Tax competition, location, and horizontal foreign direct investment[*]

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Abstract

We develop a model of capital tax competition in which imperfectly competitive firms choose both the number of plants they operate and their location. When compared to models with single-plant firms, the presence of multinationals reverses some standard results. First, instead of being subsidized, capital may actually be taxed in equilibrium, which shows that the presence of taxable 'multinational rents' relaxes tax competition. Second, even when firms are subsidized, their subsidy-inclusive profits may be decreasing in subsidies, due to fiercer price competition by more multinationals. Third, multinationals may give rise to multiple equilibria in the tax game, one of which can be a 'subsidy trap' characterized by many multinationals, high subsidy levels, and low welfare.

Keywords: capital tax competition; international trade; multi-plant location problem; foreign direct investment; imperfect competition

JEL Classification: F12; F23; H27; H73; R12

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1   Introduction

It is a widely documented fact that tax incentives are a pervasive instrument used by governments and developers to attract mobile capital and foreign direct investment (henceforth, FDI; see UNCTAD, 2000, for a recent survey). Although market access, trade costs, and strategic considerations surely explain the larger part of firms' decisions to adopt a multinational structure, it has been recognized that taxes and tax incentives matter a lot in influencing the subsequent destinations and levels of FDI flows (see, e.g., Cummins and Hubbard, 1995; Devereux and Freeman, 1995). Given the empirical importance of financial and fiscal incentive packages, it is natural to ask whether government competition creates windfall gains for mobile firms, thus leading to inefficient outcomes by putting strain on governments' budgets. In recent years, such fears have been increasingly voiced by politics and the public, given the apparent inflation of subsidies granted to firms and investors. Many incentive packages are indeed perceived as being 'overly generous', with total state contribution matching up to 30% of company investments.[1] This may be regarded as yet another illustration of the well-known 'race to the bottom', where tax rates are set to inefficiently low levels or where capital is subsidized at inefficiently high rates.

Although the tax competition literature has extensively analyzed the role of FDI, it has often overlooked the multinational firms' objective to gain access to spatially separated market (see, e.g., Oates 1972; Wilson 1986; Wildasin 1988). Recently, several contributions have linked more closely tax competition, international trade, and spatial issues in models of imperfect competition. Building on the 'new economic geography' paradigm, several authors have investigated tax issues when firms and workers are allowed to agglomerate in some locale in response to economic opportunities (e.g., Ludema and Wooton, 2000; Baldwin and Krugman, 2004; Ottaviano and van Ypersele, 2005; Borck and Pflüger, 2005). Yet, such contributions still fail to fully capture the impact of horizontal FDI and firms' location choices on tax competition outcomes, because firms are not allowed to alter their production structures and duplicate plants to overcome trade barriers.[2]

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[1] For example, total state financial inventives for the Smart automobile plant of Mercedes-Benz and Swatch in Hambach, France, amounted to US$111 million, for a total company investment of US$370 million (Raff, 2004, p.2747). See also Görg and Greenaway (2002) for other illustrative examples.

[2] Baldwin and Krugman (2004), Haufler and Pflüger (2004), and Borck and Pflüger (2005) develop 'new economic geography' models of capital tax competition with product differentiation, trade costs, and imperfect competition. Contrary to us, they do not allow for multinational firms and their framework abstracts from pro-competitive effects. Ottaviano and van Ypersele (2005) develop a model similar to ours featuring pro-competitive effects. Yet, they do not allow for multinationals by assuming that firms operate in a single country only. Most other contributions deal with multinationals by focussing on the location and investment choices of a single monopolist (see, e.g., Haufler and Wooton, 1999, 2005; Devereux and Hubbard, 2003; Raff 2004). The 'new economic geography' literature dealing with multinationals usually

Given the growing importance of multinationals it is, in our opinion, important to develop a framework in which the relationship between production structure, location, and tax competition can be more fully analyzed. Doing so may especially allow us to better understand why the empirical literature finds only somewhat mixed evidence about the importance of international tax competition. Indeed, it is known that, although average statutory taxes have fallen in OECD countries during the 1980s and 90s, tax bases have increased at the same time so that effective marginal tax rates and tax revenues have remained rather stable (Devereux, Griffith and Klemm, 2002). Stated differently, the empirical facts hardly support the thesis of a strong 'race to the bottom'. Also, whereas many empirical contributions support the view that capital is mobile between alternative foreign locations, they generally find no significant capital mobility between domestic and foreign locations (e.g., Devereux and Freeman, 1995). The explanation we propose in this paper is that firms need not relocate capital from the domestic to the foreign country in response to tax changes, but that they may rather raise capital to establish additional plants in foreign locations, thereby *de facto reducing capital mobility*. More precisely, we show that tax competition can be drastically weakened when firms have the option to establish an additional plant rather than simply relocating their unique existing one.

To illustrate this point concisely, we develop a two-country model of capital tax competition in which mobile firms endogenously choose both their location and production structure. All firms sell their products in both countries and face trade costs, so that they choose to operate a single plant in one of the two countries when transport costs or trade barriers are small, whereas they build a second plant and incur additional fixed costs in the opposite case. As in Ottaviano and van Ypersele (2005), utilitarian governments offer subsidies (or tax credits) to attract firms and to increase their residents' consumption surplus and profit claims. Whereas single-plant firms generally constitute a highly mobile tax base, multinational firms represent a more immobile one. The reason is that being multinational may give rise to taxable rents (via trade cost savings), so that marginal changes in subsidies do not alter firms' location choices. By consequence, when many firms engage in horizontal FDI the tax bases are relatively less elastic and subsidy competition is likely to fall. There would, for example, be no scope for tax competition in a world where all firms are 'McDonald-type' multinational corporations which need to build and operate an outlet in each locale to sell their 'hamburgers' there.

Previewing our main results, we show that the outcome of the tax competition game crucially depends on the level of trade costs and the cost of capital before subsidies. If, on the one hand, trade costs are low relative to the cost of capital, firms will always choose to serve both markets from a single production site. In that case, the tax base is fully mobile

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disregards the issue of tax competition and the impacts this may have on firms' choices (e.g., Markusen and Venables, 1998, 2000; Navaretti and Venables, 2004).

and firms react in the usual way to differences in taxes by changing locations: tax rates are strategic complements and capital will always be subsidized in any non-cooperative tax equilibrium (see, e.g., Wilson, 1999; Haufler and Wooton, 1999; Ottaviano and van Ypersele, 2005). If, on the other hand, trade costs are high relative to the cost of capital, firms will always choose to run multiple production plants and serve each market locally through subsidiary sales. In that case, multinationals are more profitable than single-plant exporters so that governments may actually tax away firms 'organizational rents'. This in turn destroys the incentives for harmful tax competition and lowers subsidies to capital since the tax base is immobile.[3] Finally, when trade costs take intermediate values relative to the cost of capital, both multinationals and single-plant firms coexist in equilibrium. In that case, the tax base is partially immobile and capital may be either taxed or subsidized, depending on firms' profitability before subsidies.

Our analysis further reveals that not only the difference in tax rates between countries matters in the presence of multinationals, but also their *absolute level across countries*. Indeed, an increase in the average level of subsidies decreases the cost of capital and leads to the entry of multinational firms, which makes the global economy more competitive by reducing mark-ups. As we will show, this has important implications for the nature of tax competition by creating complementarities in firms' locational and organizational choices.[4] Roughly speaking, by 'going multinational' firms make the global economy more competitive, thereby cutting profits of single-plant exporters by making their access to foreign markets more difficult. This in turn entices exporters to also 'go multinational', thereby further increasing competition. A by-product of this complementarity is that there may be multiple equilibria in the tax game, one of which can a 'subsidy trap' characterized by many multinationals, high subsidies, and low welfare.

The remainder of the paper is organized as follows. In Section 2, we present the model and derive the equilibrium for given tax rates and a given spatial structure. Section 3 then discusses the spatial allocation and the production structure of firms taking tax rates as given. In Section 4, we describe governments' tax choices when they play a non-cooperative tax game, and we fully characterize the equilibria as a function of the before-tax cost of capital. Section 5 discusses the robustness of our main results and presents some comparative statics. Section 6 finally concludes.

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[3]Note that this result is similar to the one obtained in models with asymmetric country sizes, in which the larger country displays a 'home market effect' and may usually tax away 'agglomeration rents' (e.g., Haufler and Wooton, 1999; Baldwin and Krugman, 2004; Ottaviano and van Ypersele, 2005). It also suggests that tax competition in non-traded goods industries (e.g., some consumer services which require firms to have 'nexus' in many locales) may be different than in traded goods industries, since firms must operate in the different markets which makes them less mobile and, therefore, easier to tax.

[4]This feature of multinational activity goes unnoticed in both the numerous models focussing on the choices of a single monopolist, as well as the models relying on CES preferences and constant mark-ups.

2 The model

Our framework builds on Ottaviano and van Ypersele (2005), who introduce capital tax competition in the 'new economic geography' model by Ottaviano and Thisse (2004).

2.1 Preferences

Consider an economy with two countries, labeled H and F. Variables associated with each country will be subscripted accordingly. Each country hosts a mass $L/2$ of immobile consumers, which have identical quasi-linear preferences over a homogeneous good and a continuum of varieties of a horizontally differentiated good. The subutility over the varieties $v \in [0, 1]$ of the differentiated good is quadratic as in Ottaviano et $al.$ (2002). The utility of a representative consumer in country $i = H, F$ is given by:

$$U_i = \sum_{j=H,F} \int_0^{N_j} q_{ji}(v)\mathrm{d}v - \frac{\beta - \gamma}{2} \sum_{j=H,F} \int_0^{N_j} [q_{ji}(v)]^2 \mathrm{d}v - \frac{\gamma}{2} \left[\sum_{j=H,F} \int_0^{N_j} q_{ji}(v)\mathrm{d}v \right]^2 + q_i^o,$$

where $q_{ji}(v)$ denotes the consumption in country i of the differentiated variety v produced in country j; q_i^o stands for the consumption of the homogeneous good in country i; N_i stands for the mass of varieties produced in country i (with $N_H + N_F = 1$); and $\alpha > 0$, $\beta > \gamma > 0$ are parameters.

