt{x_2}), c\right\}$ be the effective cost reduction obtained by firm 1, where $\theta \in [0, 1]$ is the spillover parameter.[8] So $A_1 \equiv \left\{x_1 : 0 \leq x_1 \leq (\frac{c}{\lambda} - \theta\sqrt{x_2})^2\right\}$ identifies firm 1's R&D undominated action set (and similarly for firm 2). That is, R&D levels strictly larger than $(\frac{c}{\lambda} - \theta\sqrt{x_2})^2$ are strictly dominated.

Firms are Cournot competitors in the product market. Cournot output and profit for firm 1 are

$$q_1 = \frac{a-c+\lambda\sqrt{x_1}(2-\theta)+\lambda\sqrt{x_2}(2\theta-1)}{3} \text{ and } \Pi_1 = \left(\frac{a-c+\lambda\sqrt{x_1}(2-\theta)+\lambda\sqrt{x_2}(2\theta-1)}{3}\right)^2 - x_1,$$

respectively (and similarly for firm 2). The validity of the previous formulas is granted by Assumption 1, stated below.

Assumption 1: $a > 2c$

Assumption 1 restricts the attention to the case in which the product market is

[8] The specification of the R&D technology used here is equivalent to that introduced by d'Aspremont and Jacquemin, according to which, in the first stage of the game, firm 1 chooses a cost reduction level y_1, facing an R&D cost of $\frac{\gamma}{2}y_1^2$, $\gamma > 0$. Given the pair of actions (y_1, y_2), the effective cost reduction of firm 1 is equal to $y_1 + \beta y_2$, where β is the spillover parameter. Therefore, the R&D production function $y_1 = \lambda\sqrt{x_1}$ represents the inverse mapping of the R&D cost function used by d'Aspremont and Jacquemin, with $x_1 = \left(\frac{1}{\lambda}y\right)^2$ and $\lambda = \sqrt{\frac{2}{\gamma}}$.

large relative to the marginal cost of production. It ensures that the Nash equilibrium of the market subgame is unique, with production levels being strategic substitutes and both firms active in the market. This allows to emphasize the role of R&D with respect to firm asymmetry.

Two different scenarios for the behavior of firms in the R&D stage are considered:

Nash competition (N): Firms act simultaneously and non cooperatively. Each firm sets its own R&D expenditure given the action of the rival. Attention is restricted to subgame perfect equilibria (SPNE). Therefore, the game is solved going backward from the second stage to the first, where the maximization problem faced by each firm is defined conditionally on the second stage equilibrium payoffs.

Firm 1 solves:

$$\max_{x_1 \in A_1} \left\{ \Pi_1^N(x_1,x_2) \equiv \left(\tfrac{a-c+\lambda\sqrt{x_1}(2-\theta)+\lambda\sqrt{x_2}(2\theta-1)}{3} \right)^2 - x_1 \right\}, \tag{1}$$

and similarly for firm 2. Since the Cournot equilibrium in the market stage is unique, every Nash equilibrium of the R&D game with payoffs as in (1), gives rise to a SPNE of the whole game. For the sake of brevity, in the remainder of the paper mention is made to the Nash equilibria of the R&D game only.

Joint lab (J): Firms manage a joint lab in order to maximize the overall combined profit net of the innovative expenditure x^J, sharing in both R&D benefits and efforts. More precisely, the total cost reduction is independent of the spillover rate and equals $2\lambda\sqrt{x^J}$. *Ex post* symmetry between members is generated by construction.[9] So, the whole game induces a mixed cooperative and non cooperative symmetric equilibrium. As for the N case, second stage profits are taken conditionally on the unique Cournot

[9] The joint lab scenario was formally introduced by Amir (2000). Though independent of the spillover rate, it is equivalent to the *cartelized joint venture* discussed in Kamien *et al.* (1992).

equilibrium of the market subgame.

The joint lab solves:

$$\max_{0\leq x\leq(\frac{c}{\lambda})^2}\left\{\Pi^J(x)\equiv 2\left(\frac{a-c+\lambda\sqrt{x}}{3}\right)^2-x\right\}. \qquad (2)$$

The solution to (2) is $x^J=\frac{4(\lambda(a-c))^2}{(9-2\lambda^2)^2}$,[10] which generates aggregate profit $\Pi^J=\frac{2(a-c)^2}{9-2\lambda^2}$, total cost reduction $TCR^J=\frac{4(a-c)\lambda^2}{9-2\lambda^2}$, and consumer surplus $C^J=18(\frac{a-c}{9-2\lambda^2})^2$.

2.2 Equilibria with R&D competition

In this subsection the reaction functions in R&D space are characterized and the equilibria for the non cooperative scenario are derived. Throughout the analysis, the following assumption is made.

Assumption 2: $\lambda^2\frac{a}{c}<\frac{9}{(2-\theta)(1+\theta)}$.

