Intertemporal equilibrium with a resource bequest motive

Thierry Bréchet[*], Stéphane Lambrecht[†]

November 10, 2005

Abstract

In this paper we question the role of a joy-of-giving bequest motive of a privately-owned renewable resource for sustainability. We model an overlapping generations economy in which individuals are endowed with a renewable resource. This resource can be exploited at no cost by the young households and provided to production or bequeathed to the next generation. We highlight two main results. First, the mere existence of a bequest motive does not guarantee a sustainable outcome. Second, when the resource is preserved in equilibrium, its level does not necessarily coincide with the efficient one. Whether the resource stock is too high or too low the capital stock should be lower than the golden rule level.

Keywords: overlapping generations, joy-of-giving, natural resource, sustainability

JEL codes: D91; Q20; D64

Acknowledgements: Preliminary versions of this paper were presented at the SURED Conference on Sustainability and Natural Resource Dynamics (Ascona, Switzerland), at the EAERE Annual Conference (Budapest, Hungary) and at the Environmental Workshop at CORE. The authors thank Robert Cairns, Geir Asheim, Claude d'Aspremont, Paul Belleflamme, Pierre-André Jouvet, Philippe Michel, Gilles Rotillon, Katheline Schubert, Marc Germain and Vincent van Steenberghe for fruitful comments. The usual disclaimer applies.

[*]Center for Operations Research and Econometrics (CORE) and IAG, Chair Lhoist Berghmans in Environmental Economics and Management, Université catholique de Louvain, Voie du Roman Pays 34, B-1348 Louvain-la-Neuve, Belgium, e-mail: brechet@core.ucl.ac.be (corresponding author)

[†]Laboratoire MEDEE, Faculté des Sciences Economiques et Sociales, Université des Sciences et Technologies de Lille, 59655 Villeneuve d'Ascq, France, and Center for Operations Research and Econometrics (CORE), Université catholique de Louvain, Belgium; e-mail: stephane.lambrecht@univ-lille1.fr

1 Introduction

In this article we ask whether a privately-owned renewable productive resource can be conserved and managed optimally when households have a joy-of-giving (Andreoni, 1989) resource bequest motive.

In matter of environmental issues, it is probably exaggerate to say that all individuals are purely selfish. On the other hand, it may be equally unrealistic to assume that they have a perfectly universal concern for the entire posterity. It seems however reasonable to assume that they enjoy the idea to accomplish their duty regarding future generations because they experience a "warm glow" or a "joy-of-giving" from fulfilling their duty. Andreoni (1989) used this idea to model the so-called joy-of-giving bequest motive and applied it to charities giving and transfers inside the family. In this article, we investigate the micro-foundations of a growth model and build a framework in which individuals have some dose of interest for future generations. We use the joy-of-giving bequest motive to model the bequest of resource from one generation to the other. We model an overlapping generations (OLG) economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. However, the joy-of-giving bequest motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The exploited resource is combined with man-made capital and labor to produce a consumption/investment good. The issue of substitution between the forms of capital is addressed, as well as the issue of selfishness versus altruism and their implications on the opportunity sets left to future generations.

Despite the importance of bequests (Löfgren, 1991) there are few studies that incorporate them into the analysis. Hultkrantz (1992) examines the implications of bequests in an OLG economy with forest and timber bequest occurring in the form of unharvested forest stock. Amacher et al. (1999) introduce a more complex forest management with harvesting and silviculture investment. The representative landowner derives utility both from consuming and leaving a timber bequest for the next generation, the indirect utility of the next generation entering its own utility function. In all these papers (see also Jouvet et al., 2000), however, bequest motives always rely on the assumption of altruism à la Barro (1974). As Becker (1993) himself admits, this form of intergenerational concern requires human capacities that are beyond the capacities of the most prescient. The joy-of-giving bequest motive does not make this implicit assumption.

The paper is organized as follows. In section 2 we describe an OLG economy with physical capital and a renewable resource, in which households have a resource bequest motive. Section 3 and section 4 present the main two results of our analysis. Section 5 concludes.

2 An economy with a resource bequest motive

The economy is of the Diamond's (1965) type with a constant population, but with the two extensions of an extracted resource and a joy-of-giving bequest motive. The young households at time $t = 0$ hold the global stock of resource Z_{-1}. This stock is shared equally between the N first young: $z_{-1} = Z_{-1}/N$. This section presents the natural resource dynamics, the agents' and the firms' behavior and characterize the equilibrium.

