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Information revelation in markets with pairwise meetings : dynamic case with constant entry flow

Tanguy ISAAC[1]

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Abstract

We study information revelation in markets with pairwise meetings. We focus on the one sided case and perform a dynamic analysis of a constant entry flow model. The same question has been studied in an identical framework in Serrano and Yosha (1993) but they limit their analysis to the stationary steady states. Blouin and Serrano (2001) study information revelation in a one-time entry model and obtain results different than Serrano and Yosha (1993). We show that there is dramatically loss when restricting the analysis of a constant flow entry model to stationary steady states. Nevertheless, we show that this loss might not explain completely the difference in the results presented in the two papers.
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[1]Department of Economics, Université catholique de Louvain, Belgium.
E-mail: isaac@ires.ucl.ac.be.

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Introduction

We can illustrate the general issue of information revelation in market with pairwise meetings by a parallelism with what we can observe on all the places of interest in Egypt. On all these places, one observes bargaining between Egyptians and Tourists. The Egyptians try to sell a guided tour of the place. The Tourists are the potential buyers. There is neither a central institution nor a unique public price. The phase of bargaining happens after a matching between one seller and one buyer. When an Egyptian reaches an agreement with a Tourist, the two quit the market to effectuate the tour. In case of disagreement, the two separate and are matched anew with an agent of the opposite type.

The asymmetric information concerns the interest of the place. It is not obvious, for a Tourist, if the place has a long history, if there is a lot of anedoctes about the site. Some Tourists can be uninformed about the interest while some other ones are informed, for instance, because they know some friends who previously visited the same place. Of course, all the Egyptians know the exact interest of the place.

The interest of the place has an influence on the value and the cost of the guided tour. It is more intersting to have a guide when there is a lot of things to say about the site. At the same time, it is more costly for an Egyptian to guide when the place is interesting, at least because it takes more time.

We can expect that the *good* price is higher when the place is of high interest. It is also natural that the uninformed Tourists try to extract information from their matches with different partners. This learning is expensive because there is a waste of time. Naturally, sellers try to exploit their information's advantage by misrepresenting. By misrepresenting, sellers incur also a cost for the same reason, i.e. the waste of time.

The main issue will be to determine if the trading process will imply an information revelation. Especially when the agents become infinitely patient, i.e. the market becomes approximately frictionless.

In market with pairwise meetings, the information revelation litterature began with the seminal paper Wolinsky (1990) [1]. The model studied in this paper is more general than ours because there are also some uninformed

[1] Concerning the market with pairwise meetings with perfect information, there is a significant literature studying following the seminal works of Gale, Rubinstein and Wolinsky. For a review, see Osborne and Rubinstein (2000).

sellers. In our egyptian story, it would mean that some Egyptians are not aware of the place's interest. The main result of Wolinsky (1990) is that some trades occur at a wrong price according to the state even when market becomes approximately frictionless.

Gale (1989) conjectures the great importance of the assumption that uninformed agents are present in the two sides of the market because a noise is created if the cost of learning decreases. Indeed, the decreasing of the cost causes the probability - of an uninformed agent to meet another uninformed agent - to increase. This requires, however, the information power of meeting to decrease when the cost of learning declines.

Serrano and Yosha (1993) show that Gale's conjecture is correct. They use the same model than Wolinsky (1990), but they assume that all sellers are informed. The noise force disappears, since uninformed buyers always meet informed sellers. Finally, Serrano and Yosha (1993) establish that all transactions occur at the right price whenever the market becomes approximately frictionless.

Wolinsky (1990) and Serrano and Yosha (1993) use a constant flow entry model. At each period, a certain number of new agents enter the market. To simplify the analysis, these papers consider only the stationary steady states. In other words, they consider the situations where the number of agreements is exactely equal to the entry flow. Blouin and Serrano (2001) study the same question of information revelation but in a one-time entry model[2]. At the first period, all the agents are present and nobody enters the market in the following periods. They obtain a dramatically different result in the one sided case. They conclude, in this case, that some transactions occur at wrong prices even when the market is frictionless. The two sided analysis provides results similar to Wolinsky (1990).

The question is to know if it is due to the difference of hypothesis or due to the restriction of the analysis to the steady states in the case of a constant entry flow model. In the case of a constant flow entry, we can imagine that there exist a kind of externality between the different generations of agents, which imply the difference of the results. Concerning the restriction to the steady states, it is not unreasonable to believe that a part of the story is missing and explain the differences. The loss of a part of the story is also important in an approach of market design. In the sense that a decentral-

[2]For a discussion of these two hypothesis (constant entry flow and one time entry) in the perfect information case see Gale (1987). Generally, the implicit economy in the constant entry flow model is not well defined. Nevertheless, the constant entry flow model remains interesting at least because they may correspond better to some real markets.

ized market with pairwise meetings is a possible market design that can be compared to others designs. To compare correctly, it might be important to not loose a part of the story.

In this paper, we show that the steady states analysis looses effectively a part of the story. But, we also show that when markets become approximately frictionless, there is an equilibrium with full information revelation in the case of the constant flow entry model even if the analysis is not restricted to steady states.

In the first section, we present the model. The second is devoted to characterize equilibria and we show in the third section that some equilibria are not well described by a steady state analysis. The fourth section concerns steady states. The last section concludes.

1 The model

We consider the model of Serrano and Yosha (1993) without modifing it but we study the outcomes without assuming an *a priori* stationarity of the equilibrium.

Times runs discretely from 0 to ∞[3]. Each period is identical. On one side, there are sellers who have one unit of indivisible good to sell. On the other side, there are buyers who want to buy one unit of this good. In each period, a continuum of measure M of new sellers and the same quantity of buyers enter on the market. The sellers' number which arrive on the market is equal to the buyers' one. The agents quit the market when they have traded. Hence, the number of sellers is always equal to the number of buyers.

There exist two possible states of the world which influe on the payoff of the agents. If the state is low (L), the cost of production (c_L) for the sellers but also the utility (u_L) of the buyers are low. If the state is high (H), the corresponding parameters $(c_H$ and $u_H)$ are high. The state remains identical during all the periods.

All sellers know the state of the world, whereas not all buyers are perfectly informed about the state of the world. There is a part x_B of buyers which is perfectly informed. The remaining buyers are uninformed and pos-

[3]Serrano and Yosha 1993 consider that times runs from $-\infty$ to ∞. To make the steady state analysis, it is sufficient to assume that the initial conditions are the values of the steady state. This approach is totally equivalent to the approach of Serrano and Yosha 1993.

sess a common prior belief $\alpha_H \in [0,1]$ that the state is H and $(1-\alpha_H)$ that the state is L.