Each consumer is endowed with one unit of labor, which she supplies inelastically, $\overline{q}_o > 0$ units of the homogeneous good, and an equal share of the world capital stock. Furthermore, we assume that each agent has the same claims to capital income and firms' profits. A consumer in country $i = H, F$ maximizes her utility subject to the budget constraint

$$\sum_{j=H,F} \left\{ \int_0^{N_j} p_{ji}(v) q_{ji}(v)\mathrm{d}v \right\} + p_i^o q_i^o \leq \frac{\Pi_w}{L} + w_i + p_i^o \overline{q}_o,$$

where $p_{ji}(v)$ is the consumer price in country i of variety v produced in country j; Π_w is the sum of world capital income and profits; and w_i is the wage rate in country i.

In what follows, we assume that all varieties produced in the same country are symmetric, which allows us to alleviate notation by dropping the variety index. Consumers' demand are expressed as it follows:

$$q_{ij} = a - (b + c)p_{ij} + cP_j$$

where a, b and c are positive coefficients given by

$$a \equiv \frac{\alpha}{\beta}, \qquad b \equiv \frac{1}{\beta}, \qquad c \equiv \frac{\gamma}{(\beta - \gamma)\beta},$$

and where

$$P_j \equiv N_j p_{jj} + N_i p_{ij} \qquad i \neq j$$

can be interpreted as the average price of differentiated goods in country j.

2.2 Technology and trade costs

The homogeneous good is produced under perfect competition using one unit of labor only. Profit maximization in this sector implies that $p_i^o = w_i$. We assume that this good can be costlessly traded between countries and we choose it as the numéraire, in which case $p_i^o = p_j^o = 1$ and $w_i = w_j = 1$. We furthermore assume that labor is intersectorally mobile, so that the wages in the differentiated industry are also equal to 1.

Turning to the differentiated sector, we assume that each firm owns some firm-specific asset (e.g., a patent right) which grants it monopoly power over a single variety. Therefore, the total mass of firms is equal to the total mass of varieties, which is fixed to 1. The total mass of firms being fixed, since there is no entry and exit, there may be pure profits in the economy. This is either because entrepreneurs are scare, or because the number of patents is limited, or because there exist significant barriers to entry in the industry.

We assume that firms do not only choose their location, but that they also choose the number of plants they operate, i.e., they make both an *organizational* and a *locational* choice. Firms can choose to be either exporters or multinationals. In the former case, firms run a single production unit and serve the foreign market through exports, whereas in the latter case they operate two production units and serve each market locally. Denote by n_i the mass of exporters based in country $i = H, F$ and by and m the mass of multinationals, respectively.[5] Since each exporter and multinational firm produces a single variety, we must have that the total number of varieties in country i, N_i, is equal to $n_i + m$. The total mass of varieties being equal to 1, we thus have

$$n_H + n_F + m = 1.$$

All firms have access to the same technology and produce their variety by using both labor and capital. Following Ottaviano and Thisse (2004), we assume that labor enters only the variable cost, whereas the fixed cost is incurred in terms of capital only. Without loss of generality, we may set the marginal labor requirement to zero since this amounts to rescaling firms' demand intercepts (see Ottaviano *et al.*, 2002). Firms require f units of capital to set up a plant in any country. Capital is assumed to be perfectly mobile

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[5]We do not need to keep track of where multi-plant firms are headquartered. This is because we assume that fixed costs are the same in both countries. When fixed costs differ, headquarters will be exclusively located in the low fixed cost country (see, e.g., Navaretti and Venables, 2004, p.54).

across sectors and countries. We assume that the rental rate r of capital is exogenously fixed. Constant rental rates may reflect the constant degree of intertemporal substitution of lenders, or simply the fact that the industry is small when compared to the rest of the economy.[6] Without loss of generality we normalize the rental rate of capital r to one.

When governments do not subsidize capital, the cost of a plant is equal to f. When governments in H and F do subsidize capital at the rates ξ_H and ξ_F, the costs of a plant are equal to $f(1 - \xi_H)$ and $f(1 - \xi_F)$, respectively. For the sake of exposition, we denote the firms' after-subsidy fixed costs by $f - s_H$ and $f - s_F$, where $s_H = \phi \xi_H$ and $s_F = \phi \xi_F$ stand for the values of subsidies. Since f is assumed to be constant, the subsidies s_H and s_F are lump sum transfers to the firms which depend on the number and location of their investments. Note that all firms in a country are equally subsidized. Such equal treatment of firms circumvents the problem of "harmful measures", which are considered to arise if tax incentives apply only to non-residents, i.e., if they are discriminatory (UNCTAD, 2000). This is illustrated by the fact that "*the EU adopted a Code of Conduct for business taxation, in which member states committed themselves to refrain from 'unfair' tax policies*" (Haufler and Wooton, 2005, p.2). Note, finally, that subsidies to local exporting firms may also be interpreted as incentives for firms not to move their plants abroad. Thus, subsidies may be seen as playing the dual role of trying to attract new firms and preventing existing firms from leaving. Note, finally, that nothing precludes subsidies a priori from being negative, in which case they are equivalent to source-based capital taxes payed in the location where the capital is used.[7]

Turning to transportation, shipping each variety of the differentiated good across countries is costly, whereas shipping it within each country is free. More specifically, shipping one unit of any variety between the two countries entails a per-unit cost of $\tau > 0$ units of the numéraire. Note that the existence of transport costs, and the absence of transfer pricing problems since we focus on capital taxation, ensure that multinationals behave like local firms in each domestic market and serve the market through subsidiary sales only. Indeed, given plant-level scale economies and transport costs the firm will never produce a fraction of demand locally while importing the rest from abroad. Hence, our model is of the 'proximity-vs-scale' type (Barba Navaretti and Venables, 2004).

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[6]We assume that the capital endowment in the two countries is large enough for agents to be able to supply any amount needed. The rest is then allocated at a rate r to the world capital market, so that we can abstract from it.

[7]In this paper, we do not consider residence-based taxes. Under perfect information, residence-based taxes are non-distortionary lump sum transfers which do not generate tax competition between governments. The main discussion about residence-based taxes arises under imperfect information when tax authorities can hardly observe and collect the foreign capital incomes of their residents. This discussion is beyond the scope of the present paper. Furthermore, Keen (1993) argues that the effective taxations of multinationals is source based, independently of what tax codes effectively stipulate.

Firms and governments play a three-stage game: (i) governments set non-cooperatively their taxes (subsidies) to firms; (ii) firms choose the number and location of their production plants; and (iii) given the previous choices, firms they set profit maximizing prices. We solve the game by backward induction for its subgame-perfect Nash equilibria.

In the following sub-section, we present the market outcome. We then discuss firms' locational and organizational choices in Section 3 and present the tax equilibria in Section 4.

2.3 Price equilibrium

Since our framework features a continuum of firms, each firm is negligible to the market and sets its own prices taking all other variables as given. Considering n_H, n_F and m, as well as the subsidies s_H and s_F, as fixed, firms maximize their profits with respect to prices. In accord with empirical evidence (see, e.g., Head and Mayer, 2000; Haskel and Wolf, 2001), we assume that *international markets are segmented*. Firms are hence free to set prices specific to each national market they sell their product in.

In what follows, we superscript variables pertaining to exporters by s and to multinationals by m. The profit before subsidy of an exporter established in country $i = H, F$ is given by

$$\Pi_i^s = \frac{L}{2} p_{ii} q_{ii} + \frac{L}{2} (p_{ij} - \tau) q_{ij} - f,$$

whereas the profit before subsidy of a multinational is given by

$$\Pi^m = \frac{L}{2} p_{HH} q_{HH} + \frac{L}{2} p_{FF} q_{FF} - 2f.$$

Since multinationals serve each market locally only, their local pricing decisions are identical to those of domestic firms operating in same market. Firms maximize their profits with respect to own prices taking the average prices P_H and P_F as given. Substituting the solution in the average prices and solving for the price equilibrium yields :

$$p_{ii}^* = \frac{2a + c n_j \tau}{2(2b + c)} \qquad p_{ji}^* = p_{ii}^* + \frac{\tau}{2} \tag{1}$$

$$q_{ii}^* = (b + c) p_{ii}^* \qquad q_{ji}^* = (b + c)(p_{ji}^* - \tau), \tag{2}$$

with symmetric expressions for country-F firms. Because $n_H + n_F + m = 1$, the equilibrium price in country H can also be expressed as follows:

$$p_{HH}^* = \frac{2a + c\tau - c(m + n_H)\tau}{2(2b + c)}.$$

This reveals that prices in country H decrease with the mass of plants located in that country (n_H). Note that the mass of multinationals appears in both countries' equilibrium prices. Hence, contrary to exporters which have only an impact on prices in the country they are located in, *multinationals put downward pressure on prices in both countries*.

To simplify the analysis and avoid a proliferation of sub-cases, we assume that trade costs are sufficiently low such that international trade is always feasible. It is readily verified that $q_{HF}^* > 0$ and $q_{FH}^* > 0$ for all allocations (n_H, n_F, m) provided that

$$\tau < \tau^{\text{trade}} \equiv \frac{2a}{2b+c}\,. \tag{3}$$

Condition (3), which is henceforth referred to as *trade feasibility condition*, also makes sure that international prices net of transport costs always remain strictly positive. Put differently, when (3) holds international trade occurs regardless of the choices during the first two stages of the game.

Using the profit-maximizing prices (1) and quantities (2), the profits can be expressed as follows:

$$\Pi_i^s = \frac{L(b+c)}{2} \left[(p_{ii}^*)^2 + \left(p_{jj}^* - \frac{\tau}{2} \right)^2 \right] - f \tag{4}$$

$$\Pi^m = \frac{L(b+c)}{2} \left[(p_{HH}^*)^2 + (p_{FF}^*)^2 \right] - 2f. \tag{5}$$

Finally, the consumer surplus of a resident in country $i = H, F$ is given by

$$\begin{aligned} S_i = {} & \frac{a^2}{2b} - a \left[(n_i + m)\, p_{ii}^* + n_j p_{ji}^* \right] + \frac{b+c}{2} \left[(n_i + m)\, (p_{ii}^*)^2 + n_j (p_{ji}^*)^2 \right] \\ & - \frac{c}{2} \left[(n_i + m)\, p_{ii}^* + n_j p_{ji}^* \right]^2. \end{aligned} \tag{6}$$

We now turn to the issue of subsidy competition between governments when firms are geographically mobile and can decide on the number of production plants they operate in the global economy.

3 Firms' structure, location, and profits

Subsidies affect firms' location, production structure, and profits. We begin by investigating which types of equilibrium configurations (n_H^*, n_F^*, m^*) may arise for any *given* couple of subsidies (s_H, s_F). We then discuss how subsidies affect firms' profits in the different configurations.