2.1 The natural resource dynamics

We consider a renewable natural resource. Its stock is shared equally between the N first young individuals: $z_{-1} = Z_{-1}/N$. Let us first, in this section, describe the resource own dynamics, i.e without human exploitation. Tthe extraction decision will be studied in the next section. The equation which governs the evolution of each individual endowment in the renewable resource, with zero extraction, is given by $z_t = z_{t-1} + h\left(Z_{t-1}\right) z_{t-1}$, where Z_{t-1} is the aggregate resource stock inherited from time $t-1$, z_{t-1} is the individual stock inherited from time $t-1$ by each of the N time t young individuals and where the function $h\left(.\right)$ is the resource natural return. Since the function h is assumed linear, we can also write this equation as

$$z_t = z_{t-1} + Nh\left(z_{t-1}\right) z_{t-1} \tag{1}$$

We make the following hypotheses on the function h: $\exists z > 0 : h\left(z\right) = -1/N$ and $h''\left(z\right) < 0$, $\forall z$. We adopt the notation $H\left(.\right)$ for RHS of (1). Hence these dynamics of the individual resource stock in the absence of extraction are given by

$$z_t = H(z_{t-1}) z_{t-1} \tag{2}$$

Example 1 ***The quadratic specification*** - *Let us consider a quadratic specification for the total natural return which is added to the existing stock each period*

$$z_t = z_{t-1} + N(\mu - \nu z_{t-1}) z_{t-1} \tag{3}$$

These dynamics can be represented by a bell-shape curve. Their properties are the following. The stock z_{t-1} *must belong to the interval* $(0, z_{\max})$, *where* $z_{\max}$ *is the threshold value of* z_{t-1} *such that the total natural return is negative and annihilates all the existing stock:* $z_{\max} + N(\mu - \nu z_{\max}) z_{\max} = 0$, *i.e.* $z_{\max} = (N\nu)^{-1}(1 + N\mu)$. *The maximum of these dynamics is reached when* $z_{t-1} = \bar{z} = (2N\nu)^{-1}(1 + N\mu)$. *The steady state is given by* $z^{ne} = \mu/\nu$, *where the upper-script "ne" stands for "no extraction" of the resource. At the steady state* z^{ne}, *the total natural return is equal to zero. The steady state may be on any side of the bell-shape dynamics. If* $z^{ne} \leq \bar{z}$ *then the slope of the dynamics at the steady state is positive. If* $z^{ne} > \bar{z}$ *then the slope at the steady state is negative. The resource own dynamics are explosive only when the slope at the steady state is smaller than* -1. *This occurs under the following condition* $1 + N\mu - 2N\nu z^{ne} < -1 \Leftrightarrow \mu > 2/N$.

Insert here figure 1: Phase diagram for the natural resource.

2.2 Households' behavior

Each individual lives for two periods: youth and old age. He is endowed with one unit of labor which he supplies inelastically during his first period of life for a real wage w_t. He is also endowed with the total available individual resource stock $H(z_{t-1}) z_{t-1}$, composed of his parents' bequest z_{t-1} augmented by it natural return $Nh(z_{t-1}) z_{t-1}$. He decides how much to extract of this inherited stock. Extraction is costless. He provides the amount extracted e_t to the production process for a real price q_t. There are two possible uses for his first-period total income, $w_t + q_t e_t$: consumption c_t and savings s_t. When old, the individual bequeathes the unextracted resource stock z_t to his heir, invests his savings in productive capital and receives capital income $R_{t+1} s_t$, where $R_{t+1} = 1+r_{t+1}$ is the interest factor. He consumes all his second-period income and then dies. This is summarized by the youth and old-age budget constraints

$$w_t + q_t e_t = c_t + s_t \tag{4}$$

$$R_{t+1} s_t = d_{t+1} \tag{5}$$

and by the equation of motion of the individual resource stock with extraction

$$H\left(z_{t-1}\right) z_{t-1} = e_t + z_t \tag{6}$$

The individual's preferences are defined on youth and old-age consumption, c_t and d_{t+1}, and on the level of the unextracted resource stock bequeathed to his heir, z_t. They are represented by the following additively separable utility function

$$U_t = (1-\beta)\log c_t + \beta \log d_{t+1} + \gamma \log z_t \tag{7}$$

The parameter $\beta \in (0,1)$ reflects the weight attached to consuming when old while $\gamma > 0$ is the degree of the joy-of-giving bequest motive.

One interpretation of the joy-of-giving bequest motive is that the individual has the feeling of doing his duty by abstaining from consuming the whole family good. He feels he has to preserve the resource for the sake of his heir. By doing so he makes sure that he does not threaten the opportunities of his descendant. It should be emphasized that the bequest motive we assume here is substantially different from a concern for the resource or the environment as a whole. Indeed, not only the individual does not care about the other individuals' resource stocks, but also he gets utility only from his own bequest.

