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INEFFICIENCY OF COMPETITIVE EQUILIBRIUM
WITH HIDDEN ACTION AND FINANCIAL
MARKETS*

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ABSTRACT

In this paper, we study a pure exchange economy with idiosyncratic uncertainty, hidden action and multiple consumption goods. We consider two different market structures: contingent markets on the one hand, and financial and spot markets on the other hand. We propose a competitive equilibrium concept for each market structure. We show that the equilibrium with contingent markets is efficient in an appropriate sense, while the equilibrium with financial and spot markets is inefficient, provided that assumptions on preferences more general than those usually considered in the literature hold.

KEYWORDS: Hidden action, enforcement, constrained efficiency.

JEL CLASSIFICATION: D53, D61, D82.

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1. INTRODUCTION

In this paper, we study a pure exchange economy with idiosyncratic uncertainty, hidden action and multiple consumption goods. For this economy, we consider two different market structures. On the one hand, we assume that there exists a complete set of markets for commodities with delivery contingent on idiosyncratic states. On the other hand, we assume that there exists a complete set of financial markets, together with spot markets for consumption goods.

For each market structure considered, we propose a suitable equilibrium concept and we study its efficiency properties, in order to understand if differences in market structure result in differences in efficiency of equilibrium.

It is well known that when there is no hidden action, assuming complete contingent markets or complete financial and spot markets is irrelevant as far as efficiency is concerned. This result follows because the two market structures allow to reach the same allocations of consumption goods, provided that prices are related in an appropriate way.

When there is hidden action, the incentive compatibility constraint becomes an essential element of the equilibrium. With multiple consumption goods, that constraint depends on optimal bundles, which in turn depend on the market structure. The latter therefore becomes relevant for the efficiency of competitive equilibria.

In this paper, we first show that any equilibrium with contingent markets is efficient in an appropriate sense. Subsequently, we show that the equilibrium with financial and spot markets is inefficient, provided that assumptions on preferences more general than those usually considered in the literature hold.

The paper is organized as follows: in section 2 the economy is described and in section 3 the appropriate efficient allocation is characterized. Subsequently, in section 4 the competitive equilibrium with contingent markets is introduced, and its efficiency properties are studied in section 4.3. Subsequently, in section 5 the competitive equilibrium with financial markets is introduced, and its efficiency properties are studied in sections 5.3-5.5. Finally, in section 6 the relevant related literature is discussed.

2. THE ECONOMY

Consider a pure exchange economy with $L \geq 1$ consumption goods. The economy is populated by a large number of consumers all ex ante equal, and lasts two periods $t = 0, 1$.

There is no consumption at $t = 0$, when consumers face an idiosyncratic uncertainty which realizes in the second period. At $t = 1$, each consumer may be in one out two states $s \in \mathcal{S} = \{1, 2\}$.

Uncertainty affects only the amount of each good consumers are endowed with at the beginning of the second period. Individual endowments are denoted by $w = (w_1, w_2) \in \mathbb{R}_+^{2L}$.¹ It is assumed that $w_1 \gg w_2$, so that state $s = 1$ will be referred to as the good state.²

Consumers can choose an action $a \in \mathcal{A} = [0, 1]$ affecting, on the one hand, the probabilities of the idiosyncratic states, and, on the other hand, the utility consumers get from consumption.

Preferences are represented by the following expected utility function $v : \mathbb{R}_+^{2L} \times \mathcal{A} \rightarrow \mathbb{R}$:

$$v(x_1, x_2, a) := \pi_1(a) u(x_1, a) + \pi_2(a) u(x_2, a),$$

where $\pi_s(a) : \mathcal{A} \rightarrow (0, 1)$ is a function giving the probability of state s when action a is chosen.

According to the above general formulation, a is assumed to affect the utility of every consumption good once the state is realized. This case will be referred to as non-separable preferences.

A polar case arises when a does not affect the utility of any consumption good once the state is realized. This case will be referred to as separable preferences and it is formally stated in the following:

Assumption S. (SEPARABLE PREFERENCES) *There exist utility indexes $\tilde{u} : \mathbb{R}_+^L \rightarrow \mathbb{R}$ and $c : \mathcal{A} \rightarrow \mathbb{R}$ such that $u(x_s, a) := \tilde{u}(x_s) - c(a)$.*

Intermediate cases are also possible. In particular it may be the case that a affects the utility of some, but not all consumption goods. A particularly simple case is considered in the following:

Assumption H. (HEMI-NON-SEPARABLE PREFERENCES) *There exist utility indexes $f : \mathbb{R}_+ \rightarrow \mathbb{R}$ and $g : \mathbb{R}_+^{L-1} \times \mathcal{A} \rightarrow \mathbb{R}$ such that $u(x_s, a) := f(x_{sj}) + g(x_s^{-j}, a)$ for some consumption good j ,*

where $x_s^{-j} \in \mathbb{R}_{++}^{L-1}$ is a bundle consumption goods obtained by eliminating good sj .

