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Asymmetric information, word-of-mouth and social networks: from the market for lemons to efficiency

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Abstract

I analyze a market where there is a homogeneous good, which quality is chosen, and therefore known, by a single producer. Consumers do not know the quality of the good but they use their acquaintances in order to obtain information about it. Information transmission exhibits decay and consumers assign a common initial willingness-to-pay before information transmission takes place. I define an equilibrium concept for this type of situation and characterize the set of resulting equilibria for any possible social network. The main conclusion from this characterization is that, if there is a maximal level of quality (given by technological knowledge) that can be chosen, then, the producer may choose lower levels of quality as the population of consumers is getting more internally connected, due to free-riding on information by consumers when quality levels are low. This "adverse-selection" effect vanishes if consumers are expected to coordinate on the most favorable equilibria for the producer, if there is zero initial willingness-to-pay or if there are no technological constraints.

Keywords: Networks, word-of-mouth communication, asymmetric information.
JEL Classification: D4,D8,L1.

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1 Introduction

This paper analyzes incentives for provision of quality in markets where the quality level is unknown to consumers, but where the network of social relationships consumers are engaged in is used for transmitting information about quality. I study the effects of the particular social network structure on the buying decisions of consumers, and how this affects the choice of price and quality by producers.

Examples that fit this type of situation are experience goods which service quality level is not perceived before purchasing takes place and where consumers share opinions truthfully. Think of provision of services like doctors, lawyers, car diagnosis, etc. Empirical evidence on the use of word-of-mouth by consumers can be found in the marketing literature. For instance, Arndt (1965) found that a positive word-of-mouth about a new food product makes it more likely to be purchased. Engel, Blackwell & Kegerreis (1969) conducted a survey on arriving clients to a new center for car diagnosis. They found that 60% of the respondents who recalled the reason to try the new establishment named word-of-mouth. On another level, Freick & Price (1987) find evidence of the existence of individuals who are key in information transmission through casual conversations with their friends.

The model is as follows. There is one good provided by a single producer choosing quality and price. This quality level is parameterized in terms of willingness-to-pay on the consumers' side and there is a maximum quality level available, given by technological knowledge. Consumers have a common (meaning equal) willingness-to-pay in the absence of information transmission. This willingness-to-pay prior to the word-of-mouth communication can be interpreted as the level of reputation, or as being provided by some public device, available to all consumers. Nowadays, one can think of that public device to include the internet.

There are social relations among consumers, through which word-of-mouth information about the quality of the good is transmitted. This means that the specific structure of the social network has an effect on the quan-

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tity of information held by a consumer as it is the channel through which consumers transmit word-of-mouth information. I assume the process of information transmission happens on a sequence of bilateral meetings, since a social structure is commonly understood as a set of bilateral relationships.[1] Each of these meetings correspond to a couple of agents that are directly linked in the social network. The process of information transmission is one of a mechanical nature. Consumers are not strategically sharing information since information is transmitted through casual conversations taking place in a social interaction context. Information transmission exhibits decay, reflecting the fact that information acquired from a direct friend has a higher impact on the choice of the consumer than the one acquired from a friend of a friend, or a friend of a friend of a friend, etc.

This setting on the bilateral information transmission process implies the following structure of the individual's willingness-to-pay. The willingness-to-pay for any consumer on the social network will be a convex combination of the initial one and the real one. The weights depend on the particular structure of the network and the relative position of the consumer in it, and on the decay of information transmission through the network. The longer the information travels, the stronger is the decay of information and the closer the willingness-to-pay is to the initial one. Formally, this situation can be described as a discount applied to the real value each time the information passes through a consumer. As this discount tends to 1, the value of information coming directly or indirectly is the same, and as the discount tends to 0, the situation is equivalent to the one where there are no network relations, or, equivalently, to the one where there is no information transmission, among consumers. Consumers rely on the information coming from the shortest path. Alternative specifications as for example when consumers willingness-to-pay depend on the number of acquaintances that send the information are possible, but they arrive to similar conclusions complicating the model.[2]

An equilibrium concept is defined where the producer chooses first quality and price, and, afterwards, consumers choose simultaneously to buy or not, given the price and the initial willingness-to-pay, and taking into account word-of-mouth effects, where the social structure with the decay of information associated to it plays a role. In this new equilibrium concept, where the key notion is network-consistent beliefs on the consumers' side,

[1]Note that multilateral relationships can be written as a set of bilateral relationships in which everybody is related with all the rest. There is no loss of generality by the bilateralism assumption.

[2]See the concluding remarks for a detailed discussion on that.

consumers are simultaneously taking their purchasing decision, but their beliefs about the choice of all the other consumers to whom they are directly or indirectly connected are correct in equilibrium. Their willingness-to-pay for the quality of the good is consistent with the purchasing decisions of all the other consumers, the decay of information and the social structure. Hence, I am using a concept of rationality inspired by notions of "consistent beliefs" or "rational expectations", appearing in game theory or macroeconomic theory contexts. It is inspired and not equivalent to these notions, since an equivalent notion would result in a perfect inference of the quality level by consumers. The problem of information would be irrelevant in such a case. In this model, this could be obtained by imposing the parameter of decay being equal to 1. By allowing the information transmission process to have some decay when transmitting the information, I obtain that consumers in equilibrium may be positively or negatively surprised after buying the good (in equilibrium).

A very intuitive interpretation of the equilibrium concept takes the point of view of the producer. Think for example of a dentist. Higher quality service means that he has to become a better dentist by paying a better university, making higher efforts, etc. This means that he becomes a better dentist at a cost. Once he got his title he has to decide the fee he will charge to his potential clients (or patients). These two decisions take place before his clients come to his office. When taking expectations about how many clients he may have per choice of quality and price, he is aware of the social structure and of the fact that the word-of-mouth has an effect on the decisions of his potential clients, but he does not know in which order they may need his services. In summary, this equilibrium concept is an attempt to write down the possible final (or total) demand that the producer may expect to arise when he takes into account word-of-mouth effects on his potential consumers' decisions.

I first characterize the equilibrium in terms of the choice of quality and price by the producer, and the set of consumers who are buying in equilibrium (given the quality and price chosen by the producer). On the consumers' side, several cases are distinguishable depending on whether quality is higher or lower than the initial willingness-to-pay and how this is related to the price. If quality is higher than the initial willingness-to-pay all consumers will buy the product when price is lower than the reputation level. When price is higher than the reputation level, but smaller than the maximal willingness-to-pay that can be generated in the model, only connected consumers might buy the product, or in other words, disconnected consumers never buy the product. In this case, there is a coordination problem. On one hand, if there is one connected consumer who buys, then all her acquaintances will buy due to the good news about quality transmitted through the social structure. On the other hand, no one buying at all is also a possible equilibrium, as no purchase implies no information transmission. If quality is lower than the reputation level the problem of fixing a price for the producer is equivalent to choosing distances on the network. For each fixed distance, several configurations may appear, which intuitively would correspond to different orders of consumers.

Given consumers' behavior, a profit-maximizing producer chooses quality and prices in the following way. About quality, he chooses to provide either the maximum quality level available (the efficient situation), the initial willingness-to-pay for consumers (reputation level), or zero quality level (a market for lemons). I identify the conditions for each of these three possibilities. More specifically, his decision can be characterized in terms of threshold levels for the maximal quality level available. If the maximum level of quality is above a certain threshold the producer always chooses the maximum level available, provided the marginal effect of a higher quality on costs is lower than the decay of information transmission. Below the threshold level, the producer either provides the quality level equivalent to the reputation, if the marginal effect of quality on costs is low, or provides zero quality level (as in a market for lemons), if the marginal effect of quality on costs is high, but still smaller than the decay of information. If the marginal effect of quality on costs is higher than the decay of information, the producer always provides zero quality, therefore obtaining a result in the spirit of the market for lemons.

With respect to the price, the profit-maximizing producer chooses prices depending on the structure of the social network. When the producer chooses the maximum quality level available, the corresponding price makes agents at distance one indifferent between buying the good or not. When he chooses the quality equal to the reputation level the price is exactly equal to the quality provided and therefore, to the reputation level. Finally, when he chooses a zero quality level the price is set above the quality level and at most equal to the reputation level. In this case, the producer chooses a strictly positive price, as long as the initial willingness-to-pay or reputation level is strictly positive. The (optimal) profit for the producer in this case can be understood as a measure of the monopoly power and can be used to do comparative statics, meaning comparing equilibrium quality levels for different given social structures.

When comparing the choice of quality for two different social structures or networks, one finds that the producer may choose lower levels of quality for social networks that are internally more connected. The intuition behind this result works as follows. When quality levels are lower than the reputation, information is costly in the sense that (i) in order to obtain information about the real quality level consumers need to buy the good, and (ii) consumers are paying a price higher than the actual level of quality, therefore losing utility when they buy the good. This latest point is due to the fact that prices in equilibrium are a convex combination of the real quality and the reputation. Hearing about quality through the network once there is a consumer who buys results in a "free-riding" of information, specially when quality levels are very low with respect to the price. This is so since consumers who hear through the network that quality levels are low and decide not to buy the good 'free-ride" information from the consumer who buys since the latter is the only one who paid the price (needed to learn the information). When new links are created among agents that are very centered in the network of social structure, a consumer who did not "free-ride" may start doing it with this new connection. The producer benefits from that, as this free-riding from a centered consumer results in peripheral consumers receiving less information, as compared to the situation when the centered consumer did not free ride, and due to the decay of information transmission. This yields the peripheral consumers buying the good, and not free-riding information as before. If peripheral consumers are more numerous than core or centered agents the producer gets higher demand at low quality levels in denser networks.

The mechanism for this negative effect works intuitively in a very similar way to the one in Bramoullé & Kranton (2005) and Galeotti (2005). The former find, in a context where results from privately financed experimentation diffuse along social links, that new links may damage overall welfare by reducing individual's incentives to conduct experiments due to free-riding effects of information. Galeotti (2005) finds, in a context of price searching by consumers, and where consumers share information on prices privately found through a social network, that consumers may search less intensively in equilibrium for denser networks again due to free-riding of information.

There is a literature studying effects of word-of-mouth communication on consumers behavior. For example, Ellison & Fudenberg (1995), Vettas (1997), Corneo & Jeanne (1999) and Banerjee & Fudenberg (2004). Contrary to what I do here, these papers explore the causes of herding behavior and the arise of fashion on a population.[3] In all these papers, the informa-

[3]An interesting question for further research would be to introduce competition in the model, and study not only the choice of quality by producers, but the herding behavior

tion transmission takes place in a different way than it does here. First, they are all dynamic settings, while mine is static, and second, the way information transmission is modeled is not bilateral in spirit, except for Corneo & Jeanne (1999). In this last paper, each bilateral meeting implies a transmission of consumption skills, and meetings are random, while here consumers are meeting all people who are already socially related to them. In Ellison & Fudenberg (1995) each consumer hears of the current experiences of a random sample of the other players, in Vettas (1997) the knowledge about the quality is an increasing function of past purchases and, finally, in Banerjee & Fudenberg (2004), as in Ellison & Fudenberg (1995), consumers consult a sample from the rest of the population, with the difference that those consulted people report not only what they themselves have chosen, but they may also send signals that are correlated with the payoffs from the choices (an indication of how satisfied the consulted consumers are with the alternative). This paper differs from the papers cited above in the following features. First, I introduce a static model where the concept of equilibrium includes the effect of the word-of-mouth communication through the network. Second, I model the social structure specifically as a network of bilateral relations. Finally, as mentioned before, their aim is to characterize herding behavior by consumers, while mine is to eventually study the effect of the specifics of the social structure on the choice of quality by the producer.

This paper is organized as follows. In Section 2 the model is presented. Section 3 discusses the results of the paper and presents an example. Section 4 makes some concluding remarks.

2 The Model

There is one homogeneous good produced by a single agent, referred to as the producer. The quality of this good can be parameterized by $\theta \in [0, \theta^{max}]$, where θ^{max} denotes the maximal level of quality given by the present level of technological knowledge. This parametrization corresponds to monetary units, or, in other words, the common willingness-to-pay for consumers. Let $N = \{1, ..., n\}$ denote the finite set of (potential) consumers. Throughout the paper, I will refer to θ as the "quality" when I am dealing with the producer, and as the "willingness-to-pay" when I am dealing with consumers.

The quality parameter θ is known to (and eventually chosen by) the producer, but unknown to consumers. Consumers have a common, initial

by consumers.

willingness-to-pay $\bar{\theta}$, based on a public information device about the real value of θ. Note that $\bar{\theta}$ could be equal to 0. There is a network of social relations among consumers in N, and the structure of this network will have an effect on the quantity of information owned by a consumer. For the sequel, the network of social relations will be referred to as the social structure.

The timing of the process is as follows. The producer chooses quality θ and then price given θ. Then consumers choose simultaneously to consume or not, given the prices and the social structure.

2.1 Production

For the sake of simplicity, the good with quality θ in $[0, \theta^{max}]$ is produced at a marginal cost of $c\theta$, for $0 < c < 1$, with zero fixed costs.