3.1 Structure and location

As shown, in the next section, only two equilibrium configurations are relevant in the subsidy game between governments. Thus, for the sake of brevity, we only discuss these

configurations in this section.

(i) Pure exporter configurations (i.e., $m^* = 0$) require that *subsidy-inclusive* profits of exporters are equalized across countries. Evaluating (4) and (5) at the equilibrium prices (1), the profit differential between exporters in countries H and F is equal to

$$\Pi_H^s + s_H - (\Pi_F^s + s_F) = K(n_F - n_H) + s_H - s_F, \quad K \equiv \frac{c(b+c)}{4(2b+c)}L\tau^2 > 0.$$

Equating this profit differential to zero, the equilibrium masses of exporters and plants in countries H and F are given by

$$n_H^* = \frac{1}{2} + \frac{1}{2K}(s_H - s_F) \qquad \text{and} \qquad n_F^* = \frac{1}{2} - \frac{1}{2K}(s_H - s_F). \tag{7}$$

Hence, by making capital cheaper, a country's subsidy attracts *exporters* from the other country. In a pure exporter configuration, subsidy competition is of a 'win-lose' type in terms of number of local firms and, therefore, in term of access to products.

Feasibility of a pure exporter configuration further requires that exporters are more profitable than multinationals:

$$\Pi^m + s_j - \Pi_i^s \leq 0, \;\; i = H, F \quad \Longleftrightarrow \quad s_H + s_F \leq 2B - K \tag{8}$$

$$n_H^* \geq 0 \quad \Longleftrightarrow \quad s_F \leq s_H + K \tag{9}$$

$$n_F^* \geq 0 \quad \Longleftrightarrow \quad s_F \geq s_H - K, \tag{10}$$

where all profits are computed using the equilibrium prices (1), evaluated at $n_H = n_H^*$, $n_F = n_F^*$ and $m = 0$, and where

$$B \equiv f - \frac{(b+c)[4a - (2b+c)\tau]}{8(2b+c)}L\tau.$$

The set of subsidies supporting this configuration, delimited by (8)–(10), is depicted by zone (i) in Figure 1.[8]

Insert Figure 1 about here.

(ii) Mixed configurations, where exporters and multinationals may coexist (i.e., $m^* > 0$, $n_H^* \geq 0$ and $n_F^* \geq 0$), require that subsidy-inclusive profits of all types of firms are equalized across countries: $\Pi_H^s + s_H = \Pi_F^s + s_F = \Pi^m + s_H + s_F$. Note that this configuration includes the limiting case where all firms are multinational, but are just indifferent between shutting down one plant and adopting an export structure.

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[8]The parameter values underlying Figure 1 are as follows: $\alpha = 1$, $\beta = 1$, $\gamma = 0.5$, $L = 4$, $\tau = 0.1$, and $\phi = 0.132$.

The profit differential between an exporter in country H and a multinational can be expressed as follows:

$$\Pi^m + s_F - \Pi_H^s = \frac{L}{2}(b+c)\tau\left(p_{FF}^* - \frac{\tau}{4}\right) - f + s_F,$$

a symmetric expression holding for country F. Equating the above profit differentials to zero, we readily obtain the following equilibrium masses of *plants* in countries H and F:

$$n_H^* + m^* = \frac{K - 2B + s_H}{K} \qquad \text{and} \qquad n_F^* + m^* = \frac{K - 2B + s_F}{K}.$$

By making capital cheaper, a country's subsidy increases the number of *plants* in its market. Since $n_H + n_F + m = 1$, we get the equivalent conditions

$$n_H^* = \frac{B - s_F}{K}, \qquad n_F^* = \frac{B - s_H}{K}, \qquad m^* = \frac{K - 2B + s_H + s_F}{K}. \tag{11}$$

It is important to observe that the mass of exporters in country H *is independent of the subsidy s_H set by this country*. This subsidy only induces exporters in country F to build a second plant in country H.

Finally, feasibility of the configuration requires that

$$n_H^* \geq 0 \iff s_F \leq B \tag{12}$$

$$n_F^* \geq 0 \iff s_H \leq B \tag{13}$$

$$m^* > 0 \iff s_H + s_F > 2B - K. \tag{14}$$

The set of subsidies supporting this configuration, delimited by (12)–(14), is depicted by zone (ii) in Figure 1.

Proposition 1 *The location of production is sensitive to the difference in subsidies in the pure exporter configuration, whereas the difference in subsidies plays no role for firms' location in a mixed configuration where exporters and multinational firms coexist in all countries.*

As shown in Figure 1 and in the Appendix 1, other configurations exist but are never equilibria of the subsidy game between governments. For instance, there is a configuration (iii) where only multinationals exit and configurations (iv) and (v) (resp. (iv') and (v')) where no exporters are located in country F (resp. country H).

3.2 Profits

Because subsidies affect both firms' structure and location, *their impact on subsidy-inclusive profits is a priori ambiguous.* A more careful analysis of this point will prove

useful in guiding intuition as to which additional effects may arise in the presence of multinationals.

In the pure exporter configuration (i), a subsidy increase directly raises the profit of firms located in that country and, by triggering relocation towards it, also raises profits in the other country by making competition there less fierce. Indeed, it is easy to show that in (the interior of the domain of) configuration (i), we have

$$\frac{\mathrm{d}\left(\Pi_H^s + s_H\right)}{\mathrm{d}s_H} = \frac{b + n_H^* c}{2b + c} > 0, \qquad \frac{\mathrm{d}\left(\Pi_H^s + s_H\right)}{\mathrm{d}s_F} = \frac{b + n_F^* c}{2b + c} > 0.$$

Yet, this result does not hold in the presence of multinationals, since subsidies affect the number of plants rather than firms' location. The result being that a subsidy increases competition in the country where new plants are built, whereas it leaves competition in the other country unchanged. Thus, competition in the global economy increases and profits fall in both countries. We can indeed show that in configuration (ii) we have

$$\frac{\mathrm{d}\left(\Pi_H^s + s_H\right)}{\mathrm{d}s_H} = \left(1 - \frac{2p_{HH}^*}{\tau}\right) < 0, \qquad \frac{\mathrm{d}\left(\Pi_H^s + s_H\right)}{\mathrm{d}s_F} = \left(1 - \frac{2p_{FF}^*}{\tau}\right) < 0,$$

since $p_{HH}^* > \tau/2$ and $p_{FF}^* > \tau/2$ under the trade feasibility condition (3). Hence, the impact of subsidies on profits is drastically modified in the presence of multinationals, due to the possibility of firms' endogenously changing their production structure.

Proposition 2 (firms' profits) *Suppose that governments increase their subsidies (reduce their taxes). In the pure exporter configuration (i), profits after subsidy increase; whereas in the mixed configuration (ii), profits after subsidy decrease.*

It is easy to show that profits are minimized in two cases. Firstly, when firms are heavily taxed (i.e., $(s_H, s_F) \to (-\infty, -\infty)$ in a pure exporter configuration, which yields negative profits below some threshold); and secondly, when firms are highly subsidized in the mixed configurations (i.e., $(s_H, s_F) = (B, B)$, which yields a local minimum for profits). In what follows, we restrict our attention to the meaningful situation in which firms' subsidy-inclusive profits are positive so that production takes place. This imposes, firstly, that taxes are not too high so that firms make positive profits in configuration (i); and, secondly, that profits are positive at the point $(s_H, s_F) = (B, B)$, which implies that firms' profits are always positive for any optimal subsidies in configuration (ii). Because $\Pi^m + s_H + s_F = \Pi^m + 2B$ when $s_H = s_F = B$, the formal condition for this is that

$$\frac{1}{4} L \left(b + c\right) \left(\tau^{\text{trade}} - \tau\right)^2 > 0,$$

which is always satisfied since $\tau < \tau^{\text{trade}}$.

To sum up, our analysis reveals that subsidies have a strong impact on firms' location, structure, and profits. In particular, when subsidies are small relative to fixed costs, the industry includes exporters only, whereas it also includes multinationals when they are larger. In the absence of multinational firms, the location of production is quite sensitive to the difference in subsidies. By contrast, in the presence of multinational firms, the difference in subsidies plays no role. Finally, firms may gain or lose from higher subsidies, depending on their organizational structure. In particular, *profits may well decrease with larger subsidies as the global economy becomes more competitive due to an increasing mass of multinationals.* We now explore the impact of endogenous multinationals on governments' subsidy competition.

4   Subsidy competition

Ottaviano and van Ypersele (2005) have shown that tax competition leads to subsidies to capital funded by taxes on labor. Stated differently, governments subsidize capital to attract mobile firms to their country. What happens when firms can not only choose location but also their number of plants? How does this affect the insights gained from tax competition models?

In this section, we assume that firms and capital are wholly owned by residents of countries H and F ('fully diversified portfolio' assumption).[9] More precisely, country H's residents get one-half of firms' profits after subsidy and one-half of the capital rents. Country F's residents get the same shares of profits and capital rentals.

The government of country H chooses s_H to maximize its residents' welfare given by

$$
\begin{aligned}
W_H \;=\; & \frac{L}{2} S_H - (n_H + m)\, s_H \\
& + \frac{1}{2} \Big[n_H \left(\Pi_H^s + s_H + f \right) + n_F \left(\Pi_F^s + s_F + f \right) \\
& + m \left(\Pi^m + s_H + s_F + 2f \right) \Big],
\end{aligned}
\tag{15}
$$

which consists of local consumer surplus minus the direct cost of subsidizing capital, plus one-half of cash-flows to creditors, f, and cash-flows to firms' owners, $\Pi_i^s + s_i$ and $\Pi^m + s_H + s_F$. Differentiating W_H with respect to s_H, we can express the impact of a

---

[9]Ottaviano and van Ypersele (2005) use a similar assumption. Still, Raff (2004) has shown that the ownership structure of firms and the distribution of profits is not innocuous. In Section 5, we will discuss the robustness of our results under alternative assumptions on the distribution of profit and capital rents.

marginal increase in subsidy on local welfare as follows:

$$
\begin{aligned}
\frac{\mathrm{d}W_H}{\mathrm{d}s_H} &= \frac{1}{2}\frac{\partial n_H}{\partial s_H}\left(L\frac{\partial S_H}{\partial n_H} + \Pi_H^s + f - s_H\right) + \frac{1}{2}\frac{\partial n_F}{\partial s_H}\left(L\frac{\partial S_H}{\partial n_F} + \Pi_F^s + f + s_F\right) \\
&\quad + \frac{1}{2}\frac{\partial m}{\partial s_H}\left(L\frac{\partial S_H}{\partial m} + \Pi^m + 2f - s_H + s_F\right) \\
&\quad + \frac{1}{2}\frac{\partial p_{HH}}{\partial n_F}\frac{\partial n_F}{\partial s_H}\left(L\frac{\partial S_H}{\partial p_{HH}} + n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} + n_F\frac{\partial \Pi_F^s}{\partial p_{HH}} + m\frac{\partial \Pi^m}{\partial p_{HH}}\right) \\
&\quad + \frac{1}{2}\frac{\partial p_{FF}}{\partial n_H}\frac{\partial n_H}{\partial s_H}\left(n_H\frac{\partial \Pi_H^s}{\partial p_{FF}} + n_F\frac{\partial \Pi_F^s}{\partial p_{FF}} + m\frac{\partial \Pi^m}{\partial p_{FF}}\right) - \frac{1}{2}\left(n_H + m\right). \quad (16)
\end{aligned}
$$

In this expression, the last term captures the direct cost of a subsidy paid to all firms. The first three terms represent the indirect effects of the subsidy through relocation of firms at constant prices; whereas the intermediate terms finally indicate the indirect effect of a subsidy through price changes.