It is assumed that the action chosen by a consumer remains unknown to anyone else in the economy, so that information is asymmetric. On the other hand, it is assumed that endowments are verifiable at $t = 1$, so that it is possible to determine in which idiosyncratic state consumers are.

¹ \mathbb{R}_+^L denotes the positive orthant of dimension L .

²Given two vectors $x, y \in \mathbb{R}_+^L$, $x \gg y$ means $x_l > y_l$ for all l .

3. CONSTRAINED EFFICIENT ALLOCATION

Suppose there exists a benevolent planner perfectly informed on the fundamentals of the economy, in particular on preferences and endowments. Suppose in addition that he is subject to the same limits on information as any other economic agent, so that he cannot verify the level of action chosen by consumers.

The role of the planner is to assign a feasible distribution of consumption goods and to prescribe an action so as to maximize consumers' utility. Because of limits on information, the planner can only prescribe actions that are optimal given the assigned consumption bundle. In the standard jargon, this means that the planner can only choose among the set of incentive compatible actions.

Implicit in this reasoning is the fact that the planner observes actual consumption at $t = 1$, so that he can enforce a given distribution of consumption bundles. It is therefore excluded that consumers re-trade after having received their prescribed bundles. This in turn implies that the choice of a is the only hidden action consumers make.

From the above discussion, it follows that the constrained efficient allocation (x_1, x_2, a) is the solution of the following:

$$\begin{aligned} \max_{x_1, x_2, a} \quad & v(x_1, x_2, a) \\ \text{s.t.} \quad & \pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) \leq 0, \end{aligned} \tag{1a}$$

$$a = \arg \max v(x_1, x_2, a). \tag{1b}$$

In the above problem, (1a) is the feasibility constraint, which holds in expected value thanks to assumption of large number of consumers, while (1b) is the incentive compatibility constraint. It states that the only admissible actions in the planner's choice set are those maximizing the consumers' utility given the prescribed consumption bundle.

In what follows, \mathcal{B} will denote the set of (x_1, x_2, a) satisfying (1a) and (1b).

The solution of the planner's problem will be the benchmark for evaluating the efficiency of the different types of competitive equilibrium to be introduced in the following sections.

4. THE CONTINGENT MARKET (CM) MODEL

In this section, we suppose there exists a complete set of markets for commodities with delivery contingent on idiosyncratic states. Consumers trade

in these markets, which are open at $t = 0$, by selling their state-contingent endowments and buying a bundle of consumption goods to be delivered only if the corresponding state is realized.

Once bundles have been chosen, markets close. Consumers then choose their preferred action a . At $t = 1$, the uncertainty is resolved, and consumption goods are eventually delivered according to purchases.

In order to define an equilibrium concept for this economy, we have to introduce an appropriate price system. Given the complete set of contingent-commodity markets, SL prices has to be quoted. Suppose for a moment that there is no hidden action. In this case, we know from Malinvaud (1972) that there exist restrictions on the price system such that a well defined equilibrium concept displaying satisfactory efficiency properties can be defined.

These restrictions essentially require the price of one unit of good l delivered in state s to be properly related to the actual probability of that state. In particular, suppose that p_l is the price of one unit of good l with delivery at $t = 1$ no matter what the state is. Following Malinvaud (1972), we may call it the price for *sure delivery* of good l .

Given the price for sure delivery, and given that state s has probability π_s , we let the price at $t = 0$ of one unit of good l with delivery at $t = 1$ in state s be equal to $\pi_s p_l$.³

The introduction of hidden action does not alter the above reasoning in an essential way. In this case π_s depends on a . Hence we shall naturally assume that the price of one unit of good l with delivery contingent on state s when action a is chosen is equal to $\pi_s(a) p_l$.