The profit to the producer is given by

$$\pi(\theta, p) = q^d(\theta, p)[p - c\theta], \tag{1}$$

where $q^d(\theta, p)$ is the expected number of consumers buying one unit of the good (and therefore the expected demand) when the producer chooses quality θ and price p. The way this expected value is computed by the producer is formalized in the equilibrium concept.

2.2 Consumers: Utility

The good with quality θ is available for consumers at a market price denoted by p. Consumers are risk neutral, need at most one unit of the good and prefer higher quality and lower price. Thus, the utility function for each consumer i can be written as

$$U_i(\theta) = \begin{cases} \theta - p, & \text{if } i \text{ buys the good,} \\ 0, & \text{otherwise.} \end{cases} \tag{2}$$

Consumers do not know the value of θ. They have two sources of information: (1) as I said before, there exists a common (to all consumers) willingness-to-pay in the absence of word-of-mouth communication, denoted $\bar{\theta}$, and (2) the word-of-mouth communication among consumers takes place through bilateral meetings across the existing social structure. We assume that the accuracy of the information transmitted through the network exhibits decay. This will be formalized in the equilibrium concept defined in 2.4 below.

2.3 Consumers: The Social Structure

I proceed to formally describe the social structure since some notions on networks are needed prior to the definition of the equilibrium concept in 2.4. The network of social relations among consumers in N can be represented by an undirected graph g, which is a set of unordered pairs (i, j), where $i, j \in N$, and $i \neq j$. Throughout the paper, each unordered pair (i, j) will be referred to as a link. A link in the network (or social structure) means that those two consumers have casual conversations containing relevant information about the quality of the good θ. In what follows, the set N is considered to be fixed. Figure 1 shows different social structures for $n = 5$.

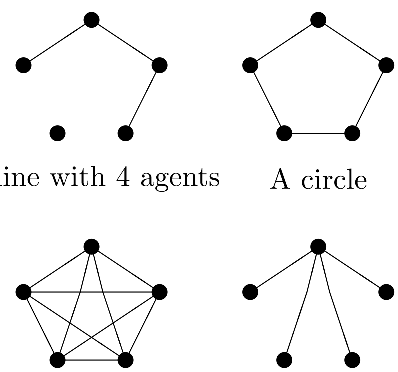

FIGURE 1

Given a graph g, $N(g) \subseteq N$ denotes the set of consumers having at least one link in g. Let $n(g) = |N(g)|$. In the examples above, $n(g) = 4$ in the line with four agents, while $n(g) = 5$ in the rest of networks shown in Figure 1. A group of consumers $S \subseteq N$ is called a component of g if: (1) for every two consumers in S, there is a path, that is, a set of consecutive links in g connecting them, and (2) for any consumer i in S and any consumer j not in S, there is no path in g which connects them. Let $\mathcal{C}(g)$ be the set of components of g. Note that $\mathcal{C}(g)$ is a partition of N. Denote by $\varsigma(g)$ the number of elements in $\mathcal{C}(g)$. In Figure 1 above, the line with four agents has two components, one where there is one isolated consumer and the other one with four of them together. The other graphs have only one component.

A graph g is *connected* if $\mathcal{C}(g) = \{N\}$, or, in other words, if $\varsigma(g) = 1$. Therefore, in the examples in Figure 1 above only the line with four agents is not connected.

Two consumers are said to be connected in g if they belong to the same component of g. Given a graph g and two connected consumers i and j in N, a geodesic joining i and j in g is a shortest path in g going from i to j, i.e., it is a path with minimum number of links. The number of links in any geodesic joining two consumers is called the geodesic distance. If i and j are not connected in g then the geodesic distance is fixed to be ∞. The maximum geodesic distance between two **connected** consumers in g is called the diameter of g and is denoted by $D(g)$. In the examples just shown above in Figure 1, the line with four agents has a diameter equal to three, the circle and the star have both a diameter equal to two and the complete graph has a diameter equal to one.

2.4 Equilibrium concept

In the definition of the equilibrium concept the consumers' side is separated from the producer's side.

2.4.1 Consumers' side

The social structure plays a role in the formation of beliefs about the quality of the good on the consumers' side. I introduce first the notion of "network-consistent beliefs" prior to the notion of a "continuation equilibrium" for a given choice of θ and p by the producer.

Definition 2.1 *A consumer is said to have network-consistent beliefs if, given any θ and p chosen by the producer, and given the choices of all the other consumers in N, her willingness to pay for θ can be written as*

$$\delta^{d(i,j;g)}\theta + \left(1 - \delta^{d(i,j;g)}\right)\bar{\theta}, \tag{3}$$

where $\delta \in (0,1)$ is the parameter measuring the decay of information and $d(i,j;g)$ is the geodesic distance between i and the closest consumer in $N\backslash\{i\}$ who buys the product in equilibrium, denoted j.

The fact that consumers build their beliefs about θ (equivalent to building a willingness-to-pay in our model) as in (3) captures the following process of information transmission:

1. A consumer who buys the product in equilibrium knows the value of θ. He sends a true message $m_1 = \theta$ to her direct friends.

2. Although people are telling the truth, a consumer includes what she hears through the social network in her new willingness-to-pay only up to a parameter $\delta \in (0,1)$. This parameter reflects the idea of how different a consumer thinks she is from another one, since θ is a willingness-to-pay extracted from experiencing the good. The more a consumer thinks her personal experience is going to be different than the one of any of her neighbors, the closer is δ to 0. A consumer who is not buying in equilibrium, but who has a direct friend in the network who did, hears from this friend θ, includes this information in his idea of what is worth paying for the good according to δ, and transmits the message $m_2 = \delta\theta + (1-\delta)\,\bar{\theta}$.

3. In general, any consumer i hears messages coming from different people on the social network before buying. "Updating" a consumer's willingness-to-pay according to a heard message m results in $\delta m + (1-\delta)\,\bar{\theta}$, which in turn is $\delta^{d(i,j;g)}\theta + \left(1-\delta^{d(i,j;g)}\right)\bar{\theta}$, where j is a consumer in $N \backslash \{i\}$ who is buying in equilibrium.

By simplicity, consumers rely on the message that has crossed the minimum number of links. Therefore, j in the expression $\delta^{d(i,j;g)}\theta + \left(1-\delta^{d(i,j;g)}\right)\bar{\theta}$ denotes the closest consumer to i who is buying in equilibrium. But, since, in general, consumer i could hear messages coming from several different paths in the network, one could assume that the way consumers update the information about $\bar{\theta}$ depends not only on the distance to the closest consumer who is buying, but on the total number of consumers to whom they are directly or indirectly connected, and each of the respective distances. This article would look in such a case more complex formally (and therefore heavier to follow and read) but the spirit of results and main point of it would remain equivalent. See the section of Concluding Remarks for a deeper discussion on this point.

By allowing the network having some frictions when transmitting the information (due to trust or to correlation on tastes), I obtain that consumers in equilibrium may be positively or negatively surprised after buying the good, as compared to the belief they have before acquiring the good.

Definition 2.2 *Given any choice of θ and p by the producer, a subset of consumers $S \subseteq N$ is called a continuation equilibrium after θ and p if:*

1. *Each consumer in S maximizes her utility given all the other consumer's choices, the chosen price p and her belief about the chosen quality θ.*

2. *All consumers have network-consistent beliefs (see Definition 2.1) about quality θ.*
3. *When a consumer is indifferent between buying or not, she is assumed to buy.*

Given the social structure g and the parameter of decay δ, network-consistent beliefs imply that: (1) for any consumer i who is buying in equilibrium it has to be that

$$\delta^{d(i,j;g)}\theta + \left(1 - \delta^{d(i,j;g)}\right)\bar{\theta} \geq p,$$

where j is the closest consumer to i who is buying in equilibrium; and (2) for any consumer i who is not buying in equilibrium it holds that

$$\delta^{d(i,h;g)}\theta + \left(1 - \delta^{d(i,h;g)}\right)\bar{\theta} < p,$$

where h is the closest consumer to i in g who is buying.

Consumers use the information coming through the network from the closest consumer who decides to buy the good, when deciding to buy or not, but they take their purchasing decision simultaneously. The notion of rationality used in this equilibrium implies, therefore, that consumers have correct beliefs about (i) the choice of all the other consumers to whom they are directly or indirectly connected, and (ii) the quality of the good, assuming that this quality is spread through the network (before the purchasing takes place) with some decay. Hence, I can say that consumers have "consistent beliefs", given the network structure and the parameter of decay. Note that, as the decay tends to 1, connected consumers (formally, the ones in $N(g)$) have "rational expectations" in equilibrium, in the sense that the expected value they assign to the quality is equal to the real one (provided there is at least one consumer who actually buys the good in each component of the network).

Let $Q(\theta, p)$ denote the set of all possible continuation equilibria $S \subseteq N$ if the producer chooses quality θ and price p. Since consumers are choosing simultaneously there may be multiplicity of equilibria on the consumers side. This implies that $Q(\theta, p)$ consists of several subsets of consumers in general.

In order to illustrate how the equilibrium on the consumer's side looks like for a given social structure g, consider the following examples.

Example 1. Let $n = 5$ and let g be as given by Figure 2.

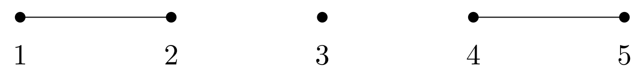

FIGURE 2

Different cases have to be distinguished.

1. If $\theta > \bar{\theta}$ and $p \leq \bar{\theta}$ then any consumer with no information at all will buy, since the price is not bigger than the initial willingness-to pay. So there is at least one consumer who buys. If this consumer is connected to someone else on the social structure then some information about the quality of the good is transmitted, and we have to take into account network-consistent beliefs. Imagine consumer 1 buys the good. Then, by network-consistent beliefs, consumer 2 also buys since her (network-consistent) willingness-to-pay is equal to $\delta\theta + (1-\delta)\,\bar{\theta}$, which is greater than the price $\bar{\theta}$, as $\bar{\theta} < \theta$. The same applies if we reverse the roles of 2 and 1, and if we consider consumers 4 and 5. Therefore, everybody buys since consumers maximize utility (and in case of indifference, they are assumed to buy). We therefore write $Q(\theta, p) = \{\{1, 2, 3, 4, 5\}\}$.

2. Assume now that $\theta > \bar{\theta}$ and $\bar{\theta} < p \leq \delta\theta + (1-\delta)\,\bar{\theta}$. Imagine consumer 1 buys the good. Then, by network-consistent beliefs, consumer 2 also buys since her (network-consistent) willingness-to-pay is equal to $\delta\theta + (1-\delta)\,\bar{\theta}$, which is equal to the price, and we have assumed that in case of indifference consumers buy. The same applies if we reverse the roles of 2 and 1, and if we consider consumers 4 and 5. This means that any equilibrium in which 1 buys, 2 has to buy too (and vice versa), and any equilibrium in which 4 buys, 5 has to buy too. Finally, 3 will never buy as the price is higher than the initial willingness-to-pay. Note that there may be no one in each component buying at all, since the initial willingness-to-pay is smaller than the price. If no one buys, everybody has willingness-to-pay equal to the initial one, since no purchase implies no information transmission about the quality of the good, and no one wants to buy either. So the possible different configurations in a continuation equilibrium are 1 and 2 buying and no one else, or 1 and 2 with 4 and 5 buying, or only 4 and 5 buying, or no one buying at all. We thus write $Q(\theta, p) = \{\{1, 2\}, \{4, 5\}, \{1, 2, 4, 5\}, \{\emptyset\}\}$.

3. Let $\theta \leq \bar{\theta}$ and $p \leq \delta\theta + (1 - \delta)\,\bar{\theta}$. Since the price is smaller than the initial willingness-to-pay, we know that there will be at least one consumer per component buying the good. Imagine those consumers buying (one per component) are 1, 3 and 4. Using the same argument as before, if 1 buys, 2 buys in equilibrium (and vice versa), and if 4 buys, 5 buys in equilibrium (and vice versa). Therefore, we obtain that all consumers buy the good in equilibrium and then $Q\,(\theta, p) = \{\{1, 2, 3, 4, 5\}\}$.

4. Finally, let $\theta \leq \bar{\theta}$ and $\delta\theta + (1 - \delta)\,\bar{\theta} < p \leq \bar{\theta}$. As the price is smaller or equal to the initial willingness-to-pay, we know that there will be at least one consumer per component buying the good. But since $\theta < \bar{\theta}$, $\delta\theta + (1 - \delta)\,\bar{\theta} < \bar{\theta}$, and no one connected to a buyer would like to buy the good in equilibrium. So we obtain that, in equilibrium, only one consumer in each component buys or $Q\,(\theta, p) = \{\{1, 3, 4\}, \{2, 3, 4\}, \{1, 3, 5\}, \{2, 3, 5\}\}$.

From all the above, one can summarize the analysis as follows.