Assumption 2 rules out the circumstance in which, in the non cooperative scenario, the intersection of firms' R&D reaction functions identifies the pair of R&D levels $\left((\frac{c}{\lambda(1+\theta)})^2,(\frac{c}{\lambda(1+\theta)})^2\right)$, in which case both firms would achieve a full cost reduction in equilibrium.[11]

Let $x_i^*(x_j)\equiv\left(\frac{(2-\theta)\lambda(a-c+\lambda\sqrt{x_j}(2\theta-1)}{9-[(2-\theta)\lambda]^2}\right)^2$ denote the unique root of $\frac{\partial\Pi_i^N(x_i,x_j)}{\partial x_i}=0$, $i,j=1,\ 2$, and $i\neq j$, *i.e.* the interior part of firm i's reaction curve in R&D space. Under Assumption 1, $\Pi_i^N(x_i,x_j)$ is strictly concave in x_i. So the first order

[10] The first order condition for (2) is $\frac{4\lambda[a-c+\lambda\sqrt{x}]}{18\sqrt{x}}-1=0$. Given Assumption 1, stated below, the second order condition, $\frac{\lambda(c-a)}{9x^{3/2}}<0$, is satisfied.

[11] Such an equilibrium is excluded if $x_i^*\left((\frac{c}{\lambda(1+\theta)})^2\right)<(\frac{c}{\lambda(1+\theta)})^2$, where $x_i^*(.)$, as defined in the remainder of this subsection, denotes the interior part of firm i's reaction curve in R&D space. This condition is equivalent to the requirement of Assumption 2. With zero spillovers, Assumption 2 reduces to $2\lambda^2\frac{a}{c}<9$, that prevents the case of dominant strategies with either firm choosing a maximal R&D level (see Amir & Wooders (1998)).

condition for (1) is sufficient.[12] In addition, firm i's reaction function in R&D space is continuous and single valued, and is given by

$$r_i(x_j) \equiv \arg\max\left\{\Pi_i^N(x_i,x_j) : x_i \in A_i\right\} = \min\left\{x_i^*(x_j), (\frac{c}{\lambda} - \theta\sqrt{x_j})^2\right\}. \tag{3}$$

Given the specification here of the spillover mechanism, it is convenient to linearize the R&D reaction functions by taking the square root of the R&D level to be the decision variable. According to (3), the linearized reaction function of firm i is piecewise linear, corresponding to the isocost line $\sqrt{x_i} = \frac{c}{\lambda} - \theta\sqrt{x_j}$ whenever the first order condition of the program induces a higher R&D level than what is sufficient to realize a full cost reduction. This line is decreasing with slope $-\theta$. Along it, firm i achieves a full cost reduction for any investment level of the opponent. Larger investment levels are strictly dominated.

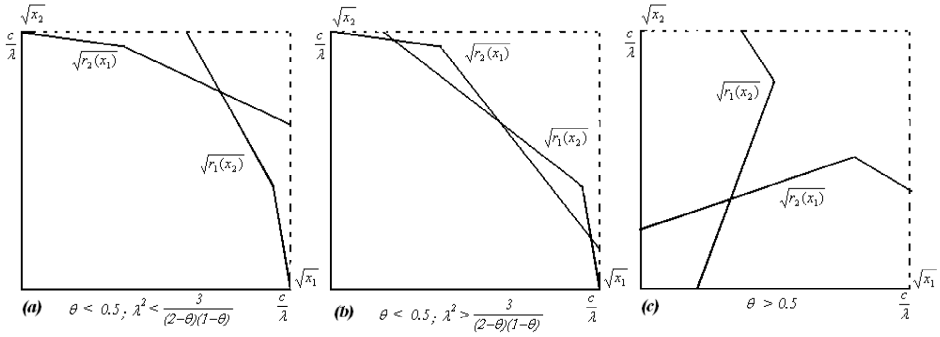
Since R&D is cost reducing, for zero or sufficiently small spillovers R&D investments have the same strategic relationship as quantities. Namely, when $\theta < 0.5$, R&D levels are strategic substitutes and the R&D reaction functions slope down. When $\theta > 0.5$, the part of firm i 's reaction function stemming from the first order condition slopes up. When this latter hits the boundary identified by the line $\sqrt{x_i} = \frac{c}{\lambda} - \theta\sqrt{x_j}$, the reaction function slopes downward and strategic substitutability is restored. I can now state the following.