Two decisions characterize the individual's problem: the saving decision and the extraction decision. Considering prices as given, the individual chooses s_t and e_t in order to maximize his utility. By substituting c_t, d_{t+1} and z_t by their respective expressions, we get the following maximization problem:

$$\max_{\{s_t, e_t\}} (1-\beta)\log\left(w_t + q_t e_t - s_t\right) + \beta\log\left(R_{t+1}s_t\right) + \gamma\log\left(H\left(z_{t-1}\right)z_{t-1} - e_t\right) \tag{8}$$

and the first-order conditions write

$$\frac{1-\beta}{w_t + q_t e_t - s_t} = \frac{\beta}{s_t} \tag{9}$$

$$\frac{(1-\beta)\, q_t}{w_t + q_t e_t - s_t} \leq \frac{\gamma}{H\,(z_{t-1})\, z_{t-1} - e_t} \tag{10}$$

with equality if $e_t \geq 0$. Solving the first equation for s_t as a function of e_t yields: $s_t = \beta\,(w_t + q_t e_t)$. If extraction is unconstrained, the solution to the maximization problem is given by

$$e_t = \frac{H\,(z_{t-1})\, z_{t-1}}{1+\gamma} - \frac{\gamma}{1+\gamma}\frac{w_t}{q_t} \tag{11}$$

$$s_t = \frac{\beta}{1+\gamma}\,[w_t + q_t H\,(z_{t-1})\, z_{t-1}] \tag{12}$$

If optimal extraction is constrained, the saving decision simply writes $s_t = \beta w_t$. It is never optimal for the individual to extract all the resource since we would then get an infinitely low utility.

The extraction decision depends on two elements. First, it is increasing in the inherited stock. Second, it is decreasing in the relative price of labor with respect to the price of the resource, w_t/q_t. The condition of non-negativity of e_t is given by

$$\gamma \leq \frac{q_t H\,(z_{t-1})\, z_{t-1}}{w_t} \tag{13}$$

The right-hand side of the non-negativity constraint on e_t is the ratio of the inherited resource stock valued at price q_t on the wage income. This ratio reflects the relative importance of the two individual's sources of income when young. It increases as the individual's dependence on the bequeathed resource increases.

2.3 Firms' behavior

There is a representative firm which produces the consumption/investment good. The technology of production displays constant returns to scale of the three production factors: capital K, labor L and extracted resource E. It is represented by a linearly homogeneous production function: $F\,(K_t, L_t, E_t)$. The profit of the representative firm is $\pi_t = F\,(K_t, L_t, E_t) - R_t K_t - w_t L_t - q_t E_t$. The firm maximizes its profit with respect to K_t, L_t and E_t considering prices as given. The first-order conditions are given by: $F'_K\,(K_t, L_t, E_t) = R_t, F'_L\,(K_t, L_t, E_t) = w_t$ and $F'_E\,(K_t, L_t, E_t) = q_t$. We shall assume a CES specification for the production function,

$$F\,(K_t, L_t, E_t) = A\left(\alpha_K K_t^{-\rho} + \alpha_L L_t^{-\rho} + \alpha_E E_t^{-\rho}\right)^{-1/\rho} \tag{14}$$

In intensive terms, the FOC's read as follows

$$\frac{\alpha_K}{A^\rho}\left[\frac{f(k_t,e_t)}{k_t}\right]^{1+\rho} = R_t \tag{15}$$

$$\frac{\alpha_E}{A^\rho}\left[\frac{f(k_t,e_t)}{e_t}\right]^{1+\rho} = q_t \tag{16}$$

$$\frac{\alpha_L}{A^\rho} f(k_t,e_t)^{1+\rho} = w_t \tag{17}$$

where $f(k_t,e_t) = A\left(\alpha_K k_t^{-\rho} + \alpha_L + \alpha_E e_t^{-\rho}\right)^{-1/\rho}$.

2.4 The competitive equilibrium

We first study the equilibrium of period t. What is given in period t is the inherited resource stock z_{t-1} and the productive capital k_t. We determine the following time t variables: the prices w_t, R_t and q_t, the individuals' resource supply, the bequeathed stock and consumptions: e_t, z_t, c_t and d_t, and the representative firm's factor demands and output supply K_t, L_t, E_t and Y_t. The labor market equilibrium implies $L_t = N$. Hence, $k_t = K_t/N$ and $e_t = E_t/N$ in equilibrium and the equilibrium expressions of factor prices are given by the marginal productivities valued at these k_t and e_t.

Proposition 1 *(i) In equilibrium, the individual's optimal extraction is unconstrained.*
(ii) Individual's optimal extraction does not depend on capital:

$$e_t = e\left(\underset{+/-}{z_{t-1}}, \underset{-}{\gamma}, \underset{-}{\rho}\right) \tag{18}$$

(iii) An increase in the inherited resource stock z_{t-1} increases the extraction if the inherited stock is low enough ($z_{t-1} < \bar{z}$). Beyond the threshold value $\bar{z}$, an increase in the inherited resource stock decreases extraction.
(iv) An increase in the degree of bequest motive γ decreases extraction.
(v) As substitutability between factors decreases (increasing ρ), extraction decreases.

Proof. See appendix "Extraction in equilibrium". ■

The fact that the equilibrium extraction does not depend on capital is due, at first, to the fact that the relative price w_t/q_t is independent of k_t in equilibrium. Indeed the ratio of the marginal productivities of labor and resource only depends on e_t. Second, the additive separability of the log-linear utility function is also responsible for this feature.