1. If $\theta > \bar{\theta}$ and $p \leq \bar{\theta}$ then $Q\,(\theta, p) = \{\{1, 2, 3, 4, 5\}\}$.
2. If $\theta > \bar{\theta}$ and $\bar{\theta} < p \leq \delta\theta + (1 - \delta)\,\bar{\theta}$ then $Q\,(\theta, p) = \{\{1, 2\}, \{4, 5\}, \{1, 2, 4, 5\}, \{\emptyset\}\}$.
3. If $\theta \leq \bar{\theta}$ and $p \leq \delta\theta + (1 - \delta)\,\bar{\theta}$ then $Q\,(\theta, p) = \{\{1, 2, 3, 4, 5\}\}$.
4. If $\theta \leq \bar{\theta}$ and $\delta\theta + (1 - \delta)\,\bar{\theta} < p \leq \bar{\theta}$ then $Q\,(\theta, p) = \{\{1, 3, 4\}, \{2, 3, 4\}, \{1, 3, 5\}, \{2, 3, 5\}\}$.
5. Otherwise, $Q\,(\theta, p) = \{\{\emptyset\}\}$.

Example 2. Let $n = 5$ and let g be as given by Figure 3.

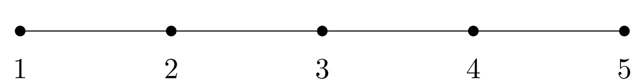

FIGURE 3

Solving for the continuation equilibria as before:

1. If $\theta > \bar{\theta}$ and $p \leq \bar{\theta}$ then $Q(\theta, p) = \{\{1,2,3,4,5\}\}$.

2. If $\theta > \bar{\theta}$ and $\bar{\theta} < p \leq \delta\theta + (1-\delta)\bar{\theta}$ then $Q(\theta, p) = \{\{1,2,3,4,5\}, \{\emptyset\}\}$.

3. If $\theta \leq \bar{\theta}$ and $p \leq \delta\theta + (1-\delta)\bar{\theta}$ then $Q(\theta, p) = \{\{1,2,3,4,5\}\}$.

4. If $\theta \leq \bar{\theta}$ and $\delta\theta + (1-\delta)\bar{\theta} < p \leq \delta^2\theta + \left(1-\delta^2\right)\bar{\theta}$ then $Q(\theta, p) = \{\{1,3,5\}, \{1,4\}, \{2,4\}, \{2,5\}\}$.

5. If $\theta \leq \bar{\theta}$ and $\delta^2\theta + \left(1-\delta^2\right)\bar{\theta} < p \leq \delta^3\theta + \left(1-\delta^3\right)\bar{\theta}$ then $Q(\theta, p) = \{\{1,4\}, \{1,5\}, \{2,5\}, \{3\}\}$.

6. If $\theta \leq \bar{\theta}$ and $\delta^3\theta + \left(1-\delta^3\right)\bar{\theta} < p \leq \delta^4\theta + \left(1-\delta^4\right)\bar{\theta}$ then $Q(\theta, p) = \{\{1,5\}, \{2\}, \{3\}, \{4\}\}$.

7. If $\theta \leq \bar{\theta}$ and $\delta^4\theta + \left(1-\delta^4\right)\bar{\theta} < p \leq \bar{\theta}$ then $Q(\theta, p) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}\}$.

8. Otherwise, $Q(\theta, p) = \{\{\emptyset\}\}$.

The main difference with the previous example (Figure 2) lies in cases 4, 5 and 6. As an illustration consider case 4 (the argument for the other two cases work very similarly). Since $\theta < \bar{\theta}$, the price is smaller than the initial willingness-to-pay, and there is at least one consumer (i.e., one consumer per component, but there is only one component for this case) who buys in equilibrium. Assume this consumer is agent 1. As the price is strictly greater than $\delta\theta + (1-\delta)\bar{\theta}$, consumer 2, who is at a distance 1, is not willing to pay the price (by network-consistent beliefs), so she is not buying in that equilibrium. If 3 buys, then 4 (and 2 either) does not want to buy, but 5 does. In general, for two consumers who are at a distance 1 there will be at most one of them who buys. Or, in other words, two consumers who are buying in equilibrium are at a distance of at least 2 (since $p \leq \delta^2\theta + \left(1-\delta^2\right)\bar{\theta} < \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$, for any $k > 2$). A configuration (in the network) of consumers who are buying in equilibrium must then form a set such that: (i) two consumers in the set are at a distance of at least 2 in g, and (ii) any consumer who is not in the set is at a distance of at most 1 to at least one consumer in the set.[4]

Next subsection presents the equilibrium on the producer's side.

[4] Remember the configuration equilibria are written in terms of consumers who actually buy the good.

2.4.2 Producer's side

Definition 2.3 *The producer has consistent beliefs if for any choice of θ and p the expected number of consumers $q^d(\theta, p)$ can be written as the convex combination*

$$\sum_{S \in Q(\theta,p)} \rho(S, Q(\theta, p))\, |S|, \tag{4}$$

where $\rho(S, Q(\theta, p)) > 0$, for all $S \in Q(\theta, p)$, and $\sum_{S \in Q(\theta,p)} \rho(S, Q(\theta, p)) = 1$.

Consistent beliefs for the producer mean that the producer is assigning positive probability only to sets of consumers who are rational in the sense of network-consistency. Note that the probability assigned to each of the possible equilibria S on the consumers side, for given θ and p, denoted $\rho(S, Q(\theta, p))$, depends only on S itself and on $Q(\theta, p)$. This means that if there are two choices (θ, p) and (θ', p') the beliefs for the producer are the same if $Q(\theta, p) = Q(\theta', p')$. In other words, the beliefs do not depend on the particular choices of θ or p, or on the graph g, as far as the resulting set of possible equilibria $Q(\theta, p)$ is the same.

The intuition behind the notion of $Q(\theta, p)$ as a set of continuation equilibria for the producer works as follows. Imagine a situation where the producer has a fixed establishment or location, in other words, a shop, where once the product is offered, quality and price are fixed. When taking the decision of which quality to offer and which price to ask, he takes into account the fact that (potential) consumers make use of word-of-mouth information about the quality the good. He knows the network structure, but he does not know the order at which consumers are arriving at the shop. Each of the resulting sets of what I have called the continuation equilibrium represents a final set of buying consumers for at least one particular order (on all the potential consumers). The equilibrium notion presented here is an attempt to writing down this type of situation when there is at most one consumer who buys at a time. Notice that the producer is optimistic in the sense that when offering a higher quality level he may expect getting all consumers buying the product, even when the price is higher than the initial willingness-to-pay.[5]

I proceed to formally introduce the concept of equilibrium.

[5]Intuitively, at least one consumer suspects that quality maybe good since price is higher than the initial willingness-to-pay and tries the product.

Definition 2.4 *An equilibrium is a triple $\mathcal{E} = (\theta^*, p^*, Q^*)$, where θ^* and p^* are non-negative real numbers and Q^* consists of subsets of consumers such that*

1. *The producer chooses quality θ^* and price p^* maximizing expected profit given his beliefs about the final demand, for each possible quality and price. His beliefs are consistent (see Definition 2.3).*

2. $Q^* = Q(\theta^*, p^*)$.

It is clearly seen from the definition of the equilibrium that $\mathcal{E}$ is a function of the parameters of the model: the impact of quality on marginal costs c and the system of beliefs ρ, on the producer side, and the prior $\bar{\theta}$, the parameter of decay δ and the social network g, on the consumers side. To avoid abuse of notation I simply write $\mathcal{E}$ instead of $\mathcal{E}\left(c, \rho, \bar{\theta}, \delta, g\right)$, as these parameters are considered fixed. Eventually, the social structure g will be allowed to vary. In that case, I will specify the argument that varies in the expression of $\mathcal{E}$.

To finish the section, consider the example provided in Figure 2 above. Assuming the producer's beliefs are uniform for any choice of θ and p we obtain the following.[6] As we have already seen, for $\theta > \bar{\theta}$ the producer may choose a price equal to $\bar{\theta}$, or a price equal to $\delta\theta + (1-\delta)\bar{\theta}$. If he chooses $\bar{\theta}$ he will obtain a profit of $5\left(\bar{\theta} - c\theta\right)$, while if he chooses $\delta\theta + (1-\delta)\bar{\theta}$ he will obtain a (expected) profit equal to $2\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right]$. For $\theta \leq \bar{\theta}$ he could also choose a price equal to $\bar{\theta}$ or to $\delta\theta + (1-\delta)\bar{\theta}$, with the difference that in this case $\bar{\theta} > \delta\theta + (1-\delta)\bar{\theta}$. If he chooses a price equal to $\bar{\theta}$ he will obtain a profit of $3\left[\bar{\theta} - c\theta\right]$. If he chooses a price equal to $\delta\theta + (1-\delta)\bar{\theta}$ he will then obtain a profit of $5\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right]$.

As he is maximizing profit, the reader may check that the equilibrium for this example looks as follows.

1. Let $\delta \leq \frac{2}{5}$. Then:

   (a) The producer chooses quality level θ^{max} and a price equal to $\delta\theta^{max} + (1-\delta)\bar{\theta}$ if $c \leq \delta$ and $\theta^{max} \geq \frac{3+2\delta-5c}{2(\delta-c)}\bar{\theta}$.

   (b) The producer chooses quality level $\bar{\theta}$ and a price equal to $\bar{\theta}$ if $c \leq \delta$ and $\bar{\theta} \leq \theta^{max} < \frac{3+2\delta-5c}{2(\delta-c)}\bar{\theta}$.

[6] By uniform beliefs I mean that for any continuation equilibrium each of the possible configurations has equal probability of occurrence.

(c) The producer chooses quality level 0 and a price equal to $(1-\delta)\,\bar{\theta}$, otherwise.

2. Let $\delta > \frac{2}{5}$. Then:

   (a) The producer chooses quality level θ^{max} and a price equal to $\delta\theta^{max} + (1-\delta)\,\bar{\theta}$ if $c \leq \frac{2}{5}$ and $\theta^{max} \geq \frac{3+2\delta-5c}{2(\delta-c)}\bar{\theta}$, or, if $\frac{2}{5} < c < \delta$ and $\theta^{max} \geq \frac{2\delta+1}{2(\delta-c)}\bar{\theta}$.

   (b) The producer chooses quality level $\bar{\theta}$ and a price equal to $\bar{\theta}$ if $c \leq \frac{2}{5}$ and $\bar{\theta} \leq \theta^{max} < \frac{3+2\delta-5c}{2(\delta-c)}\bar{\theta}$.

   (c) The producer chooses quality level 0 and a price equal to $\bar{\theta}$, otherwise.

Next section characterizes the equilibrium for any possible social structure g and some conclusions are drawn.

3 Results

Theorem 3.2 characterizes the equilibrium as a function of the parameters, for different ranges of the latter. As a previous step, Lemma 3.1 presents the continuation equilibrium for any possible choice of quality and price by the producer. Lemma 3.1 not only helps proving the results stated in Theorem 3.2 but it also helps understanding the equilibrium behavior for consumers.

I need the following definitions in order to introduce next lemma. Recall that, for given social structure g, the set $\mathcal{C}(g)$ denotes the set of components of g. Consider the components in $\mathcal{C}(g)$ which are not singleton and the union of any number of components in $\mathcal{C}(g)$ which are not singleton. The set containing all these elements and the empty set is denoted by $\mathcal{VC}(g)$. So, an element in $\mathcal{VC}(g)$ is either the empty set, or a component of g with at least two consumers, or a union of any number of components of g with at least two consumers.

For every $k \in \{1, 2, ..., \infty\}$ define $\mathcal{N}_k(g)$ as the set of maximal (with respect to inclusion) subsets of consumers such that the geodesic distance in g between any two consumers in the set is at least k. Formally, a group of consumers $S \subseteq N$ is in $\mathcal{N}_k(g)$ if: (1) for every two consumers in S their geodesic distance is at least equal to k, and (2) for any consumer i not in S there exists at least one consumer j in S such that their geodesic distance is strictly less than k. It is easily seen that (a) $\mathcal{N}_1(g) = \{N\}$ and (b) if $k > D(g)$ any set in $\mathcal{N}_k(g)$ consists of exactly one consumer per component of g. We will denote such a set by $\mathcal{N}_\infty(g)$.

Lemma 3.1 *Let θ and p be the choices of quality and price made by the producer. Then the continuation equilibrium $Q(\theta, p)$ and the expected number of consumers $q^d(\theta, p)$ for the producer (for a given system of consistent beliefs ρ) looks as follows:*

1. *Let $\theta > \bar{\theta}$ and $p \leq \bar{\theta}$. Then,*
$$Q(\theta, p) = \{N\} \text{ and } q^d(\theta, p) = n.$$

2. *Let $\theta > \bar{\theta}$ and $\bar{\theta} < p \leq \delta\theta + (1-\delta)\bar{\theta}$. Then,*
$$Q(\theta, p) = \mathcal{VC}(g) \text{ and } q^d(\theta, p) = \sum_{S \in \mathcal{VC}(g)} \rho(S, \mathcal{VC}(g))\,|S|.$$

3. *Let $\theta < \bar{\theta}$ and $\delta^{k-1}\theta + \left(1-\delta^{k-1}\right)\bar{\theta} < p \leq \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$ for some $k \leq D(g)$. Then,*
$$Q(\theta, p) = \mathcal{N}_k(g) \text{ and } q^d(\theta, p) = \sum_{S \in \mathcal{N}_k(g)} \rho(S, \mathcal{N}_k(g))\,|S|.$$

4. *Let $\theta < \bar{\theta}$ and $\delta^{D(g)}\theta + \left(1-\delta^{D(g)}\right)\bar{\theta} < p \leq \bar{\theta}$. Then,*
$$Q(\theta, p) = \mathcal{N}_\infty(g) \text{ and } q^d(\theta, p) = \varsigma(g).$$

For all the remaining cases, $Q(\theta, p) = \{\{\emptyset\}\}$ and $q^d(\theta, p) = 0$.