To complete the reasoning it is only required to take into account the appropriate level of action. The only possible candidate is the incentive compatible one, which satisfies:

$$a = \arg \max v(x_1, x_2, a).$$

From the above expression it is apparent that the incentive compatible level of a is well defined only once the the whole consumption bundle (x_1, x_2) is considered. Therefore the price of one unit of good l in state s cannot be quoted unless the consumption level of all the other goods is specified.

Moreover, the price one unit of good l in state s may well be different when consumed in conjunction with different quantities of other consumption goods. For this reason, the price system just constructed is sometime referred to as non-linear.

³The price system described in the text is a simple generalization to the case of multiple consumption good of the well known *fair price* system.

4.1. CM CONSUMERS' PROBLEM

From the above discussion, it follows that at $t = 0$ – taking $p \in \mathbb{R}_{++}^L$ as given – consumers choose (x_1, x_2, a) so as to solve the following:

$$\begin{aligned} \max_{x_1, x_2, a} \quad & v(x_1, x_2, a) \\ \text{s.t.} \quad & \pi_1(a)(p \cdot (x_1 - w_1)) + \pi_2(a)(p \cdot (x_2 - w_2)) \leq 0, \end{aligned} \quad (2a)$$

$$a = \arg \max v(x_1, x_2, a). \quad (2b)$$

In the above problem, (2a) is the budget constraint – a single inequality because of the market structure, while (2b) is the incentive compatibility constraint.

In what follows, $\mathcal{C}(p)$ will denote the set of (x_1, x_2, a) that satisfy (2a)–(2b).

4.2. CM EQUILIBRIUM

In a contingent market (CM) equilibrium, consumers optimize and markets clear. These properties are collected in the following:

Definition 4.1. *A CM equilibrium is (x, a, p) such that:*

1. $(x, a) = \arg \max \{v(x, a) \mid (x, a) \in \mathcal{C}(p)\}$,
2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0$.

The market clearing condition in the above definition is expressed in expected value because of the assumption of large number of consumers.

4.3. EFFICIENCY OF CM EQUILIBRIUM

In this section, we analyze the efficiency of CM equilibrium, where the former is defined as usual: an equilibrium allocation of consumption goods, and the associated incentive compatible action, is constrained efficient if there does not exist another feasible allocation of consumption goods, and an associated incentive compatible action, such that no consumer is worse off, and at least one consumer is better off.

In the economy under analysis all consumers are ex ante equal. Hence the last requirement implies that the utility of the (representative) consumer is higher at the alternative allocation than at the equilibrium one.

Given this definition of efficiency, it is straightforward to verify the following:

Proposition 1. *Every CM equilibrium is constrained efficient.*

Proof. Take a CM equilibrium (x, a, p) and suppose that it is not constrained efficient. Then there exists $(\bar{x}, \bar{a}) \in \mathcal{B}$ such that $v(\bar{x}, \bar{a}) > v(x, a)$. Since $(\bar{x}, \bar{a}) \in \mathcal{B}$ implies $\bar{a} = \arg \max v(\bar{x}, a)$, if the following inequality holds:

$$\pi_1(\bar{a})(p \cdot (\bar{x}_1 - w_1)) + \pi_2(\bar{a})(p \cdot (\bar{x}_2 - w_2)) \leq 0,$$

then $(\bar{x}, \bar{a}) \in \mathcal{C}(p)$ and (x, a) cannot be an equilibrium choice, for it is not utility maximizing. It follows that:

$$\pi_1(\bar{a})(p \cdot (\bar{x}_1 - w_1)) + \pi_2(\bar{a})(p \cdot (\bar{x}_2 - w_2)) > 0,$$

which implies that:

$$p \cdot (\pi_1(\bar{a})(\bar{x}_1 - w_1) + \pi_2(\bar{a})(\bar{x}_2 - w_2)) > 0,$$

hence $(\bar{x}, \bar{a}) \notin \mathcal{B}$, a contradiction that concludes the proof. ■

The above proposition extends to the case of hidden action the result of efficiency of competitive equilibrium for economies with a complete set of contingent-commodity markets.

4.4. CHARACTERIZATION OF CM EQUILIBRIUM

In this section, we propose a characterization of the CM equilibrium through first order conditions that will be used later on in the paper.