Proposition 1: The pair (x^s, x^s) *is a Nash equilibrium of the R&D game, where* $x^s = \left(\frac{\lambda(2-\theta)(a-c)}{9-(2-\theta)\lambda^2(1+\theta)}\right)^2$. *If* $\lambda^2 < \frac{3}{(2-\theta)(1-\theta)}$, *then* (x^s, x^s) *is unique and stable. If* $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$, *then* (x^s, x^s) *is unstable, and two other locally stable Nash equilibria*

[12] In fact, Assumption 1 implies $\frac{\partial^2 \Pi_1^N(x_1,x_2)}{\partial x_1^2} = \frac{(\theta-2)\lambda(a-c+(2\theta-1)\sqrt{x_2}\lambda)}{18x_1^{3/2}} < 0$. In addition, $\lim_{x_1\to 0} \frac{\partial \Pi_1^N(x_1,x_2)}{\partial x_1} = \infty$. So a zero R&D level violates a marginal condition for (1).

of the form $(\overline{x},\, r(\overline{x}))$, $(r(\overline{x}),\, \overline{x})$ *obtain, each satisfying* $\sqrt{\overline{x}} = \frac{c}{\lambda} - \theta\sqrt{r(\overline{x})}$ *and* $\overline{x} = \left(\frac{1}{\lambda}\frac{9c-\lambda^2(2-\theta)(2c(1-\theta)+a\theta)}{9-2\lambda^2(2-\theta)(1-\theta^2)}\right)^2 > r(\overline{x}) = \lambda^2\frac{(2c(\theta-1)+a)^2(\theta-2)^2}{(2\lambda^2(2+\theta^3-2\theta^2-\theta)-9)^2}$.

A proof is provided in the appendix.



Linearized R&D reaction curves

The figure describes three possible equilibria for the R&D stage of the game.[13] Both examples (a) and (c) relate to the standard case discussed in the literature in which the unique symmetric equilibrium is stable under Cournot best reply dynamics, with strategic substitutability and complementarity, respectively.

Example (b) illustrates the case of multiple equilibria addressed in this paper. According to Proposition 1 this occurs when the productivity of private R&D is large relative to the benefits from imitation $\left(i.e.,\ \lambda^2 > \frac{3}{(2-\theta)(1-\theta)}\right)$. In this case, the symmetric equilibrium of the R&D game is unstable under Cournot best reply dynamics,[14]

[13] As reported, the R&D reaction functions have been linearized by taking the square root of the R&D levels to be the decision variable.

[14] Henriques (1990) studies the stability of the AJ model for the specific case of $\lambda = \sqrt{2}$. In her setting the model is unstable when $\theta < 0.17$.

and two other asymmetric and locally stable equilibria arise, each involving full innovation by the larger firm. More precisely, the two asymmetric equilibria are mirror images of each other, both inducing an R&D expert-novice configuration where the more R&D intensive firm achieves a full cost reduction.

Note that the minimal value of λ observable under the asymmetric equilibria increases with the spillover rate. It eventually approaches the upper bound identified by Assumption 2, above which the equilibrium is unique, symmetric, and involves a full cost reduction for either firm. Namely, asymmetry is sustainable in equilibrium for small spillovers only, with symmetric R&D profiles prevailing whenever $\theta \geq 0.2$.[15] The intuition is that the external effect of R&D is a substitute for an autonomous investment. By raising the opportunity cost of private research, large spillover rates prevent a firm from fully investing in the first stage.

It is worth emphasizing that, given the restrictions imposed by Assumptions 1 and 2, the parameter a does not play any role as to whether or not the equilibrium market structure is symmetric. As usual in the case of linear demand, a can be thought of as a market size parameter. Similar results have been reported in the literature. In a Cournot model with fixed cots and free entry, Neumann *et al.* (2001) show that a vertical market expansion (a rise of the parameter a) does not influence firms' size but only causes entry. In a similar context, Götz (2004) finds that firms' technological choices do not depend on vertical market size. In line with these results, Proposition 1 here says that the equilibrium distribution of market shares depends on the technological parameters λ and θ only, while it is not affected by the size of the market.

[15] Assumptions 1 and 2 imply $\lambda^2 < \frac{4.5}{(1+\theta)(2-\theta)}$. This, along with $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$, implies $\theta < 0.2$. So firm asymmetry obtains only if R&D investments are strategic substitutes.

3 Comparing asymmetric R&D competition and the joint lab

In this section, asymmetric R&D competition and cooperation via joint lab are compared.[16] The case of zero spillovers is discussed first. This case has been already addressed in Amir and Wooders, where it is shown that total cost reduction and hence consumer surplus are larger under the joint lab than with the asymmetric Nash equilibrium, whereas joint profits are sometimes larger with R&D competition. Proposition 2 below states the superiority of the joint lab in terms of social surplus.

Proposition 2: If $\theta = 0$, then social surplus with the joint lab is greater than with the asymmetric Nash equilibrium with R&D competition.

A proof is provided in the appendix.

As mentioned, in the case of spillovers, welfare results are ambiguous. In what follows two offsetting effects of R&D spillovers on industry performance are discussed, concerning consumer surplus and joint profits respectively.