At a steady state equilibrium the economy reproduces itself each period. Extraction is equal to the natural return which is added each period to steady stock, *i.e.*, per capita:

$$e(z,\gamma,\rho) = Nh(z)\,z \tag{19}$$

The dynamics of the economy is as follows. At each period, we solve for e_t as a function of z_{t-1} and we determine z_t and k_{t+1}. The dynamics of z_t and k_{t+1} are given by

$$z_t = H(z_{t-1})\,z_{t-1} - e(z_{t-1},\gamma,\rho) \tag{20}$$

$$\begin{aligned} k_{t+1} = {} & \frac{\beta}{(1+\gamma)\,A^{\rho}} \Big[\alpha_L f\left[k_t, e(z_{t-1},\gamma,\rho)\right]^{1+\rho} \\ & +\alpha_E f\left[k_t, e(z_{t-1},\gamma,\rho)\right]^{1+\rho} \frac{H(z_{t-1})\,z_{t-1}}{e(z_{t-1},\gamma,\rho)^{1+\rho}}\Big] \end{aligned} \tag{21}$$

The dynamics of z_t are independent of capital. Given the initial conditions, *i.e.* given z_{-1} and k_0, we determine the intertemporal equilibrium. We illustrate the equilibrium by using a Cobb-Douglas production function.

Example 2 ***The Cobb-Douglas-quadratic example***
Assume the Cobb-Douglas special case of the CES production function : $f(k_t,e_t) = Ak_t^{\alpha_K}e_t^{\alpha_E}$. *Then equilibrium prices read* $R_t = \alpha_K A k_t^{\alpha_K-1}e_t^{\alpha_E}$, $w_t = \alpha_L A k_t^{\alpha_K}e_t^{\alpha_E}$ *and* $q_t = \alpha_E A k_t^{\alpha_K}e_t^{\alpha_E-1}$. *Together with a quadratic resource dynamics* $e_t + z_t = [1+N(\mu-\nu z_{t-1})]\,z_{t-1}$, e_t, z_t *and* k_{t+1} *write as follows*

$$e_t = \varepsilon(\gamma)\left[1+N(\mu-\nu z_{t-1})\right]z_{t-1} \tag{22}$$

$$z_t = \left[1-\varepsilon(\gamma)\right]\left[1+N(\mu-\nu z_{t-1})\right]z_{t-1}$$

$$k_{t+1} = \frac{\beta}{1-\beta}\varepsilon(\gamma)^{\alpha_E}\left(\alpha_L + \frac{\alpha_E}{\varepsilon(\gamma)}\right)Ak_t^{\alpha_K}\left[\left(1+N(\mu-\nu z_{t-1})\right)z_{t-1}\right]^{\alpha_E}$$

where $\varepsilon(\gamma) = \alpha_E(\alpha_E+\alpha_E\gamma+\alpha_L\gamma)^{-1} \in (0,1)$.

At a steady state equilibrium we have $N(\mu - \nu z) z = \varepsilon(\gamma)[1 + N(\mu - \nu z)] z$*. We can solve for* z *and deduce* e

$$z = \frac{\mu}{\nu} - \frac{1}{N\nu}\frac{\alpha_E}{\gamma(1-\alpha_k)} \tag{23}$$

$$e = \frac{\alpha_E}{(1-\alpha_K)} z \tag{24}$$

The steady state equilibrium value of k *is the solution of* $s = k$ *where* $s = \beta(1-\alpha_k) A k^{\alpha_K} e^{\alpha_E}$*:*

$$k = \left(\beta(1-\alpha_K) A e^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} \tag{25}$$

Two issues can be stressed out. First, it may happen that, despite the bequest motive towards the natural resource, this resource collapses, thus compromising the ability of forthcoming generations to fulfill their own needs. Second, the possibility for reaching the maximum steady state consumption level through the competitive equilibrium is not guaranteed. These issues are discussed in the two following sections.

3 Resource extinction despite altruism

The possibility to reach a trivial equilibrium where the resource stock is equal to zero cannot be ruled out.

Proposition 2 *Let the resource own dynamics be quadratic,*

$$z_t = z_{t-1} + N(\mu - \nu z_{t-1}) z_{t-1}.$$

(i) in the case where factors are strong substitutes ($\rho \in (-1, 0)$), the resource extinction never occurs, whatever $\gamma > 0$;
(ii) in the case where factors are poor substitutes ($\rho > 0$), the resource extinction never occurs only if the concern for the bequeathed resource is higher than the following threshold

$$\underline{\gamma} = \frac{1}{N\mu} \tag{26}$$

Proof. See appendix "Dynamics of z_t". ■

The mere existence of a taste for bequeathing the resource is not always sufficient to avoid the extinction of the resource. More importantly, when factors are poor substitutes the taste for bequest must not only be positive, but larger than the minimum threshold $\underline{\gamma}$ to guarantee preservation. This minimum value depends on the technology of production (ρ), the resource's productivity (μ) and the population level (N). The reason for the $\underline{\gamma}$ threshold is the following. As the resource stock tends to zero the equilibrium price of the extracted resource tends to infinity when factors are high substitutes, whereas it tends to a finite value when factors are complementary.

