Social desirability of earnings tests

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Abstract

In many countries pension systems involve some form of earnings test; i.e., an individual’s benefits are reduced if he has labor income. This paper examines whether or not such earning tests emerge when pension system and income tax are optimally designed. We use a simple model with individuals differing both in productivity and their health status. The working life of an individual has two “endings”: an official retirement age at which he starts drawing pension benefits (while possibly supplementing them with some labor income) and an effective age of retirement at which professional activity is completely given up. Weekly work time is endogenous, but constant in the period before official retirement and again constant (but possibly at a different level), after official retirement. Earnings tests mean that earnings are subject to a higher tax after official retirement than before. We show under which conditions earnings tests emerge both under a linear and under a nonlinear tax scheme. In particular, we show that earning tests will occur if heterogeneities in health or productivity are more significant after official retirement than before.
1 Introduction

In many countries pension benefits are subject to earnings tests. In a nutshell this means that an individual whose earnings exceed a certain ceiling faces a reduction in pension benefits. Some countries do not have any such test. Others have them, but in variable forms. In the United States they have recently been suppressed for individuals passed the full retirement age. But for a long time they only applied to retirees between the ages of 65 and 70. Observe that such earnings tests are typically only based on labor income; they do not apply to capital and rental incomes or to other pensions. The ceiling and the percentage of reduction also vary from country to country. In Belgium, for instance, earnings above 7000 euro are subject to a 100% for regular retirees. Naturally, earnings below that amount are subject to income taxation. Workers who have exited the labor force before the normal age of retirement, through disability or unemployment insurance, are subject to even stricter restrictions. In their case, no earnings are allowed at all.

Why do we have earning tests? Are they socially desirable? Before answering these two questions we note that earning tests are rarely questioned in the case of unemployment insurance. Also, in a number of welfare programs benefits are subject to means testing that concerns not only earnings, but also all other sources of income.

There are three potential rationales for earnings tests in pension systems. The first one is redistributive: in a second-best world it may (or may not) be desirable to tax earnings after the statutory age of retirement at a higher rate than other sources of income including pension benefits. The second argument is related to the idea that forcing elderly workers into retirement fosters employment of the young. This belief is as widespread as it is ill-founded. The third argument applies to heavily redistributive pension systems and is the same argument one finds behind means testing for welfare benefits: benefits are granted because people are without any other resources; if they are shown to have resources, they are not entitled to assistance. Consequently, this argument should not be used for contributory (Bismarckian) pension systems.

Quite clearly, only the first argument could apply to the majority of social security

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1 See Boldrin et al. (1999) and Fenge and Pestieau (2005).
systems. But as we show below, it could lead to recommend fostering rather than penalizing labor after retirement. In other words, instead of overtaxing earnings, one could end up taxing them less heavily than earning before retirement.

Before proceeding with our theoretical investigation, let us mention some of the available evidence. Friedberg (2000), as well as Disney and Smith (2002), suggests that the abolition of the earnings tests where they existed had a small but significant impact on labour supply. More recently Gustman and Steinmeier (2004) simulate the effect of abolishing the remainder of social security earnings test in the US (namely that between ages 62 and 67). They show that the earnings test entails a reduction of about ten percent in the number of married men at full time work in that age bracket. All these empirical papers take a purely positive perspective. They do not address the issue of the social desirability of earnings test even though they implicitly view an increase in labor supply as a good thing.

The purpose of this paper is to examine whether or not an earning test is part of the design of an optimal pension and contribution scheme. We adopt a rather simple model where individuals differ both in productivity and health status (disutility of labor). The working life of an individual has two endings: an “official” retirement age at which he starts drawing pension benefits. He can continue to work after that age and earn a different (most likely lower) wage until the “effective” age of retirement when he completely withdraws from the labor market. Weekly worktime is endogenous, but constant in the period before official retirement and still constant, but hopefully lower after official retirement. The pension and contribution schedule is first restricted to be linear (Section 4); then a general, nonlinear scheme is considered (Section 5). Within both settings, we show earning test will occur when health status and/or productivities tend to be “more dispersed” after official retirement than before. In other words, pension benefits (for all or some of the individuals) should be subject to earnings tests when the heterogeneity between individuals is “larger” after the formal retirement than before. This could be the case, for instance, when health differences are exacerbated by aging and/or when wages in the second (post retirement) career spell are more unequally distributed than in the first career spell.
In the literature on social security and retirement, the concept of “implicit taxation” is widely used, more often than that of earning test. When an individual who is eligible for retirement benefits decides to work one more year, he foregoes pension benefits and continues to contribute. On the other hand, he may be entitled to an increase in his future pension benefits. Implicit taxation arises when the third effect does not compensate the first and second one. In that case, the contribution and benefit scheme provides incentives for “early” retirement.

Earnings tests and early retirement incentives are two sides of the same reality. However, in our setting, implicit taxation of formal retirement on the one hand and that of effective retirement (end of the second career) are not exactly symmetrical. An implicit tax on the effective retirement age is in itself a form of earnings test; it discourages post retirement work at the extensive margin rather than just at the intensive margin. The implicit tax on formal retirement, on the other hand, is not an earnings test per se. However, it also has the effect of discouraging labor supply “around” the normal age of retirement. To be more precise, the implicit tax discourages labor supply before while the earning test discourages it after this formal age of retirement. Our setting and particularly the nonlinear case allows us to deal with both issues in an integrated way.