The first effect relates to the remarkable fact that in the equilibrium with R&D competition the larger firm achieves a full cost reduction. As long as asymmetry is sustainable in equilibrium, this feature is persistent under positive spillovers with the R&D expenditure of the larger firm- say $\sqrt{x_1}$- lying on the isocost reduction line $\sqrt{x_1} = \frac{c}{\lambda} - \theta\sqrt{x_2}$. As is summarized in Proposition 3, the combined effect of full innovation and increasing spillovers makes industry-wide cost reduction rise.

Proposition 3: The aggregate level of cost reduction in the asymmetric Nash equilibrium with R&D competition is an increasing function of the spillover rate.

[16] The analysis focuses on the parameter region within which the symmetric equilibrium under R&D competition is unstable. Recall from footnote 15 that a necessary condition for the symmetric equilibrium to be unstable is $\theta < 0.2$. Also notice that Assumptions 1 and 2 implicitly impose an upper bound on the productivity of R&D, *i.e.* $\lambda < 1.5$. Moreover, $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$ together with Assumption 1 and Assumption 2, implies $\frac{3}{(2-\theta)(1-\theta)}\frac{a}{c} < \frac{9}{(2-\theta)(1+\theta)}$, and hence $2 < \frac{a}{c} < \frac{3(1-\theta)}{(1+\theta)}$.

A proof is provided in the appendix.

The following argument gives an intuition for Proposition 3. Along the isocost line $\sqrt{x_1} = \frac{c}{\lambda} - \theta\sqrt{x_2}$, the best response of the more R&D intensive firm to a given expenditure of the less intensive one reduces as θ increases. This is due to the mentioned fact that on the boundary part of the R&D expert's reaction function in R&D space, the effective cost reduction going to that firm is constant and hence private and spillover effects are substitute. On the other hand, at the asymmetric equilibrium, the marginal benefit to the smaller firm of its own R&D increases with the spillover rate,[17] *i.e.* the smaller firm's R&D reaction function shifts up as the spillover increases. So, a raise of θ causes the small firm to increase and the large firm to reduce the respective R&D levels. Since at the asymmetric equilibrium $r'(\overline{x}) < -1$, this makes the effective cost reduction of the less R&D intensive firm rise. Given that the effective cost reduction of the R&D expert is constant (and equal to c), the overall effect of increasing θ on industrywide cost reduction is positive.

Consumer surplus is increasing in aggregate cost reduction. So, for the non cooperative scenario, increasing spillovers are advantageous for consumers. On the other hand, the innovative performance of the joint lab is independent of the spillover rate. It follows that the difference between aggregate cost reduction and hence consumer surplus in the competitive and the collusive scenario increases with θ. As stated in Proposition 4, and contrary to the case of zero spillovers discussed in Amir and Wood-

[17] Let firm 2 be the R&D novice and denote $\Lambda \equiv \partial^2\Pi_2^N/\partial x_2\partial\theta$. At the asymmetric equilibrium $\Lambda_{eq} = \left(1/9\sqrt{r(\overline{x})}\right)\lambda\left(\lambda\left(c - \theta\sqrt{r(\overline{x})}\right)(5-4\theta) + 2\lambda\sqrt{r(\overline{x})}(\theta-2) - a + c\right)$, where $r(\overline{x})$ is defined as in Proposition 1. Recall that $a < \frac{3c(1-\theta)}{(1+\theta)}$ (see footnote 16), $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$, and $0 < \lambda\sqrt{r(\overline{x})} < c$. This implies $\Lambda_{eq} > \frac{\lambda\left(c\left(\sqrt{\frac{3}{(2-\theta)(1-\theta)}}(5-4\theta)+\frac{3(\theta-1)}{1+\theta}+1\right)+\lambda\sqrt{r(\overline{x})}\left(4\theta^2-3\theta-4\right)\right)}{9\sqrt{r(\overline{x})}} > \frac{\lambda\left(\sqrt{\frac{3}{(2-\theta)(1-\theta)}}(5-4\theta)+\frac{3(\theta-1)}{1+\theta}+1+\left(4\theta^2-3\theta-4\right)\right)}{9\sqrt{r(\overline{x})}}$, which is positive for any admissible θ.

ers (1998), if θ is not *too small*, then aggregate cost reduction and consumer surplus are both larger with asymmetric R&D competition.

Proposition 4: There exists $\theta_* (\lambda)$ such that, if $\theta > \theta_* (\lambda)$ then the aggregate levels of cost reduction and consumer surplus in the asymmetric Nash equilibrium with R&D competition are larger than the respective levels under the joint lab.

A proof is provided in the appendix.[18]

Proposition 3 implicitly establishes a central property of the asymmetric Nash equilibria. Given a full cost reduction by the larger firm, growing levels of the aggregate cost reduction clearly involve reduced firm heterogeneity, as measured by the difference of firms' marginal costs or market shares. This induces a second effect of spillovers on social surplus. By reducing asymmetry in the product market, increasing spillovers bring combined profits down. The idea is that a rise in the spillover rate reduces the market share going to the more efficient firm with R&D competition. Analogously to what mentioned relative to consumer surplus, joint profits with the joint lab are independent of the spillover rate. So, the difference between aggregate profits with R&D competition and R&D cooperation increases with θ. As reported in the introduction, Amir and Wooders (1998) show that with zero externalities joint profits are greater under the joint lab than with the asymmetric equilibrium if demand is relatively large.[19] Proposition 5 here claims that R&D spillovers weaken the condition under which the superiority of the joint lab in terms of aggregate profits is restored.