4.1 Goverments' best responses

We now determine the subsidies government H may potentially choose in the subsidy game for a given value of s_F. This allows us then to derive and discuss governments' best responses associated with different spatial structures. All standard calculations are relegated to Appendix 2.

To begin with, it is of interest to note that government H will always choose subsidies in the configurations (i) or (ii). Assume, indeed, that the subsidy s_H is sufficiently large so that country F has no exporters, which corresponds to either configuration (iii), (iv) or (v). Then every firm operates a plant in country H and possibly one in country F. If government H increases its subsidy, this does not alter the distribution of exporters across the two countries. Furthermore, because the subsidy s_H has no impact on the decision of building a plant in country F, this action does not alter firms' organizational structure. Hence, the only effect of such a subsidy is to raise firms' profits, which cannot be welfare improving since some profits accrue to foreign shareholders.[10] Formally, one can check in configurations (iii), (iv) and (v) that $\partial n_H^*/\partial s_H = \partial n_F^*/\partial s_H = \partial m/\partial s_H = 0$, so that marginal welfare (16) reduces to

$$
\frac{dW_H}{ds_H} = -\frac{1}{2}\left(n_H^* + m^*\right) = -\frac{1}{2} < 0. \quad (17)
$$

By consequence, government H will never set a subsidy compatible with configurations (iii), (iv) and (v); whereas, by symmetry, government F will never set a subsidy compatible

---

[10] As shown in Section 5, this result holds for all possible ways of distributing profits between domestic and foreign shareholders and, therefore, does not depend on our assumption of an equal redistribution.

with configurations (iii), (iv') and (v'). Tax competition equilibria therefore necessarily occur only in configurations (i) and (ii) (including their borders), which we now analyze in more detail.

We now firstly determine government H's best response function for configurations (i) and (ii) separately. We then combine the pieces of those best response functions and derive the equilibria of the subsidy game.

(i) Pure exporter configurations: Suppose that the industry consists of only exporters, as in configuration (i) above. Straightforward calculation shows that changes in prices and locations are given by

$$\frac{\partial p_{HH}}{\partial n_F} = \frac{\partial p_{FF}}{\partial n_H} = \frac{c\tau}{2(2b+c)} \qquad \text{and} \qquad \frac{\partial n_H^*}{\partial s_H} = -\frac{\partial n_F^*}{\partial s_H} = \frac{1}{2K} \ .$$

The marginal welfare (16) can hence be rewritten as follows:

$$
\begin{aligned}
\frac{dW_H}{ds_H} &= -\frac{1}{2}n_H - \frac{\partial n_H}{\partial s_H}s_H + \frac{\partial n_H}{\partial s_H}\frac{L}{2}\left(\frac{\partial S_H}{\partial n_H} - \frac{\partial S_H}{\partial n_F}\right) \\
&+ \frac{1}{2}\frac{\partial p_{HH}}{\partial n_F}\frac{\partial n_F}{\partial s_H}\left[L\frac{\partial S_H}{\partial p_{HH}} + n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} + n_F\frac{\partial \Pi_F^s}{\partial p_{HH}}\right] \\
&+ \frac{1}{2}\frac{\partial p_{FF}}{\partial n_H}\frac{\partial n_H}{\partial s_H}\left(n_H\frac{\partial \Pi_H^s}{\partial p_{FF}} + n_F\frac{\partial \Pi_F^s}{\partial p_{FF}}\right).
\end{aligned}
\tag{18}
$$

The first term in (18) stands for the cost of the subsidy paid to all firms located in country H. Indeed, when government H increases its subsidy s_H it pays an additional amount to all firms established in H, with one-half of this subsidy being recouped by local shareholders through profit redistribution. The second term captures the cost of the subsidy due to firms relocating to country H or opening a second production plant there. Since profits across countries are equal in equilibrium, shareholders are unaffected by such a relocation and the cost of the subsidy is simply equal to s_H. The third term captures the impact of firms' relocation on local consumer surplus. An increase in the subsidy s_H attracts plants to country H, so that more varieties are produced and sold locally at the price p_{HH} which allows consumers to save on transport costs. The fourth term is identically equal to zero. This is because when profits are evenly redistributed across countries in the model by Ottaviano *et al.* (2002), an increase in p_{HH} reduces the consumer surplus of home residents and augments their share in total profits in exactly the same proportions. The last term finally captures the effect of a change in foreign prices due to the subsidy. An increase in s_H entices firms to leave country F, which increases prices and profits there and thus benefits to local shareholders. To sum up, the government balances the costs of the subsidy and its associated profit effects with the benefits in local consumer surplus and foreign profits.

In Appendix 2, we show that dW_H/ds_H is a decreasing function of s_H and that $dW_H/ds_H = 0$ if and only if

$$s_H = \widehat{s}_H(s_F) \equiv \frac{8b + 3c}{16b + 7c} s_F \tag{19}$$
$$+\tau L (c + b) \frac{4a (b + c) - \tau (2b^2 + 4bc + c^2)}{2 (2b + c) (16b + 7c)},$$

which is an affine function of s_F, with positive intercept and a positive slope less than 1. When government F increases it subsidy s_F, government H responds by raising its subsidy s_H but by less than the full amount. Hence, in a pure exporter configuration, subsidies are *strategic complements*, which is the standard result of the literature. Note that $\widehat{s}_H$ shifts upwards when the demand for the differentiated good (i.e., a or L) increases. This is because in such a case there is a larger volume of imports so that government H has stronger incentives to attract firms in order to save on transport costs incurred by its residents.

The graph of the best response $\widehat{s}_H(s_F)$ is depicted in Figure 2. Note that when it lies above (resp. below) of the domain of configuration (i), the best response is given by the upper border (9) (resp. lower border (10)) of the configuration. Indeed, when the demand is sufficiently large, on the one hand, $\widehat{s}_H$ may lie wholly above the domain of configuration (i). An increase in s_H then raises local welfare ($dW_H/ds_H > 0$) because the cost of the subsidy is always less than the savings in transport costs. In this case, the optimal subsidy s_H lies on the upper border of the domain of configuration (i). When the demand is smaller, on the other hand, $\widehat{s}_H$ intersects the domain of configuration (i) for a particular range of subsidies s_F. If s_F belongs to this range, the optimal subsidy is of course given by $\widehat{s}_H(s_F)$. If s_F lies below any value in this particular interval, then configuration (v) applies. Since, as argued above, such a value of $\widehat{s}_H$ cannot be an optimal subsidy, government H will reduce its subsidy until it reaches the border (9) between configurations (i) and (v), which then yields the optimal subsidy choice. Finally, if s_F lies above any value of this particular interval, the optimal subsidy lies either on the border (8) or on the border (10). This case is illustrated by Figure 2 (borders are denoted by their equation numbers). To sum up, in configuration (i) government H's optimal subsidy is given by a combination of the function $\widehat{s}_H(s_F)$ and the borders (8), (9) and (10) (see Figure 2).[11]

Insert Figure 2 about here.

(ii) Mixed configurations: Suppose that the industry comprises all types of firms as is the case in the mixed configuration (ii). Expression (11) shows that changes in location

---

[11]The parameter values underlying Figure 2 are as follows: $\alpha = 1$, $\beta = 1$, $\gamma = 0.85$, $L = 6$, $\tau = 0.07$, and $\phi = 0.33$.

are given by

$$\frac{\partial n_H^*}{\partial s_H} = 0 \qquad \text{and} \qquad \frac{\partial n_F^*}{\partial s_H} = -\frac{\partial m^*}{\partial s_H} = -\frac{4\left(2b+c\right)}{L\left(b+c\right)\tau^2 c} < 0.$$

Hence, an increase in country H's subsidy has as sole effect to entice exporters in country F to become multinationals. Stated differently, the subsidy does not alter the number of plants and, therefore, the competition in country F. Still, in contrast to Ottaviano and van Ypersele (2005), the subsidy *affects the number of varieties produced locally in the subsidizing country, since it attracts plants.* In other words, tax competition is not a 'zero-sum game' in terms of locally produced varieties.[12]

The marginal welfare of the subsidy (16) can be expressed as follows:

$$\begin{aligned}
\frac{dW_H}{ds_H} &= -\frac{1}{2}\left(n_H + m\right) + s_H \frac{\partial n_F}{\partial s_H} - \frac{1}{2}\frac{\partial n_F}{\partial s_H}\left(L\frac{\partial S_H}{\partial m} - L\frac{\partial S_H}{\partial n_F}\right) \\
&\quad + \frac{1}{2}\frac{\partial p_{HH}}{\partial n_F}\frac{\partial n_F}{\partial s_H}\left(L\frac{\partial S_H}{\partial p_{HH}} + n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} + n_F\frac{\partial \Pi_F^s}{\partial p_{HH}} + m\frac{\partial \Pi^m}{\partial p_{HH}}\right)
\end{aligned} \qquad (20)$$

As in a pure exporter configuration, the first term of (20) stands for the cost of the subsidy paid to all firms located in country H when one-half of the subsidy is recouped by local shareholders. The second term captures again the cost of the subsidy paid to firms opening a plant in country H. The third term captures the impact of firms' relocation on local consumer surplus. An increase in s_H entices exporters located in F to become multinationals, which intensifies competition in H, thus decreasing local prices and raising consumer surplus. Finally, the fourth term is identically equal to zero, as in the pure exporter case and for the same reasons.