Proof of Lemma 3.1. Consider first the continuation equilibrium when the producer chooses a θ such that $\theta > \bar{\theta}$. Note that in such a case,

$$\bar{\theta} < ... < \delta^k\theta + \left(1-\delta^k\right)\bar{\theta} < \delta^{k-1}\theta + \left(1-\delta^{k-1}\right)\bar{\theta} < ... < \theta, \qquad (5)$$

as $\bar{\theta} < \theta$.

Assume the producer chooses a price p such that $p \leq \bar{\theta}$. As consumers maximize utility and we have assumed that, in case of indifference, they would buy, we obtain that all disconnected consumers and at least one connected consumer buy the product. By equation (5) any connected consumer who is directly or indirectly connected to a consumer who is also buying is willing to pay the price, since the willingness-to-pay will be higher than $\bar{\theta}$. As a result, $Q(\theta, p) = \{N\}$ and the expected number of consumers is equal to n.

Assume now that the producer chooses a price $\bar{\theta} < p \leq \delta\theta + (1-\delta)\,\bar{\theta}$. Then, from (5) and using a similar argument to the one used before, in any continuation equilibrium in which at least one connected consumer i buys the product, the set of consumers buying in equilibrium includes the component of N in g that contains i. But as $p > \bar{\theta}$, there may be components in which nobody buys, or nobody buys at all. Therefore, for all these prices $\bar{\theta} < p \leq \delta\theta + (1-\delta)\,\bar{\theta}$ the continuation equilibrium is any set of consumers such that in each component where there is at least one consumer buying, all consumers in that component buy or no consumers at all buy. Formally, $Q\,(\theta, p)$ is equal to $\mathcal{VC}(g)$ in such a case. Since the continuation equilibrium is the same for all this range of prices, the expected number of consumers is the same, namely $\sum\limits_{S\in\mathcal{VC}(g)} \rho\,(S, \mathcal{VC}(g))\,|S|$. This is so by consistency of beliefs on the producer side.

Consider now the case when $\bar{\theta} > \theta$. Note that, in this case,

$$\theta < \delta\theta + (1-\delta)\,\bar{\theta} < ... < \delta^k\theta + \left(1-\delta^k\right)\bar{\theta} < \delta^{k+1}\theta + \left(1-\delta^{k+1}\right)\bar{\theta} < ... < \bar{\theta}. \quad (6)$$

Assume the producer chooses a price p such that $\delta^{k-1}\theta + \left(1-\delta^{k-1}\right)\bar{\theta} < p \leq \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$, for some $k \in \{1, 2, ..., D(g)\}$. Hence, by (6), the set of consumers is a maximal set such that any two consumers are at a distance more or equal than k. This is so since, by network-consistent beliefs, any consumer who is at a distance smaller than or equal to $k-1$ to a consumer who is actually buying in equilibrium has a willingness-to-pay smaller than p, as it is seen from equation (6). Any two consumers who are actually buying in equilibrium cannot be at a distance smaller than k, since, by contradiction, they would then have a willingness-to-pay smaller than the price by, as before, network-consistent beliefs and equation (6). This is a contradiction since they are maximizing utility. Therefore, $Q\,(\theta, p) = \mathcal{N}_k(g)$ and

$$q^d\,(\theta, p) = \sum_{S\in\mathcal{N}_k(g)} \rho(S, \mathcal{N}_k(g))|S|, \quad (7)$$

where $\rho(S, \mathcal{N}_k(g)) > 0$, for all $S \in \mathcal{N}_k(g)$, and $\sum\limits_{S\in\mathcal{N}_k(g)} \rho(S, \mathcal{N}_k(g)) = 1$.

Assume now the producer chooses a price p such that $\delta^{D(g)}\theta + \left(1-\delta^{D(g)}\right)\bar{\theta} < p \leq \bar{\theta}$. Then, the expected number of consumers is $\varsigma(g)$ for two reasons. First, there is at least one consumer per component who buys the product, as $p \leq \bar{\theta}$, and second, any consumer who is directly or indirectly connected to a consumer who buys the product has a willingness-to-pay which is smaller than the price p, by network-consistent beliefs and equation (6). Therefore, for all prices p such that $\delta^{D(g)}\theta + \left(1 - \delta^{D(g)}\right)\bar{\theta} < p \leq \bar{\theta}$ it is true that $Q(\theta, p) = \mathcal{N}_\infty(g)$ and $q^d(\theta, p) = \varsigma(g)$.

The remaining cases are the ones when the price is higher than the maximum willingness-to-pay that could be generated in equilibrium and than the initial willingness-to-pay $\bar{\theta}$. Therefore, there are no consumers willing to pay the price, and the expected numbers of consumers is 0. This completes the proof of Lemma 3.1. □

Note that Lemma 3.1 intuitively says that the decision problem for the producer can be reduced to low or high quality (as compared to the initial willingness-to-pay $\bar{\theta}$) and, if it is low, price is equivalent to choosing distances in the network among consumers, parameterized as k. If quality is high, choosing a price consists of taking the risk of having no consumers at all at a price higher than the initial willingness-to-pay, or have all consumers with certainty at a price equal to the initial willingness-to-pay.

Once the equilibrium behavior on the consumers' side is solved for any possible choice of θ and p, I can state the equilibrium behavior on the producer's side. This is done in the following theorem.

Recall that there is a maximum level of θ, denoted θ^{max}, given by the frontier of technological knowledge. I make use of the following definitions. Let

$$vc(g) = \sum_{S \in \mathcal{VC}(g)} \rho\left(S, \mathcal{VC}(g)\right)|S|, \tag{8}$$

and let

$$n_k(g) = \sum_{S \in \mathcal{N}_k(g)} \rho\left(S, \mathcal{N}_k(g)\right)|S|, \tag{9}$$

where, recall, $\mathcal{N}_k(g)$ consists of maximal sets of consumers such that the distance in between two consumers in the set is at least k. Note that, by Lemma 3.1, the expected (with respect to consistent beliefs) number of consumers for the producer, denoted by $q^d(\theta, p)$, is equal to $vc(g)$ if he chooses a quality level $\theta > \bar{\theta}$ and a price in the range $\bar{\theta} < p \leq \delta\theta + (1 - \delta)\bar{\theta}$, and, it is equal to $n_k(g)$ if he chooses a quality level $\theta \leq \bar{\theta}$ and a price in the range $\delta^{k-1}\theta + \left(1 - \delta^{k-1}\right)\bar{\theta} < p \leq \delta^k\theta + \left(1 - \delta^k\right)\bar{\theta}$. As the producer is maximizing profit, if there is an equilibrium in which the expected number of consumers is $n_k(g)$ it has to be that the price is maximum in the range, i.e., $p = \delta^k \theta + \left(1 - \delta^k\right) \bar{\theta}$.

Assume the producer chooses $\theta = 0$. Since $\bar{\theta} > 0$ we are in the last two cases of Lemma 3.1. As stated just above, if the expected number of consumers is equal to $n_k(g)$, for some k in $\{1, 2, ...\}$, it has to be that p is equal to $\left(1 - \delta^k\right) \bar{\theta}$. The producer would choose a price that maximizes $n_k(g)\left(1 - \delta^k\right) \bar{\theta}$, for k in $\{1, 2, ...\}$ if θ is fixed to be equal to 0.

Let π^0 be defined as

$$\pi^0 = \max_{k \in \{1,2,...\}} n_k(g) \left(1 - \delta^k\right).$$

Note that, by definition, $\pi^0 \bar{\theta}$ is the expected profit (for the producer) in the continuation equilibrium for $\theta = 0$ and its corresponding profit-maximizing price, given the social structure g.

Let k^0 be the minimum distance between any two consumers who are actually buying in the continuation equilibrium for $\theta = 0$ and its corresponding profit-maximizing price, i.e.,

$$k^0 = argmax_{k \in \{1,2,...\}} n_k(g) \left(1 - \delta^k\right).$$

Let $\bar{c} = \frac{n - \pi^0}{n}$. Note that $\frac{n - \pi^0}{n}$ is always not greater than δ. This is so since $n_1(g) = n$, and therefore, if $\pi^0 \neq n(1 - \delta)$ this means that there is another $k > 1$ such that $n_k(g)\left(1 - \delta^k\right) > n(1 - \delta)$. Since π^0 is the solution of the maximization problem stated above, it is clear then that $\pi^0 \geq n(1 - \delta)$, or, $\frac{n - \pi^0}{n} \leq \delta$.

It is also easily seen that $\bar{c} \geq 0$. Note that π^0 is no greater than n as the producer will never get a profit higher than $n\bar{\theta}$ when choosing $\theta = 0$. Getting $n\bar{\theta}$ implies that he charges a price equal to $\bar{\theta}$ (the highest he can charge in order to have any consumers at all) and all consumers buy. In general, due to this word-of-mouth effect, he will get less consumers acquiring the good. All of them will buy either only because the are disconnected, i.e. the social structure is the empty graph, or $\delta = 0$.

Finally, let

$$\theta_1^T = \frac{n(1 - c) - vc(g)(1 - \delta)}{vc(g)(\delta - c)} \bar{\theta}, \tag{10}$$

and

$$\theta_2^T = \frac{\pi^0 - vc(g)(1 - \delta)}{vc(g)(\delta - c)} \bar{\theta}. \tag{11}$$

Note that θ_1^T and θ_2^T are the quality levels that, being above $\bar{\theta}$, yield an expected profit equal to the ones corresponding to $\theta = \bar{\theta}$ and to $\theta = 0$, respectively, assuming $c < \delta$.[7] The following theorem characterizes the equilibrium behavior for any possible social network structure g.

Theorem 3.2 *Let c, θ^{max}, $\bar{\theta}$, δ and g be given. Let $\mathcal{E} = (\theta^*, p^*, Q^*)$, $\bar{c}$, θ_1^T and θ_2^T be as defined above. Assume that θ^{max} is at least $\bar{\theta}$, so that the level of quality believed by consumers in the absence of information transmission is technologically feasible. Then:*

1. *Let $c \leq \bar{c}$. Then, $\theta^* = \theta^{max}$, $p^* = \delta\theta^{max} + (1 - \delta)\,\bar{\theta}$ and $Q^* = \mathcal{VC}(g)$ if and only if $\theta^{max} \geq \theta_1^T$. On the other hand, $\theta^* = \bar{\theta}$, $p^* = \bar{\theta}$ and $Q^* = \{N\}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \theta_1^T$.*

2. *Let $\bar{c} \leq c \leq \delta$. Then, $\theta^* = \theta^{max}$, $p^* = \delta\theta^{max} + (1 - \delta)\,\bar{\theta}$ and $Q^* = \mathcal{VC}(g)$ if and only if $\theta^{max} \geq \theta_2^T$. On the other hand, $\theta^* = 0$, $p^* = \left(1 - \delta^{k^0}\right)\bar{\theta}$ and $Q^* = \mathcal{N}_{k^0}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \theta_2^T$.*

3. *Let $\delta \leq c$. Then, $\theta^* = 0$, $p^* = \left(1 - \delta^{k^0}\right)\bar{\theta}$ and $Q^* = \mathcal{N}_{k^0}$.*

The proof of this theorem is in the appendix.

Theorem 3.2 states the following. If θ^{max} is big enough, the producer prefers charging a price making consumers at distance 1 in the network indifferent between buying or not, and risking having no consumers at all with some probability, as this price, namely $\delta\theta^{max} + (1 - \delta)\,\bar{\theta}$, is increasing in θ^{max}. For small values of θ^{max} this choice above is not the best one anymore since the price corresponding to $\delta\theta^{max} + (1 - \delta)\,\bar{\theta}$ is not great enough to compensate for the possibility of having no consumers at all. In this case, he may choose $\bar{\theta}$ or 0. Choosing $\bar{\theta}$ implies an optimal price for the producer equal to $\bar{\theta}$ and getting all consumers. This is better than choosing 0 quality level as far as the impact on marginal costs, c, is not too big. Note that 0 may be the most profitable choice if c is still small than the decay δ for θ^{max} small enough. Finally, if c is bigger than δ the producer never chooses to provide positive quality levels, as the effect of the word-of-mouth communication on consumers' decisions, modeled as δ, is not high enough to compensate for higher costs for him, namely c.