Proposition 5: Let $w \equiv \frac{a}{c}$. There exists $w_* (\lambda)$ such that, if $\frac{a}{c} \geq \left(1 - \theta^2\right) w_* (\lambda)$

[18] $\theta_* (\lambda)$ is defined in the appendix. Calculations show that the total cost reduction is greater in the asymmetric equilibrium whenever $\theta > 0.07$.

[19] A sufficient condition for that is $\frac{a}{c} \geq w_* (\lambda)$. For a definition of $w_* (\lambda)$ see the proofs of Propositions 1 and 5 in the appendix.

then combined profits with the joint lab are larger than combined profits at the asymmetric Nash equilibrium with R&D competition.

A proof is provided in the appendix.[20]

4 Conclusion

In a standard model of strategic R&D, endogenous firm asymmetry under R&D competition obtains for sufficiently small spillovers only. By raising the opportunity cost of private R&D, large spillovers induce Cournot stability in the R&D stage, leading to symmetric investment profiles in equilibrium. In addition, spillovers promote the diffusion of R&D results among asymmetric competitors, so reducing firm heterogeneity in terms of R&D propensities and hence marginal costs and market shares. These results lend support to a traditional interpretation of spillovers as a barrier to firm heterogeneity and a stimulus to industrywide diffusion of technological progress.[21]

When R&D is perfectly appropriable, a symmetric joint lab dominates asymmetric R&D competition in terms of social surplus. As shown in the literature, while consumer surplus is larger with the joint lab, joint profits are sometimes larger with asymmetric R&D competition. By contrast, with not *too small* spillovers, asymmetric R&D competition is relatively advantageous for consumers, but not for firms. As a consequence, an ambiguous scenario for policy prescriptions obtains.

[20] Calculations show that a sufficient condition for combined profits to be larger under the joint lab is $\theta \geq 0.05$.

[21] See, for instance, Röller and Sinclair-Desgagnè (1996), and Spence (1984). An opposite interpretation of spillovers is due to Eeckhout and Jovanovic (2002). According to the authors, spillover rates, by inducing followers to free ride, create permanent inequality among firms. A second example of externalities as a source of firms' diversity is provided by Amir and Wooders (1999). In their framework spillovers are unilateral, *i.e.* they flow from the more R&D intensive firm to the opponent only. Given this specification, each firm's reaction function in R&D space is discontinuous along the diagonal. As a consequence, only asymmetric investment profiles are sustainable in equilibrium.

Appendix

Proof of Proposition 1

Recall that $\Pi_i^N(x_i, x_j)$ as defined in (1) and $r_i(x_j)$ as defined in (3) are concave in x_i and continuous and single valued, respectively. Observe first that the pair (x^s, x^s) satisfying $\frac{\partial \Pi_i^N(x^s,x^s)}{\partial x_i} = 0$, $i \neq j$, $i = 1, 2$ is a Nash equilibrium of the R&D game. Solving the previous system yields $x^s = \left(\frac{\lambda(2-\theta)(a-c)}{9-(2-\theta)\lambda^2(1+\theta)}\right)^2$. Linearize now the reaction functions in R&D space by taking the square root of the R&D level to be the decision variable. Stability of the equilibrium under best reply dynamics requires $|r_1'(x_2)\, r_2'(x_1)| < 1$, where $r_i'(.)$ denotes the slope of firm i's reaction function, $i = 1, 2$. By Assumption 2, the slope of each firm's linearized reaction function at the symmetric equilibrium is given by $\frac{\lambda^2(2-\theta)(2\theta-1)}{9-[(2-\theta)\lambda]^2}$. Given Assumption 2, the stability requirement is written $\frac{\lambda^2(2-\theta)(2\theta-1)}{9-[(2-\theta)\lambda]^2} > -1$, so (x^s, x^s) is stable if and only if $\lambda^2 < \frac{3}{(2-\theta)(1-\theta)}$. Further, the absolute value of the slope of the iso cost reduction part of each reaction function is equal to $\theta \leq 1$. Given this, if $\lambda^2 < \frac{3}{(2-\theta)(1-\theta)}$, then (x^s, x^s) is unique from the contraction mapping theorem. If $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$, then (x^s, x^s) is unstable. Since the reaction functions are piecewise linear, two other equilibria of the form $(\overline{x}, r(\overline{x}))$, $(r(\overline{x}), \overline{x})$ obtain, each satisfying $\sqrt{\overline{x}} = \frac{c}{\lambda} - \theta\sqrt{r(\overline{x})}$. Solving with respect to $\overline{x}$ yields $\overline{x} = \left(\frac{1}{\lambda}\frac{9c-\lambda^2(2-\theta)(2c(1-\theta)+a\theta)}{9-2\lambda^2(2-\theta)(1-\theta^2)}\right)^2$ and $r(\overline{x}) = \lambda^2\frac{(2c(\theta-1)+a)^2(\theta-2)^2}{(2\lambda^2(2+\theta^3-2\theta^2-\theta)-9)^2}$. Since $\theta\frac{\lambda^2(2-\theta)(2\theta-1)}{9-[(2-\theta)\lambda]^2} < 1$, the two asymmetric equilibria are locally stable. In addition, $\sqrt{\overline{x}} - \sqrt{r(\overline{x})} = \frac{9c-\lambda^2 a(2-\theta)(1+\theta)}{\lambda(9-2\lambda^2(1-\theta^2)(2-\theta))}$. Given Assumptions 1 and 2, the denominator of the previous expression is positive. Consider the numerator and notice that Assumption 2 implies $9c - \lambda^2 a(2-\theta)(1+\theta) > 9c\left(1 - \frac{(2-\theta)(1+\theta)}{(2-\theta)(1+\theta)}\right) = 0$. Hence $\overline{x} > r(\overline{x})$. ■