At each period, all the agents are randomly matched with an agent of the other type. At each meeting, the agents can announce one of two prices : p^H and p^L. If both agents announce same price, trade occurs at this price. If a seller announces a lower price, trade occur at an intermediate price p^M. If a seller announces a higher price, trade does not occur. The different parameters are assumed to be ordered such that :

$$c_L < p^L < u_L < p^M < c_H < p^H < u_H \tag{1}$$

Staying on the market implies a zero payoff. The instantaneous payoff when a trade occurs is the price minus the cost for a seller and the utility minus the price for a buyer. All agents discount the future by a constant factor δ.

In state H, we call p^H the *good* price because trade at other prices implies a loss for the sellers. Similarly, the price p^L is the *good* price in state L because trade at other prices involves loss for the buyers.

After each meeting with a seller who announces p^H, a buyer will actualise his belief α_H according to Bayes'rule. If a uniformed buyer meets a seller who announces p^L, he knows that state is L but it does not really matter since this buyer will trade and leave the market.

It is convenient to say that a seller (resp. a buyer) plays *soft* when he announces p^L (resp. p^H) and *tough* when he announces the p^H (resp. p^L). When an agent plays *soft*, he is ensured to trade and to quit the market. So, to describe completely the strategy of an agent, it is sufficient to give the number of periods in which he plays *tough*. The strategy of an agent might depend on the time of entry on the market. We note $n_{SH}(t)$ the number of periods during which a seller plays *tough* when he enters in time t on a market which is in state H. Similarly, we define $n_{SL}(t)$, $n_{BH}(t)$, $n_{BL}(t)$ and $n_B(t)$. Naturally, the strategy of an uninformed buyer $n_B(t)$ is independent of the state of the world.

We define now the proportions of agents who play *tough*. The proportion of the total number of buyers in the market who in state H at period t announce p^L is called $B_H^l(t)$. We define in the same way $B_L^l(t)$, $S_H^h(t)$ and $S_L^h(t)$. These values are known to all agents. The total number of buyers in the market is noted $K^H(t)$ and $K^L(t)$ according to the state. It is also the number of sellers in the market since sellers are as numerous as buyers.

An equilibrium is a profile of strategies where each agent is maximizing his expected payoff, given the strategies of the other agent. All parameters $(p^H, p^M, p^L, c_H, c_L, u_H, u_L, x_B, \delta, \alpha_H)$ are common knowledge.

2 Characterisation and existence of an equilibrium

Our approach is not to study all the dynamic cases, we retrict our analysis to equilibria where uninformed buyers play always *soft*. It is clearly a simplifying assumption[4], unfortunately, as we will see later, this simplification has a cost.

In a first step, we establish the strategy of sellers and informed buyers in state H and strategy of informed buyers in state L. Then we give conditions which constraint uninformed buyer's strategy. The next step is the characterization of the evolution of the market at equilibrium. Especially, we characterize the sequences $S^h_L(t)$ and $B^l_L(t)$ at equilibrium. Actually, our characterization of $B^l_L(t)$ gives us an iterative rule to build from an initial condition the unique sequence $B^l_L(t)$ compatible with optimal seller's behaviour. Finally, we see that for any sequence $B^l_L(t)$ satisfying the characterization there exists at least one set of strategies $n_{SL}(t) : t \in [0, \infty]$ which implies this sequence.

2.1 Trivial or constrained strategies

In the following claim, we characterize the equilibrium strategies of informed buyers and of sellers in state H.

Claim 1 *In any equilibrium* $n_{SH}(t) = \infty$, $n_{BL}(t) = \infty$ *and* $n_{BH}(t) = 0$ $\forall t$.

Proof An informed seller in state H knows that his payoff will be negative if he trades at an other price than p^H. Since the payoff of perpetual disagreement is 0, he will always prefer to play *tough* even if it implies a long delay before trading. The reasoning is identical for an informed buyer in state L. An informed buyer in state H will understand that $n_{SH}(t) = \infty$ and thus he will never trade while he plays *tough*. Playing *tough* only delays the payoff. So, it is better for this kind of buyer to play immediately *soft*.

For the sake of simplicity, we will consider only situations where $n_B(t)$ is always equal to zero. Claim 2 will give sufficient conditions to ensure that $n_B(t) = 0$ is an optimal strategy. To establish this claim, we define ΔV_B

[4] In this model, we must determine the optimal behaviour of an infinite number of agents. The stationary steady state assumption largely reduces the number of different strategies. The strategies cannot depend on the time of entry on the market, so, there remain only five different strategies (i.e. $(n_{SL}, n(SH, nBH, nBL, nB)$ which do not depend on time).

which is the difference of gain between playing *soft* tomorrow and playing *soft* today for an uninformed buyer.

$$\begin{aligned}
\Delta V_B &= \Delta V_B(S_L^h, S_L^h(+1)) \\
&= \alpha_H(u_H - p^H)\delta \\
&+ (1-\alpha_H)[(1-S_L^h)(u_L - p^L) + \delta S_L^h[(u_L - p^M) + S_L^h(+1)(p^M - p^H)]] \\
&- [\alpha_H(u_H - p^H) + (1-\alpha_H)[(u_L - p^M) + S_L^h(p^M - p^H)]] \qquad (2)
\end{aligned}$$

The last line corresponds to the payoff involved by playing *soft* today. The payoff in state H which is equal to $(u_H - p^H)$ is multiplied by the probability that the state is H. The term in brackets, which is multiplied by the probability that the state is L, is naturally the payoff in state L. This payoff can be written $(1-S_L^h)(u_L - p^M)$ (i.e. the probability to meet a *soft* seller times the payoff involved by this meeting) plus $S_L^h(u_L - p^H)$ (i.e. the probability to meet a *tough* seller times the payoff involved). The two first lines correspond to playing today *tough* and tomorrow *soft*. The meaning of the first line is obvious. It is just important not to forget the discount factor δ. Indeed, if the state is H, a buyer who announces p^L does not trade. In the case where the state is L, there is a probability $(1-S_L^h)$ that a buyer meets a *soft* seller and obtains today $(u_L - p^L)$. If a buyer does not have this luck, which happens with probability S_L^h, he will have tomorrow an expected payoff equal to the expression in brackets. Once again, we must not forget the discount factor.