2 The model

Consider an individual with lifespan normalized to 1. We denote $t \in [0, 1]$ the time subscript, and instantaneous consumption $c_t$. During his first career of length $z_1$, he supplies $\ell_1$ units of labor. During his second career of length $z_2 - z_1$, he supplies $\ell_2$ units of labor. After $z_2$, the individual’s labor supply is zero. Denoting $r(t)$ the instantaneous intensity of labor disutility (with $r'(t) > 0$ and $r(0) = 0$), instantaneous utilities are written as:

\begin{align*}
  U_{1t} &= u(c_t) - V(\ell_1) r(t) \text{ if } t \leq z_1, \quad (1) \\
  U_{2t} &= u(c_t) - V(\ell_2) r(t) \text{ if } z_1 < t \leq z_2, \quad (2) \\
  U_{3t} &= u(c_t) \text{ if } z_2 < t \leq 1, \quad (3)
\end{align*}
where \(z_1\) is interpreted here as the “normal” age of retirement i.e. the age at which the individual begins to draw retirement benefits. Past that age, the individual may continue working but this may translate into a reduction of his retirement benefits. As \(r(t)\) increases over time, a given level of labor supply creates an increasing disutility over the life; consequently, one expects that the individual chooses a lower labor supply during the second career (see the end of this section).\(^2\)

Assuming the interest rate and the discount rate both equal to 0, the life cycle utility \(U\) and the budget constraint are written as:

\[
U = \int_0^{z_1} U_1 dt + \int_{z_1}^{z_2} U_2 dt + \int_{z_2}^{1} U_3 dt,
\]

\[
\int_0^{1} c dt \leq \int_0^{z_1} w_1 \ell_1 dt + \int_{z_1}^{z_2} w_2 \ell_2 dt,
\]

where \(w_1\) and \(w_2\) are respectively the labor productivities during the first and the second career. For the time being, we do not make any assumption on the relationship between \(w_1\) and \(w_2\).

Given that the interest and the discount rates are both equal to zero, the individual smooths consumption over the life cycle such that \(c_t = c\). Using this and (1), (2) and (3), one can rewrite the life cycle utility and the budget constraints as:

\[
U = u(c) - V(\ell_1) R(z_1) - V(\ell_2) [R(z_2) - R(z_1)],
\]

\[
c = z_1 w_1 \ell_1 + (z_2 - z_1) w_2 \ell_2.
\]

where \(R(z) = \int_0^z r(t) dt\) so that \(R'(z) > 0\) and \(R''(z) > 0\).

We can think of the \(R\) function as reflecting the health status of the an individual. The healthy individual is characterized by a low instantaneous intensity of labor disutility denoted \(r^L\). The individual with bad health is characterized by a high \(r\), denoted \(r^H\) so that \(r^L(t) \leq r^H(t)\) for every \(t\). Recall that both of these functions are increasing \(^2\)Alternatively, \(\ell\) could be considered as a continuous function of time. In this case, the optimal labor supply would be decreasing over time. Our assumption is mainly made for simplicity. However, on can also think of it as reflecting the restriction that an individual is constrained to choose a constant \(\ell\) for a given period of time, which is not unrealistic.
over time. By integration of $r$, it follows that $R^L(z) \leq R^H(z)$. Additionally, we assume that the productivity $w_i$ in each career spell ($i = 1, 2$) can take two values $w_i^H$ and $w_i^L$ with $w_i^H \geq w_i^L$. Observe that $w_i^j$ ($j = H, L$) is not restricted to be equal to $w_i^j$; consequently, the wage heterogeneity can differ between career spells.$^3$

We assume that the economy is composed of two types of individuals, $G$ and $B$, who differ both in their health status and their productivities. The individuals of type $G$ is in good health and has a high productivity in the first and the second periods: $r^G(.) = r^H(.)$ and $w_i^G = w_i^H$ for $i = 1, 2$. The individual $B$ is in bad health and has low productivities: $r^B(.) = r^H(.)$ and $w_i^B = w_i^L$ for $i = 1, 2$.

So far, the individuals budget constraint has been specified absent any government intervention. Specifically, there is no pensions system (neither benefits nor contributions) and no income taxation. The following section analyses the problem of the individual for a given policy of the government.

3 Individual choices and the First Best

3.1 Individual choices

Assume the individual faces a general (possibly) nonlinear tax function that depends upon his first and second career incomes and his first and second career lengths. We denote this function $T(y_1, y_2, z_1, z_2)$ where $y_i = w_i \ell_i$ for $i = 1, 2$. This tax function encompasses both the income tax system as well as the pension scheme (see expression (9) below for an example of a linear specification); see also Cremer et al. (2004) for further discussion.

An individual solves:

$$\max_{c, y_1, y_2, z_1, z_2} U = u(c) - V\left(\frac{y_1}{w_1}\right)R(z_1) - V\left(\frac{y_2}{w_2}\right)[R(z_2) - R(z_1)]$$

s.t $: \quad c = y_1 z_1 + y_2 [z_2 - z_1] - T(y_1, y_2, z_1, z_2)$

$^3$Throughout the paper, we use subscript $i$ for the career and upperscript $j$ for the type of individual.
This leads to the following tradeoffs:

\[
MRS_{cy1} = \frac{V'(\ell_1) R(z_1)}{w_1 u'(c)} = z_1 - T_{y1} \tag{5}
\]

\[
MRS_{cy2} = \frac{V'(\ell_2)[R(z_2) - R(z_1)]}{w_2 u'(c)} = (z_2 - z_1) - T_{y2} \tag{6}
\]

\[
MRS_{cz1} = \frac{R'(z_1)[V(\ell_1) - V(\ell_2)]}{u'(c)} = y_1 - y_2 - T_{z1} \tag{7}
\]

\[
MRS_{cz2} = \frac{R'(z_2)V(\ell_2)}{u'(c)} = y_2 - T_{z2} \tag{8}
\]

where \(MRS_{ab}\) denotes the marginal rate of substitution between \(a\) and \(b \in \{c, y_1, y_2, z_1, z_2\}\) and \(T_a\) denotes the marginal tax with respect to variable \(a\).

These tradeoffs have to be compared to their non-distorted counterparts obtained by setting all \(T_a = 0\). For example, the tradeoff between \(c\) and \(y_1\) will be downward distorted if there is a positive marginal tax on the first career intensive income that is if \(T_{y1} > 0\). A similar reasoning applies to the other choice variables.