Proof of Proposition 2

[Amir and Wooders, 1998, Prop.2, pg.69, Prop.3, pg.70] show that, in the absence of spillovers, consumer surplus under the joint lab is greater than consumer surplus at the asymmetric equilibrium with R&D competition, and that combined profits with the joint lab are larger than at the asymmetric equilibrium if $\frac{a}{c} \geq \frac{45}{2(9+\lambda^2)}$. Hence, given Assumption 1, welfare analysis can be restricted to the case in which $2 < \frac{a}{c} < \frac{45}{2(9+\lambda^2)}$. Let $w \equiv \frac{a}{c}$, and define $w_*(\lambda) \equiv \frac{45}{2(9+\lambda^2)}$. Also let $\overline{\Pi}^N$ and $\underline{\Pi}^N$ denote the equilibrium profits to the more and the less R&D intensive firm under R&D competition, respectively. With zero spillover effects, combined profits and consumer surplus at the asymmetric equilibrium with R&D competition are equal to $\overline{\Pi}^N + \underline{\Pi}^N = \left(\frac{a(3-2\lambda^2)+3c}{9-4\lambda^2}\right)^2 + \frac{(a-2c)^2}{9-4\lambda^2} - \left(\frac{c}{\lambda}\right)^2$ and $C^N = \frac{1}{2}\left(\frac{2a(3-\lambda^2)-3c}{9-4\lambda^2}\right)^2$, respectively. Equilibrium combined profits and consumer surplus with the joint lab are equal to $\Pi^J = 2\frac{(a-c)^2}{9-2\lambda^2}$, and $C^J = 18\left(\frac{a-c}{9-2\lambda^2}\right)^2$, respectively.

Define the difference between welfare levels with the asymmetric equilibrium and with the joint lab as $\Delta W \equiv \Pi^N + C^N - \left(\Pi^J + C^J\right)$. ΔW has the same sign as $\Delta^* \equiv \frac{h(w,\lambda)}{2\lambda^2(9-4\lambda^2)^2(9-2\lambda^2)^2}$, where

$$h(w,\lambda) \equiv w\lambda^2\left(5832 - 7452\lambda^2 + 2448\lambda^4 - 176\lambda^6\right) +$$
$$w^2\lambda^4\left(1548\lambda^2 - 528\lambda^4 + 48\lambda^6 - 1296\right)$$
$$+\lambda^2\left(19683 - 8748\lambda^2 + 1548\lambda^4 - 128\lambda^6\right) - 13122.$$

Taking derivatives with respect to w yields

$$\frac{\partial h(w,\lambda)}{\partial w} = \lambda^2\left(5832 - 7452\lambda^2 + 2448\lambda^4 - 176\lambda^6\right) +$$
$$2w\lambda^4\left(1548\lambda^2 - 528\lambda^4 + 48\lambda^6 - 1296\right).$$

Given Assumptions 1 and 2, and imposing $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$ (*i.e.* imposing that the symmetric equilibrium with R&D competition is unstable), it must be that $\sqrt{1.5} <$

$\lambda < 1.5$ (see footnotes 15 and 16). Since $\left(1548\lambda^2 - 528\lambda^4 + 48\lambda^6 - 1296\right) > 0$ for any admissible value of λ, it follows that $\frac{\partial h(w,\lambda)}{\partial w} < \left.\frac{\partial h(w,\lambda)}{\partial w}\right|_{w=w_*(\lambda)} < 0$, for $\sqrt{1.5} < \lambda < 1.5$. Hence, Δ^* is a decreasing function of w. The result follows by observing that $\Delta^* < \left.\Delta^*\right|_{w=2} < 0$, for $\sqrt{1.5} < \lambda < 1.5$. ■