Claim 2 *The following conditions are sufficient to ensure* $n_B(t) = 0\ \forall t$.

$$\delta \geq \frac{p^H - p^M - u_L + p^L}{p^M - u_L} \quad \textit{and} \quad \alpha_H > \frac{p^M - p^L}{(1-\delta)(u_H - p^H) + (p^M - p^L)} \qquad (3)$$

or

$$\delta \leq \frac{p^H - p^M - u_L + p^L}{p^M - u_L} \quad \textit{and} \quad \alpha_H > \frac{p^H - u_L - \delta(u_L - p^M)}{u_H - u_L + \delta(p^H - u_H + u_L - p^M)} \qquad (4)$$

Proof Clearly, $\Delta V_B < \Delta V_B(S_L^h, 0)$. It is easy to see that $\Delta V_B(S_L^h, 0)$ is a linear function in S_L^h. It implies that either $\Delta V_B(1,0)$ or $\Delta V_B(0,0)$ is the maximum value that ΔV_B can take. $\Delta V_B(0,0) < 0$ is equivalent to the second inequality of (3). The first inequality is the condition such that $\Delta V_B(0,0)$ is the maximal value of ΔV_B. Second inequality of (4) is equivalent to $\Delta V_B(1,0) < 0$. Obviously, the first inequality (4) is the condition such that the maximal value of ΔV_B is equal to $\Delta V_B(1,0)$. It is sufficient because if an uninformed buyer remains in the market his belief α_H can never decrease[5].

2.2 Dynamic of the market

In what follows, we will assume that at least one of the conditions of claim 2 is satisfied. The market is then at a stationary steady-state when the state of the world is H

$$K^H = M \quad (5)$$
$$B^l_H = 0 \quad (6)$$
$$S^h_H = 1 \quad (7)$$

Since $n_{SH}(t) = \infty \quad \forall t$, all sellers play *tough* in each period. So, the proportion of sellers who in state H announce p^H is equal to one. The proportion of buyer who announce p^L is always equal to 0. Indeed, $n_{BH}(t) = 0$ and by assumption $n_B(t) = 0$. All agents announce the same price p^H which implies that all matches involve a trade and that all the agents quit the market. The number of agents on the market is thus equal to the number which has just entered in the market.

If the state of the world is L, the variables evolve according the following rules

$$K^L(+1) = K^L B^l_L S^h_L + M \quad (8)$$
$$B^l_L(+1) = \frac{K^L B^l_L S^h_L + x_B M}{K^L B^l_L S^h_L + M} \quad (9)$$

$B^l_L(0) = x_B$. $S^h_L(t)$ is chosen such that the payoffs of sellers are maximized. By claim 1, $n_{BL}(t) = \infty \quad \forall t$ which implies $B^l_L \neq 0$. If $S^h_L \neq 0$, there will be in each period $K^L B^l_L S^h_L$ matches which will finish on disagreement. The concerned agents will remain on the market. The total number of agents in the market will thus be equal to the sum of agents who did not reach an agreement in the previous period and of agents who are newly entered the market. Buyers who did not reach an agreement in the previous period are obligatorily informed since by assumption $n_B(t) = 0$. Considering $n_{BL}(t) = \infty$, all these buyers will continue playing *tough*. The informed buyers who arrive in the market will also announce p^L. Hence, the total

[5]For a recall, α_H is updated according to the Bayes' rule $\alpha_H/(\alpha_H + (1-\alpha_H)S^h_L)$.

number of *tough* buyers is effectively equal to the numerator of expression (9).

We can show that for each B_L^l there is only one possible K^L. It is more explicit if we write from (9) S_L^h as a function of $B_L^l(+1)$ and that then we introduce this new expresion (10) in (8) to obtain (11)

$$S_L^h = \frac{(x_B - B_L^l(+1))M}{(B_L^l(+1) - 1)B_L^l K^L} \tag{10}$$

$$K^L = \frac{1 - x_B}{1 - B_L^l} M \tag{11}$$

We can rewrite (9) as

$$B_L^l(+1) = \frac{(1 - x_B)B_L^l S_L^h + x_B(1 - B_L^l)}{(1 - x_B)B_L^l S_L^h + (1 - B_L^l)} \tag{12}$$

Obviously, the right term is increasing in S_L^h. It implies that if $S_L^h \in [0, 1]$ then

$$x_B \leq B_L^l(+1) \leq \frac{x_B + (1 - 2x_B)B_L^l}{1 - x_B B_L^l} \equiv p(B_L^l) \tag{13}$$

So, $B_L^l(+1) \in [x_B, p(B_L^l)]$.

2.3 Characterisation of $S_L^h(t)$ at equilibrium

We define $\Delta V_{SL}(B_L^l(t), B_L^l(t + 1))$ which is the difference of gain between playing *soft* tomorrow and playing *soft* today for an informed seller in state L. This difference depends on time because $B_L^l(t)$ may be non-stationary. Remark that $\Delta V_{SL}(B_L^l(t), B_L^l(t + 1) < 0$ does not imply that the best solution is to stop in t.

$$\begin{aligned}
\Delta V_{SL}(B_L^l, B_L^l(+1)) &= (1 - B_L^l)(p^H - c_L)... \\
&... +B_L^l \delta[((1 - B_L^l(+1))(p^M - c_L) + B_L^l(+1)(p^L - c_L)]... \\
&... -[((1 - B_L^l)(p^M - c_L) + B_L^l(p^L - c_L)] \\
&= B_L^l \Big[(-p^H + p^M - p^L + c_L) + \delta(p^M - c_L) + ... \\
&... +\delta B_L^l(+1)(p^L - p^M)\Big] + (p^H - p^M) \\
&\equiv B_L^l[X - B_L^l(+1)Y] + Z
\end{aligned} \tag{14}$$

Clearly, Y and Z are positive. The sign of X is indetermined. In the first equality, the two first lines correspond to playing *tough* today and *soft* tomorrow while the third one corresponds to playing *soft* today.[6]

Assume that a seller stops today playing *tough*. ΔV_{SL} is a measure of gain for a seller if he decides to play *tough* one period more. The measure of gain for a seller if he decides to play *tough* T periods more is given by the sum of succesive ΔV_{SL} balanced in order to take account of discount factor δ. If there exists a T such that this sum is positive, then playing *tough* T periods more gives a higher expected payoff than playing *soft* today. If this sum is negative for all T, then the maximum expected payoff is reached by playing *soft* today. If the sum is null for a T, then the seller is indifferent between playing *soft* today or playing *tough* T periods more.