When there is no distortion this problem naturally leads to a higher labor supply in the first career than in the second one. To see this, set \(T_a = 0\) for every \(a\) and use (5) and (6) to obtain:

\[
\frac{V'(\ell_1)}{V'(\ell_2)} = \frac{w_1 (R(z_2) - R(z_1)) / (z_2 - z_1)}{w_2 R(z_1) / z_1},
\]

and with \(w_1 > w_2\), \(\ell_1 > \ell_2\) follows by convexity of \(R(.)\) and \(V(.)\). Because labor becomes more and more painful as age increases, labor supply will be lower when old.

### 3.2 First-best

Throughout the paper social welfare is given by

\[
W = \sum_{j=G,B} n^j \Phi(U^j),
\]

where \(U^j\) denotes utility of type \(j\), while \(\Phi\) is an increasing and concave function. Assume, for the time being that individual characteristics are publicly observable. The first-best then simply consists of two “consumption bundle” \((c^j, y_1^j, y_2^j, z_1^j, z_2^j)\), one for each type which are determined to maximize social welfare subject to the resource
constraint. This yields the following Lagrangian problem:

\[
\max_{c^j, y_1^j, y_2^j, z_1^j, z_2^j} \mathcal{L} = \sum_{j=G,B} n_j^j \Phi \left( u(c^j) - V \left( \frac{y_1^j}{w_1^j} \right) R^j \left( z_1^j \right) - V \left( \frac{y_2^j}{w_2^j} \right) \left[ R^j \left( z_2^j \right) - R^j \left( z_1^j \right) \right] \right) \\
+ \mu \sum_{j=G,B} n_j^j \left( y_1^j z_1^j + y_2^j \left( z_2^j - z_1^j \right) - c^j \right)
\]

where \( n_j^j (j = G, B) \) is the fraction of type \( j \) individual in the population while \( \mu \) is for the Lagrange multiplier of the resource constraint.

The solution of this problem is simple; the tradeoffs between the different variables are the same as in the laissez-faire and are given by (5)–(8), with all \( T \)'s set to zero. To decentralize such a solution, personalized lump-sum taxes are sufficient; there is no need to interfere with the choice of \( \ell_i \) or \( z_i \). Furthermore we have

\[
\Phi(U^G)u'(c^G) = \Phi(U^B)u'(c^B),
\]

which implies identical consumption levels \( c^G = c^B \) when \( \Phi \) is linear (utilitarian case). When \( \Phi \) is strictly concave, consumptions levels are not equal; consumption is used to compensate for difference in disutility of labor in order to mitigate differences in utilities. In the extreme (quasi-linear) case when \( u'(c) \) is constant we obtain \( U^G = U^B \).

4 Linear pension scheme

Let us now assume that the pension scheme is restricted to be linear. Specifically, we assume that both contributions and benefits are linear functions of income and retirement age. This yields the following specification for \( T \).

\[
T(y_1, y_2, z_1, z_2) = \tau_1 y_1 z_1 + \tau_2 y_2 (z_2 - z_1) - (1 - \alpha z_1) p.
\]  

where \( \tau_1 \) is the marginal tax rate applied to earning in the first career spell, i.e., before retirement. Once the individual receives pension benefits, his (second career earnings) are taxed at a constant marginal rate of \( \tau_2 \). Pension benefits are related to retirement age through the term \( (1 - \alpha z_1^j) p \), where \( \alpha \) is an exogenous parameter which characterizes the benefit formula. When \( \alpha = 0 \), (lifetime) pension benefits are flat (equal to \( p \)) and are independent of the age of retirement. On the other hand when \( \alpha = 1 \), instantaneous
pension benefits are independent of retirement age (which of course implies that total benefits decrease with retirement age). In what follows we concentrate on the case where \( \alpha = 0 \) but we shall briefly sketch the implication of a positive \( \alpha \) below.

This specification is simple but yet sufficiently flexible to account for the idea of earnings tests. Specifically, the formula involves an earnings test if income after retirement is taxed at a higher rate than income before retirement, i.e., when \( \tau_2 > \tau_1 \). With this linear specification, the FOC (5) to (8) can be rewritten as:

\[
\begin{align*}
\left[V \left( \ell_j^2 \right) - V \left( \ell_j^1 \right) \right] R^j \left( z_j^1 \right) + u' \left( c^j \right) \left[ \ell_j^1 w_j^1 \left( 1 - \tau_1 \right) - \ell_j^2 w_j^2 \left( 1 - \tau_2 \right) - \alpha p \right] &= 0 \\
- V' \left( \ell_j^1 \right) R^j \left( z_j^1 \right) + u' \left( c^j \right) w_j^1 \left( 1 - \tau_1 \right) z_j^1 &= 0 \\
- V \left( \ell_j^2 \right) R^j \left( z_j^2 \right) + u' \left( c^j \right) \ell_j^2 w_j^2 \left( 1 - \tau_2 \right) &= 0 \\
- V \left( \ell_j^2 \right) \left[ R^j \left( z_j^2 \right) - R^j \left( z_j^1 \right) \right] + u' \left( c^j \right) w_j^2 \left( 1 - \tau_2 \right) \left( z_j^2 - z_j^1 \right) &= 0.
\end{align*}
\]

for every \( j = G, B \). From these FOC we obtain the demand functions, expressing the endogenous variables \( \ell_j^1, \ell_j^2, z_j^1, z_j^2 \) and \( c^j \) as functions of the tax instruments \( \tau_1, \tau_2 \) and \( p \). Substituting into (4) yields an indirect utility function:

\[
\tilde{U}^j \left( \tau_1, \tau_2, p \right)
\]

with derivatives

\[
\begin{align*}
\tilde{U}_{\tau_1}^j &= -u' \left( c^j \right) \ell_j^1 w_j^1 z_j^1 \\
\tilde{U}_{\tau_2}^j &= -u' \left( c^j \right) \ell_j^2 w_j^2 \left( z_j^2 - z_j^1 \right) \\
\tilde{U}_{p}^j &= u' \left( c^j \right) \left( 1 - \alpha \right) z_j^1.
\end{align*}
\]