Interestingly, this proposition sheds light on the interplay between sustainability concepts and factors substitutability in an equilibrium analysis. When factors are substitutes within the production process, the resource is not essential to production. From a technological point of view it would be possible to maintain consumption opportunities of future generations even if the resource is exhausted. Such a technology is compatible with a *weak sustainability* criterion (Hartwick, 1977). However, our analysis concludes that the intertemporal competitive equilibrium will never lead to such an outcome. A positive resource stock will be maintained in the long run irrespective of the degree γ of the bequest motive. Thus, in equilibrium a *strong sustainability* criterion will also be satisfied. When factors are poor substitutes, the resource is essential to production, but what does matter is the value of γ. The equilibrium will lead to a sustainable outcome with a preserved resource stock only if $\gamma > \underline{\gamma}$. The teaching of this result is that the technological possibilities of substitution between production factors is only one part of the story. We show that it is misleading to exclusively rely on them to study sustainability. What matters in the interaction between technology and preferences at equilibrium.

Insert here figure 2. Phase diagram with extraction.

4 A preserved but misused resource

In this section we assume there exists a single long run positive equilibrium level of the resource stock. What ensures that this preserved resource stock maximizes consumption level in the long run? Let us first study the productive efficiency stationary path and then the conditions on preferences to decentralize this path.

4.1 Dynamic efficiency of steady state

Dynamic efficiency in the long run consists in maximizing the net stationary production defined as the difference between production per head and investment in capital per head, *i.e.* $\phi(k,z) = f\left[k, H(z)z - z\right] - k$. This problem writes:

$$\max_{\{k,z\}} \phi(k,z) = f\left[k, H(z)z - z\right] - k \tag{27}$$

Under suitable conditions on the limit properties of capital marginal productivity f'_k (*i.e.* $\lim_{k\to+\infty} f'_k(k,.) = 0$ and $\lim_{k\to 0} f'_k(k,.) = +\infty$) there exists an interior solution to the consumption maximization problem. The first-order conditions for an interior maximum are the following:

$$f'_k\left(k^*, H(z^*)z^* - z^*\right) = 1 \tag{28}$$

$$H'(z^*)z^* + H(z^*) = 1 \tag{29}$$

The first equation in k and z is the equivalent of the standard condition defining the Golden rule capital stock. The choice of efficient capital stock is determined by the usual trade-off between the marginal productivity of capital and the population growth rate (here 1). The second equation only depends on z and always has an interior solution. At z^*, the steady exploitation e is maximized. The trade-off for the extracted resource is similar to the one for capital. The marginal natural return $(H'(z^*)z^* + H(z^*))$ must equal the marginal effort to leave the resource stock unchanged next period (*i.e.* 1). We illustrate these properties with a simple example.

Example 3 ***The Cobb-Douglas-quadratic example* -** *Assume a Cobb-Douglas production function,* $y = Ak^{\alpha_K}e^{\alpha_E}$, *with* $A > 0$ *the multi-factor productivity index and* α_K *and* α_E *the elasticities of capital and extracted resource intensities; assume further that the resource evolves according to the quadratic function* $z = [1 + N(\mu - \nu z)]z - e$, *with* $\mu > 0$ *and* $\nu > 0$. *In this case, we have*

$$z^* = \frac{\mu}{2\nu} \tag{30}$$

$$e^* = \frac{N\mu^2}{4\nu} \tag{31}$$

$$k^* = \left(\alpha_K A \left(e^*\right)^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} \tag{32}$$

We adopt the following definition:

Definition 1 *An economy is said to be resource-conservative (resp. resource consuming) in steady state if its resource stock is larger (resp. smaller) than the stock* z^* *which maximizes net production.*

A resource-conservative economy could increase the consumption of all generations, including the present one, by just raising resource extraction. The unextracted resource closely parallels the unconsummed numeraire: it is invested to restore the next period stock. For this reason, we could label this case "over-accumulation" of resource. The inverse holds for a resource-consuming economy. Resource exploitation should be temporarily reduced to let the resource reach the higher z^* level. At that level, exploitation is eventually run at a higher level than initially. This case could be labelled "under-accumulation" of resource. In this case, the economy could reach a higher a level of consumption per head, but at the expense of the current generation.

4.2 Efficiency of the equilibrium

The following proposition establishes conditions on preferences to decentralize the efficient stationary path steady state with a Cobb-Douglas production function.