In Appendix 2, we show that $\mathrm{d}W_H/\mathrm{d}s_H$ is a decreasing function of s_H and that $\mathrm{d}W_H/\mathrm{d}s_H = 0$ in a mixed configuration if and only if

$$s_H = \widetilde{s} = \frac{3}{4}f - \frac{1}{32}L\tau\frac{b+c}{2b+c}\left(4a - 2\tau b + \tau c\right). \qquad (21)$$

Therefore, the subsidy that maximizes local welfare in country H is independent of the other country's subsidy s_F. We thus have shown the following proposition.

Proposition 3 *Whereas subsidies are strategic complement in the pure exporter configuration, they are strategically independent in the mixed configuration where multinationals and exporters coexist.*

---

[12]This effect seems to be supported empirically. Indeed, Devereux and Freeman (1995) have reported that tax policy hardly affects the investor's choice between domestic and foreign investment. Even when tax policy affects this choice, it often does not take the form of a simple relocation but consists in the opening of an additional production unit.

In the presence of multinationals, each government disregards the action of its rival when it sets its own subsidy. In other words, *in the presence of multinationals, tax competition may entirely disappear.*

The graph of the best response $\widetilde{s}$ is also depicted in Figure 2. Note that when it lies above (resp. below) of the domain of configuration (ii), the best response is given by the upper border (13) (resp. lower border (14)) of the configuration. Indeed, as is the case of $\widehat{s}_H$ discussed before, the subsidy $\widetilde{s}$ shifts upwards when the demand for the differentiated good increases (i.e., a or L increases). The intuition is the same as previously. When demand is large, $\widetilde{s}$ may also lie above the domain of configuration (ii). In that case, an increase in s_H always raises local welfare ($\mathrm{d}W_H/\mathrm{d}s_H > 0$), so that the optimal subsidy s_H lies on the upper border (13) of the domain of this configuration, i.e., $s_H = B$. The optimal subsidy is again independent of s_F and there is no subsidy competition. When the demand is small, $\widetilde{s}$ intersects the domain of configuration (ii) for all subsidies $2B - K - \widetilde{s} \leq s_F \leq B$. When s_F belongs to this interval, the optimal subsidy s_H is of course given by $\widetilde{s}$. However, when $s_F < 2B - K - \widetilde{s}$, marginal local welfare is negative for every subsidy supported by configuration (ii). The intuition underlying this result is that when s_F is small enough, many exporters settle in country H whereas the number of multinationals is small. Still, when there simultaneously exist exporters and multinationals, a larger subsidy s_H affects only the number of multinationals while it is paid to all firms in the country. As a result, the government prefers to reduce its spending on subsidies up to the point where no multinationals survive. In Figure 2, one can see that government H decreases s_H below the smallest admissible value in configuration (ii), i.e., the border (14) which coincides with the border (8) of configuration (i).

In the previous paragraphs, we have described the pieces of the best response functions for configuration (i) and (ii). Yet, *each government may set subsidies that induce different configurations, so that its best response to the other country's subsidy may imply changes of firms' production structures.* Put differently, governments' best response function will consist of the combination of the pieces derived above and will include transitions between configurations which may be continuous or discontinuous, depending on parameter values. As shown below, discontinuous transitions are upward jumps in this model, which may give rise to multiple equilibria in the subsidy game.

4.2 Subsidy equilibria

We now characterize the equilibrium configurations when governments simultaneously and non-cooperatively set their subsidies. Note from the outset that the existence of pure-strategy Nash equilibria in this subsidy game is guaranteed, because subsidies are defined over a compact subset of $\mathbb{R}$ and because governments' subsidy reaction functions

are upward sloping (with a possible upward jump; see Vives, 1999, p.41). We first present the possible equilibrium configurations and then discuss the conditions under which they occur. As suggested by the analysis of the constrained best replies, subsidy equilibria may yield a configuration with only exporters, a mixed configuration including both organizational strutures, or a configuration with only multinationals. We start by describing the former.

(i) A pure exporter equilibrium exhibits the traditional features of tax competition. When the tax base is internationally mobile, subsidies are strategic complements since each government raises its subsidy in reaction to an increase in the other country's subsidy, to counteract the relocation of mobile firms. Such 'subsidy competition' may lead to an excessive inflation of subsidies or, equivalently, excessively low taxes. Formally, the equilibrium subsidies are such that $s_H^* = \widehat{s}_H(s_F^*)$ and $s_H^* = \widehat{s}_F(s_H^*)$, which yields

$$s_H^* = s_F^* = \tau L \left(b + c\right) \frac{4a\left(b + c\right) - \left(2b^2 + 4bc + c^2\right)\tau}{8\left(2b + c\right)^2} > 0. \qquad (22)$$

First, as can be seen from expression (22), equilibrium subsidies increase with demand (a and L). When demand is large, both governments have incentives to attract firms in order for local consumers to save on transport costs. Second, the equilibrium subsidies are always positive. Indeed, as usual in the tax competition literature, governments impose a negative externality on each other and offer too large subsidies or set too low tax rates. Finally, subsidies are independent of firms' fixed costs f. This is because in a pure exporter configuration, firms have to pay the whole amount of fixed costs in either country anyway, so that governments may neglect this aspect when deciding on their optimal subsidies.

(ii) A mixed equilibrium is characterized by the following equilibrium subsidies:

$$s_H^* = s_F^* = \widetilde{s}, \qquad (23)$$

where $\widetilde{s}$ is given by (21). This illustrates the impact on tax competition of firms being able to endogenously choose their production structure. *When exporters and multinationals co-exist, governments' subsidy choices are independent.* Contrary to the pure exporter case, the equilibrium subsidies now decrease with larger demand (i.e., a and L), whereas they increase with respect to the cost of a plant. The latter effect is due to the fact that firms now may choose not the pay the cost of the second plant when it is too high; whereas the former effect stems from the fact that increasing demands raise operating profits of multinationals (and therefore, in equilibrium, of all firms), which allows governments to subsidize less (resp., to tax more) without triggering a structural change in firms' organization.

As a limit case of the mixed equilibrium (ii), the subsidy game yield a **pure multinational equilibrium**. In Figure 2, this corresponds to the case where the graph of $\widetilde{s}$ lies above the domain of configuration (ii) and where the best response function thus lies on the border between configurations (ii) and (iii), where only multinationals operate in the economy. This case naturally occurs for small fixed costs f of a plant. On the one hand, when the economy consists of multinationals only, as in configuration (iii), these firms constitute an immobile tax base which governments tend to tax as much as possible without triggering organizational changes. Hence, the subsidy equilibrium must lie at the point at which any further decrease in subsidies would entice some multinationals to modify their structure. On the other hand, a lower subsidy leading to configuration (ii) cannot improve local welfare as its costs for local consumers are higher than its benefits. Therefore, subsidy competition yields an equilibrium where all firms own two plants but some of them are almost willing to shut one down. In such a situation, governments set equilibrium subsidies equal to

$$s_H^* = s_F^* = B = f - \frac{(b+c)[4a - (2b+c)\tau]}{8(2b+c)} L\tau. \tag{24}$$

As can be seen from expression (24), these are decreasing in demands (a and L) and in transport costs (τ), for the same reasons as in case (ii). Note, finally, that the equilibrium subsidies increase one-to-one with respect to f. This is because any increase in f decreases multinationals' profits by exactly that amount, which must be offset by an increase in subsidies for firms to remain multinational in equilibrium.

After the description of the possible equilibria, it is of importance to discuss the precise *conditions under which they actually occur*. As will become clear, there exist sets of parameter values for which multiple equilibria are possible. It turns out to be convenient to discuss the different types of equilibria as a function of the fixed cost f of a plant. When this cost varies, the subsidy equilibria can include exporters only (i), multinationalss only (iii), or a mix of both types of firms (ii). In Appendix 3, we define five thresholds f_i (for $i = 1, 2, \ldots 5$), which are all positive under the trade feasibility condition and which satisfy $f_1 < f_2$, $f_3 < f_4 < f_2$ and $f_5 < f_2$. Using those thresholds, we establish the following conditions for subsidy equilibria.

Proposition 4 *(i) A pure exporter configuration is always an equilibrium of the subsidy game if $f > f_4$ and is never so if $f < f_3$.*
(ii) A mixed configuration is always an equilibrium of the subsidy game if $f \in (f_1, f_5)$ and is never so if $f \notin [f_1, f_2]$.
(iii) A pure multinational configuration is an equilibrium of the subsidy game if and only if $f \leq f_1$.

Proof. See Appendix 3. ∎

As expected, a pure multinational equilibrium of the tax game occurs when the fixed cost of a plant f is small, when product demand (a or L) and transport cost τ are large. When those conditions are not fulfilled, the equilibrium includes at least some exporters.

Since the conditions in Proposition 4 are not mutually exclusive, different equilibria can exist for the same set of parameter values. Collecting the previous results, we have the following corollary.

Corollary 5 (multiple equilibria) *In the subsidy game, there exist simultaneously a pure exporter and a mixed equilibrium if* $\max\{f_1, f_4\} < f < f_5$. *There exist simultaneously a pure exporter and a pure multinational equilibrium if* $f_4 < f < f_1$.

The conditions in Corollary 5 illustrate the interesting situation in which there are *multiple equilibria in the subsidy game.* Indeed, both a pure exporter equilibrium with low subsidies, and a mixed (resp., a pure multinational) equilibrium with high subsidies can be sustained. An illustration of this case is depicted in Figure 2, where the bold solid lines are the optimal subsidies for country H. As can be seen, the best response has an 'upwards jump', which leads in this case to the existence of a *high subsidy equilibrium* characterized by the existence of few exporters and many multinational firms; and a *low subsidy equilibrium* characterized by the existence of only exporters. The economy may get stuck in either one of the two equilibria. In the example depicted in Figure 2, the high subsidy equilibrium yields higher welfare than the low subsidy equilibrium. This is because more multinationals decrease prices and costly transportation in the economy, whereas the subsidy is not too costly since it comes partly back to the shareholders via firms' profits. Yet, we argue in the next section that the high subsidy equilibrium may actually be worse than the low subsidy equilibrium when a share of profits goes to the rest of the world. In that case, subsidy competition leads to excessively high subsidies, which are costly since they feed back into profits that accrue to shareholders in the rest of the world. This suggests that small open countries may be less able to profitably subsidize firms to attract investment, because subsidies are more costly for them ('subsidy trap').

We conclude this section by a remark on the level of subsidies. In configuration (i) with only exporters, subsidies are always positive and independent of the cost of capital. However, in configuration (ii) including multinationals, subsidies may be positive or negative depending on the cost of a plant. *Capital may be taxed in the equilibrium of the subsidy game* when fixed costs are small or when product demand is large. Stated differently, multinational firms represent an immobile tax base which governments may profitably tax.