The maximization of utilitarian welfare can then be expressed with the following Lagrangian:

\[
\begin{align*}
\max_{\tau_1, \tau_2, p} \mathcal{L} &= \sum_{j=G,B} n^j \Phi \left[ \tilde{U}^j \left( \tau_1, \tau_2, p \right) \right] + \mu \sum_{j=G,B} n^j \left[ \tau_1 z_j^1 w_j^1 \ell_j^1 + \tau_2 \left( z_j^2 - z_j^1 \right) w_j^2 \ell_j^2 - p \right],
\end{align*}
\]

where \( \mu \) is the Lagrange multiplier associated with the revenue constraint. We denote \( y_j^1 = w_j^1 \ell_j^1 \) and define \( \beta^j = \Phi'(U^j)u'(c^j) \) as the marginal social utility of income of
individual $j$. Furthermore, we use the operator $E$ for $\sum n^j$ and denote compensated effects by a tilde. The FOCs are stated in Appendix A. Observe that compensated effects (denoted by a tilde) arise because an increase in marginal taxes brings about a change in pension benefits; see equations (A1) and (A2). Rearranging these expressions yields

$$ -\text{cov}(\beta, y_1z_1) + \mu \tau_1^1 \left( Ew_1z_1 \frac{\partial \hat{\ell}_1}{\partial \tau_1} + Ey_1 \frac{\partial \bar{z}_1}{\partial \tau_1} \right) + \tau_2 \left( Ew_2(z_2 - z_1) \frac{\partial \hat{\ell}_2}{\partial \tau_1} + Ey_2 \left( \frac{\partial \bar{z}_2}{\partial \tau_1} - \frac{\partial \bar{z}_1}{\partial \tau_1} \right) \right) = 0, $$

$$ -\text{cov}(\beta, y_2(z_2 - z_1)) + \mu \tau_1^2 \left( Ew_1z_1 \frac{\partial \hat{\ell}_1}{\partial \tau_2} + Ey_1 \frac{\partial \bar{z}_1}{\partial \tau_2} \right) + \tau_2 \left( Ew_2(z_2 - z_1) \frac{\partial \hat{\ell}_2}{\partial \tau_2} + Ey_2 \left( \frac{\partial \bar{z}_2}{\partial \tau_2} - \frac{\partial \bar{z}_1}{\partial \tau_2} \right) \right) = 0, $$

where $\text{cov}(x, y) = Exy - ExEy$ is the covariance. Using a self-explanatory notation, we rewrite these equations to obtain

$$ -\text{cov}(1) + \mu \tau_1 \Delta_1^1 + \mu \tau_2 \Delta_2^1 = 0, $$

$$ -\text{cov}(2) + \mu \tau_1 \Delta_1^2 + \mu \tau_2 \Delta_2^2 = 0, $$

from which we get our tax formulas:

$$ \tau_1 = \frac{\Delta_1^2 \text{cov}(1) - \Delta_1^1 \text{cov}(2)}{\mu \left[ \Delta_1^1 \Delta_2^2 - \Delta_1^2 \Delta_1^2 \right]}, $$

$$ \tau_2 = \frac{\Delta_1^1 \text{cov}(2) - \Delta_1^1 \text{cov}(1)}{\mu \left[ \Delta_1^1 \Delta_2^2 - \Delta_1^2 \Delta_1^2 \right]}.$$  

Recall that the pension system implies an earnings test when $\tau_2 > \tau_1$. At this level of generality the comparison of $\tau_1$ and $\tau_2$ is an intricate problem. Clearly, there is no reason to expect $\tau_1 = \tau_2$ (except in special cases), but this in itself is not quite surprising. The interesting question is to study under which conditions the marginal tax rate is higher after formal retirement. To obtain some understanding of this issue, we concentrate on an extreme case which is based on assumptions. First $z_1$ is given or imposed by the pensions system at a level $\bar{z}_1$. Second, cross-derivatives (conditional

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4In other words there is a mandatory retirement age which is the same for all. Whether or not this
on a given $z_1$), namely $\partial \tilde{\ell}_2 / \partial \tau_1$, $\partial \tilde{z}_2 / \partial \tau_1$ and $\partial \tilde{\ell}_1 / \partial \tau_2$ are all zero. This is the case for instance when preferences are quasi-linear; i.e., when $u'(c)$ is constant.\footnote{To be more precise, it the the quasi-linearity, together with the separability of utility (4) which implies zero cross-derivatives when $z_1$ is fixed.} We then have:

$$
\tau_1 = \frac{\text{cov} (\beta, y_1 z_1)}{\mu E w_1 z_1 \frac{\partial \tilde{\ell}_1}{\partial \tau_1}},
$$

(13)

$$
\tau_2 = \frac{\text{cov} (\beta, y_2 (z_2 - \bar{z}_1))}{\mu \left( E w_2 (z_2 - \bar{z}_1) \frac{\partial \tilde{\ell}_2}{\partial \tau_2} + E y_2 \frac{\partial \tilde{z}_2}{\partial \tau_2} \right)}.
$$

(14)

Note that the two covariances are negative and that the own compensated derivatives are also negative. As usual in models with linear tax schedules, the covariance terms in the numerator express the redistributive effect (of an increase in $\tau_i$ when the proceeds are distributed in a uniform way through $p$). The denominator on the other hand reflects the distortions associated with the marginal tax. A set of sufficient condition to obtain $\tau_2 > \tau_1$ is first that wage and health inequalities increase after the formal age of retirement, yielding $|\text{cov} (\beta, y_2 (z_2 - \bar{z}_1))| > |\text{cov} (\beta, y_1 \bar{z}_1)|$ while, second, labor supply is not more elastic after than before the formal age of retirement (so that the denominator of (14) is not larger—in absolute value—than the denominator of (13)). While the first of these conditions does not appear to be implausible, the second one is rather strong. After retirement adjustments are both at the extensive and the intensive margin ($\tilde{\ell}_2$ and $z_1$) while only the intensive margin is relevant before retirement. More significantly, one would expect more flexibility in labor supply in the second career than before formal retirement. To sum up, in the most simple case earnings tests arise when after inequality increases while the elasticity of labor supply does not increase too significantly. Only empirical evidence can tell us which of the possibly conflicting effects dominate. From a purely theoretical perspective, even a negative earnings test (an earnings credit with $\tau_1 > \tau_2$) is not impossible.