Proposition 1 *Optimal strategies are such that the sequence $S^h_L(t) \in [0,1]$ satisfies*

$$S^h_L(t) = 1 \implies \exists\, T \ s.t. \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) \geq 0 \quad (15)$$

$$S^h_L(t) < 1 \implies \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) \leq 0 \quad \forall\, T \quad (16)$$

$$\exists\, T \ s.t. \sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) > 0 \implies S^h_L(t) = 1 \quad (17)$$

$$\sum_{i=0}^{T} \delta^i \Delta V_{SL}(t+i) < 0 \quad \forall\, T \implies S^h_L(t) = 0 \quad (18)$$

2.4 Characterisation of $B^l_L(t)$ at equilibrium

We define in a first step some functions and sets that we use to state a proposition characterising the sequence $B^l_L(t)$ at equilibrium. The functions will give in certain circumstances the next element of the sequence $B^l_L(t)$ at equilibrium. The sets will be used to define these circumstances.

[6] If a seller plays *soft* today, he has a probability $(1 - B^l_L)$ to meet a *soft* buyer and consequently to obtain a payoff $(p^M - c_L)$, otherwise (i.e. with probability B^l_L) he will get $(p^L - c_L)$ due to a meeting with a *tough* buyer. If a seller announces p^H, he will reach an agreement only if he is matched with a *soft* buyer. It occurs with a probability $(1 - B^l_L)$ and the payoff is then $(p^H - c_L)$. Otherwise, with a probability B^l_L, he will remain in the market. In the next period, if he plays *soft*, he has an expected payoff equal to the expression between brackets which must be multiplied by the discount factor δ because trade occurs one period later.

$z(B_L^l)$ We define this function such that $\Delta V_{SL}(B_L^l, z(B_L^l)) = 0$.

$p(B_L^l)$ This function is define as follows

$$p(B_L^l) = \frac{x_B + (1-2x_B)B_L^l}{1 - x_B B_L^l} \tag{19}$$

Clearly, $p(B_L^l) > B_L^l$. Remark that if $B_L^l(+1) = p(B_L^l)$ then $S_L^h = 1$[7]. The function $p(B_L^l)$ is increasing in its argument. If we define a sequence $B_L^l(t)$ by $B_L^l(+1) = p(B_L^l)$, we observe $\lim_{t\to\infty} B_L^l(t) = 1$. In other words, $\forall \gamma \in [x_B, 1[$ and $\forall B_L^l(0) \in [x_B, 1]$ there exists a $\bar{t}$ such that $B_L^l(t) > \gamma \forall t \geq \bar{t}$.

$$\begin{aligned} \Delta V_{SL}(B_L^l, p(B_L^l)) &= B_L^l[X - Y\frac{x_B + (1-2x_B)B_L^l}{1 - x_B B_L^l}] + Z \quad (20) \\ \frac{\partial \Delta V_{SL}(B_L^l, p(B_L^l))}{\partial B_L^l} &= \frac{-Y[x_B + 2(1-2x_B)B_L^l - (1-2x_B)x_B(B_L^l)^2]}{(1 - B_L^l x_B)^2} \cdots \\ &\cdots + \frac{X(1 - x_B B_L^l)^2}{(1 - B_L^l x_B)^2} \end{aligned}$$

This derivative is negative if $X < Yx_B$[8]. Remark that there exists some configurations of the model's parameters such that this inequality is satisfied for all δ. The negativity of this derivative will be useful to ensure the convexity of the set P that we will define later.

$l(B_L^l)$ The definition of this function is given by

$$B_L^l[X - Yl(B_L^l)] + Z = -\max_T(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i))) \tag{21}$$

with $B_L^l(1) = l(B_L^l)$ and $B_L^l(i+1) = p(B_L^l(i))$ for $i > 1$. The left-hand term is the instantaneous ΔV_{SL} if we go from B_L^l to $l(B_L^l)$. The right-hand term, is the balanced sum of ΔV_{SL} when all sellers continue to play *tough* (i.e. $S_L^h = 1$) in the T following periods from $l(B_L^l)$. T is chosen to maximize this sum. Remark that the left-hand term is a decreasing continuous function of $l(B_L^l)$ while the right-hand term is an increasing one. So, if there exists one

[7] It can be clearer when we remember the comments below (13)

[8] X and Y are defined by (14).

$l(B_L^l)$, it is unique.

We define the following numbers :
β is such that $\Delta V_{SL}(\beta, \beta) = \beta[X - \beta Y] + Z = 0$
σ_1 is such that $\Delta V_{SL}(\sigma_1, p(\sigma_1)) = \sigma_1[X - p(\sigma_1)Y] + Z = 0$
$\sigma_2 = p(\sigma_1) = z(\sigma_1)$.
We will meet β in the steady state analysis. Actually, in one steady state of the model, B_L^l takes this value. If the derivative below (20) is negative, σ_1 is the greatest value that B_L^l can take and σ_2 is the highest value for $p(B_L^l)$ if we would like $\Delta V_{SL}(B_L^l, p(B_L^l)) \geq 0$.

We will now define some sets. A set O that B_L^l can never reached because there exist some informed buyers. A set P where ΔV_{SL} is positive or null even when all the sellers play *tough*. When $B_L^l \in P$, $p(B_L^l)$ is in P or in another set A. This last set includes β. The set A is divided in two subsets A_1 of elements lower or equal to β and A_2 of elements higher or equal to β. These sets are formally defined as follows

$$\begin{aligned} O &= [0, x_B] \\ P &=]x_B, \sigma_1[\\ A_1 &= [\sigma_1, \beta] \\ A_2 &= [\beta, \sigma_2] \\ A &= A_1 \cup A_2 \end{aligned}$$

If $x_B > \sigma_1$, $P = \emptyset$ and σ_1 is replaced by x_B in definition of A_1. When $x_B > \beta$, A_1 is also an empty set and x_B replaces β in A_2's definition. The set A_2 is also empty if $x_B > \sigma_2$.

We give now some technical claims. The proofs are relegated to an appendix section. The first claim establishes formally the order that we had implicitly assumed for σ_1, σ_2 and β when we defined the sets. The other ones are usefull to proof proposition 2.

Claim 3 *$\sigma_1 < \beta < \sigma_2$.*

Claim 4 *(a) If $\gamma > \lambda$ then $z(\gamma) < z(\lambda)$.*
(b) If $\gamma > \lambda$ then $l(\gamma) < l(\lambda)$.
(c) If $\gamma > \lambda$ then $p(\gamma) > p(\lambda)$.