5 Discussion and robustness

We now discuss some extensions of our model and analyze the robustness of our main findings.

5.1 Market integration and trade barriers

How does international market integration affect the nature of the tax equilibrium and the values of the subsidies? In Krugman (1991), market integration is presented as a reduction of trade barriers or, equivalently, an increases in the freeness of trade. In this section we show that a fall in trade barriers may intensify or attenuate subsidy competition.

First, from (8)–(10) it can be seen that decreasing trade barriers makes the occurence of pure-single plant equilibria more likely by shifting (8) outwards and by shifting (9) and (10) inwards. Note that when τ is close to 0, the equilibrium involves only exporters and subsidy competition vanishes ($s_H^* = s_F^* = 0$). This is because when trade is costless, consumers in both countries have the same access to all varieties independently of their place of production, so that it is pointless to attract firms using subsidies.

It is also readily verified that

$$\frac{\partial s_H^*}{\partial \tau} \gtreqless 0 \quad \Longleftrightarrow \quad \tau \lesseqgtr \overline{\tau} \equiv \frac{2a(b+c)}{2b^2 + 4bc + c^2},$$

where the threshold $\overline{\tau} < \tau_{\text{trade}}$. Hence, when trade costs fall below some threshold, any further decrease will reduce the equilibrium subsidies. This is because the gain of having local firms falls with smaller trade costs, which reduces governments' incentives to offer subsidies to attract firms. Yet, beyond some threshold of trade barriers, demands for imports become small whereas subsidies are still costly. This entices governments to reduce their subsidies when trade barriers get larger. Therefore, equilibrium subsidies will be largest for intermediate values of trade barriers since the gain from having local production is largest there.

Concerning the mixed configuration, one can see from (12)–(14) that decreasing trade barriers make the boundaries (12)–(13) shift inwards to the origin, whereas (14) shifts outwards, so that the area supporting this configuration shrinks. It is readily verified that

$$\frac{\partial \widetilde{s}_H}{\partial \tau} = -\frac{L(b+c)(2a - 2b\tau + c\tau)}{16(2b+c)} < 0$$

where the last inequality holds because of the trade feasibility condition (3). Hence, a fall in trade barriers raises the level of subsidies. The intuition is that smaller trade barriers decrease the returns to being multinational and induces fewer firms to operate

a second plant. This in turn leads to governments having larger incentives to subsidize capital, since the tax base becomes more mobile and since residents eventually cash in the dividends of more profitable firms.

Proposition 6 (trade barriers) *In a pure exporter equilibrium, a fall in trade barriers relaxes subsidy competition when trade costs are sufficiently low, whereas it intensifies it for large trade barriers. In particular, subsidy competition will be the strongest, and equilibrium subsidies will be highest, for intermediate levels of integration.*

5.2 Profit distributions

Until now we have considered that all profits are redistributed to the two countries and that this redistribution is equal across countries. We now assume that a share $0 \leq \lambda_H < 1$ of profits and capital rents accrue to country H's agents, whereas a share $0 \leq \lambda_F < 1$ goes to agents in country F, with $\lambda_H + \lambda_F \leq 1$ and a strict inequality when a part of the profits and capital rents go to absentee shareholders. Without loss of generality, we consider that country-H residents have a claim to a share $\lambda_H \geq \lambda_F$, i.e., country H gets a larger share of the redistributed profits. Redefine welfare (15) in country H as follows,

$$
\begin{aligned}
W_H &= \frac{L}{2} S_H + \lambda_H \left[n_H \left(\Pi_H^s + f + s_H \right) + n_F \left(\Pi_F^s + f + s_F \right) \right. \\
&\quad \left. + m \left(\Pi^m + 2f + s_H + s_F \right) \right] - \left(n_H + m \right) s_H.
\end{aligned}
$$

Computations analoguous to those in the previous section show that the constrained best reply in the pure exporter configuration (i) is given by

$$
\begin{aligned}
\widehat{s}_H(s_F) &= \frac{8b + c(5 - 4\lambda_H)}{16b + c(9 - 4\lambda_H)} s_F \\
&\quad + \frac{L(b + c)\tau \left(2a(b + c) - (b^2 + c(3 - 2\lambda_H)b + c^2(1 - \lambda_H)) \tau \right)}{(2b + c)(16b + c(9 - 4\lambda_H))},
\end{aligned}
$$

which has still a positive slope less than unity and a positive intercept. A symmetric expression holds for country F. The equilibrium of the subsidy competition game can then be readily computed. Of course, this equilibrium is no longer symmetric because governments' subsidies are not equally costly due to the differing profit claims.

In the case of a mixed configuration (ii), the equilibrium subsidy is given by

$$
\widetilde{s}_H = \frac{3c + 4b(1 + \lambda_H)}{8b + c(5 - 2\lambda_H)} f - L(b + c)\tau \frac{2a - (b + c(\lambda_H - 1))\tau}{4(8b + c(5 - 2\lambda_H))}
$$

which is again independent of s_F. This shows that the strategic independence of subsidies in the mixed congiguration is not an artifact of our equal profit redistribution assumption. We can further show that $\partial s_H^* / \partial \lambda_H > 0$ and $\partial \widetilde{s}_H / \partial \lambda_H > 0$, the same conditions applying to country F.

Proposition 7 (profit shares) *An increase in country i's profit share λ_i raises equilibrium subsidies in both the pure exporter and the mixed configurations.*

Proof. See Appendix 4. ∎

When all firms are exporters, subsidies are used to attract firms and increase firms' profits after subsidy. Moreover, subsidies are partly recouped by the shareholders residing in the countries that grant the subsidies. Therefore, if absentee shareholders own only a small share of the industry profits (large value of $\lambda_H + \lambda_F$), the cost of subsidizing capital is small and each government tends to subsidize more.

It is of interest to compare the difference in equilibrium subsidies when shares in capital and profits differ across countries. We can show the following result.

Proposition 8 (subsidy gap) *The country with the larger share of claims to profits subsidizes more (or taxes less) in equilibrium. Furthermore, the subsidy gap $s_H^* - s_F^*$ is always increasing with trade costs, whereas the subsidy gap $\widetilde{s}_H - \widetilde{s}_F$ is decreasing when $\tau < 2a/(10b + 3cN) < \tau_{\text{trade}}$ and increasing otherwise.*

Proof. See Appendix 4. ∎

The intuition underlying Proposition 8 is that part of the subsidy is recouped through firms' profits, which makes the subsidy less costly for the country having higher profit claims and, therefore, leads to an increase in it. Furthermore, in the pure exporter case international integration reduces the tax gap between countries. This is because, contrary to Baldwin and Krugman (2004), there are no agglomeration economies that can be taxed away in our model and because not all firms are agglomerated in one country. Yet, in the presence of multinational firms, the tax base becomes partly immobile, so that for some range of parameter values a deepening international integration may actually raise the equilibrium subsidies.

To conclude this section, the important thing to note is that the general shape of governments' best responses does not depend on how profits are shared within the two countries and between the two countries and the rest of the world. Of course, the equilibria become asymmetric and take different values when $\lambda_H \neq \lambda_F$, yet the general nature of the regimes remains the same. This suggests that our findings are robust.

6   Conclusions

We have developed a model of subsidy competition with multinational firms and we have shown that when firms are able to choose their location and organizational structure, the impact of subsidies on prices and profits differ. In particular, when the cost of a second

plant is large relative to the costs or exporting goods, firms prefer to operate a single plant. When the subsidy in a country rises, firms relocate their plant to that country, which increases competition there and relaxes it in the other country. Profits before subsidy increase in the latter locale, whereas they decrease before subsidy, but increase after subsidy, in the former one. On the contrary, when the cost of a second plant is small relative to the costs of exporting goods, some firms prefer to operate a plant in every country. In that case, when the subsidy in a country rises some exporters choose to open a second plant in this country. This increases competition there without relaxing it in the other country. Competition increases globally in the economy, which reduces prices and profits before subsidies. Yet, in contrast to the case with exporters, the additional subsidy does not offset the fall in profits so that profits after subsidy actually decrease. Thus, subsidies may trigger a rise or a fall in profits, depending on firms' changes in operational structure in response to the subsidy.

The different impacts of subsidies on firms' locational and organizational choices affect governments' equilibrium subsidies. In the absence of multinationals, firms locate according to subsidy differences and competition for mobile capital entices governments to inflate subsidies. This is the traditional result of the tax competition literature. However, higher subsidies reduce the cost of capital and may hence affect firms' organization as the latter may choose to set up several plants. When many firms run plants in all countries, a reduction in a country's subsidy induces some firms to shut down the plant located in that country whereas the government in that country is able to save subsidies on all multinational firms that keep their plants there. This may explain why competition for mobile capital is less fierce once a sufficient mass of multinationals operate in the global economy, and why the prediction of 'race to the bottom' lacks empirical support.

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Appendix 1: Firms' structure and location

In addition to configurations (i) and (ii) discussed in Section 3, the following configurations may arise (but are irrelevant for the subsidy game):

(iii) Pure multinational configurations (i.e., $m^* = 1$ and $n_H^* = n_F^* = 0$) are supported for subsidies such that $\Pi^m + s_H + s_F > \Pi_H^s + s_H$ and $\Pi^m + s_H + s_F > \Pi_F^s + s_F$, i.e., for $s_F > B$ and $s_H > B$. Under these conditions, no firm has an incentive to shut-down a plant given the current level of subsidies. The set of subsidies supporting this configuration is depicted in by zone (iii) in Figure 1.

(iv) Mixed configurations with agglomeration in H (i.e., $n_F^* = 0$, $n_H^* > 0$, and $m^* > 0$) require that subsidy-inclusive profits of exporters in H and multinationals are equalized: $\Pi_H^s + s_H = \Pi^m + s_H + s_F$. This yields

$$n_H^* = \frac{B - s_F}{K} \qquad \text{and} \qquad m^* = N - n_H^*.$$

Feasibility of this configuration further requires that $n_H^* \geq 0$, $m^* \geq 0$ and $\Pi_H^s + s_H > \Pi_F^s + s_F$, which implies that $B - NK \leq s_F \leq B$ and $s_H > B$. The set of subsidies supporting this configuration is depicted by zone (iv) in Figure 1.