As a first step, we replace the incentive compatibility constraint (2b) with the corresponding first order condition:

$$\partial_a v(x_1, x_2, a) = 0.$$

The CM consumers' problem is then rewritten as follows:

$$\begin{aligned} \max_{x_1, x_2, a} \quad & v(x_1, x_2, a) \\ \text{s.t.} \quad & \pi_1(a)(p \cdot (x_1 - w_1)) + \pi_2(a)(p \cdot (x_2 - w_2)) \leq 0. \end{aligned} \quad (3a)$$

$$\partial_a v(x_1, x_2, a) = 0. \quad (3b)$$

An interior solution of the above problem is characterized by the following system of equations:⁴

⁴To save on notation, we write $\partial_{ax_1} v$ for $\partial_{x_1} \partial_a v$, and π_s for $\pi_s(a)$.

$$\partial_{x_1} v - \hat{\gamma} \pi_1 p - \hat{\mu} \partial_{ax_1} v = 0, \quad (4a)$$

$$\partial_{x_2} v - \hat{\gamma} \pi_2 p - \hat{\mu} \partial_{ax_2} v = 0, \quad (4b)$$

$$\hat{\gamma} \hat{\sigma} + \hat{\mu} \partial_{aa} v = 0, \quad (4c)$$

$$\pi_1 (p \cdot (x_1 - w_1)) + \pi_2 (p \cdot (x_2 - w_2)) = 0, \quad (4d)$$

$$\partial_a v(x_1, x_2, a) = 0, \quad (4e)$$

where $\hat{\gamma}$ is the multiplier for constraint (3a), $\hat{\mu}$ is the multiplier for constraint (3b) and:⁵

$$\hat{\sigma} := \partial_a \pi_1 (p \cdot (x_1 - w_1)) + \partial_a \pi_2 (p \cdot (x_2 - w_2)).$$

Together with the market clearing conditions, (4a)–(4e) characterize the CM equilibrium.

We shall make explicit at this point the equilibrium restrictions on the gradient of the (Bernoulli) utility index u , since they will be useful later on. Using (3b), direct calculation gives:

$$\partial_{ax_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u.$$

Substitute the above equation in (4a)–(4b) and collect terms to get:

$$\left(\frac{\pi_s - \hat{\mu} \partial_a \pi_s}{\pi_s} \right) \partial_{x_s} u = \hat{\gamma} p + \hat{\mu} \partial_{ax_s} u. \quad (5)$$

We remark for future reference that any consumption bundle x_s which is part of a CM equilibrium must satisfy (5).

5. THE FINANCIAL MARKET (FM) MODEL

In this section, we consider a market structure different from the previous one. At $t = 0$, instead of trading on contingent markets, consumers trade on financial market so as to transfer income from one state to the other.

The asset traded at $t = 0$ may be equivalently interpreted as an insurance contract, or as a portfolio of S Arrow-securities, each paying one unit of account if and only if state s happens.

⁵In deriving (4c) we have used (3b).

In what follows, $\tau = (\tau_1, \tau_2) \in \mathbb{R}^2$ will denote the state-contingent payoffs of the asset bought by consumers.

After having bought the asset, consumers choose their preferred action. Subsequently, at $t = 1$, uncertainty resolves, and trading on spot markets takes place. Consumers use income obtained from selling endowments and from asset payoff to buy goods with immediate delivery.

To define a suitable equilibrium concept for the economy just described, $S + L$ prices must be considered, respectively for the financial asset and the consumption goods.

Given the structure of trades, asset prices depend on the action chosen by consumers, which in turn depends on the optimal consumption at $t = 1$. The incentive compatibility constraint must then be adapted to consider the effects of trading on the spot markets.

5.1. FM CONSUMERS' PROBLEM

From the above discussion, it follows that at $t = 0$ – taking $p \in \mathbb{R}_{++}^L$ as given – consumers choose (x, a, τ) so as to solve:

$$\begin{aligned} \max_{x_1, x_2, a, \tau_1, \tau_2} \quad & v(x_1, x_2, a) \\ \text{s.t.} \quad & \pi_1(a)\tau_1 + \pi_2(a)\tau_2 \leq 0, \end{aligned} \tag{6a}$$

$$p \cdot (x_1 - w_1) \leq \tau_1, \tag{6b}$$

$$p \cdot (x_2 - w_2) \leq \tau_2, \tag{6c}$$

$$a = \arg \max v(x_1, x_2, a), \tag{6d}$$

$$x_1 = \arg \max \{u(x_1, a) \mid p \cdot (x_1 - w_1) \leq \tau_1\}, \tag{6e}$$

$$x_2 = \arg \max \{u(x_2, a) \mid p \cdot (x_2 - w_2) \leq \tau_2\}. \tag{6f}$$

In the above problem, (6a) is the budget constraint for the financial asset, with prices actuarially fair given the action chosen, while (6b) and (6c) are the budget constraints for the consumption goods. As for (6d)–(6f), they constitute the (extended) incentive compatibility constraint. They imply, in particular, that any admissible level action must be optimal given the optimal consumption at $t = 1$.