Proposition 3 *With a Cobb-Douglas production function and a quadratic resource dynamics, there exists a system of individuals' preferences* (β^*, γ^*) *which maximizes net stationary production. This system of preferences is such that*

$$\gamma = \gamma^* \equiv \frac{2\alpha_E}{N\mu\left(1-\alpha_K\right)} \tag{33}$$

$$\beta = \beta^* \equiv \frac{\alpha_K}{1-\alpha_K} \ \in (0,1) \tag{34}$$

This implies that $\alpha_K \in (0, \frac{1}{2})$

Proof. *See appendix "Conditions on preferences".* ■

Most of the time, the preferences will drive the economy to another long run equilibrium. Assume an inefficient competitive equilibrium in which the natural resource is under-accumulated ($z < z^*$). This may come from a taste for bequeathing the resource lower than γ^*. Whatever the level of capital per head, the net product is not maximized. What is the capital stock k which maximizes the consumption per head? According to the Hartwick rule, one should expect k to be larger than k^*. The following proposition shows that it is never the case.

Proposition 4 *Whenever an economy under-accumulate or over-accumulate its natural resource, the level of capital which maximizes net production is always lower than k^*.*

Proof. See appendix: "Conservationists" *vs* "exploitationists". ■

What explains this result is that, as long as the resource stock is not equal to z^*, extracted resource is not maximized ($e < e^*$). Indeed, only z^* leads to the *maximum sustainable yield*. Since e is lower than e^*, the marginal productivity of the capital stock k^* is lower than the marginal cost of reproducing k^* each period. As a consequence, only a lower capital stock can make it.

The Hartwick rule addresses the issue of the role of substitutability between natural and man-made capital stocks (see *e.g.* Asheim *et al.* (2003)). This question, in fact, should be considered from two points of view: from a technological perspective, as done above, but also from the point of view of the contribution of each stock to sustainability, *i.e.* from their capacity to raise revenues.

Let consider a world where people are eager to fulfill their duty towards future generations by preserving a high level of the natural resource. Call this a *conservative* economy (*i.e.* conservative with respect to the natural resource). Formally, this economy is such $\gamma > \gamma^*$. The previous proposition shows that, when people are too conservative ($\gamma > \gamma^*$ and $z > z^*$), it follows that $e < e^*$ and that it is not necessary to maintain a capital stock as high as k^*: a smaller capital stock would maximise the consumption level ($\beta < \beta^*$). Let us now consider a world where people are relatively selfish or short-sighted in the sense that they neglect their duty towards the future generations. Call this an *exploitationist* economy (agents over-exploit the natural resource for their own welfare). Formally we have $\gamma < \gamma^*$. In this case, one should expect that it might be helpful to maintain a capital stock higher than k^* as a compensation. Actually, our proposition reveals that an excessive selfishness ($\gamma < \gamma^*$ and $z < z^*$), and thus a low resource stock, would recommend also a lower capital stock (through $\beta < \beta^*$). Be the economy *conservative* or *exploitationist*, the capital stock which maximizes the net product is always lower than k^*.

5 Conclusion

We model an overlapping generations economy in which individuals are privately-endowed with a renewable resource. This resource can be extracted at no cost by the young households and provided to production as a source of revenue. However, a joy-of-giving bequest motive motivates the transfer of the unexploited resource to the heirs so as to let them the opportunity to raise their own revenues from the resource. The purpose was to analyze whether a decentralized decision-making process with environmental constraint may fulfill the necessary condition for sustainability. The main findings are the following.

In the long run, the bequest motive does not systematically guarantee sustainability. When production factors are high substitutes and thus when extracted resource is inessential to production, any degree of the bequest motive is compatible with a preserved resource. So, both weak (consumption preservation) and strong sustainability (resource stock preservation) are satisfied. On the contrary, when factors are poor substitutes, *i.e.* when the resource is essential to production, strong sustainability (resource preservation) is required in order to have weak sustainability. We derive a condition on the degree of the bequest motive for strong sustainability to hold.

There exists a system of preferences which decentralizes the target of the consumption-maximizing path in the long run. But in most cases preferences will differ from this and the economy will converge to a sub-optimal long run equilibrium. As we showed, resource-conservative economies, which run a high steady resource stock, should compensate with a lower capital stock to maximize the second-best consumption level (substitutability result). On the contrary, resource-consuming economies, which run a low level of steady resource stock, should also keep a lower capital stock to maximize second-best consumption per head (complementarity result).

This paper illustrated the insights of an intertemporal equilibrium analysis for the study of the issue of sustainability. In particular, we studied the implications for sustainability of a joy-of-giving bequest motive applied to a privately-owned renewable resource. Despite this bequest motive there is room for corrective public policies. This requires further research.