The discussion so far was for the case where compensated labor supplies where independent (between career spells). Reintroducing the cross-derivatives makes the age is exogenously given or chosen to maximize welfare does not matter for the argument presented here.
interpretation of the expression even more complicated. First, the distortions are now interdependent and may reinforce or neutralize each other. This is not unusual in linear tax models. Second, and more interestingly, redistributive terms are then also interdependent in that cov(2) enters the expression for $\tau_1$ and cov(1) that for the $\tau_2$. This means essentially that the pre-tax inequality in the first career (and hence the potential redistributive benefit of an increase in $\tau_1$) depends on $\tau_2$. Here, even more than before only empirical evidence can resolve the issue. In the meantime, one can only speculate that direct effects are likely to outweigh indirect ones in which case one essentially returns to the analysis with separability.

To conclude the discussion of the linear case, let us revisit the assumption that $\alpha = 0$, so that the (total) pension is independent of retirement age. Allowing for a positive value of $\alpha$ does not only complicate the expressions, but it also brings in the concept of “implicit taxation” which is closely related to that of earnings tests.

In the literature on social security and retirement, the concept of implicit taxation is widely used, more often than that of earning test. The relation between the concept of earning test and that of implicit tax can be made explicit in the linear case. We have an earning test if $\tau_2 > \tau_1$. As to the implicit tax, there is one for $z_1$ and one for $z_2$. Denoting it with $\theta^j_i$ we have:

$$\theta^j_1 = y^j_1 \tau_1 - y^j_2 \tau_2 + \alpha p \quad \text{and} \quad \theta^j_2 = y^j_2 \tau_2.$$

The implicit tax varies with the type of individuals. With $\alpha > 0$ it tends to be higher for the formal retirement as then there is a double burden: the tax and the forgone benefits. In any case, even if $\tau_1 = \tau_2$, implicit tax rates differ between individuals. In our setting, the parameter $\alpha$ does not affect the implicit tax on $z_2$. With the linear specification we have adopted there is always such an implicit tax on $z_2$, except when $\tau_2 = 0$. To see this recall that retirement benefits are not affected by $z_2$; consequently, there is an implicit tax as long as the individual faces a positive contribution rate on the second career income.\(^6\)

\(^6\)Similarly, when $\alpha = 0$ is assumed as above, there is necessarily an implicit tax on $z_1$ as long as $\tau_1 > 0$.  

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Earnings tests and early retirement incentives are two sides of the same reality. However, in our setting, implicit taxation of formal retirement \((z_1)\) and of the end of the second career \((z_2)\) are not exactly symmetrical. The implicit tax on \(z_2\) is in itself a form of earnings test, discouraging post retirement work at the extensive margin rather than just at the intensive margin (on which we have concentrated above). The implicit tax on \(z_1\), on the other hand is not an earnings test per se. However, it also has the effect of discouraging labor supply “around” the normal age of retirement. To be more precise, the implicit tax discourages labor supply before while the earning test discourages it after this formal age of retirement. We shall revisit this issue below in the nonlinear case.

5 Nonlinear pension scheme

Let us now turn to the more general problem of the design of a nonlinear pension and contribution scheme. We suppose that health status and productivities are not publicly observable while incomes \(y_i\) \((i = 1, 2)\) and retirement ages \(z_i\) \((i = 1, 2)\) are observable. The problem to determine the optimal utilitarian allocation is the same as in the first-best except that an incentive constraint has to be added. This constraint ensures that the individual of type \(G\) does not mimic the individual of type \(B\).\(^7\) The problem can then be stated as follows:

\[
\max_{c^j, y^1_j, y^2_j, z^1_j, z^2_j} \sum_{j=G,B} n^j \Phi \left( u(c^j) - V \left( \frac{y^1_j}{w^1_j} \right) R^j \left( z^1_j \right) - V \left( \frac{y^2_j}{w^2_j} \right) \left[ R^j \left( z^2_j \right) - R^j \left( z^1_j \right) \right] \right) 
\]  

(15)

s.t.

\[
\sum_{j=G,B} n^j \left( y^1_j z^1_j + y^2_j \left( z^2_j - z^1_j \right) \right) - c^j
\]  

(16)

\[
uG(c^G) - V \left( \frac{y^G}{w^1_1} \right) R^G \left( z^G_1 \right) - V \left( \frac{y^G}{w^2_2} \right) \left[ R^G \left( z^G_2 \right) - R^G \left( z^G_1 \right) \right] \geq \]

\[
uB(c^B) - V \left( \frac{y^B}{w^1_1} \right) R^B \left( z^B_1 \right) - V \left( \frac{y^B}{w^2_2} \right) \left[ R^B \left( z^B_2 \right) - R^B \left( z^B_1 \right) \right].
\]  

(17)

\(^7\)Our assumptions on productivities and health status ensure that this constraint will effectively be binding.
Let $\mu$ and $\lambda$ denote the Lagrange multipliers respectively associated with the resource and the incentive constraints. The first-order conditions are stated in Appendix B.