Note that the market surplus in any equilibrium outcome is equal to the number of consumers who are actually buying times $(1 - c)\,\theta^*$. This

[7] See Lemma 3.1.

implies that, if $c < 1$, the best equilibrium (in terms of maximizing total surplus) is the one in which the producer provides maximum quality level and consumers coordinate on the equilibrium where the number of buyers is maximum. In this sense, even when the impact on marginal costs c is small enough, one still needs a threshold level to give the producer the incentives to provide the highest quality level possible.

On the other hand, as δ tends to 0, the situation is equivalent to the one in which there is no network. The profit function in the continuation equilibrium tends to $n\left(\bar{\theta} - c\theta\right)$, whenever $\theta < \frac{\bar{\theta}}{c}$. Note that this function is decreasing, and so the level of quality offered by the producer will tend to 0. On the contrary, as δ tends to 1 k^0 tends to ∞ and Theorem 3.2 becomes for this case:

1. Let $c \leq \frac{n-\varsigma(g)}{n}$. Then, $\theta^* = \theta^{max}$, $p^* = \theta^{max}$ and $Q^* = \mathcal{VC}(g)$ if and only if $\theta^{max} \geq \frac{n}{vc(g)}\bar{\theta}$. On the other hand, $\theta^* = \bar{\theta}$, $p^* = \bar{\theta}$ and $Q^* = \{N\}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \frac{n}{vc(g)}\bar{\theta}$.

2. Let $\frac{n-\varsigma(g)}{n} \leq c \leq 1$. Then, $\theta^* = \theta^{max}$, $p^* = \theta^{max}$ and $Q^* = \mathcal{VC}(g)$ if and only if $\theta^{max} \geq \frac{\varsigma(g)}{vc(g)(1-c)}\bar{\theta}$. On the other hand, $\theta^* = 0$, $p^* = \bar{\theta}$ and $Q^* = \mathcal{N}_\infty(g)$ if and only if $\bar{\theta} \leq \theta^{max} \leq \frac{\varsigma(g)}{vc(g)(1-c)}\bar{\theta}$.

3. Let $1 \leq c$. Then, $\theta^* = 0$, $p^* = \bar{\theta}$ and $Q^* = \mathcal{N}_\infty(g)$.

Therefore, even in the case when the quality is observed through the network with no decay, a threshold level is still needed in order to guarantee that the producer has the right incentives, i.e., the ones to provide the highest quality level, even for c small enough.

Finally, as the network structure g gets denser, every unordered pair of consumers is in the network. Assume thus that, $vc(g) \to \bar{\rho}n$, for some $\bar{\rho} \in (0,1)$, $D(g) \to 1$ and $\varsigma(g) \to 1$. Then, the corresponding "limiting" equilibrium depends on how big is δ.

1. Let $\delta \leq \frac{n-1}{n}$.
   (a) If $0 \leq c \leq \delta$, then, $\theta^* = \theta^{max}$, $p^* = \delta\theta^{max} + (1-\delta)\bar{\theta}$ and $Q^* = \{N, \emptyset\}$ if only if $\theta^{max} \geq \frac{(1-c)-\bar{\rho}(1-\delta)}{\bar{\rho}(\delta-c)}\bar{\theta}$. On the other hand, $\theta^* = \bar{\theta}$, $p^* = \bar{\theta}$ and $Q^* = \{N\}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \frac{(1-c)-\bar{\rho}(1-\delta)}{\bar{\rho}(\delta-c)}\bar{\theta}$.
   (b) Let $\delta \leq c$. Then, $\theta^* = 0$, $p^* = (1-\delta)\bar{\theta}$ and $Q^* = \{N\}$.

2. Let $\frac{n-1}{n} \leq \delta \leq 1$.

(a) If $0 \leq c \leq \frac{n-1}{n}$, then, $\theta^* = \theta^{max}$, $p^* = \delta\theta^{max} + (1-\delta)\,\bar{\theta}$ and $Q^* = \{N, \emptyset\}$ if and only if $\theta^{max} \geq \frac{(1-c)-\bar{\rho}(1-\delta)}{\bar{\rho}(\delta-c)}\bar{\theta}$. On the other hand, $\theta^* = \bar{\theta}$, $p^* = \bar{\theta}$ and $Q^* = \{N\}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \frac{(1-c)-\bar{\rho}(1-\delta)}{\bar{\rho}(\delta-c)}\bar{\theta}$.

(b) If $\frac{n-1}{n} \leq c \leq \delta$, then, $\theta^* = \theta^{max}$, $p^* = \delta\theta^{max} + (1-\delta)\,\bar{\theta}$ and $Q^* = \{N, \emptyset\}$ if and only if $\theta^{max} \geq \frac{1-\bar{\rho}n(1-\delta)}{\bar{\rho}n(\delta-c)}\bar{\theta}$. On the other hand, $\theta^* = 0$, $p^* = \bar{\theta}$ and $Q^* = \{\{1\}, \{2\}, ..., \{n\}\}$ if and only if $\bar{\theta} \leq \theta^{max} \leq \frac{1-\bar{\rho}n(1-\delta)}{\bar{\rho}n(\delta-c)}\bar{\theta}$.

(c) If $\delta \leq c$, then, $\theta^* = 0$, $p^* = \bar{\theta}$ and $Q^* = \{\{1\}, \{2\}, ..., \{n\}\}$.

Comparing the limiting cases for δ tending to 1 with the one where the network structure tends to the complete one, one can observe that the fact that the network is getting denser and denser has a stronger effect (in providing the correct incentives for the producer) than the decay of information becoming 1, specially as the population is getting large. This is so since the ratio $\frac{n-1}{n}$ tends to 1 as n is getting larger. Then, as n is getting larger, $\delta < \frac{n-1}{n}$, and the producer never provides quality equal to 0 for any $c < \delta$.

It is also important to remark that, as it is seen from Theorem 3.2, as $\bar{\theta}$ tends to 0, the equilibrium tends to $(\theta^*, p^*, Q^*) = (\theta^{max}, \delta\theta^{max}, \mathcal{VC}(g))$, if $c \leq \delta$, and to $(\theta^*, p^*, Q^*) = (0, 0, \{N\})$, otherwise. This means that the producer will provide the highest quality level independently of the values of the parameters, or the population size, as far as $c \leq \delta$. So $\bar{\theta} > 0$ always works against consumers.

Next theorem identifies sufficient conditions for the choice of quality by the producer to be monotonic on the density of the social structure. In order to do that, I need the following notation. Let $\theta^*(g)$ and $\theta^*(g')$ be the choice of quality in equilibrium when the social network structure is given by g and g', respectively, for given c, θ^{max}, $\bar{\theta}$ and δ. Finally, let $\pi^0(g)$ and $\pi^0(g')$ denote, respectively, the maximum expected profit for the producer when the social network structure is given by g or g' and he chooses quality 0.

Theorem 3.3 *Let g and g' be two social network structures on N such that $g' \subseteq g$. If $\pi^0(g) \leq \pi^0(g')$ and $vc(g) \geq vc(g')$, then $\theta^*(g) \geq \theta^*(g')$.*

The proof of this theorem is in the appendix. The importance of the result stated in Theorem 3.3 lies on the fact that it may not always be true that the network getting denser works in favor of consumers, in the sense that a network with more links will give the producer incentives to provide higher quality levels. In the following theorem I prove that this "negative" effect for consumers vanishes when consumers are believed to coordinate on the most favorable equilibrium for the producer.

In the light of the statement of Theorem 3.3, one can argue that the only reason why the provision of quality is not monotonic in the density of the network are the beliefs of the producer. By imposing some (additional) reasonable condition on the beliefs of the producer, one would obtain that the provision of quality is always monotonic. The second condition in Theorem 3.3, namely that $vc(g') \leq vc(g)$, plays a role in determining the threshold levels θ_1 and θ_2. Intuitively, as $vc(g')$ does not get smaller, the threshold levels do not get greater (ceteris paribus). It is easily seen that for "reasonable" beliefs that condition holds without any problem. The first condition, namely that $\pi^0(g') \geq \pi^0(g)$, may not be true even for reasonable beliefs, given a problem of free-riding of information on the consumers' side when quality levels are bad. Next example illustrates this phenomenon.

Example 3. Let $n = 8$ and consider the two network structures in Figure 4.

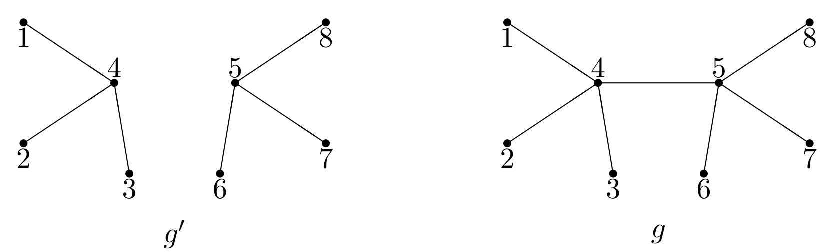

FIGURE 4

As it is seen from Figure 3, and assuming the producer assigns uniform probabilities everywhere:[8]

1. $vc(g') = 4$, $D(g') = 2$, $n_1(g') = 8$, $n_2(g') = 4$ and $\varsigma(g') = 2$.

[8]The corresponding sets are $\mathcal{N}_2(g') = \{\{1,2,3,6,7,8\},\{1,2,3,5\},\{4,6,7,8\},\{4,5\}\}$, on one hand, and $\mathcal{N}_2(g) = \{\{1,2,3,6,7,8\},\{1,2,3,4\},\{4,6,7,8\}\}$, and $\mathcal{N}_3(g) = \{\{1,6\},\{1,7\},\{1,8\},\{2,6\},\{2,7\},\{2,8\},\{3,6\},\{3,7\},\{3,8\},\{4\},\{5\}\}$, on the other hand.

2. $vc(g) = 4$, $D(g) = 3$, $n_1(g) = 8$, $n_2(g) = \frac{14}{3}$, $n_3(g) = \frac{20}{11}$ and $\varsigma(g) = 1$.

Note that free-riding of information gets very clearly illustrated by the possible continuation equilibria corresponding to distance 2. Consider the sets $\mathcal{N}_2(g') = \{\{1,2,3,6,7,8\}, \{1,2,3,5\}, \ \{4,6,7,8\}, \{4,5\}\}$ and $\mathcal{N}_2(g) = \{\{1,2,3,6,7,8\}, \{1,2,3,4\}, \{4,6,7,8\}\}$, which correspond to the continuation equilibria at g' and g, respectively, when the producer chooses a $\theta < \bar{\theta}$ and a price p such that $\delta\theta + (1-\delta)\bar{\theta} < p \leq \delta^2\theta + (1-\delta^2)\bar{\theta}$. In g', when agents 4 and 5, who are both centers of their respective four-agent stars, are disconnected, there is an equilibrium where only 4 and 5 buy for $\theta < \bar{\theta}$ and $\delta\theta + (1-\delta)\bar{\theta} < p \leq \delta^2\theta + (1-\delta^2)\bar{\theta}$. This equilibrium configuration is the worst for the producer, as it implies less quantity sold. Furthermore, in this equilibrium configuration the maximum number of agents free-ride information, since in the other possible equilibrium configurations there are at least four consumers who buy. By free-riding of information in this situation I mean the following. Recall that the only way to obtain information about the actual quality level θ is acquiring the good and experiencing it yourself. Therefore, the price of the good can be interpreted as the cost of acquiring information. The cost of information is too high if the price chosen by the producer is higher than the actual quality level. This is happening when quality levels are bad, meaning lower than the initial willingness-to-pay. Free-riding means that some agents buy the good, experience the quality level and tell their acquaintances that quality level is below the price level. These latter will not buy the good, therefore not experience a loss of utility, since they do not acquire the information themselves. In the equilibrium configuration where only consumers 4 and 5 acquire the good, consumers 1, 2 and 3 learn from 4 that it is not worth it paying the price without acquiring the good themselves, or, in other words, without paying the cost for information (i.e., the bad price). The same applies to 6, 7 and 8 with respect to 5. When considering g, this equilibrium configuration disappears, while the others stay the same. This means that, at the denser structure g, if agent 4 buys, 5 starts free-riding from her, as do 1, 2 and 3, and vice versa. As 5 starts free-riding, then consumers 6, 7 and 8 don't get first-hand information, but second-hand information already, and therefore decide to buy the good. So any beliefs on the producers' side that assign a positive probability for $\{4,5\}$ in $\mathcal{N}_2(g')$ may result in higher expected profit for g' than for g at $\theta = 0$ if $k^0 = 2$ for g.