6 Appendices

6.1 Extraction in equilibrium

Proof of point (i) - At an unconstrained-extraction time t equilibrium, there is a unique finite positive quantity e_t which equalizes the prices from the inverted resource supply and demand functions on the factor market and which is inferior to $H(z_{t-1})z_{t-1}$. From the expression of aggregate resource supply

$$Ne_t = N(1+\gamma)^{-1} H(z_{t-1})z_{t-1} - N(1+\gamma)^{-1}\gamma q_t^{-1}w_t$$

and from the equilibrium value of the real wage rate $w_t = (\alpha_L/A^\rho) f(k_t,e_t)^{1+\rho}$, we derive the inverted resource supply

$$q_t = \frac{\gamma(\alpha_L/A^\rho) f(k_t,e_t)^{1+\rho}}{H(z_{t-1})z_{t-1} - (1+\gamma)e_t} \tag{35}$$

and the inverted resource demand verifies

$$q_t = \frac{\alpha_E}{A^\rho}\frac{f(k_t,e_t)^{1+\rho}}{e_t^{1+\rho}} \tag{36}$$

Equating the above two expressions of the price q_t yields:

$$\frac{\gamma\alpha_L}{H(z_{t-1})z_{t-1} - (1+\gamma)e_t} = \frac{\alpha_E}{e_t^{1+\rho}} \tag{37}$$

The LHS tends to $\alpha_L\gamma/H(z_{t-1})z_{t-1}$ as e_t tends to 0, while the RHS tends to $+\infty$ as e_t tends to 0. The LHS is increasing in e_t until the value

$$(1+\gamma)^{-1} H(z_{t-1})z_{t-1}\,(< H(z_{t-1})z_{t-1})$$

at the limit of which it tends to $+\infty$; from the other side, as e_t tends to $(1+\gamma)^{-1} H(z_{t-1})z_{t-1}$, the LHS tends to $-\infty$. Beyond $(1+\gamma)^{-1} H(z_{t-1})z_{t-1}$, as e_t increases the LHS increases until 0 at the limit; but this is economically meaningless, since extraction cannot be larger than the stock. The RHS decreases as e_t increases and tends to 0 as e_t tends to $+\infty$. As a result, there always exists a finite positive $e_t \leq H(z_{t-1}) z_{t-1}$, such that the two curves cross.

Proof of point (ii) - Extraction, i.e. $e_t = (1+\gamma)^{-1} H(z_{t-1}) z_{t-1} - \gamma (1+\gamma)^{-1} w_t q_t^{-1}$, in equilibrium, is given by

$$e_t - \frac{H(z_{t-1}) z_{t-1}}{1+\gamma} + \frac{\gamma}{1+\gamma} \frac{\alpha_L}{\alpha_E} e_t^{1+\rho} = 0 \tag{38}$$

which is obtained by substituting $w_t q_t^{-1}$ with its equilibrium value, i.e.

$$\frac{\alpha_L A^{-\rho} f(k_t, e_t)^{1+\rho}}{\alpha_E A^{-\rho} f(k_t, e_t)^{1+\rho} e_t^{-(1+\rho)}} = \frac{\alpha_L}{\alpha_E} e_t^{1+\rho} \tag{39}$$

This equation in e_t is independent of capital. Its solution is a function $e(z_{t-1}, \gamma, \rho)$.

Proof of point (iii) - The solution of this equation is a function of z_{t-1}, γ and ρ: $e_t = e(z_{t-1}, \gamma, \rho)$. Let us study the derivative of this function w.r.t. z_{t-1}:

$$\frac{de_t}{dz_{t-1}} = \frac{(1+\gamma)^{-1} [H'(z_{t-1}) z_{t-1} + H(z_{t-1})]}{1 + \gamma (1+\gamma)^{-1} \alpha_L \alpha_E^{-1} (1+\rho) e_t^{\rho}} \tag{40}$$

or

$$\frac{de_t}{dz_{t-1}} = \varepsilon(z_{t-1}, \gamma, \rho) \left[H'(z_{t-1}) z_{t-1} + H(z_{t-1})\right] \tag{41}$$

where

$$\varepsilon(z_{t-1}, \gamma, \rho) = \frac{\alpha_E}{\alpha_E + \alpha_E \gamma + \alpha_L \gamma (1+\rho) e(z_{t-1}, \gamma, \rho)^{\rho}} \tag{42}$$

belongs to the interval $(0,1)$. Thus the derivative de_t/dz_{t-1} has the same sign as the derivative of the dynamics without extraction $z_t = \phi(z_{t-1},)$, i.e. $\phi'(z_{t-1}) = H'(z_{t-1}) z_{t-1} + H(z_{t-1})$, i.e. first increasing for for values $z_{t-1} \in (0, \overline{z}]$ and then decreasing for $z_{t-1} \in (\overline{z}, H(z_{t-1}) z_{t-1})$.

Proof of point (iv) - The derivative of $e(z_{t-1}, \gamma, \rho)$ w.r.t. γ is given by:

$$\frac{de_t}{d\gamma} = -\frac{H(z_{t-1}) z_{t-1} (1+\gamma)^{-2} + \alpha_L \alpha_E^{-1} e_t^{1+\rho} (1+\gamma)^{-2}}{1 + \gamma (1+\gamma)^{-1} \alpha_L \alpha_E^{-1} (1+\rho) e_t^{\rho}} < 0 \tag{43}$$

Proof of point (v) - From (38) it is straightforward that the higher $\rho \in (-1, +\infty)$, the lower e_t.