Substituting $(A3)$ in $(A5)$ and $(A7)$ and rearranging yields:

$$MRS_{cy}^B = \left[ \frac{\Phi' (U^B) - \frac{\lambda}{n_B} MRS_{cy1}^B}{\Phi' (U^B) - \frac{\lambda}{n_B} MRS_{cy1}^B} \right] z_1^B$$  \hspace{1cm} (18)

and

$$MRS_{cy}^B = \left[ \frac{\Phi' (U^B) - \frac{\lambda}{n_B} MRS_{cy2}^B}{\Phi' (U^B) - \frac{\lambda}{n_B} MRS_{cy2}^B} \right] (z_2^B - z_1^B)$$  \hspace{1cm} (19)

where an upper bar denotes variables of the “mimicker” (i.e., of individual $G$ consuming the consumption bundle of individual $B$). Given that $w_j^B \leq w_j^G$ for $j = 1, 2$ and $R_B (z) \geq R_G (z)$ for every $z$ and making use of the convexity of $V$, one has.

$$\frac{MRS_{cy}^G}{w_1^B w'(c)} \leq \frac{V' \left( \frac{y_B^B}{w_1^B} \right) R_B \left( z_1^B \right)}{w_1^B w'(c)} = MRS_{cy1}^B$$  \hspace{1cm} (20)

Similarly,

$$\frac{MRS_{cy}^G}{w_1^B w'(c)} \leq \frac{V' \left( \frac{y_B^B}{w_2^B} \right) [R_B (z_2^B) - R_B (z_1^B)]}{w_2^B w'(c)} = MRS_{cy2}^B$$  \hspace{1cm} (21)

Consequently, (18) and (19) imply:

$$MRS_{cy1}^B \leq z_1^B,$$

$$MRS_{cy2}^B \leq z_2^B - z_1^B,$$

so that there is a marginal downward distortion on $y_1^B$ and $y_2^B$. Using equations (5) and (6), this is equivalent to saying that individual $B$ faces a positive marginal contribution rate in both of his career spells. Though an important intermediary point to make, this result in itself is of course hardly surprising. Even less surprising is that marginal taxes for individual $G$ are zero in both periods. To see this combine (A4) with (A6) and (A8) to obtain

$$MRS_{cy1}^G = z_1^G,$$

$$MRS_{cy2}^G = z_2^G - z_1^G.$$
Returning to the main question of this paper namely the existence of an earnings test, we now have to examine whether the pension schedule involves a higher marginal tax on the second career income than on the first career income. We already know that for individuals of type $G$ this is not the case; their marginal tax rate is zero at any time. For type $B$ the question is more interesting and less trivial. To compare $T^B_{y_1}$ and $T^B_{y_2}$ we use (5) and (6) to derive:

$$[1 - T^B_{y_1}] = \frac{MRS^B_{cy_1}}{z_1^B}, \quad (22)$$

$$[1 - T^B_{y_2}] = \frac{MRS^B_{cy_2}}{(z_2^B - z_1^B)}. \quad (23)$$

Combining these expressions with (18) and (19) yields:

$$T^B_{y_2} \geq T^B_{y_1} \iff \frac{MRS^G_{cy_1}}{MRS^B_{cy}} \leq \frac{MRS^G_{cy_2}}{MRS^B_{cy_2}}, \quad (24)$$

where $MRS^G_{cy_1}/MRS^B_{cy} \leq 1$ as shown in (20) and (21).

To understand this expression, observe that $\frac{MRS^G_{cy_1}}{MRS^B_{cy}}$ is the ratio between the marginal rate of substitution (between $c$ and $y_i$) of the mimicker and that of the individual of type $B$. When this ratio is smaller after formal retirement ($i = 2$) than before ($i = 1$), the marginal tax rate will be larger after retirement. Now since $MRS^G_{cy_1}/MRS^B_{cy} \leq 1$, a smaller ratio means a larger degree of heterogeneity between mimicking and mimicked individuals. Thus we get the same qualitative result as in the linear case namely that a higher degree of heterogeneity is associated with a larger marginal tax rate. The intuition is once again based on an argument of redistribution and distortions but the effect at work is more complicated in a non-linear setting. Here the distortion (on type $B$) is created to discriminate between the mimicking individual and the mimicked (in order to relax an otherwise binding incentive constraint). This is more effective the larger is the degree of heterogeneity between these two individuals.\(^8\)

To facilitate interpretation of (24) we use the definitions of the $MRS$, equations (5)

---

\(^8\)The relationship between the ratio $\frac{MRS^G_{cy_1}}{MRS^B_{cy}}$ and the distortion can also be directly seen from (18) and (19).
and (6) and rearrange. This yields:

\[ T_{y_2}^B \succ T_{y_1}^B \]

\[ \frac{w_1^B V'(\frac{y_B}{w_1})}{w_1^G V'(\frac{y_G}{w_1})} R^G(z_1) \Leftrightarrow \frac{w_2^B V'(\frac{y_B}{w_2})}{w_2^G V'(\frac{y_G}{w_2})} \left[ R^G(z_2^B) - R^G(z_1^B) \right] \]

\[ \frac{w_1^B}{w_2^G} \left( \frac{V'(\ell_{1}^B)}{V'(\ell_{2}^G)} \right) \left[ \frac{R^B(z_2^B) - R^B(z_1^B)}{R^B(z_1^B)} \right] \Leftrightarrow \frac{w_2^B V'(\ell_{2}^G)}{w_2^G V'(\ell_{2}^B)} \left[ \frac{R^G(z_2^B) - R^G(z_1^B)}{R^G(z_1^B)} \right] \]

To understand this condition we consider the special cases where \( V \) is a constant elasticity function of the type \( V(\ell) = k\ell^{\alpha+1}/(\alpha + 1) \) where \( k, \alpha > 0 \), so that \( V'(\ell) = k\ell^\alpha \).

Then one easily obtains

\[ T_{y_1}^B \succeq T_{y_2}^B \]

\[ \frac{w_1^B/w_1^G}{w_2^B/w_2^G} \left[ \frac{R^B(z_2^B) - R^B(z_1^B)}{R^B(z_1^B)} \right] \left[ \frac{R^G(z_2^B) - R^G(z_1^B)}{R^G(z_1^B)} \right] \geq 1 \]

The first term measures how heterogeneity in terms of productivities evolves over time (across career spells). When \( 1 > w_1^B/w_1^G > w_2^B/w_2^G \), heterogeneity increases over time and this tends to make earnings tests desirable \( (T_{y_2}^B > T_{y_1}^B) \). This is very much in line with the results obtained in the linear case. Note that when the elasticity of \( V \) is not constant, this term has a more complicated structure but an increase in wage heterogeneity continues to favor earnings test.