Proof of Proposition 3

Let $w \equiv \frac{a}{c}$. Given Assumptions 1 and 2 and imposing $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$ (*i.e.* imposing that the symmetric equilibrium with R&D competition is unstable), it must be that $w > 2$ and $\theta < 0.2$ (see footnote 15). The aggregate level of cost reduction at the asymmetric equilibrium is equal to $TCR^N = c + \lambda\sqrt{r(\overline{x})} + \theta\lambda\sqrt{\overline{x}}$, where c and $\lambda\sqrt{r(\overline{x})} + \theta\lambda\sqrt{\overline{x}}$ are the effective cost reductions obtained by the more and the less R&D intensive firm respectively, and $\overline{x}$ and $r(\overline{x})$ are defined as in Proposition 1 above. Notice that

$$TCR^N = (1+\theta)\left(c + \lambda\sqrt{r(\overline{x})}\,(1-\theta)\right) =$$
$$(1+\theta)\left(c + \lambda^2 \tfrac{(2c(\theta-1)+a)(\theta-2)}{(2\lambda^2(2+\theta^3-2\theta^2-\theta)-9)}\,(1-\theta)\right).$$

Taking derivatives with respect to θ yields

$$\tfrac{\partial TCR^N}{\partial \theta} = 9\tfrac{\lambda^2\left(a(\theta(3\theta-4)-1)+2c\left(\theta\left(2\theta^2-5\theta+4\right)-1\right)\right)+9c}{(9-2\lambda^2(2-\theta)(1-\theta^2))^2}.$$

The numerator of the previous expression has the same sign as

$$\lambda^2\left(w\left(\theta\left(3\theta-4\right)-1\right) + 2\left(\theta\left(2\theta^2-5\theta+4\right)-1\right)\right) + 9.$$

It is easy to check that for the parameters under consideration,

$$\left(w\left(\theta\left(3\theta-4\right)-1\right) + 2\left(\theta\left(2\theta^2-5\theta+4\right)-1\right)\right) < 0.$$

So, $\lambda^2\left(w\left(\theta\left(3\theta-4\right)-1\right)+2\left(\theta\left(2\theta^2-5\theta+4\right)-1\right)\right)+9 > z\left(w,\,\theta\right)+9$, for Assumption 2, where $z\left(w,\,\theta\right)=\frac{9\left(w(\theta(3\theta-4)-1)+2\left(\theta\left(2\theta^2-5\theta+4\right)-1\right)\right)}{w(1+\theta)(2-\theta)}$. Taking derivatives with respect to w yields $\frac{\partial z(w,\,\theta)}{\partial w}=18\frac{2\theta^3-5\theta^2+4\theta-1}{w^2(1+\theta)(\theta-2)}>0$, for $\theta<0.2$. The result follows by observing that $z\left(w,\,\theta\right)+9> z\left(w,\,\theta\right)|_{w=2}+9=9\left(2\frac{\theta^2(\theta-1)-1}{(2-\theta)(1+\theta)}+1\right)>0$, for $\theta<0.2$. ■

Proof of Proposition 4

Let $w\equiv\frac{a}{c}$. Given Assumptions 1 and 2 and imposing $\lambda^2>\frac{3}{(2-\theta)(1-\theta)}$ (*i.e.* imposing that the symmetric equilibrium with R&D competition is unstable), it must be that $2<w<\frac{3(1-\theta)}{1+\theta}$ and $\theta<0.2$ (see footnotes 15 and 16).

Recall from section 2 and the proof of Proposition 3 above that total cost reductions at the asymmetric Nash equilibrium and with the joint lab are equal to $TCR^N=(1+\theta)\left(c+\lambda^2\frac{(2c(\theta-1)+a)(\theta-2)}{(2\lambda^2(2+\theta^3-2\theta^2-\theta)-9)}(1-\theta)\right)$and $TCR^J=\frac{4(a-c)\lambda^2}{9-2\lambda^2}$, respectively. Further, TCR^N-TCR^J has the same sign as

$$\Delta^{TCR}\equiv3\frac{\lambda^2\left(w\left(\left(4+2\theta^3-4\theta^2-2\theta\right)\lambda^2+3\theta^3-6\theta^2-6\right)+3\left(\theta+8\theta^2-4\theta^3-6\right)\right)+27(1+\theta)}{(9-2\lambda^2(2+\theta^3-2\theta^2-\theta))(9-2\lambda^2)}.$$

It is easy to check that, within the admissible region of parameters, the denominator of the previous expression is positive. Consider next the numerator and note that $\left(4+2\theta^3-4\theta^2-2\theta\right)\lambda^2+3\theta^3-6\theta^2-6>0$ if $\lambda^2>\frac{3}{(2-\theta)(1-\theta)}$. Hence, $\Delta^{TCR}>0$ if and only if $w>\underline{w}\left(\lambda,\theta\right)\equiv3\frac{4\lambda^2\theta^3-\lambda^2\theta-8\lambda^2\theta^2-9+6\lambda^2-9\theta}{\lambda^2((4+2\theta^3-4\theta^2-2\theta)\lambda^2+3\theta^3-6\theta^2-6)}$.