Note that if the producer chooses a $\theta = 0$ and the structure is equal to g' it is always better for him to choose a distance equal to 1 than to 2 (i.e., distance 2 is dominated by 1 at g'). The reader may check that if

$\frac{3}{4} < \delta < \frac{2\sqrt{7}}{7}$ then $\pi^0(g') = 2 < \frac{14}{3}(1-\delta^2) = \pi^0(g)$, meaning that the producer chooses a distance equal to ∞ (having one consumer buying per component) when $\theta = 0$ at g', while he chooses a distance equal to 2 at g. In such a case, for $\frac{3}{4} \leq c \leq \delta$ and $\theta_2^T(g') < \theta^{max} < \theta_2^T(g)$, $\theta^*(g') = \theta^{max}$ and $\theta^*(g) = 0$. This means that for intermediate values of δ and c, the producer switches earlier from 0 to θ^{max} at g' than at g. This is due to the fact that the worst configuration equilibrium at distance 2 for bad quality levels does not appear at g. This makes distance 2 too profitable at g for the lowest quality level in this range of the parameters.[9]

In summary, free-riding on information has a negative effect on consumers' welfare (they are better the higher the quality level) as follows. When quality levels are lower than the reputation (note that this is possible only if the reputation level is strictly positive), information is costly. Consumers free-ride information in the sense that they rely on the information coming from the network instead of paying for first-hand information (situation equivalent to consuming the good). So more links may lead to core agents in the network to free-ride (as compare to the network without those links). The peripheral agents connected to the core agents who are now free-riding decide thus to buy instead of free-riding (this comes also from the decay of information) in the denser network. When new links are created among agents that are already very connected to other people, one agent who did not "free-ride" may start doing it with this new connection. The producer benefits from that, as this free-riding effect results in more agents buying low quality levels for denser networks. This eventually may result in low quality levels being too profitable for the producer for denser networks. Next theorem states one possible way of getting rid of this puzzle.

In order to state next theorem, I need the following definitions. Let $e(\theta, p) = max_{S \in\ Q(\theta,p)} |S|$ be the maximum number of consumers in a continuation equilibrium after θ and p. Then, one can define a measure of optimism for the producer (or an ability for coordination on the consumers' side) as $\tilde{\rho}(\theta, p) = \sum_{S \in Q(\theta,p):\ |S| = e(\theta,p)} \rho(S, Q(\theta, p))$, or, in words, $\tilde{\rho}(\theta, p)$ is the probability with which the producer believes consumers are coordinating in the most favorable equilibria for him.

Theorem 3.4 *Let $\tilde{\rho}(\theta, p)$ be close enough to 1 for all choices of θ and p. Then, $\theta^*(g') \leq \theta^*(g)$ for any g, g' such that $g' \subseteq g$.*

[9] For the sake of completeness, $\theta_2^T(g') = \frac{2\delta - 1}{2(\delta - c)} \bar{\theta}$ and $\theta_2^T(g) = \frac{6\delta + 1 - 7\delta^2}{6(\delta - c)} \bar{\theta}$.

Theorem 3.4 identifies the coordination problem as one of the consequences of the failure of the social structure to provide higher quality levels, as consumers are getting more and more acquaintances. The proof can be found in the appendix.

Note that if (i) θ^{max} tends to infinity, i.e., if there are no technological constraints, or if (ii) $\bar{\theta} = 0$, i.e., there is no reputation level, then, by Theorem 3.2, the producer always chooses θ^{max} for $c < \delta$ and 0 for $c > \delta$; so that the choice is independent of the social structure.

These results give us the insight that the friction on the consumers side, namely the coordination problem, are causing the choice of quality by the producer to be non monotonic on the density of the network. Note that $\bar{\theta}$ being strictly positive has a similar effect too. Both facts can again be interpreted as the result of free-riding of information in the consumers' side. When the producer is optimistic, he assigns a probability close to 1 to the continuation equilibria where less free-riding of information is happening, and free-riding is relevant only for quality levels below $\bar{\theta}$.

4 Concluding Remarks

I have presented a model to study the way information sharing by consumers through word-of-mouth gives incentives for provision of quality in the context of a market with asymmetric information.[10] The most striking result lies on the fact that the choice of quality by the producer is not monotonic on the density of the network, due to free-riding on information on the consumers' side. As final remarks, I would like to comment on two issues on the definition of consumer's continuation equilibria. First, the fact that consumers update their willingness-to-pay based only on their distance to the closest consumer who is buying in equilibrium, and second, the fact that the equilibrium concept is a static notion, creating what has been called "problems of coordination".

With respect to the first issue recall that, in the definition of network-consistent beliefs, the willingness-to-pay for a consumer in equilibrium has to be

$$\delta^{d(i,j;g)}\theta + \left(1 - \delta^{d(i,j;g)}\right)\bar{\theta}, \tag{12}$$

[10]Here, asymmetric information means that one side of the market has more information than the other one, as the producer knows the quality, but consumers don't.

where $\delta \in (0, 1)$ is the parameter measuring the decay of information and $d(i, j; g)$ is the geodesic distance between i and the closest consumer in $N \backslash \{i\}$ who buys the product, denoted j. A way of generalizing this idea could be to substitute the expression in (12) by

$$\lambda_{i,g}\theta + (1 - \lambda_{i,g})\,\bar{\theta},$$

where $\lambda_{i,g}$ depends not only the distance to the closest buyer in equilibrium, but on the number of buyers to whom consumer i is connected in equilibrium and the corresponding distances. Or even on the number of different paths through which she would obtain information in equilibrium and their respective lengths.

Note that the expression for network-consistent beliefs in (12) is what reduces the producer's pricing decision to choosing distances on the network when quality levels are low. Specifying some other structure for the effect of the word-of-mouth reduces the producer's pricing decision to choosing objects on the network, as before, depending not only on distances, but also on the number of connections. This also separates prices into different intervals, as in Lemma 3.1 prices are separated in terms of distances, depending on the possible λ's that could arise on the given network structure. So the problem will be solved similarly, but with a structure depending on the particular structure for λ that we would choose.

About problems of free-riding, notice that this effect would still hold, but it would arise for more restricted intervals of the parameters. Think again of Example 3 and consider distance 2. Recall that $\mathcal{N}_2(g') = \{\{1, 2, 3, 6, 7, 8\}, \{1, 2, 3, 5\}, \{4, 6, 7, 8\}, \{4, 5\}\}$ and $\mathcal{N}_2(g) = \{\{1, 2, 3, 6, 7, 8\}, \{1, 2, 3, 4\}, \{4, 6, 7, 8\}\}$. Any choice of quality and price where the set $\{1, 2, 3, 5\}$ is a continuation equilibrium for g' would result in the rest of $\mathcal{N}_2(g')$ being a continuation equilibrium too, even if we consider more sophisticated specifications for $\lambda_{i,g}$ in the definition of network-consistent beliefs. The reason works as follows. If $\{1, 2, 3, 5\}$ are buying for a given choice of θ and p this means that this choice of quality and price generates the following features in equilibrium:

1. Consumers who do not have any source of information would buy, as 5 does

2. Consumers who have at least one direct friend who buy do not want to buy, as 6, 7 and 8 do not buy when 5 does, and 4 does not buy when 1, 2 and 3 do (getting three people buying makes the information to be more accurate, so if one friend distance one is enough to prevent a consumer from buying, three friends with distance one will prevent more intensively a consumer from buying)

3. Consumers who have at least two indirect friends who buy still want to buy (as 1 buys when 2 and 3 do, etc)

The case for $\{4, 6, 7, 8\}$ is equal, and the case for $\{1, 2, 3, 6, 7, 8\}$ uses the two last arguments above. Finally, the set $\{4, 5\}$ uses the first two arguments above.

If agents 4 and 5 get connected, as it happened in Example 3, then when 4 buys, by the second feature of the equilibrium that is generated 5 will not buy once she is connected to 4. It remains to determine whether 6, 7 and 8 would buy in such a case. But, by the last feature above, we know that at least two indirect friends buying does not prevent from buying, and therefore at least two consumers out of 6, 7 and 8 would buy. This means that the worst equilibrium for the producer, namely $\{4, 5\}$, given a particular choice of quality and price, disappears due to free-riding effects, as in the case when we only consider $\lambda_{i,g}$ to depend only on the distance to the closest consumer who is buying. Depending on how we specify $\lambda_{i,g}$ and how the maximal and minimal of those λ's relate with c we can still obtain non monotonicity of the choice of quality by the producer on the density of g due to free-riding on information by consumers when quality levels are low.

Finally, note that the equilibrium concept is a static one, and this generates multiplicity of equilibria. As said before, this can be interpreted as each different possible equilibrium configuration corresponding to different orders of consumers arriving at the shop or office, once the quality and price are fixed.[11] Some future research could be done on different dynamical processes, where consumers would learn about the quality of the good from previous clients and afterwards they would decide to acquire the good or not. Note that in such a case, all consumers will behave the same in equilibrium. The reason works as follows. At time 0 consumers have a common initial willingness-to-pay. If price is higher than such initial willingness-to-pay nobody acquires the good, as information transmission takes place after acquiring the good. Then, next period all consumers start with the same common initial willingness-to-pay, as no purchase implies no information transmission. If price is lower than the initial willingness-to-pay, then everybody will buy the good and the quality level is known for all of them. In both cases, consumers jump to next period of time having all of them the same piece of information. The only way to get heterogeneity about the consumers' willingness-to-pay is by specifying different updating rules, when

[11] Note that two different orders could give the same final configuration.

there is a common initial willingness-to-pay, or different initial willingness-to-pay.[12] In both cases, the problem of multiplicity is ruled out, but at the cost of specifying different updating rules or initial willingness-to-pay. The question of whether the producer is aware of all that different information from consumers may be difficult to justify.

An alternative model that is more adjusted to the example for service providers would be as follows. Assume that consumers have same updating rules and initial willingness-to-pay, but that with a probability q a consumer i needs the good and then the utility function structure of the model applies, and with probability $1-q$ he does not. Think, for example, of a lawyer. With some probability q any consumer in the network may have an on-the-job accident and need a lawyer against the company he works for. By simplicity, assume these probabilities are equal for all consumers and the event of "needing a lawyer" is independent across consumers. Then, we obtain equilibrium configurations on the network with "holes", meaning that we obtain individuals at distance two purchasing, or at distance three, etc, in the network. We will not get rid of multiplicity but this specification will provide a richer structure on the producer's beliefs. Further research may be done on that, as more structure on beliefs may allow for the introduction of competition on the producer's side.

For any of the two alternatives, it seems intuitively clear that free-riding of information will still happen on the consumers' side when quality levels are low. It is left to determine whether the effects of it are so important as to have an impact on the choice of quality levels, as it happens in the model developed in this paper here.

Appendix

Proof of Theorem 3.2.

In order to present the proof of this result, I simplify the analysis by assuming that the producer takes a two-step decision: first, he decides the level of quality θ^*, and afterwards, he chooses price. Note that the final resulting decision by the producer is the same as when he chooses both at the same time. The profit function in equilibrium $\pi^*(\theta)$ is built by fixing the quality level and letting the producer choose the price maximizing profit given this

[12] By different updating rules I mean that their functional form or specification differ from one consumer to another.

quality level and consistent beliefs. In other words,

$$\pi^*(\theta) = \max_p q^d(\theta, p)(p - c\theta). \tag{13}$$

I make use of the following Claim.

Claim 1. The expected profit function $\pi^*(\theta)$ is continuous and such that:

1. Let $c < \delta$. Then, there is a **unique** interior local maximum at $\bar{\theta}$, and a local minimum at the right of $\bar{\theta}$, after which $\pi^*(\theta)$ is strictly increasing.

2. Let $\delta < c < 1$. Then, $\pi^*(\theta)$ is continuous and decreasing for all θ.

Proof of Claim 1. Consider first the continuation equilibrium when the producer chooses a θ such that $\theta > \bar{\theta}$. If he chooses a price p such that $\bar{\theta} < p \leq \delta\theta + (1-\delta)\bar{\theta}$, by Lemma 3.1, all these prices would yield the same continuation equilibrium, namely $\mathcal{VC}(g)$. As the producer maximizes profit, if there is any equilibrium with $Q(\theta, p) = \mathcal{VC}(g)$ it has to correspond to the highest price p such that the expected number of consumers is $vc(g)$. This price corresponds to $p = \delta\theta + (1-\delta)\bar{\theta}$.

If $p \leq \bar{\theta}$, then, by Lemma 3.1 again, the number of consumers is equal to n. But then, if there is one equilibrium with $Q(\theta, p) = \{N\}$, it has to be that $p^* = \bar{\theta}$, as the producer maximizes profit.

Therefore, for a given $\theta \geq \bar{\theta}$, the continuation equilibrium is such that $p = \bar{\theta}$ and $Q\left(\theta, \bar{\theta}\right) = \{N\}$ if and only if

$$n\left[\bar{\theta} - c\theta\right] \geq vc(g)\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right]. \tag{14}$$

Note that equation (14) holds if $\bar{\theta} \leq \theta \leq \frac{n - vc(g) + \delta vc(g)}{c(n - vc(g)) + \delta vc(g)}\bar{\theta}$.