Let us now turn to the second term, the ratio in brackets. Each term of this fraction gives the growth rates of \( R \) for type \( B \) (numerator) and type \( G \) (denominator). Denoting these growth rates \( \varepsilon^j_R(z) = dR^j(z)/R^j(z) \) for \( j = G, B \), there is room for earnings tests if \( \varepsilon^G_R(z) < \varepsilon^B_R(z) \) that is to say if the disutility for the retirement age increases faster with age for an individual with bad health (type \( B \)) than for an individual with good health. Roughly speaking this is equivalent to saying that differences in health status are exacerbated over time or that heterogeneity in health increases over time.\(^9\)

\(^9\)The interpretation of the second term does not depend on the specification of \( V \).
Summing up, as far as the desirability of earnings tests is concerned, the results for the nonlinear pension scheme are consistent with those obtained in the linear case: in both setting, earnings tests are desirable when heterogeneity (in health or (and) productivity) increases after retirement.

6 Earnings test and implicit taxation

With nonlinear pensions and contribution schemes, the issue of implicit taxation is more interesting than in the linear case. This is because the specification is now sufficiently flexible so that both implicit taxation and earnings test are truly endogenous. Implicit taxation arises when \( T_{z_1} \) and/or \( T_{z_2} \) are positive. Regarding \( z_2 \), we have

\[
MRS^B_{cz_2} = \left[ \frac{\Phi' (U^B) - \frac{\lambda}{n_B}}{\Phi' (U^B) - \frac{\lambda}{n_B} MRS^G_{cz_2}} \right] y^B_2 \tag{26}
\]

where

\[
MRS^G_{cz_2} = \frac{V \left( \frac{y^B_2}{w^G_2} \right) R^{Gz_2} (z^B_2)}{u' (c^B)} \leq \frac{V \left( \frac{y^B_2}{w^G_2} \right) R^{Bz_2} (z^B_2)}{u' (c^B)} = MRS^B_{cz_2} \tag{27}
\]

follows because with \( V \) increasing one has \( V \left( \frac{y^B_2}{w^G_2} \right) < V \left( \frac{y^B_2}{w^B_2} \right) \) while, by assumption \( R^{Gz_2} (z^B_2) < R^{Bz_2} (z^B_2) \). Using (27) and (8), (26) implies that \( T_{z_2} > 0 \). Consequently, there is a downward distortion on \( z_2 \). As argued in Section 4 this distortion can be seen as an earnings test, discouraging post retirement age at the extensive margin. Interestingly, this earnings test always occurs. Unlike for the intensive margin results based on the marginal tax rate, no extra assumptions on relative heterogeneity (between career spells) are required.

Turning to \( z_1 \), we have

\[
MRS^B_{cz_1} = \left[ \frac{\Phi' (U^B) - \frac{\lambda}{n_B}}{\Phi' (U^B) - \frac{\lambda}{n_B} MRS^G_{cz_1}} \right] (y^B_1 - y^B_2) \tag{28}
\]

where

\[
MRS^G_{cz_1} = \frac{R^{Gz_1} (z^B_1)}{u' (c^B)} \left[ V \left( \frac{y^B_1}{w^G_1} \right) - V \left( \frac{y^B_2}{w^G_2} \right) \right] \leq \frac{R^{Bz_1} (z^B_1)}{u' (c^B)} \left[ V \left( \frac{y^B_1}{w^G_1} \right) - V \left( \frac{y^B_2}{w^G_2} \right) \right] = MRS^B_{cz_1}. \tag{29}
\]
In words, here the comparison between the marginal rate of substitution of the mimicker and that of the mimicked is ambiguous and consequently, we cannot sign the distortion on $z_1$. This ambiguity arises because prolonging the first career implies a change in the marginal disutility proportional to the term $V\left(\frac{y_1^B}{w_1^1}\right) - V\left(\frac{y_2^B}{w_2^1}\right)$ which can be larger as well as smaller for $j = B$ than for $j = G$. In other words, working an additional year implies an “opportunity cost” as measured by the disutility of working after the first career and the comparison of this opportunity cost between types is ambiguous. Note that in the case for the tradeoff between $c$ and $z_2$ there is no opportunity cost related to the prolongation of the second career because the individual stops working.\textsuperscript{10}

7 Conclusion

This paper has developed a simple optimal income tax problem where each individual chooses how much to work in a first and a second career the lengths of each being also optimally determined. When the government cannot observe health or productivity of agents, redistribution involves (at the bottom) positive marginal income taxes on the first and the second career intensive incomes. Moreover, the marginal income taxes differ between the first and the second careers. We show that this difference depends on heterogeneity as measured by marginal rates of substitution between the consumption good and intensive incomes between the mimicker and the mimicked. It turns out that an earnings test may be part of an optimal system for agents with low productivity and (or) bad health status.

References


\textsuperscript{10} Consequently, for $z_2$ we are essentially back to the setting studied by Cremer et al. (2004).


Appendix

A First-order conditions for problem (10)

The first-order conditions with respect to $\tau_1, \tau_2$ and $p$ are given by:

\[
\frac{\partial L}{\partial \tau_1} = - \sum_{j=G,B} n^j \Phi'(U^j) u'(c^j) y_{1}^j z_{1}^j + \mu \sum_{j=G,B} n^j \left[ z_{1}^j y_{1}^j + \tau_1 w_{1}^j z_{1}^j \frac{\partial \ell_{1}^j}{\partial \tau_1} \right] + \tau_2 w_{2}^j \left( z_{2}^j - z_{1}^j \right) \frac{\partial \ell_{1}^j}{\partial \tau_1} A + \tau_2 w_{2}^j \frac{\partial \ell_{2}^j}{\partial \tau_2} = 0,
\]