In what follows, $\mathcal{D}(p)$ will denote the set of (x, a, τ) that satisfy (6a)–(6f).

5.2. FM EQUILIBRIUM

In a FM equilibrium consumers optimize and markets clear. These properties are summarized in the following:

Definition 5.1. *A FM equilibrium is (x, a, τ, p) such that:*

1. $(x, a, \tau) = \arg \max \{v(x, a) \mid (x, a, \tau) \in \mathcal{D}(p)\}$,
2. $\pi_1(a)(x_1 - w_1) + \pi_2(a)(x_2 - w_2) = 0$.

We remark that the market clearing condition in above definition is expressed in expected value because of the assumption of large number of consumers.⁶

5.3. EFFICIENCY OF FM EQUILIBRIUM

In this section, we analyze the efficiency of FM equilibrium. Efficiency is defined as before: an equilibrium allocation of consumption goods, and the associated level of action, is constrained efficient if there does not exist another feasible allocation of consumption good, and an associated incentive compatible action, such that no consumer is worse off, and at least one consumer is better off.

In the introduction, we claimed there exist cases in which the FM equilibrium is constrained inefficient. A quick observation of choice sets $\mathcal{C}(p)$ and $\mathcal{D}(p)$ reveals where the problem lies.

Since $(x, a, \tau) \in \mathcal{D}(p)$ implies $(x, a) \in \mathcal{C}(p)$, it seems possible to prove efficiency of FM equilibrium as follows: take a CM equilibrium (\hat{x}, \hat{a}, p) . Construct transfers $\hat{\tau} := p \cdot (\hat{x} - e)$. Assume that $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(p)$, but that $(\hat{x}, \hat{a}, \hat{\tau})$ is *not* a FM equilibrium. In this case, there exist $(\bar{x}, \bar{a}, \bar{\tau}) \in \mathcal{D}(p)$ such that $v(\bar{x}, \bar{a}) > v(\hat{x}, \hat{a})$. As $(\bar{x}, \bar{a}) \in \mathcal{C}(p)$, we get a contradiction with (\hat{x}, \hat{a}) being a CM equilibrium. Hence we shall conclude that (\hat{x}, \hat{a}) must be a FM equilibrium, which is then efficient, since every CM equilibrium so is.

This reasoning holds provided that $(\hat{x}, \hat{a}, \hat{\tau}) \in \mathcal{D}(p)$. The relevant question is then whether a CM equilibrium, together with implied financial transfers, satisfies the choice set of the FM consumers' problem.

Inspection of the set $\mathcal{D}(p)$ reveals that it is possible that the required inclusion is not satisfied. The crux of the problem lies in equations (6e) and (6f). They require \hat{x} be optimal at $t = 1$ – when trading on consumption goods takes place – given prices p , transfers $\hat{\tau}$ and the action \hat{a} chosen at $t = 0$.

⁶In the interpretation of τ as a portfolio of Arrow-securities, the market clearing condition on the asset market is automatically satisfied at the optimum.

If there is only one consumption good, these equations do not add any relevant constraint on FM equilibrium, since there is no trading at $t = 1$. If preferences are separable, the level of action does not affect the level of ex post utility, hence even in this case we may suspect that these equation do not impose any further constraint on FM equilibrium. In this two cases, we therefore expect the above inclusion to be satisfied, and the conclusion on the constrained efficiency of FM equilibrium to be correct.

When preferences are not separable, consumers who are given enough income to buy \hat{x} may still to choose a different bundle. This happens when \hat{x} does not satisfies (6e)–(6f), where a is taken as fixed. In this case, here exists \tilde{x} which costs not more that \hat{x} at prices p and gives higher utility in at least one state s . Since \tilde{x} would be feasible when action is \hat{a} , and since it gives higher utility in a least one state, it would be a possible CM equilibrium candidate. Yet there is no guarantee that (\tilde{x}, \hat{a}) satisfies the incentive compatibility constraint, hence that it is an actual CM equilibrium candidate.