On the contrary, if $\theta \geq \frac{n - vc(g) + \delta vc(g)}{c(n - vc(g)) + \delta vc(g)}\bar{\theta}$, then,

$$n\left[\bar{\theta} - c\theta\right] \leq vc(g)\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right], \tag{15}$$

which implies that the producer chooses a price equal to $\delta\theta + (1-\delta)\bar{\theta}$, and consumers coordinate on a configuration in $\mathcal{VC}(g)$. All this implies that the expected profit in the continuation equilibrium for a given $\theta \geq \bar{\theta}$ is

$$\pi^*(\theta) = \begin{cases} n\left[\bar{\theta} - c\theta\right], & \text{if } \bar{\theta} \leq \theta \leq \frac{n - vc(g) + \delta vc(g)}{c(n - vc(g)) + \delta vc(g)}\bar{\theta} \\ vc(g)\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right], & \text{otherwise.} \end{cases} \tag{16}$$

This is to say that $\pi^* (\theta)$ is decreasing in a small neighborhood at the right of $\bar{\theta}$. Furthermore, if $c < \delta$, $\pi^* (\theta)$ is increasing at the right of $\frac{n-vc(g)+\delta vc(g)}{c(n-vc(g))+\delta vc(g)}\bar{\theta}$, and therefore there is a local minimum at $\frac{n-vc(g)+\delta vc(g)}{c(n-vc(g))+\delta vc(g)}\bar{\theta}$. On the other hand, if $c > \delta$, then, the function is always decreasing at the right of $\bar{\theta}$.

Consider now the case when $\bar{\theta} > \theta$. If the producer chooses a price p such that $\delta^{k-1}\theta + \left(1-\delta^{k-1}\right)\bar{\theta} < p \leq \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$, for some $k \in \{1, 2, ..., D(g)\}$, then, by Lemma 3.1, the expected number of consumers is $n_k(g)$. Recall that $n_k(g)$ denotes the expected number of consumers if $Q(\theta, p) = \mathcal{N}_k(g)$, i.e.,

$$n_k(g) = \sum_{S \in \mathcal{N}_k(g)} \rho(S, \mathcal{N}_k(g))|S|, \tag{17}$$

where $\rho(S, \mathcal{N}_k(g)) > 0$, for all $S \in \mathcal{N}_k(g)$, and $\sum_{S \in \mathcal{N}_k(g)} \rho(S, \mathcal{N}_k(g)) = 1$. Note that, by definition, $\varsigma(g) \leq n_k(g) \leq \max_{S \in \mathcal{N}_k(g)} |S|$. As for all prices p with $\delta^{k-1}\theta + \left(1-\delta^{k-1}\right)\bar{\theta} < p \leq \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$ the expected number of consumers is the same, if there is any equilibrium with $Q(\theta, p) = \mathcal{N}_k(g)$ it has to correspond to the highest price p such that the expected number of consumers is $n_k(g)$. It is easily seen that this price is $p = \delta^k\theta + \left(1-\delta^k\right)\bar{\theta}$. Furthermore, by definition, $n_k(g) = \varsigma(g)$ if and only if $k > D(g)$. Therefore, for any price p such that $\delta^{D(g)}\theta + \left(1-\delta^{D(g)}\right)\bar{\theta} < p \leq \bar{\theta}$, the expected number of consumers is $\varsigma(g)$. As the producer is maximizing, if there is any continuation equilibrium with $\theta < \bar{\theta}$ and with $\varsigma(g)$ as the expected number of consumers, it has to correspond to $p = \bar{\theta}$, or, in other terms, $p = \delta^\infty\theta + (1-\delta^\infty)\bar{\theta}$. All this implies, given $\theta < \bar{\theta}$, that

$$\pi^* (\theta) = \max_{k \in \{1, \ldots, D(g), \infty\}} n_k(g) \left[\left(\delta^k - c\right)\theta + \left(1-\delta^k\right)\bar{\theta}\right]. \tag{18}$$

Hence, the function $\pi^* (\theta)$ for $\theta < \bar{\theta}$ is continuous as it is an upper envelope of affine functions. Furthermore, $\pi^* (\theta)$ is always positive since $c\theta < \theta < \bar{\theta}$, for $c < 1$, and the producer could always choose a price equal to $\bar{\theta}$, getting at least one buyer.

Note that, from (16), the function $\pi^* (\theta)$ is decreasing to the right of $\bar{\theta}$ (in a small neighborhood, if $c \leq \delta$ or all the way to the right if $c > \delta$). Consider $c > \delta$. This means that $c > \delta^k$, for any possible distance k in the social network g. From (18) we know that the function $\pi^* (\theta)$ is equal to $n_k \left[\left(\delta^k - c\right)\theta + (1-\delta)\bar{\theta}\right]$, for some distance k in the network if $\theta < \bar{\theta}$. This implies that no matter how the specific expression for $\pi^*(\theta)$ looks like, it is always decreasing on θ, as $c > \delta^k$, for all $k \in [1, 2, ..., D(g), \infty]$.

In order to see continuity for $\pi^*(\theta)$ at $\bar{\theta}$, recall, on one hand, that $\pi^*(\theta)$ tends to $n(1-c)\bar{\theta}$ as θ approaches $\bar{\theta}$ from the right. On the other hand, as θ approaches $\bar{\theta}$ from the left, the value of $\pi^*(\theta)$ approaches

$$\max_{k \in \{1,...,D(g),\infty\}} n_k(g)(1-c)\bar{\theta} = (1-c)\bar{\theta} \max_{k \in \{1,...,D(g),\infty\}} n_k(g),$$

since $\left(\delta^k - c\right)\theta + \left(1-\delta^k\right)\bar{\theta}$ tends to $(1-c)\bar{\theta}$ as θ tends to $\bar{\theta}$, for any k. But, by definition, the maximum value of $n_k(g)$ is always equal to n, the one corresponding to $k = 1$. Therefore, $\pi^*(\theta)$ tends to $n\left[(\delta - c)\theta + (1-\delta)\bar{\theta}\right]$ as θ tends to $\bar{\theta}$ from the left. This implies two things: (i) that $\pi^*(\theta)$ is continuous at $\theta = \bar{\theta}$, and (ii) that, for $c \leq \delta$, the function $\pi^*(\theta)$ is increasing in a small neighborhood to the left of $\bar{\theta}$, and decreasing in a small neighborhood to the right of $\bar{\theta}$ (for such a case, recall that the value of the function is equal to $n\left(\bar{\theta} - c\theta\right)$). In other words, $\bar{\theta}$ is a local maximum for $\pi^*(\theta)$.

Up to now it has been shown that the function $\pi^*(\theta)$ is continuous and decreasing for all θ, if $\delta < c < 1$, and that there is an interior local maximum at $\bar{\theta}$ and a local minimum $m > \bar{\theta}$ such that $\pi^*(\theta)$ is increasing for any $\theta > m$, if $c < \delta$. It remains to prove that $\bar{\theta}$ is the *unique* interior local maximum, for $c \leq \delta$. In order to do that, the following two claims are needed.

Previously to stating and proving the next two claims, I need to introduce the following notation. Recall that, for any $\theta < \bar{\theta}$,

$$\pi^*(\theta) = n_k(g)\left[\left(\delta^k - c\right)\theta + \left(1-\delta^k\right)\bar{\theta}\right],$$

for one $k \in \{1, ..., D(g), \infty\}$. Let $k(\theta)$ be such a k.

Claim 1.1. Let θ_1 and θ_2 be two quality levels such that $\theta_2 < \theta_1 < \bar{\theta}$. Then,

1. $k(\theta_1) \leq k(\theta_2)$.
2. $\pi^*(\theta_1) > \pi^*(\theta_2)$ if $c < \delta^{k(\theta_2)} \leq \delta^{k(\theta_1)}$.
3. $\pi^*(\theta_1) < \pi^*(\theta_2)$ if $\delta^{k(\theta_2)} \leq \delta^{k(\theta_1)} < c$.

Proof of Claim 1.1. I show part 1 of Claim 1.1 first. As the producer maximizes profits in the continuation equilibrium after θ_1,

$$n_{k(\theta_1)}(g)\left[\left(\delta^{k(\theta_1)} - c\right)\theta_1 + \left(1-\delta^{k(\theta_1)}\right)\bar{\theta}\right] \geq n_s(g)\left[(\delta^s - c)\theta_1 + (1-\delta^s)\bar{\theta}\right],$$

for every $s \in \{1, ..., D(g), \infty\}$. In particular, $k(\theta_2) \in \{1, ..., D(g), \infty\}$. Hence,

$$n_{k(\theta_1)}(g)\left[\left(\delta^{k(\theta_1)} - c\right)\theta_1 + \left(1 - \delta^{k(\theta_1)}\right)\bar{\theta}\right] \geq$$

$$\geq n_{k(\theta_2)}(g)\left[\left(\delta^{k(\theta_2)} - c\right)\theta_1 + \left(1 - \delta^{k(\theta_2)}\right)\bar{\theta}\right].$$

As $\theta_1 > \theta_2$, this implies that

$$\begin{aligned} &n_{k(\theta_1)}(g)\left[\left(\delta^{k(\theta_1)} - c\right)\theta_1 + \left(1 - \delta^{k(\theta_1)}\right)\bar{\theta}\right] > \\ &> n_{k(\theta_2)}(g)\left[\left(\delta^{k(\theta_2)} - c\right)\theta_2 + \left(1 - \delta^{k(\theta_2)}\right)\bar{\theta}\right], \end{aligned} \tag{19}$$

if $\delta^{k(\theta_2)} > c$. Similarly, as the producer is maximizing profit,

$$n_{k(\theta_2)}(g)\left[\left(\delta^{k(\theta_2)} - c\right)\theta_2 + \left(1 - \delta^{k(\theta_2)}\right)\bar{\theta}\right] \geq n_s(g)\left[(\delta^s - c)\theta_2 + (1 - \delta^s)\bar{\theta}\right],$$

for every $s \in \{1, ..., D(g), \infty\}$. In particular, $k(\theta_1) \in \{1, ..., D(g), \infty\}$ and therefore,

$$n_{k(\theta_2)}(g)\left[\left(\delta^{k(\theta_2)} - c\right)\theta_2 + \left(1 - \delta^{k(\theta_2)}\right)\bar{\theta}\right] \geq$$

$$\geq n_{k(\theta_1)}(g)\left[\left(\delta^{k(\theta_1)} - c\right)\theta_2 + \left(1 - \delta^{k(\theta_1)}\right)\bar{\theta}\right].$$

As $\theta_1 > \theta_2$, this implies that

$$\begin{aligned} &n_{k(\theta_2)}(g)\left[\left(\delta^{k(\theta_2)} - c\right)\theta_2 + \left(1 - \delta^{k(\theta_2)}\right)\bar{\theta}\right] > \\ &> n_{k(\theta_1)}(g)\left[\left(\delta^{k(\theta_1)} - c\right)\theta_1 + \left(1 - \delta^{k(\theta_1)}\right)\bar{\theta}\right], \end{aligned} \tag{20}$$

if $\delta^{k(\theta_1)} < c$.

Note that equations (19) and (20) are contradicting each other. Thus, if $c < \delta^{k(\theta_2)}$, then (19) holds and it cannot be that $c > \delta^{k(\theta_1)}$, or, otherwise, (20) would hold. On the other hand, if $c > \delta^{k(\theta_1)}$, then it cannot be that $c < \delta^{k(\theta_2)}$. Therefore, $c < \delta^{k(\theta_2)}$ implies that $c < \delta^{k(\theta_1)}$, and $c > \delta^{k(\theta_1)}$ implies that $c > \delta^{k(\theta_2)}$, which means that $\delta^{k(\theta_1)} \geq \delta^{k(\theta_2)}$, or, in other words, $k(\theta_1) \leq k(\theta_2)$, as $\delta \leq 1$. This completes the proof of part 1 in Claim 1.1.

I proceed to show parts 2 and 3. Note that equation (19) means that $\pi^*(\theta_1) > \pi^*(\theta_2)$, and it holds for $c < \delta^{k(\theta_2)}$. On the other hand, (20) means that $\pi^*(\theta_1) < \pi^*(\theta_2)$, and it holds for $c > \delta^{k(\theta_1)}$. This completes the proof of Claim 1.1.

Claim 1.1 then states that the function $k(\theta)$ is weakly decreasing on θ, and that the function $\pi^*(\theta)$ can be increasing or decreasing depending on the value of c relative to $\delta^{k(\theta)}$, for $\theta < \bar{\theta}$.

The following claim completes the results of monotonicity of $\pi^*(\theta)$.

Claim 1.2. If $\pi^*(\theta)$ is increasing at the right of some $\theta' < \bar{\theta}$, then, it is increasing for any θ such that $\theta' < \theta \leq \bar{\theta}$.

Proof of Claim 1.2. By part 2 of Claim 1.1, $c < \delta^{k(\theta')}$ as $\pi^*(\theta)$ is increasing at the right of θ'. On the other hand, by part 1 of Claim 1.1, $k(\theta) \leq k(\theta')$, for every θ such that $\theta' < \theta \leq \bar{\theta}$. This implies that $\delta^{k(\theta)} \geq \delta^{k(\theta')} > c$, for $\theta' < \theta \leq \bar{\theta}$. But, then, by Claim 1.1, part 2, $\pi^*(\theta)$ is increasing for every θ in $\left(\theta', \bar{\theta}\right]$. This completes the proof of Claim 1.2.

Claim 1.2 implies that, if there is a $\theta < \bar{\theta}$ being an interior local maximum then there is a θ' such that the function $\pi^*(\theta)$ is increasing at the right of θ', and there exists a θ'' at the right of θ', namely the local maximum $\theta \neq \bar{\theta}$, such that the function $\pi^*(\theta)$ is not increasing to the right of θ''. This would be contradicting Claim 1.2. We may thus conclude that there is no interior local maximum in $\left(0, \bar{\theta}\right)$. This together with (16) imply that the unique interior local maximum of $\pi^*(\theta)$, for $c < \delta$ is $\bar{\theta}$. This completes the proof of Claim 1.