Claim 5 *If $\gamma \in A$ then $z(\gamma) \leq l(\gamma) \leq p(\gamma)$.*

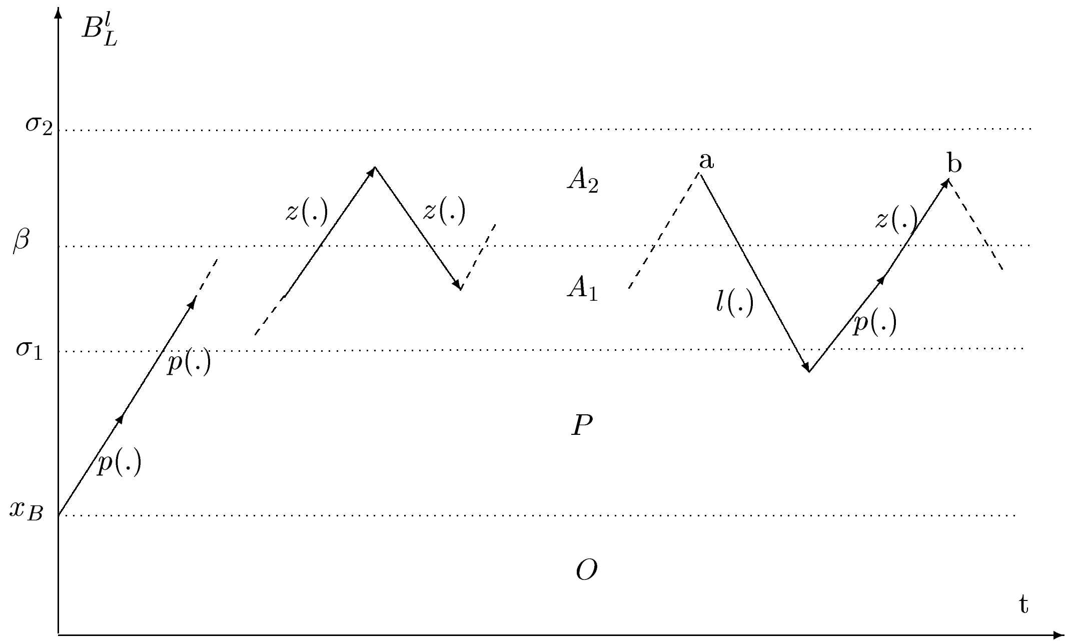


Figure 1: In P, the numbers of *soft* buyers is high. So, misrepresenting is the best action for all the sellers. In $A = A_1 \cup A_2$, the proportion of buyers who play *soft* is smaller, and misrepresenting is not always the best strategy for sellers. The function $z(.)$ is such that the payoff gain of misrepresenting compared to truth telling is null if all the following $B_L^l(t)$ are in A. So, individually, sellers are indifferent between these two actions. Otherwise, $l(.)$ ensures that the payoff implied by telling the truth in a is equal to the payoff of telling the truth in b. Hence, sellers in a are individually indifferent between misrepresenting and truth telling.

Claim 6 *If $\gamma < \beta$ then $z(\gamma) > \beta$. If $\gamma > \beta$ then $z(\gamma) < \beta$.*

Now we have the tools to charachterize B_L^l at equilibrium.

Proposition 2 *If $X < Yx_B$[9], optimal strategies are such that $B_L^l(t) \in [x_B, 1]$ evolves according to the following rules :*

$$\begin{aligned}
\textit{If } B_L^l \in P \Longrightarrow B_L^l(+1) &= p(B_L^l) \\
\textit{If } B_L^l \in A_1 \Longrightarrow B_L^l(+1) &= z(B_L^l) \\
\textit{If } B_L^l \in A_2 \Longrightarrow B_L^l(+1) &= z(B_L^l) \quad \textit{if} \quad z(B_L^l) \in A \\
B_L^l(+1) &= l(B_L^l) \quad \textit{if} \quad l(B_L^l) \in P \\
B_L^l(+1) &= x_B \quad \textit{if} \quad l(B_L^l) \in O
\end{aligned}$$

If $P = \emptyset$, $B_L^l \in A_2 \Rightarrow B_L^l(+1) = x_B$ if $z(B_L^l) \in O$.
If $P \cup A_1 = \emptyset$, $B_L^l(+1) = x_B$ in any case.

We must show that the sequence $B_L^l(t)$ described is compatible with an equilibrium and that there is no other sequence compatible with an equilibrium. Actually, by construction, the sequence $B_L^l(t)$ is compatible with an equilibrium. We will thus prove that no other sequence can be compatible with an equilibrium.

But we prove first by claim 7 that for all $B_L^l \in P \cup A$, $B_L^l(+1)$ determined by proposition 2 is included in $P \cup A$. This means that we cannot reach by iteration a point where the next element of the sequence cannot be determined by the rule of this proposition.

Claim 7 *Assume $B_L^l(t)$ evolves according to the rules given by proposition 2. If $B_L^l \in A \cup P$ then $B_L^l(+1) \in A \cup P$.*

Proof We will use the results of claims 4 and 5. $B_L^l \in P$ is equivalent to $x_B < B_L^l \leq \sigma_1$.

$$B_L^l \leq \sigma_1 \implies B_L^l(+1) = p(B_L^l) \leq p(\sigma_1) = \sigma_2 \tag{22}$$
$$B_L^l > x_B \implies B_L^l(+1) = p(B_L^l) > B_L^l > x_B \tag{23}$$

$B_L^l \in A_1$ is equivalent to $\sigma_1 < B_L^l \leq \beta$.

$$B_L^l \leq \beta \implies B_L^l(+1) = z(B_L^l) \geq z(\beta) = \beta \tag{24}$$
$$B_L^l > \sigma_1 \implies B_L^l(+1) = z(B_L^l) < z(\sigma_1) = \sigma_2 \tag{25}$$

[9] For a recall, this condition ensures the convexity of P. There exist some configurations of parameters such that this condition is satisfied for all $\delta \in [0, 1]$.

When $B_L^l \in A_2$, it is clear that all the possibilities for $B_L^l(+1)$ will be in $A \cup P$. We must prove that $B_L^l(+1)$ is defined $\forall B_L^l \in A_2$. When we combine claims 4 and 5 with claim 6, we obtain the following equivalence for all $B_L^l \in A_2$:

$$z(B_L^l) \in A_1 \Longleftrightarrow l(B_L^l) \notin P \cup O \tag{26}$$

This equivalence is sufficient to ensure that $B_L^l(+1)$ is defined in all cases.

Now, with the following claim, we prove that there is no other sequence compatible with an equilibrium where $n_B(t) = 0 \ \forall t$.