While these observations highlight the main intuition behind inefficiency of FM equilibrium, in order to verify it we have to compare the first order conditions characterizing the former with those characterizing the CM equilibrium.

5.4. CHARACTERIZATION OF FM EQUILIBRIUM

In this section, we derive the first order conditions characterizing the FM equilibrium.

Instead of working directly with the consumers' problem above defined, we first derive a reduced-form FM consumers' problem, which is closer to the CM consumers' problem. The first order conditions for the former will then be easily comparable with those for the latter.

Starting with the constraints relevant at $t = 1$, we notice that (6e) and (6f) imply that an admissible x_s satisfies:

$$\max_{x_s} u(x_s, a) \quad \text{s.t.} \quad p \cdot (x_s - e_s) - \tau_s \leq 0. \quad (7)$$

Let $x_s(\tau, a)$ denote the solution of the above problem.⁷ It is characterized by the following equations:

⁷It is apparent from (7) that consumption in state s depends only on the transfer in state s . To save on notation, we shall write $x_s(\tau, a)$ instead of $x_s(\tau_s, a)$. Moreover, since prices are always taken as given, we do not explicitly write them as an argument of the demand function.

$$\partial_{x_s} u - \lambda_s p = 0, \quad (8a)$$

$$p \cdot (x_s - e_s) - \tau_s = 0. \quad (8b)$$

Substitute $x_s(\tau, a)$ in the FM consumers' problem and rewrite it as follows:⁸

$$\begin{aligned} \max_{\tau, a} \quad & v(x_1(\tau, a), x_2(\tau, a), a) \\ \text{s.t.} \quad & \pi_1(a) (p \cdot (x_1(\tau, a) - w_1)) + \pi_2(a) (p \cdot (x_2(\tau, a) - w_2)) \leq 0, \\ & a = \arg \max v(x_1(\tau, a), x_2(\tau, a), a). \end{aligned} \quad (9a)$$

Finally, replace (9a) with the corresponding first order condition:

$$\partial_{x_1} v \cdot \partial_a x_1 + \partial_{x_2} v \cdot \partial_a x_2 + \partial_a v = 0. \quad (10)$$

Since (8a)–(8b) imply that:

$$\partial_{x_s} v \cdot \partial_a x_s = \pi_s \partial_{x_s} u \cdot \partial_a x_s = \pi_s \lambda_s (p \cdot \partial_a x_s) = 0,$$

(10) reduces to:

$$\partial_a v(x_1(\tau, a), x_2(\tau, a), a) = 0, \quad (11)$$

so that the reduced-form FM consumer's problem is given by the following:

$$\begin{aligned} \max_{\tau, a} \quad & v(x_1(\tau, a), x_2(\tau, a), a) \\ \text{s.t.} \quad & \pi_1(a) (p \cdot (x_1(\tau, a) - w_1)) + \pi_2(a) (p \cdot (x_2(\tau, a) - w_2)) \leq 0, \\ & \partial_a v(x_1(\tau, a), x_2(\tau, a), a) = 0. \end{aligned}$$

Notice that the above problem has the same structure as the CM consumers' one, the only difference being that admissible consumption bundles satisfy (6e)–(6f).

An interior solution of the above problem is characterized by the following equations:⁹

⁸Here (8b) has been used to substitute for τ_s in (6a).

⁹In deriving (13a)–(13b) we used (8a) and the fact that $\partial_{\tau_2} x_1 = \partial_{\tau_1} x_2 = 0$, while in deriving (13c) we used the fact that $p \cdot \partial_a x_s = 0$.

$$\pi_1 \lambda_1 - \gamma \pi_1 - \mu (\partial_{ax_1} v \cdot \partial_{\tau_1} x_1) = 0, \quad (13a)$$

$$\pi_2 \lambda_2 - \gamma \pi_2 - \mu (\partial_{ax_2} v \cdot \partial_{\tau_2} x_2) = 0, \quad (13b)$$

$$\gamma \sigma + \mu \partial_{aa} v + \mu (\partial_{ax_1} v \cdot \partial_a x_1 + \partial_{ax_2} v \cdot \partial_a x_2) = 0, \quad (13c)$$

$$\pi_1 (p \cdot (x_1 - w_1)) + \pi_2 (p \cdot (x_2 - w_2)) = 0, \quad (13d)$$

$$\partial_a v(x_1, x_2, a) = 0, \quad (13e)$$

where γ is the multiplier for the first constraint, μ is the multiplier for the second constraint and:

$$\sigma := \partial_a \pi_1 (p \cdot (x_1 - w_1)) + \partial_a \pi_2 (p \cdot (x_2 - w_2)).$$