\[
\frac{\partial L}{\partial \tau_2} = - \sum_{j=G,B} n^j \Phi'(U^j) u'(c^j) y_{2}^j \left( z_{2}^j - z_{1}^j \right) + \mu \sum_{j=G,B} n^j \left[ \left( z_{2}^j - z_{1}^j \right) y_{2}^j + \tau_2 w_{1}^j z_{1}^j \frac{\partial \ell_{1}^j}{\partial \tau_2} \right] + \tau_2 w_{2}^j \left( z_{2}^j - z_{1}^j \right) \frac{\partial \ell_{1}^j}{\partial \tau_1} A + \tau_2 w_{2}^j \frac{\partial \ell_{2}^j}{\partial \tau_2} = 0,
\]

\[
\frac{\partial L}{\partial p} = \sum_{j=G,B} n^j \Phi'(U^j) u'(c^j) \left( 1 - \alpha z_{1}^j \right) + \mu \sum_{j=G,B} n^j \left[ -1 + \tau w_{1}^j z_{1}^j \frac{\partial \ell_{1}^j}{\partial p} \right] + \tau_2 w_{2}^j \left( z_{2}^j - z_{1}^j \right) \frac{\partial \ell_{1}^j}{\partial \tau_1} A + \tau_2 w_{2}^j \frac{\partial \ell_{2}^j}{\partial \tau_2} = 0,
\]

where $A^j = \tau_1 y_{1}^j - \tau_2 y_{2}^j$. Combining these FOCs we obtain two expression where the tax increases are compensated by the additional benefits they generate.

\[
\frac{\partial \tilde{L}}{\partial \tau_1} = \frac{\partial L}{\partial \tau_1} + \frac{\partial L}{\partial p} E y_{1} z_{1}
\]

\[
= - E \beta y_{1} z_{1} + (E \beta)(E y_{1} z_{1})
\]

\[
+ \mu \left[ \tau_1 E w_{1} z_{1} \frac{\partial \tilde{\ell}_1}{\partial \tau_1} + \tau_2 E (z_{2} - z_{1}) w_{2} \frac{\partial \tilde{\ell}_2}{\partial \tau_2} + E A \frac{\partial \tilde{z}_1}{\partial \tau_1} + \tau_2 E y_{2} \frac{\partial \tilde{z}_2}{\partial \tau_1} \right] = 0, \quad (A1)
\]

\[
\frac{\partial \tilde{L}}{\partial \tau_2} = \frac{\partial L}{\partial \tau_2} + \frac{\partial L}{\partial p} E (z_{2} - z_{1}) y_{2}
\]

\[
= - E \beta y_{2} (z_{2} - z_{1}) + E \beta \frac{E y_{2} (z_{2} - z_{1})}{E (1 - \alpha z_{1})}
\]

\[
+ \mu \left[ \tau_1 E w_{1} z_{1} \frac{\partial \tilde{\ell}_1}{\partial \tau_2} + \tau_2 E (z_{2} - z_{1}) w_{2} \frac{\partial \tilde{\ell}_2}{\partial \tau_2} + E A \frac{\partial \tilde{z}_1}{\partial \tau_2} + \tau_2 E y_{2} \frac{\partial \tilde{z}_2}{\partial \tau_2} \right] = 0. \quad (A2)
\]
B First-order conditions for problem (15)

The first order conditions with respect to \( c^j, y_1^j, y_2^j, z_1^j, z_2^j \) are given by:

\[
\begin{align*}
n^B u^j (c^B) \Phi^j (U^B) - \mu n^B - \lambda u^j (c^B) &= 0 \quad (A3) \\
n^G u^j (c^G) \Phi^j (U^G) - \mu n^G + \lambda u^j (c^G) &= 0 \quad (A4) \\
- \frac{n^B}{w_1^B} V' (\ell_1^B) R^B (z_1^B) \Phi^j (U^B) + \mu n^B z_1^B + \frac{\lambda}{w_1^B} V' (\ell_1^B) R^G (z_1^B) &= 0 \quad (A5) \\
- \frac{n^G}{w_1^G} V' (\ell_1^G) R^G (z_1^G) \Phi^j (U^G) + \mu n^G z_1^G - \frac{\lambda}{w_1^G} V' (\ell_1^G) R^G (z_1^G) &= 0 \quad (A6) \\
- \frac{n^B}{w_2^B} V' (\ell_2^B) [R^B (z_2^B) - R^B (z_1^B)] \Phi^j (U^B) \\
+ \mu n^B (z_2^B - z_1^B) + \frac{\lambda}{w_2^B} V' (\ell_2^B) [R^G (z_2^B) - R^G (z_1^B)] &= 0 \quad (A7) \\
- \frac{n^G}{w_2^G} V' (\ell_2^G) [R^G (z_2^G) - R^G (z_1^G)] \Phi^j (U^G) + \mu n^G (z_2^G - z_1^G) \\
- \frac{\lambda}{w_2^G} V' (\ell_2^G) [R^G (z_2^G) - R^G (z_1^G)] &= 0 \quad (A8) \\
- n^B R^{B^j} (z_1^B) [V (\ell_1^B) - V (\ell_2^B)] \Phi^j (U^B) + \mu n^B (y_1^B - y_2^B) \\
+ \lambda R^{G^j} (z_1^B) [V (\ell_1^G) - V (\ell_2^G)] &= 0 \quad (A9) \\
- n^G R^{G^j} (z_1^G) [V (\ell_1^G) - V (\ell_2^G)] \Phi^j (U^G) + \mu n^G (y_1^G - y_2^G) \\
- \lambda R^{G^j} (z_1^G) [V (\ell_1^G) - V (\ell_1^G)] &= 0 \\
- n^B V (\ell_2^B) R^{B^j} (z_2^B) \Phi^j (U^B) + \mu n^B y_2^B + \lambda V (\ell_2^B) R^{G^j} (z_2^B) &= 0 \quad (A10) \\
- n^G V (\ell_2^G) R^{G^j} (z_2^G) + \mu n^G y_2^G - \lambda V (\ell_2^G) R^{G^j} (z_2^G) &= 0 \quad (A11)
\end{align*}
\]