(v) Full agglomeration in H (i.e., $n_H^* = N, n_F^* = m^* = 0$) is feasible when $\Pi_H^s + s_H > \Pi_F^s + s_F$ and $\Pi_H^s > \Pi^m + s_F$, which implies that $s_F < s_H - KN$ and $s_F < B + KN$. The set of subsidies supporting this configuration is depicted by zone (v) in Figure 1.

Note, finally, that there exist by symmetry two additional configurations (iv$'$) and (v$'$) that are the mirror cases of (iv) and (v), namely mixed configurations with exporters in F only, and full agglomeration in F. For the sake of brevity, we do not present these configurations formally (see Figure 1).

Appendix 2: Constrained best replies

In this appendix, we derive the optimal subsidies for configurations (i) and (ii). First, we compute

$$\frac{\partial S_H}{\partial n_H} = -\frac{1}{2} p_{HH} \left[2a - (b + cN)p_{HH}b + 2p_{HH}c(n_H + n_F) + cn_F\tau \right]$$

$$\frac{\partial S_H}{\partial n_F} = -\frac{1}{8} \left(\tau + 2p_{HH} \right) \left[-2(b + cN)p_{HH}b + 4p_{HH}c(n_F + n_H) \right.$$
$$\left. - cN\tau - b\tau + 4a + 2cn_F\tau \right],$$

which we will use in both cases.

Constrained best replies in configuration (i): In the pure exporter case, some straightforward calculations show that

$$n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} = n_H\left(b+cN\right)Lp_{HH}, \qquad n_F\frac{\partial \Pi_F^s}{\partial p_{HH}} = \frac{1}{2}n_FL\left(b+cN\right)\left(2p_{HH}-\tau\right)$$

$$n_H\frac{\partial \Pi_H^s}{\partial p_{FF}} = \frac{1}{2}n_HL\left(b+cN\right)\left(2p_{FF}-\tau\right) \qquad n_F\frac{\partial \Pi_F^s}{\partial p_{FF}} = n_F\left(b+cN\right)Lp_{FF}$$

and

$$\frac{\partial S_H}{\partial p_{HH}} = -aN + bp_{HH}N + \frac{1}{2}n_F\tau b.$$

Using these intermediate results, as well as the equilibrium prices (1) and the equilibrium distribution of firms (7), we can compute the following expressions:

$$-\frac{1}{2}n_H - \frac{\partial n_H}{\partial s_H}s_H = -\frac{1}{4K}\left(NK + 3s_H - s_F\right)$$

$$\frac{\partial S_H}{\partial n_H} - \frac{\partial S_H}{\partial n_F} = \frac{\tau\left(2a-\tau b\right)}{4}\frac{b+cN}{2b+cN} - \frac{s_H-s_F}{2L}$$

$$L\frac{\partial S_H}{\partial p_{HH}} + n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} + n_F\frac{\partial \Pi_F^s}{\partial p_{HH}} = (2b+cN)LN\left[p_{HH} - \frac{2a+cn_F\tau}{2(2b+cN)}\right] = 0$$

$$\frac{1}{2}n_H\frac{\partial \Pi_H^s}{\partial p_{FF}} + \frac{1}{2}n_F\frac{\partial \Pi_F^s}{\partial p_{FF}} = \frac{n_HL\left(b+cN\right)\left(2p_{FF}-\tau\right)}{4} + \frac{n_F\left(b+cN\right)Lp_{FF}}{2}$$

$$-\frac{\left(s_H-s_F\right)b}{c\tau} + \frac{NL\left(2a-b\tau\right)}{4}\frac{b+cN}{2b+cN}$$

Substituting these expressions into the marginal welfare (18) it is readily verified that $dW_H/ds_H \geq 0$ if and only if

$$-2\left(2b+cN\right)\left(16b+7cN\right)s_H + 2\left(8b+3cN\right)\left(2b+cN\right)s_F$$
$$+\tau L\left(b+cN\right)\left(4a\left(b+cN\right)-\tau\left(2b^2+4bcN+N^2c^2\right)\right) \geq 0,$$

which shows that local welfare W_H is concave in s_H and that the optimal subsidy is given by $\widehat{s}_H(s_F)$.

Constrained best replies in configuration (ii): In the mixed configurations, we readily obtain

$$L\frac{\partial S_H}{\partial p_{HH}} = \frac{Lb\tau}{2}n_F - LN\left(a-bp_{HH}\right), \qquad n_H\frac{\partial \Pi_H^s}{\partial p_{HH}} = n_H\left(b+cN\right)Lp_{HH}$$

and

$$n_F\frac{\partial \Pi_F^s}{\partial p_{HH}} = \frac{L(b+cN)}{2}n_F\left(2p_{HH}-\tau\right), \qquad m\frac{\partial \Pi^m}{\partial p_{HH}} = mL\left(b+cN\right)p_{HH}.$$

This allows us to compute

$$
\frac{L}{2}\frac{\partial S_H}{\partial p_{HH}} + \frac{n_H}{2}\frac{\partial \Pi_H^s}{\partial p_{HH}} + \frac{n_F}{2}\frac{\partial \Pi_F^s}{\partial p_{HH}} + \frac{m}{2}\frac{\partial \Pi^m}{\partial p_{HH}} = (2b+cN)LN\left[p_{HH} - \frac{2a+cn_F\tau}{2(2b+cN)}\right]
$$
$$
= 0
$$

$$
-\frac{1}{2}(n_H+m) + s_H\frac{\partial n_F}{\partial s_H} = \frac{2\,(2b+cN)}{L\,(b+cN)\,\tau^2 c}\left(f - \frac{(b+cN)[4a-(2b+cN)\tau]}{8(2b+cN)}L\tau\right.
$$
$$
\left.-3s_H\right) - \frac{N}{2}
$$

$$
L\frac{\partial S_H}{\partial m} - L\frac{\partial S_H}{\partial n_F} = \frac{L\tau\,(a-bp_{HH})}{2} - \frac{L\tau^2\,(b+cN-2n_Fc)}{8}
$$
$$
= -s_H + f
$$

Substituting these expressions into the marginal welfare (20), we obtain

$$
\frac{dW_H}{ds_H} = -8s_H\frac{2b+cN}{L\,(b+cN)\,\tau^2 c} - \frac{L\,(b+cN)\,\tau\,(4a-2\tau b + c\tau N) - 16\phi r\,(2b+cN)}{4L\,(b+cN)\,\tau^2 c},
$$
$$
+2\frac{(2b+cN)}{L(b+cN)\tau^2 c}f
$$
$$
= -8s_H\frac{2b+cN}{L\,(b+cN)\,\tau^2 c} - \frac{L\,(b+cN)\,\tau\,(4a-2\tau b + c\tau N) - 24\phi r\,(2b+cN)}{4L\,(b+cN)\,\tau^2 c}
$$

which shows that local welfare is concave in s_H and that the optimal subsidy is given by (21).

Appendix 3: Subsidy equilibria

The five thresholds $f_i > 0$ are defined as follows:

$$
f_1 \equiv \frac{L(b+cN)\tau(12a-(6b+5cN)\tau)}{8(2b+cN)}
$$
$$
f_2 \equiv \frac{L(b+cN)\tau\,(12a-(6b+Nc)\,\tau)}{8(2b+cN)}
$$
$$
f_3 \equiv \frac{L(b+cN)\tau\,[4a(3b+2cN) - (6b^2+6cNb+c^2N^2)\,\tau]}{8(2b+cN)^2}
$$
$$
f_4 \equiv \frac{L(b+cN)(3b+2cN)\tau(2a-b\tau)}{4(2b+cN)^2},
$$
$$
f_5 \equiv \frac{1}{8}L\,(b+cN)\,\tau\frac{4a\,(60b+31cN) - \tau\,(120b^2+90bcN+13c^2N^2)}{(2b+cN)\,(28b+13cN)}
$$

They will be useful in characterizing the occurence of the different types of equilibria in propositions 2–4. We now prove the three parts of Proposition 4 in three steps.

Lemma 9 (pure multi-plant equilibrium) *The subsidy game yields a pure multi-plant equilibrium (i.e., on the border between configurations (ii) and (iii)) if and only if $\phi r \leq f_1$.*

Proof. A necessary condition for a configuration with only multinationals is given by $\widetilde{s} \geq B$. Some computations show that this is equivalent to $f \leq f_1$. A sufficient condition is that $s_H = B$ is government H's best response to the subsidy $s_F = B$. When $s_F = B$, government H's subsidy yields configurations (v'), (ii) and (iii) as s_H rises (see Figure 1). However, welfare in country H is always constant in configuration (v') (since $n_H + m = 0$, so that there are no firms subsidize in country H). It is furthermore increasing in s_H in configuration (ii) and, by (17), decreasing in configuration (iii). This shows that $(s_H, s_F) = (B, B)$ is an equilibrium. ∎

Lemma 10 (necessary conditions) *In the subsidy game,*

- *if a pure exporter equilibrium exists (configuration (i)), then $\phi \geq f_3$;*

- *if a mixed equilibrium exists (configuration (ii)), then $f_1 < \phi < f_2$.*

Proof. Let us start with the pure exporter configuration. This type of equilibrium arises only if conditions (8)–(10), when evaluated at (22), hold. Because subsidies are symmetric, this requires that $\widehat{s}_H(s_F^*) \leq 2B - NK - s_F^*$, or more simply that $2s_H^* \leq 2B - NK$, holds. Some straightforward calculations show that this is the case when $f_3 \leq f$.

Concerning the mixed configurations let (s_H^1, s_F^1) be the intersection points of $s_H = \widetilde{s}$ with the line $s_H = 2B - NK - s_F$. This intersection point is given by

$$s_H^1 = \widetilde{s} \quad \text{and} \quad s_F^1 = \frac{5}{4}f - \frac{L(b + cN)\tau\left[28a - (14b + cN)\tau\right]}{32(2b + cN)} \tag{25}$$

A necessary condition for the existence of a mixed equilibrium is then given by $s_F^1 < s_H^1 < B$, which yields $f_1 < f < f_2$. ∎

Lemma 11 (sufficient conditions) *The subsidy game yields*

- *a pure exporter equilibrium if $\phi > T_4$,*

- *a mixed equilibrium if $T_1 < \phi < T_5$.*

Proof. In this appendix, we derive sufficient conditions for pure exporter and mixed equilibria to exist.

(i) Pure exporters: A sufficient condition with respect to subsidies is given by $s_H^* < B - NK$, which is equivalent to $f > f_4$. Because one can readily show that $f_4 > f_3$, this condition satisfies the necessary condition $f \geq f_3$, as presented in Proposition 10.