Multiply (13a)–(13b) by p and use (8a) to get the following system of equations:

$$\partial_{x_1} v - \gamma \pi_1 p - \mu (\partial_{ax_1} v \cdot \partial_{\tau_1} x_1) p = 0, \quad (14a)$$

$$\partial_{x_2} v - \gamma \pi_2 p - \mu (\partial_{ax_2} v \cdot \partial_{\tau_2} x_2) p = 0, \quad (14b)$$

$$\gamma \sigma + \mu \partial_{aa} v + \mu (\partial_{ax_1} v \cdot \partial_a x_1 + \partial_{ax_2} v \cdot \partial_a x_2) = 0, \quad (14c)$$

$$\pi_1 (p \cdot (x_1 - w_1)) + \pi_2 (p \cdot (x_2 - w_2)) = 0, \quad (14d)$$

$$\partial_a v(x_1, x_2, a) = 0. \quad (14e)$$

Together with the market clearing conditions, (14a)–(14e) characterize the FM equilibrium.

To highlight the role of assumptions on preferences and number of consumption goods, it is useful to rewrite the last addends in (14a)–(14c) in terms of the (Bernoulli) utility index u as follows:¹⁰

$$\partial_{x_1} v - \gamma \pi_1 p - \mu \partial_{ax_1} v + \psi_1 = 0, \quad (15a)$$

$$\partial_{x_2} v - \gamma \pi_2 p - \mu \partial_{ax_2} v + \psi_2 = 0, \quad (15b)$$

$$\gamma \sigma + \mu \partial_{aa} v + \psi_a = 0, \quad (15c)$$

¹⁰Calculations are in Appendix.

where the ψ -terms are defined as follows:

$$\psi_s := \mu\pi_s (\partial_{ax_s} u - (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s) p) , \quad (16a)$$

$$\psi_a := \mu (\pi_1 (\partial_{ax_1} u \cdot \partial_a x_1) + \pi_2 (\partial_{ax_2} u \cdot \partial_a x_2)) . \quad (16b)$$

We remark that (15a)–(15b) are $2L$ equations, while (15c) is a single equation.

5.5. COMPARING FIRST ORDER CONDITIONS

For ease of comparison, we recall here the equations characterizing a CM equilibrium:

$$\partial_{x_1} v - \gamma\pi_1 p - \mu\partial_{ax_1} v = 0 , \quad (17a)$$

$$\partial_{x_2} v - \gamma\pi_2 p - \mu\partial_{ax_2} v = 0 , \quad (17b)$$

$$\gamma\sigma + \mu\partial_{aa} v = 0 , \quad (17c)$$

$$\pi_1 (p \cdot (x_1 - w_1)) + \pi_2 (p \cdot (x_2 - w_2)) = 0 , \quad (17d)$$

$$\partial_a v(x_1, x_2, a) = 0 . \quad (17e)$$

We want to study when the above equations coincide with those characterizing a FM equilibrium:¹¹

$$\partial_{x_1} v - \gamma\pi_1 p - \mu\partial_{ax_1} v + \psi_1 = 0 , \quad (18a)$$

$$\partial_{x_2} v - \gamma\pi_2 p - \mu\partial_{ax_2} v + \psi_2 = 0 , \quad (18b)$$

$$\gamma\sigma + \mu\partial_{aa} v + \psi_a = 0 , \quad (18c)$$

$$\pi_1 (p \cdot (x_1 - w_1)) + \pi_2 (p \cdot (x_2 - w_2)) = 0 , \quad (18d)$$

$$\partial_a v(x_1, x_2, a) = 0 . \quad (18e)$$

In both systems, the last two equations are the same, hence they do not pose any problem. As for (18a)–(18c) and (17a)–(17c), they differ for the

¹¹For simplicity we neglect market clearing conditions, as they are the same in both systems.

ψ -terms. It is therefore necessary to study when $\psi_s = 0$ and $\psi_a = 0$. From (16a) and (16b), we know this happens if and only if the following system of $2L + 1$ equations is satisfied:

$$\partial_{ax_1} u - (\partial_{ax_1} u \cdot \partial_{\tau_1} x_1) p = 0, \quad (19a)$$

$$\partial_{ax_2} u - (\partial_{ax_2} u \cdot \partial_{\tau_2} x_2) p = 0, \quad (19b)$$

$$\pi_1 (\partial_{ax_1} u \cdot \partial_a x_1) + \pi_2 (\partial_{ax_2} u \cdot \partial_a x_2) = 0. \quad (19c)$$

When there is a single consumption good and when preferences are separable, the above equations are *identities*.