Claim 8 *Assume that in a sequence $B_L^l(t)$, there is a $B_L^l(+1)$ lower or higher than the one given by the rule of proposition 2, then (17) or (18) are not satisfied or $S_L^h \notin [0,1]$.*

Proof $B_L^l(+1)$ higher If $B_L^l \in P$, we know $\Delta V_{SL}(B_L^l, p(B_L^l)) > 0$ and ΔV_{SL} is increasing in its second agrument. Hence, if $B_L^l(+1) < p(B_L^l)$ we have $S_L^h < 1$ and $\Delta V_{SL}(B_L^l, B_L^l(+1)) > 0$. Assume $B_L^l \in A_1$ or $B_L^l \in A_2$ and $Z(B_L^l) \in A_1$, then $\Delta V_{SL}(B_L^l, z(B_L^l)) = 0$. Consider $B_L^l(+1) < z(B_L^l)$, clearly we have also $S_L^h < 1$ and $\Delta V_{SL}(B_L^l, B_L^l(+1)) > 0$. In the case $B_L^l \in A_2$ and $l(B_L^l) \in P$, $S_L^h < 1$ for all $B_L^l(+1) \leq l(B_L^l)$. By definition :

$$\Delta V_{SL}(B_L^l, l(B_L^l)) = -\max_T(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i))) \tag{27}$$

with $B_L^l(1) = l(B_L^l)$ and $B_L^l(i+1) = p(B_L^l(i))$ for $i > 1$. Remark that ΔV_{SL} is decreasing in its second term and the maximized term is increasing in $B_L^l(1)$. It means that if $B_L^l(+1) < l(B_L^l)$, then

$$\Delta V_{SL}(B_L^l, B_L^l(+1)) + \max_T \big(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i)))\big) > 0 \tag{28}$$

Since $S_L^h < 1$, (28) is a violation of (17). Finally, $B_L^l(+1) < x_B \Rightarrow S_L^h < 0$.

Proof $B_L^l(+1)$ lower The higher $B_L^l(+1)$, the higher S_L^h. When $B_L^l \in P$, choosing $B_L^l(+1)$ implies $S_L^h > 1$. We know that from all elements of $B_L^l(t)$ in P, the sequence will grow until it arrives in A_2 (see comment after (19) and the definition of $B_L^l(+1)$ when $B_L^l \in A_1$. From A_2, the sequence goes directly to a point in $A_1 \cup P$ and comes back more or less quickly to a point in A_2. We call *top* an element of $B^l_L(t)$ in A_2. We know[10] that, between two succesive *tops*, the sum $\sum_i \delta^i \Delta V_{SL}$ is lower or equal to zero[11]. At each *top* $S^h_L \leq 0$. At a *top*, we can take $B^l_L(+1)$ greater than given by proposition 2 but then $S^h_L > 0$ and the balanced sum of ΔV_{SL} until the next *top* is negative. According to first part of the proof, this loss can never be recovered by continuing longer because the balanced sum between two *tops* can never be positive. So, (18) is not satisfied. If $B^l_L \in A_1$, taking $B^l_L(+1) > z(B^l_L)$ implies $\Delta V_{SL} < 0$ and $S^h_L > 0$. By the line of same reasoning, we conclude that (18) is once again not satisfied.

2.5 Existence of an equilibrium

To see that there exists a profile of strategies which constitute an equilibrium we see first that all sequences $B^l_L(t)$ respecting proposition 2 implies a sequence $S^h_L(t) \in [0, 1]$. If $B^l_L \in P$, we know $B^l_L(+1) = p(B^l_L)$ which correspond to $S^h_L = 1$. When $B^l_L \in A$, by claim 5 we obtain $B^l_L(+1) \leq p(B^l_L)$. This inequality involves $S^h_L \leq 1$. By definition, $B^l_L(+1)$ can never be inferior at x_B. Hence, $S^h_L \geq 0$. It is obvious that all sequences $S^h_L(t)$ satisfaying condition of proposition 1 can be obtained by at least one profile of strategies and that all profiles which imply this kind of sequence of $S^h_L(t)$ are equilibria. Hence, we have the following proposition.

Proposition 3 *For all $B^l_L(0) \in A \cup P$ and $K^L(0)$ such that (11) is satisfied, there exists at least an equilibrium.*

3 Evolution of $B^l_L(t)$ at equilibrium

In this section, the comparison between a stationary steady state analysis and a complete analysis sheds light on some new features. Especially, it appears that the stationary steady state is not always reached in a finite number of periods. Moreover, the market does not always converge towards the stationary steady state at equilibrium.

To prove that the two analyses are identical, it must be demonstrated that all variables evolve in the same way in both analyses. On the other hand, to show a difference, it is enough that a variable does not move in an identical way. So, we study in this section only the evolution of $B^l_L(t)$ which

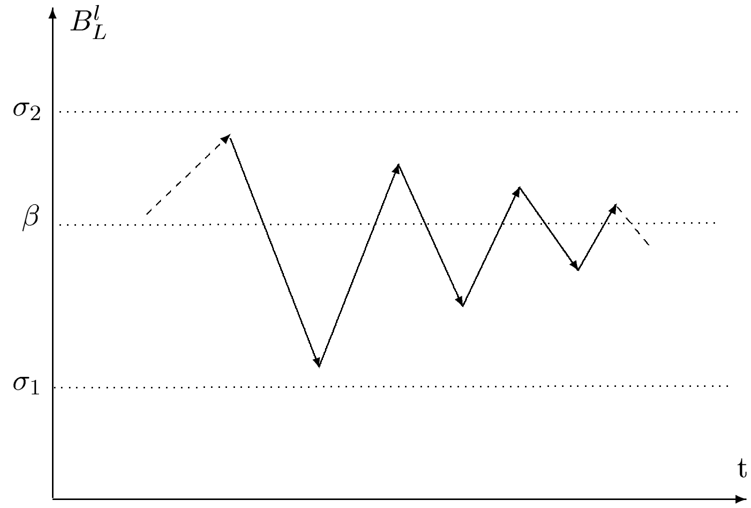
[10] In case of doubt, see the first part of the proof

[11] Remark, this balanced sum limited to a point between the two *tops* is always lower than between the two *tops*.

is sufficient to show some differences with a steady state where $B^l_L(t)$ will be constant.

We first present a case where the market converges towards a steady state.

Proposition 4 *If $X \in]0, Y x_B[$ and $x_B < \beta$, the sequence $B^l_L(t)$ converges towards β at equilibrium.*



According to the comment after $p(B^l_L)$'s definition and the characterization of $B^l_L(t)$ at equilibrium, we know that at least one element of $B^l_L(t) \in A$. The two following claims prove the proposition. The first claim proves that the distance between the sequence and the value of stationary steady state decreases strictly with time. The second claim shows that once the sequence enters in A, it will never exit this set.