By definition, the equilibrium outcome corresponds to the quality level θ maximizing the expected profit in the continuation equilibrium, $\pi^*(\theta)$, for θ in $[0, \theta^{max}]$. Fix first $c < \delta$. Since $\bar{\theta}$ is the only local maximum, I need to compare the value of the expected profit function at 0, at $\bar{\theta}$, and at θ^{max}.

Recall that $\pi^*(0) = \pi^0\bar{\theta}$, $\pi^*\left(\bar{\theta}\right) = n(1-c)\bar{\theta}$, and assume that $\theta^{max} \geq \bar{\theta}$, which implies that

$$\pi^*(\theta^{max}) = \begin{cases} n\left[\bar{\theta} - c\theta^{max}\right], & \text{if } \theta^{max} \leq \frac{n - vc(g) + \delta vc(g)}{c(n - vc(g)) + \delta vc(g)}\bar{\theta} \\ vc(g)\left[(\delta - c)\theta^{max} + (1-\delta)\bar{\theta}\right], & \text{otherwise.} \end{cases}$$

Hence, if θ^{max} lies in an interval where the function $\pi^*(\theta)$ is decreasing, then $\pi^*\left(\bar{\theta}\right) > \pi^*(\theta^{max})$.

Consider first $c < \frac{n-\pi^0}{n}$. Then, $\pi^*(0) < \pi^*\left(\bar{\theta}\right)$. Therefore, there is no equilibrium with $\theta^* = 0$. Furthermore, since the inequality $\pi^*\left(\bar{\theta}\right) < \pi^*(\theta^{max})$ can only be possible when θ^{max} lies on the increasing interval, there is a θ^T on the increasing interval at the right of $\bar{\theta}$ such that

$$n(1-c)\bar{\theta} = vc(g)\left[(\delta - c)\theta^T + (1-\delta)\bar{\theta}\right]. \tag{21}$$

This implies that $\theta^T = \frac{n(1-c)-vc(g)(1-\delta)}{vc(g)(\delta-c)}\bar{\theta} = \theta_1^T$. It is straightforward that if $\theta^{max} \geq \theta_1^T$, then, $\pi^*(\theta)$ is maximized at θ^{max}, and it is maximized at $\bar{\theta}$ otherwise. If $\theta^* = \theta^{max}$ then $p^* = \delta\theta^{max} + (1-\delta)\bar{\theta}$, and $Q^* = \mathcal{VC}(g)$. On the other hand, if $\theta^{max} = \bar{\theta}$ then $p^* = \bar{\theta}$ and $Q^* = \{N\}$.

Let $\frac{n-\pi^0}{n} < c < \delta$. Then, $\pi^*(0) > \pi^*\left(\bar{\theta}\right)$. Hence, there is no equilibrium with $\theta^* = \bar{\theta}$. Furthermore, since $\pi^*\left(\bar{\theta}\right) < \pi^*\left(\theta^{max}\right)$ can only be possible when θ^{max} lies on the increasing interval at the right of $\bar{\theta}$, there is a θ^T on the increasing interval at the right of $\bar{\theta}$ such that

$$\pi^0\bar{\theta} = vc(g)\left[(\delta-c)\,\theta^T + (1-\delta)\,\bar{\theta}\right]. \tag{22}$$

This implies that $\theta^T = \frac{\pi^0-vc(g)(1-\delta)}{vc(g)(\delta-c)}\bar{\theta} = \theta_2^T$. Again, it is clear that if $\theta^{max} \geq \theta_2^T$, then, $\pi^*(\theta)$ is maximized at θ^{max}, and it is maximized at 0 otherwise. Again, recall, that if $\theta^* = \theta^{max}$ then $p^* = \delta\theta^{max} + (1-\delta)\bar{\theta}$, and $Q^* = \mathcal{VC}(g)$. On the other hand, if $\theta^{max} = 0$ then $p^* = \left(1-\delta^{k^0}\right)\bar{\theta}$ and $Q^* = \{\mathcal{N}_{k^0}\}$.

Finally, let $c > \delta$. As the function $\pi^*(\theta)$ is continuous and decreasing for all θ in this case, the producer chooses 0 quality, price equal to $\left(1-\delta^{k^0}\right)\bar{\theta}$, and the expected demand is equal to $n_{k^0}(g)$.

This completes the proof of Theorem 3.2. □

Proof of Theorem 3.3

The proof is made by checking case by case.

Assume that $\pi^0(g) \leq \pi^0(g')$. This means that $\frac{n-\pi^0(g)}{n} \geq \frac{n-\pi^0(g')}{n}$. We distinguish the following cases.

1. Let $c < \frac{n-\pi^0(g')}{n}$ and let $\theta_1^T(g) = \frac{n(1-c)-vc(g)(1-\delta)}{vc(g)(\delta-c)}\bar{\theta}$ and $\theta_1^T(g') = \frac{n(1-c)-vc(g')(1-\delta)}{vc(g')(\delta-c)}\bar{\theta}$. Since $vc(g') \leq vc(g)$, $\theta_1^T(g) \leq \theta_1^T(g')$, and from Theorem 3.2:

   (a) If $\theta^{max} \leq \theta_1^T(g)$, then $\theta^*(g) = \theta^*(g') = \bar{\theta}$.

   (b) If $\theta_1^T(g) < \theta^{max} \leq \theta_1^T(g')$, then $\theta^*(g) = \theta^{max}$ and $\theta^*(g') = \bar{\theta}$.

   (c) If $\theta_1^T(g') \leq \theta^{max}$, then $\theta^*(g) = \theta^*(g') = \theta^{max}$.

   Therefore, $\theta^*(g) \geq \theta^*(g')$, if $c < \frac{n-\pi^0(g')}{n}$.

2. Let $\frac{n-\pi^0(g')}{n} < c < \frac{n-\pi^0(g)}{n}$ and let $\theta_1^T(g) = \frac{n(1-c)-vc(g)(1-\delta)}{vc(g)(\delta-c)}\bar{\theta}$ and $\theta_2^T(g') = \frac{\pi^0(g')-vc(g')(1-\delta)}{vc(g')(\delta-c)}\bar{\theta}$. Note that $vc(g') \leq vc(g)$ and $c > \frac{n-\pi^0(g')}{n}$ imply that $\theta_1^T(g) \leq \theta_2^T(g')$, and from Theorem 3.2:

   (a) If $\theta^{max} \leq \theta_1^T(g)$, then $\theta^*(g) = \bar{\theta}$ and $\theta^*(g') = 0$.

   (b) If $\theta_1^T(g) < \theta^{max} \leq \theta_2^T(g')$, then $\theta^*(g) = \theta^{max}$ and $\theta^*(g') = 0$.

   (c) If $\theta_2^T(g') \leq \theta^{max}$, then $\theta^*(g) = \theta^*(g') = \theta^{max}$.

   Therefore, $\theta^*(g) \geq \theta^*(g')$, if $\frac{n-\pi^0(g')}{n} < c < \frac{n-\pi^0(g)}{n}$.

3. Finally, let $\frac{n-\pi^0(g)}{n} < c$ and let $\theta_2^T(g) = \frac{\pi^0(g)-vc(g)(1-\delta)}{vc(g)(\delta-c)}\bar{\theta}$ and $\theta_2^T(g') = \frac{\pi^0(g')-vc(g')(1-\delta)}{vc(g')(\delta-c)}\bar{\theta}$. Note that $vc(g') \leq vc(g)$ and $\pi^0(g) \leq \pi^0(g')$ imply that $\theta_2^T(g) \leq \theta_2^T(g')$. Again, from Theorem 3.2:

   (a) If $\theta^{max} \leq \theta_2^T(g)$, then $\theta^*(g) = \theta^*(g') = 0$.

   (b) If $\theta_2^T(g) < \theta^{max} \leq \theta_2^T(g')$, then $\theta^*(g) = \theta^{max}$ and $\theta^*(g') = 0$.

   (c) If $\theta_2^T(g') \leq \theta^{max}$, then $\theta^*(g) = \theta^*(g') = \theta^{max}$.

Therefore, $\theta^*(g) \geq \theta^*(g')$, if $\pi^0(g) \leq \pi^*(g')$, independently from the value of c, given $c < \delta$.

On the other hand, if $c > \delta$, by Theorem 3.2, $\theta^*(g') = \theta^*(g) = 0$.
This completes the proof of Theorem 3.3. □

Proof of Theorem 3.4

I prove first that, as $\tilde{\rho}(\theta, p)$ tends to 1, for all θ and all p, $\pi^0(g') \leq \pi^0(g)$. Note that, as $\tilde{\rho}(\theta, p)$ tends to 1, the number $n_k(g) \to \max_{S \in \mathcal{N}_k(g)} |S|$. We prove the following claim.

Claim. For any g, g' with $g' \subset g$, and for any $S \in \mathcal{N}_k(g)$, there exists an $S' \in \mathcal{N}_k(g')$ such that $S \subseteq S'$.

Proof of Claim. By definition of $\mathcal{N}_k(g)$, if S is in $\mathcal{N}_k(g)$, then, (1) for all i, j in S, $d(i,j;g) \geq k$, and (2) for all i not in S there is a consumer j in S such that $d(i,j;g) < k$. Since $g' \subset g$, $d(i,j;g) \leq d(i,j;g')$ for all i, j in N and, in particular, for all i, j in S. But then (1) implies that for all i, j in S $d(i,j;g') \geq k$. If for all i not in S, there is a j in S such that $d(i,j;g') < k$, then $S \in \mathcal{N}_k(g')$. But, if this is not true, then there exist an i not in S such that for all j in S, the distance $d(i,j;g') \geq k$. Take $\tilde{S} = (S \cup i) \supset S$. It could be that either $\tilde{S} \in \mathcal{N}_k(g')$, or that there exists an h not in $\tilde{S}$ such that for all j in $\tilde{S}$, $d(h,j;g') \geq k$. Then, I add this consumer h to the set S. Eventually, I would continue adding consumers to the set S until the condition (2) for S' is satisfied. Therefore, there exists an $S' \in \mathcal{N}_k(g')$ such that $S \subseteq S'$. This completes the proof of Claim.

Since $argmax_{S\in N_k(g)}|S| \in \mathcal{N}_k(g)$, then, by the claim, there exists an $\tilde{S}$ in $\mathcal{N}_k(g')$ such that $argmax_{S\in N_k(g)}|S| \subseteq \tilde{S}$. By definition, $|\tilde{S}| \leq \max_{S'\in N_k(g')}|S'|$, and therefore, $\max_{S\in N_k(g)}|S| \leq \max_{S'\in N_k(g')}|S'|$.

Recall that, as $\tilde{\rho}(\theta,p)$ tends to 1, for all θ and p, then, $n_k(g)$ tends to $\max\limits_{S\in\mathcal{N}_k(g)}|S|$. By the claim, $n_k(g) \leq n_k(g')$, for all distances k, every time $g' \subset g$ and for $\tilde{\rho}(\theta,p)$ great enough. By definition,

$$\pi^0(g') = \max_{k\in\{1,\ldots,\infty\}} n_k(g')\left(1-\delta^k\right) = n_{k^0(g')}(g')\left(1-\delta^{k^0(g')}\right), \tag{23}$$

and

$$\pi^0(g) = \max_{k\in\{1,\ldots,\infty\}} n_k(g)\left(1-\delta^k\right) = n_{k^0(g)}(g)\left(1-\delta^{k^0(g)}\right). \tag{24}$$

Since $g' \subset g$, and for $\tilde{\rho}(\theta,p)$ big enough for all θ and p, $n_{k^0(g)}(g) \leq n_{k^0(g)}(g')$. Thus,

$$n_{k^0(g)}(g)\left(1-\delta^{k^0(g)}\right) \leq n_{k^0(g)}(g')\left(1-\delta^{k^0(g)}\right) \leq \max_{k\in\{1,\ldots,\infty\}} n_k(g')\left(1-\delta^k\right), \tag{25}$$

which, together with equations (23) and (24) implies that $\pi^0(g) \leq \pi^0(g')$.

In order to see that, as $\tilde{\rho}(\theta,p)$ tends to 1 for all (θ,p), $vc(g') \leq vc(g)$, it suffices to see that $vc(g') \to n(g')$ and $vc(g) \to n(g)$ as $\tilde{\rho}(\theta,p) \to 1$, $\forall(\theta,p)$. Note that $n(g') \leq n(g)$ if $g' \subseteq g$. By Theorem 3.3, $\theta^*(g') \leq \theta^*(g)$ as $\tilde{\rho}(\theta,p) \to 1$, $\forall(\theta,p)$, since $\pi^0(g') \geq \pi^0(g)$ and $n(g') \leq n(g)$, for $\tilde{\rho}(\theta,p)$ big enough for all θ and p. This completes the proof of Theorem 3.4. □

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