(ii) Mixed configuration: Let (s_H^2, s_F^2) be the intersection points of $\widehat{s}_H(s_F)$ with the line $s_H = 2B - NK - s_F$. One can compute that

$$s_F^2 = \frac{16b + 7cN}{12b + 5cN} f - \frac{L(b + cN)\tau\,(18a - 9b\tau - cN\tau)}{4(12b + 5cN)}$$

A sufficient condition for a configuration with all types of firm structures is given by $s_F^2 < \widetilde{s}$ and $f_1 < f < f_2$. The first condition is equivalent to

$$f < f_5 = \frac{1}{8}L\,(b + cN)\,\tau\frac{4a\,(60b + 31cN) - \tau\,(120b^2 + 90bcN + 13c^2N^2)}{(2b + cN)\,(28b + 13cN)} < f_2$$

Thus, such an equilibrium exist if $f_1 < f < f_5$. This interval is not empty if τ and cN/b are sufficiently large. ∎

Appendix 4: Discussion

Proof of Proposition 7

Some straightforward computions show that

$$\frac{\partial s_H^*}{\partial \lambda_H} = \frac{cLN(b + cN)(12b + cN(7 - 4\lambda_F))(16b + cN(9 - 4\lambda_F))\tau^2}{8(2b + cN)(12b + cN(-2\lambda_F - 2\lambda_H + 7))^2} < 0.$$

Some further computations show that

$$\frac{\partial \widetilde{s}_H}{\partial \lambda_H} > 0 \iff f > \frac{cLN(b + cN)\tau(4a - 10b\tau - 3cN\tau)}{8\,(16b^2 + 14cNb + 3c^2N^2)}. \tag{26}$$

Since the RHS of the above expression lies always below the threshold f_1, we conclude that $\partial \widetilde{s}_H / \partial \lambda_H > 0$ always hold in the mixed configuration because $f_1 \leq f \leq f_2$ must be satisfied for this configuration to arise.

Proof of Proposition 8

Some longer computations reveal that, the difference in equilibrium subsidies in the pure exporter case is equal to

$$s_H^* - s_F^* = \frac{cLN(b+cN)\lambda(\lambda_H - \lambda_F)\tau^2}{24b + 2cN(7 - 2(\lambda_H + \lambda_F))} \geq 0$$

since we assume that $\lambda_H \geq \lambda_F$. Furthermore, in the mixed equilibrium we have

$$\widetilde{s}_H - \widetilde{s}_F = \frac{(\lambda_H - \lambda_F)[8(2b+cN)(8b+3cN)r\phi - cLN(b+cN)\tau(4a - 10b\tau - 3cN\tau)]}{4(8b + cN(5 - 2\lambda_F))(8b + cN(5 - 2\lambda_H))}$$

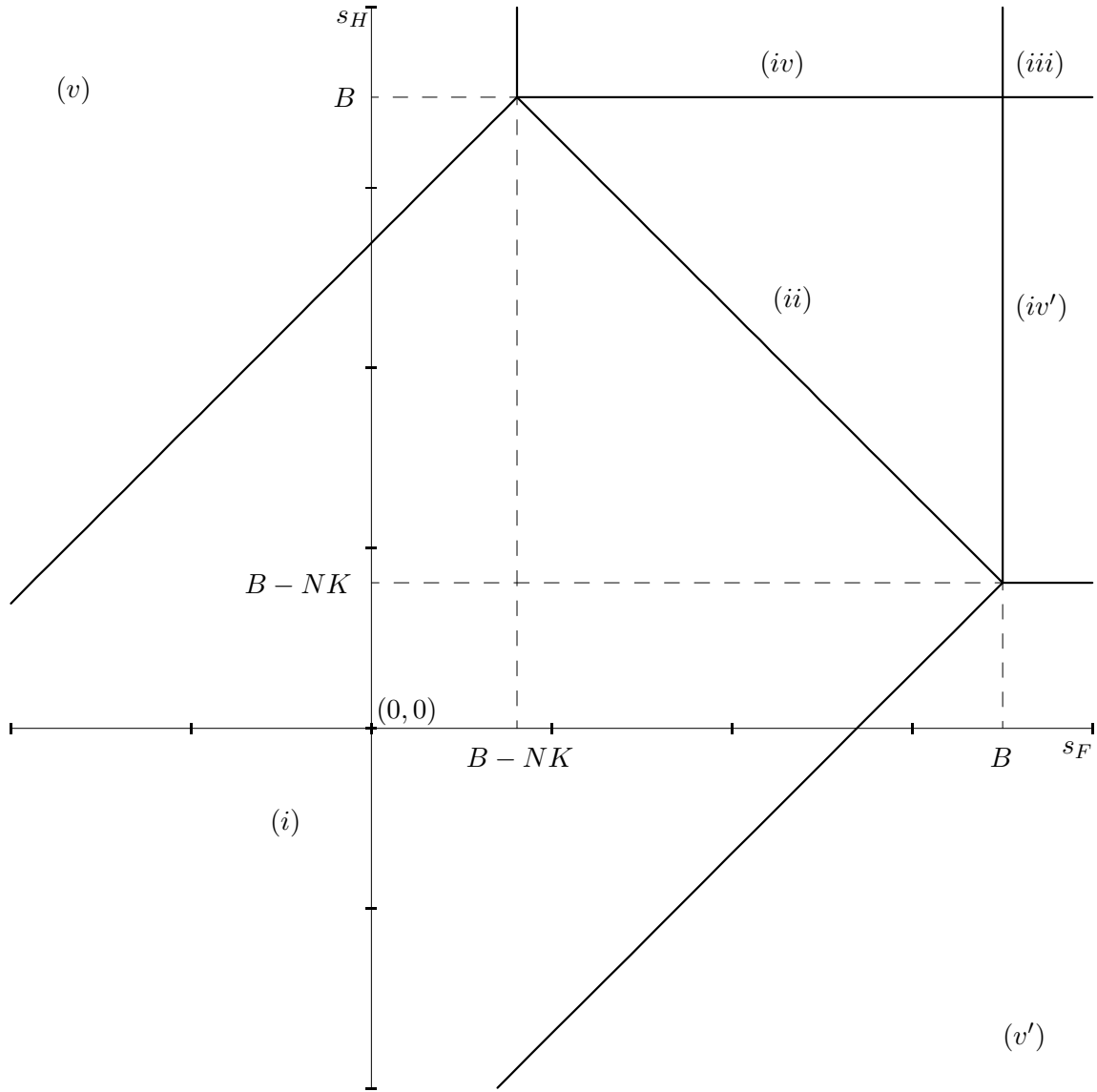the sign of which is therefore determined as in (26).
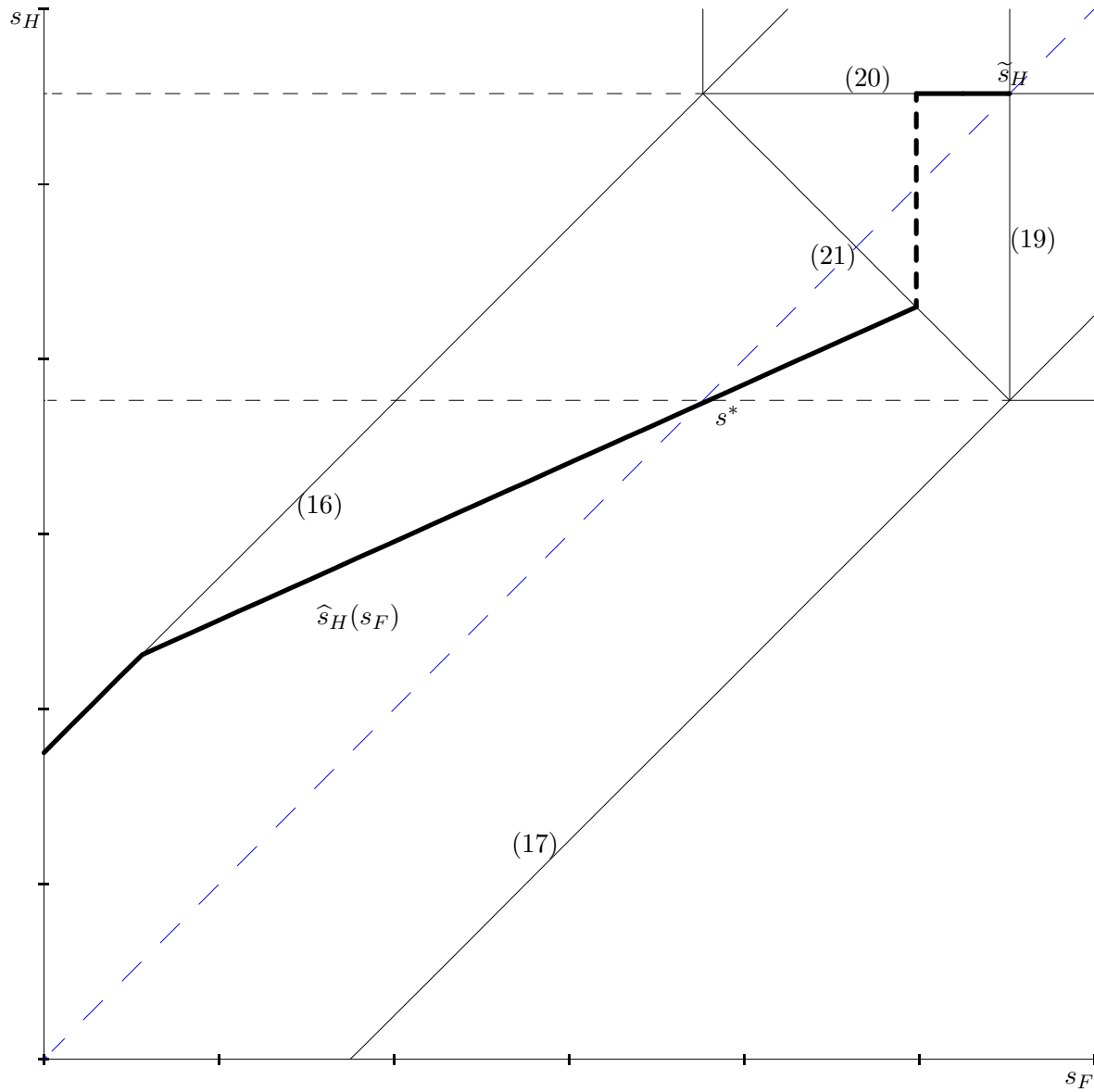
Figure 1: Spatial equilibrium structures

Figure 2: Country H's best response and multiple equilibria