Define $\theta_*\left(\lambda\right)\equiv\left(\lambda^2-1.5\right)\left(1.5-\lambda\right)\left(2.3\lambda-1.45\right)$. The conclusion follows by observing that if $\lambda^2>\frac{3}{(2-\theta)(1-\theta)}$, then $\underline{w}\left(\lambda,\theta\right)<2$ if $\theta>\theta_*\left(\lambda\right)$, with a region of the admissible parameters existing within which both $\lambda^2>\frac{3}{(2-\theta)(1-\theta)}$ and $\theta>\theta_*\left(\lambda\right)$ hold. ■

Proof of Proposition 5

Let $w \equiv \frac{a}{c}$. Given Assumptions 1 and 2 and imposing $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$ (*i.e.* imposing that the symmetric equilibrium with R&D competition is unstable), it must be that $2 < w < 3$ (see footnotes 15 and 16). In the asymmetric equilibrium with R&D competition, profits of the more R&D intensive and the less R&D intensive firm are $\overline{\Pi}^N = \frac{\left(a\left(\lambda^2\left(2+\theta^3-2\theta^2-\theta\right)-3\right)-3c(1-\theta)\right)^2}{(9-2\lambda^2(2+\theta^3-2\theta^2-\theta))^2} - \frac{\left(9c-\lambda^2(2-\theta)(2c(1-\theta)+a\theta)\right)^2}{\lambda^2(9-2\lambda^2(2-\theta)(1-\theta^2))^2}$, and $\underline{\Pi}^N = \frac{\left(9-\lambda^2\left(4+\theta^2-4\theta\right)\right)(a-2c(1-\theta))^2}{(9-2\lambda^2(2+\theta^3-2\theta^2-\theta))^2}$, respectively. Total profits under the joint lab are equal to $\Pi^J = \frac{2(a-c)^2}{9-2\lambda^2}$. Observe further that the difference between the joint profits at the asymmetric equilibrium with R&D competition and with the joint lab has the same sign as

$$\Delta^\Pi \equiv \frac{\left(w\left(\lambda^2\left(2+\theta^3-2\theta^2-\theta\right)-3\right)-3(1-\theta)\right)^2}{(9-2\lambda^2(2+\theta^3-2\theta^2-\theta))^2} - \frac{\left(9-\lambda^2(2-\theta)(2(1-\theta)+w\theta)\right)^2}{\lambda^2(9-2\lambda^2(2-\theta)(1-\theta^2))^2}$$
$$+\frac{\left(9-\lambda^2\left(4+\theta^2-4\theta\right)\right)(w-2(1-\theta))^2}{(9-2\lambda^2(2+\theta^3-2\theta^2-\theta))^2} - \frac{2(w-1)^2}{9-2\lambda^2}.$$

Taking derivatives with respect to w yields $\frac{\partial\Delta^\Pi}{\partial w} = 2\frac{k(w,\theta,\lambda)+z(\theta,\lambda)}{(2\lambda^2-9)(9-2\lambda^2(2-\theta-2\theta^2+\theta^3))^2}$, where $k(w,\theta,\lambda) \equiv 2\lambda^6\left(4-\theta\left(7\theta-8\theta^2-2\theta^3+4\theta^4-\theta^5+4\right)\right)+$

$$\lambda^4\left(\theta\left(21\theta-12\theta^2-4\theta^3+4\theta^4-\theta^5+24\right)-36\right)-9\lambda^2\left(2\theta-9\theta^2+6\theta^3-\theta^4-4\right)$$

and $z(\theta,\lambda) \equiv 9\lambda^2\left(13\theta-47\theta^2+29\theta^3-5\theta^4+12\right)+$

$$2\lambda^4\left(\theta+57\theta^2-53\theta^3-3\theta^4+16\theta^5-4\theta^6-14\right)-81\left(3\theta-\theta^2+1\right).$$

Given Assumptions 1 and 2, the denominator of the previous expression is negative. In addition, if $\lambda^2 > \frac{3}{(2-\theta)(1-\theta)}$, then $k(w,\theta,\lambda)+z(\theta,\lambda) > 0$, and hence $\frac{\partial\Delta^\Pi}{\partial w} < 0$. Define finally $w_*(\lambda) \equiv \frac{45}{2(9+\lambda^2)}$. Substituting $\left(1-\theta^2\right)w_*(\lambda)$ for w in Δ^Π, yields $\Delta^\Pi\big|_{w=(1-\theta^2)w_*(\lambda)} < 0$, for any admissible parameter. ■

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