6.2 Dynamics of z_t

The dynamics of the individual resource stock with extraction in equilibrium is $z_t - H(z_{t-1}) z_{t-1} + e(z_{t-1}, \gamma, \rho) = 0$. They have a bell shape, increasing on $(0, \overline{z})$ and decreasing on $(\overline{z}, z_{\max})$. The slope of these dynamics are given by:

$$\frac{dz_t}{dz_{t-1}} = [1 - \varepsilon(z_{t-1}, \gamma, \rho)] \left[H'(z_{t-1}) z_{t-1} + H(z_{t-1})\right] \tag{44}$$

It is therefore a fraction of $H'(z_{t-1}) z_{t-1} + H(z_{t-1})$. This last expression is the derivative of the function $\phi(z_{t-1})$ which is the dynamics of the resource without extraction. It is positive for $z_{t-1} \in (0, \overline{z})$ and negative for $z_{t-1} \in (\overline{z}, z_{\max})$. The limits are:

$$\lim_{z_{t-1} \to 0} z_t = 0 \tag{45}$$

$$\lim_{z_{t-1} \to z_{\max}} z_t = 0 \tag{46}$$

We consider the slope of the dynamics as z_{t-1} tends to 0 in the case of quadratic resource own dynamics given by $z_t = z_{t-1} + N(\mu - \nu z_{t-1}) z_{t-1}$. Since

$$\varepsilon(z_{t-1}, \gamma, \rho) = \frac{\alpha_E}{\alpha_E + \alpha_E \gamma + \alpha_L \gamma (1 + \rho) e(z_{t-1}, \gamma, \rho)^{\rho}} \tag{47}$$

and

$$H'(z_{t-1}) z_{t-1} + H(z_{t-1}) = 1 + N(\mu - 2\nu z_{t-1}) \tag{48}$$

we have

$$\lim_{z_{t-1} \to 0} \left[H'(z_{t-1}) z_{t-1} + H(z_{t-1})\right] = 1 + N\mu \tag{49}$$

$$\lim_{z_{t-1} \to 0} \varepsilon(z_{t-1}, \gamma, \rho) = \begin{cases} 0 & \text{if } \rho \in (-1, 0) \\ \frac{1}{1+\gamma} & \text{if } \rho > 0 \end{cases} \tag{50}$$

Hence, in the quadratic case, the slope of the dynamics as $z_{t-1} \to 0$ is given by

$$\lim_{z_{t-1} \to 0} [1 - \varepsilon(z_{t-1}, \gamma, \rho)] \left[H'(z_{t-1}) z_{t-1} + H(z_{t-1})\right] = \begin{cases} 1 + N\mu & \text{if } \rho \in (-1, 0) \\ \frac{\gamma(1+N\mu)}{1+\gamma} & \text{if } \rho > 0 \end{cases} \tag{51}$$

If $\rho \in (-1, 0)$ this slope $(1 + N\mu)$ is greater than 1 independently of γ. If $\rho > 0$ this slope is greater than 1 iff

$$\gamma > \frac{1}{N\mu} \tag{52}$$

Since the dynamics are continuous and concave and end up with negative slope, starting with positive slope larger than 1, there exists a non-trivial steady state z.

6.3 Conditions on preferences

We have $z = z^*$ if and only if

$$z = \frac{\mu}{\nu} - \frac{1}{N\nu}\frac{\alpha_E}{\gamma^*(1-\alpha_K)} = \frac{\mu}{2\nu} = z^* \tag{53}$$

which leads to the following condition

$$\gamma^* = \frac{2\alpha_E}{N\mu(1-\alpha_K)} \tag{54}$$

and then, taking $e = e^* = \alpha_E (1-\alpha_K)^{-1} z^*$ we have $k = k^*$if and only if

$$k = \left(\beta^* (1-\alpha_K) A (e^*)^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} = \left(\alpha_K A (e^*)^{\alpha_E}\right)^{\frac{1}{1-\alpha_K}} = k^* \tag{55}$$

which leads to the following condition

$$\beta^* = \frac{\alpha_K}{1-\alpha_K} \tag{56}$$

The condition for a positive stationary natural stock z is given by

$$\gamma^* > \underline{\gamma} \Leftrightarrow \alpha_E > \frac{1-\alpha_K}{2} \tag{57}$$

and having $\beta \in (0,1)$ requires

$$0 < \frac{\alpha_K}{1-\alpha_K} < 1 \Leftrightarrow 0 < \alpha_K < \frac{1}{2} \tag{58}$$

6.4 'Conservationists' vs 'exploitationists'

Let $\tilde{z} \neq z^*$, then by definition $\widetilde{e} < e^*$. With a CES production function we have $f''_{ke} > 0$ and so $f'_k(k^*, \widetilde{e}) < 1$. As a result, $\tilde{k}$ solution of $f'_k\left(\widetilde{k}, \widetilde{e}\right) = 1$ is such that $\tilde{k} < k^*$.

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