Claim 9 *If $X > 0$ and $x_B < \beta$, we define $B^l_L(+2)$ as $z(z(B^l_L))$* [12] *then $\forall B^l_L \in A$*

$$B^l_L > \beta \iff \beta < B^l_L(+2) < B^l_L \tag{29}$$

$$B^l_L < \beta \iff \beta > B^l_L(+2) > B^l_L \tag{30}$$

[12] If $z(\lambda) < x_B$ we impose implicitely $z(\lambda) = x_B$ which is in agreement with proposition 2. We proceed in the same way in what follows.

Proof

$$B_L^l(+2) = \frac{\frac{ZY}{\frac{Z}{B_L^l}+X} + X}{Y} \tag{31}$$

If $X > 0$, $B_L^l(+2) > B_L^l$ is equivalent to

$$0 \;>\; Y(B_L^l)^2 - XB_L^l - Z$$

By definition of β, this inequality is true if and only if $\beta > B_L^l$. The inequalities between $B_L^l(+2)$ and β are obtained from claim 6.

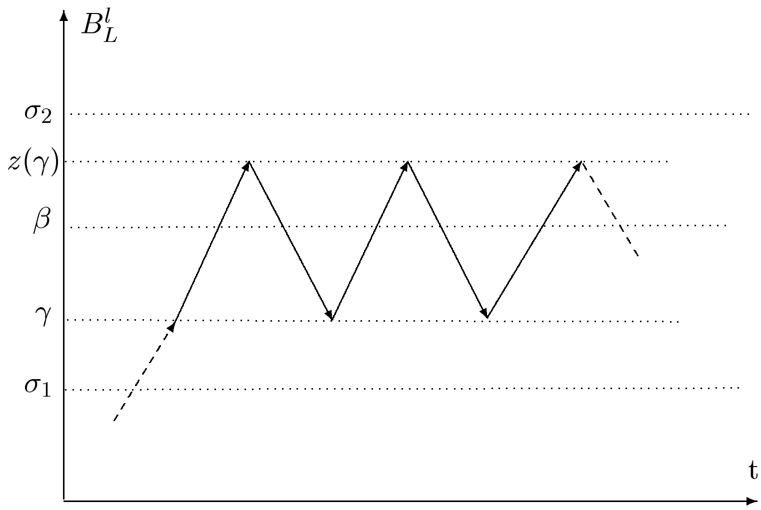
We now present the second claim which proves that once the sequence $B_L^l(t)$ enters in A, it will remain in this set during all future periods.

Claim 10 *Assume $X > 0$. If $\gamma > 0$ and $x_B < \beta$ then $z(\gamma) \in A \; \forall \gamma \in A$.*

Proof $\gamma \in A$ is equivalent to $\sigma_1 < \gamma < \sigma_2$.
By claim 4, $\gamma > \sigma_1 \Rightarrow z(\gamma) < z(\sigma_1) = \sigma_2$.
By claim 9, we know $z(z(\sigma_2)) < \sigma_2 = z(\sigma_1)$.
It implies $z(\sigma_2) > \sigma_1$. Hence $\gamma < \sigma_2 \Rightarrow z(\gamma) > z(\sigma_2) > \sigma_1$.

We now turn to a case where $B_L^l(t)$ reaches a permanent cycle in a finite number of periods.

Proposition 5 *If $X = 0$ and $x_B < \beta$, the sequence $B_L^l(t)$ increases until it reaches A. Once in A, it takes alternatively a value $\gamma \in A_1$ and a value $z(\gamma) \in A_2$. One of these two values is equal to the first element of $B_L^l(t) \in A$.*



Claim 11 *If $X = 0$ and $x_B < \beta$ then $\forall B_L^l \in A$*

$$B_L^l(+2) = B_L^l \tag{32}$$

Proof We put $X = 0$ in (31) and we obtain the claim.

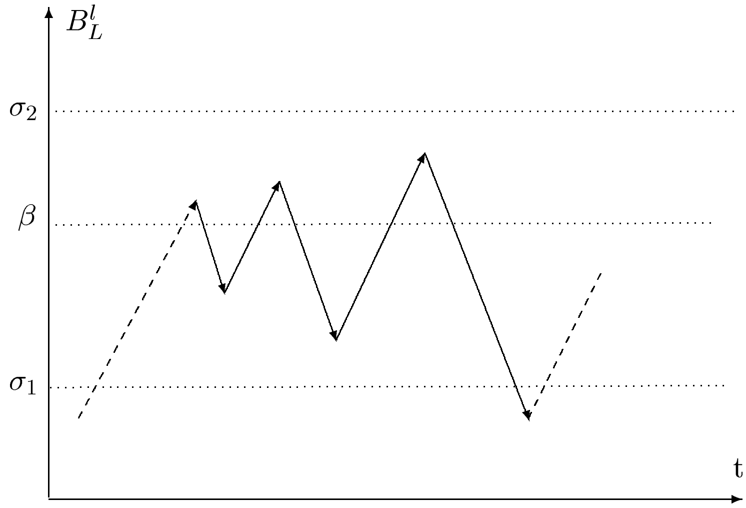
Until now, X was greater or equal to zero. A complete analysis of the situation with $X < 0$ is more complex. We restrict ourselves to showing that the behaviour cannot be identical to the previous ones. The following claim states that the sequence $B_L^l(t)$ moves away from the stationary steady state when the sequence is in A. This claim is sufficient to prove that the behaviour cannot be identical to a situation where $X \geq 0$.

Claim 12 *If $X < 0$ and $x_B < \beta$ then $\forall B_L^l \in A$*

$$B_L^l < \beta \iff \beta < B_L^l(+2) < B_L^l \tag{33}$$

$$B_L^l > \beta \iff B_L^l(+2) > B_L^l > \beta \tag{34}$$

Proof Identical to the proof of claim 9.



4 The steady states

In this section, we show that our analysis include some equilibria found by Serrano and Yosha (1993) in their steady state analysis. However, our analysis does not include all the results of Serrano and Yosha (1993) because we have imposed some restrictions on the behaviour of uninformed buyers. The last proposition relaxes this condition and establishes the existence in a dynamic analysis of an equilibrium with full information revelation when market are sufficiently frictionless.

Assume $B_l^l(0) = \beta \geq x_B$ and δ lower than the value in equation (4) of claim 2, then the rules given by proposition 2 implies a steady state. This steady state is called E3 by Serrano and Yosha (1993).

The propositions 4 and 5 concern cases with $x_B < \beta$. The sequences $B_L^l(t)$ evolve around an equilibrium $E3$. If the first element of $B_L^l(t) \in A$ is different of β, the steady state will generally not be reached in a finite number of periods. The next propositions show that some equilibria are perfectly describe by the stationary analysis because the steady state is instantaneously reached.

Proposition 6 *If $x_B > \beta$, the sequence $B_L^l(t)$ is a stationary steady state for all t.*

Proof The rule given by proposition 2 is reduced to $B_L^l(+1) = x_B$. It implies $S_L^h(t) = 0 \ \forall t$ which is supported by a unique profile for the sellers' strategies, $n_{SL}(t) = 0 \ \forall t$.

It is actually an equilibrium established by Serrano and Yosha (1993) and called $E2$. The following proposition establishes the existence of a steady state equilibrium, called E1 in Serrano and Yosha (1993), without convergence phase when δ is high. The novelty compared to Serrano and Yosha (1993) is the dynamic context of the proof.

Proposition 7 *If*

$$\alpha_H \leq \frac{p^M - p^L}{(1-\delta)(u_H - p^H) + (p^M - p^L)} \tag{35}$$

then $n_{SL}(t) = 0$ and $n_B(t) = 1 \ \forall t$ imply an equilibrium.

Proof $n_{SL}(t) = 0$ implies $S_L^h = 0$. Since no seller misrepresents, once a buyer has met a seller who announces a state H, he knows that it is useless to play *tough*. So, n_B can not be higher than one. To see that $n_B \neq 0$, it is sufficient to oberve that $\Delta V_B(0,0) > 0$. Hence, $n_B(t) = 1$ is an optimal strategy given $n_{SL}(t) = 0$.

The proposed strategies imply $B_L^l = 1$. So, $\Delta V_{SL} < 0$ and the conditions given by proposition 1 are fulfilled. Hence, no seller has an incentive to deviate.[13]

This proposition implies the existence of an equilibrium with full information revelation[14] even in a dynamic analysis. *A priori*, equilibria E1 and E3 may coexist. Hence, we did not prove that all equilibria imply complete information revelation when market becomes approximately frictionless.

[13] This result depends crucially on the fact that an individual deviation does not affect the value of S_L^h and B_L^l because agents are negligeable.

[14] All the trades occur at the *good* price.

Conclusion

By assumption, the stationary steady state analysis is unable to deal with the following intuition. At the start, sellers in state L can have an incentive to misrepresent because there exists a possibility to meet an uninformed buyer who plays soft. Consequently, the proportion of informed buyers on the market increases. The increase reduces the incentive to misrepresent. Then, there is a period in which some sellers tell the truth and the market is partially cleared. We showed that this intuition was correct.

In other words, we proved that the sequence $B_L^l(t)$ at equilibrium is rarely constant. Moreover, we discovered that the dynamic evolution does not converge with certainty to the equilibria established by the stationary analysis. Furthermore, if the sequence $B_L^l(t)$ converges towards a stationary steady state, time is needed for the transition, which requires generally an infinite number of periods.

This result is not totally general since it holds only for agents relatively impatients, i.e. δ sufficiently low. Indeed both sets of conditions in claim 2 imply an upper bound on δ. Nevertheless, it is sufficient to stress the risk to restrict our analysis to steady states. Especially, in market design, to compare this particular type of design with other ones. Always in market design, if the designer can choose the δ in a limited set, the dynamics of the equilibrium imply that the optimal δ is maybe not the higher one.

A priori we might conjecture that when δ tends to one, a convergence phase continues to exist and would imply an incomplete information revelation. Our only result for the case in which δ tends to one is proposition 7, which shows that there exists always an equilibrium without convergence phase which imply a complete information revelation. So, the differences of result between Serrano and Yosha 1993 and Blouin and Serrano 2001 can not be completely explained by the restricted analysis by Serrano and Yosha 1993.

A Additional Proofs

Proof of claim 3 Assume $\sigma_1 > \beta$. We know $\Delta V_{SL}(\sigma_1, p(\sigma_2\sigma_1)) < \sigma_1[X - \sigma_1 Y] + Z$. By definition $\Delta V_{SL}(\sigma_1, p(\sigma_1)) = 0$. This implies

$$\sigma_1[X - \sigma_1 Y] + Z \quad > \quad \beta[X - \beta Y] + Z \tag{36}$$

$$\sigma_1 \quad < \quad \beta \frac{X - \beta Y}{X - \sigma_1 Y} < \beta \tag{37}$$

There is then a contradiction. Assume $\sigma_1 = \beta$. It implies $p(\sigma_1) = \beta = \sigma_1$ but this contradicts the property $p(B_L^l) > B_L^l \quad \forall B_L^l$.
To prove the second part, we have by definition

$$\begin{aligned}
\sigma_1[X - Yp(\sigma_1)] + Z &= \beta[X - Yp(\beta)] + Z \\
\sigma_1[X - Y\sigma_2] &= \beta[X - Y\beta] \\
X - Y\sigma_2 &< X - Y\beta \\
\sigma_2 &> \beta
\end{aligned}$$

We obtain the third line by the fact that $\sigma_1 < \beta$ and the two terms in brackets are negative.

Proof of claim 4 (a) By definition,

$$\begin{aligned}
\gamma[X - Yz(\gamma)] + Z &= 0 \\
\lambda[X - Yz(\gamma)] + Z &> 0 = \lambda[X - Yz(\lambda)] + Z \\
z(\gamma) &< z(\lambda)
\end{aligned}$$

(b) As $\gamma > \lambda$ and $[X - Yl(\lambda)] < 0$, we can write the first line. The second line is obtained by $l(.)$'s definition.

$$\begin{aligned}
\gamma[X - Yl(\lambda)] &< \lambda[X - Yl(\lambda)] \\
\gamma[X - Yl(\lambda)] &< -\max_T \Big(\sum_{i=1}^{T} \delta^i \Delta V_{SL}(B_L^l(i), p(B_L^l(i))) \Big) \\
&\quad \text{with } B_L^l(1) = l(\lambda) \text{ and } B_L^l(i+1) = p(B_L^l(i)) \forall i > 1
\end{aligned}$$

The left term is decreasing in $l(.)$ while the right term is increasing. So, to find an equality, $l(\gamma)$ must be lower than $l(\lambda)$.
(c) From (19), we compute

$$\frac{\partial p(B_L^l)}{\partial B_L^l} = \frac{(1 - x_B)^2}{1 - x_B B_L^l} > 0 \tag{38}$$

Proof of claim 5 Obviously,

$$\Delta V_{SL}(\gamma, z(\gamma)) \geq \Delta V_{SL}(\gamma, l(\gamma)) \geq \Delta V_{SL}(\gamma, p(\gamma)) \tag{39}$$

For recall, ΔV_{SL} is decreasing in its second argument. So, we obtain the claim.

Proof of claim 6 It is a corollary of claim 4.

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