Indeed, if there is a single consumption good, p is normalized to one, $\partial_{\tau_s} x_s \equiv 1$ and $\partial_a x_s \equiv 0$, so that the above system is identically satisfied.¹²

If preferences are separable, then $\partial_{ax_s} u \equiv 0$, so that the above system is again identically satisfied.

In these two cases, the system of equations characterizing the CM and the FM equilibrium are therefore identical. Hence, the equilibrium allocation of goods and action level must be the same. Given that every CM equilibrium is constrained efficient, so is every FM equilibrium.

The following proposition collects these findings:

Proposition 2. *Suppose there is single consumption good or assumption S holds. Then every FM equilibrium is constrained efficient.*

We now consider the possibility that (19a)–(19c) are *not* satisfied, so that the FM equilibrium is *not* constrained efficient.

If we let $\zeta_s := (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)$, we notice that (19a)–(19b) imply that:

$$\partial_{ax_1} u = \zeta_1 p, \quad (20a)$$

$$\partial_{ax_2} u = \zeta_2 p. \quad (20b)$$

According to (20a)–(20b), for the system (19a)–(19c) to be satisfied it is necessary that $\partial_{ax_s} u$ and p are collinear, with scale factor $\zeta_s \neq 0$. If this condition is violated, the system cannot be satisfied. The following proposition formalize this intuition:

¹²Notice that in this case $\partial_{ax_s} v$ is a scalar.

Proposition 3. *Suppose there are multiple consumption goods and preferences are non-separable. Provided that $\partial_{ax_s} u \neq \zeta_s p$ for $\zeta_s \neq 0$, the FM equilibrium is not constrained efficient.*

As a final step, we verify that the condition in the above proposition is not empty:

Proposition 4. *Suppose there are multiple consumption goods and preferences satisfy assumption H. Then the FM equilibrium is not constrained efficient.*

Proof. The proof consists in verifying that $\partial_{ax_s} u \neq \zeta_s p$ for $\zeta_s \neq 0$. When assumption H holds, it follows that:

$$\partial_{ax_{s_j}} u \equiv 0, \quad \text{and} \quad \partial_{ax_{s_i}} u = \partial_{ax_{s_i}} g(x_s^{-j}, a) \quad \text{for} \quad i \neq j.$$

It follows that $\partial_{ax_s} u = \zeta_s p$ for $\zeta_s \neq 0$ implies:

$$0 = \zeta_1 p_j, \tag{21a}$$

$$\partial_{ax_{1_i}} g(x_1^{-j}, a) = \zeta_1 p_i \quad \text{for} \quad i \neq j, \tag{21b}$$

$$0 = \zeta_2 p_j, \tag{21c}$$

$$\partial_{ax_{2_i}} g(x_2^{-j}, a) = \zeta_2 p_i \quad \text{for} \quad i \neq j. \tag{21d}$$

Since the above system is inconsistent, we conclude that $\partial_{ax_s} u \neq \zeta_s p$. ■

A further intuition of the above result is obtained by comparing the restrictions on $\partial_{x_s} u$ – the gradient of the (Bernoulli) utility index u – both in FM and CM equilibrium. According to (8a), in a FM equilibrium the consumption allocation must be such that $\partial_{x_s} u$ and p are collinear. According (5), this restriction is satisfied in a CM equilibrium provided that assumption S holds, or that preferences are non-separable and $\partial_{ax_s} u = \zeta_s p$ for $\zeta_s \neq 0$.

6. RELATED LITERATURE

The idea of the CM equilibrium dates back to the seminal paper of Prescott and Townsend (1984). Formally, their model is rather different from ours. To avoid non-convexities both in the utility functions – due to a discrete action set – and in the budget constraints, lotteries on consumption goods are introduced, and consumers are allowed to choose among them. While the objects of trades are different, the market structure and the nature of

the price system are analogous to those considered here. As a result, an appropriate version of proposition 1 holds for their model too.

Closely related to the model of Prescott and Townsend (1984) is that of Kocherlakota (1998). The main formal differences are the absence of lotteries, since non-convexities are eliminated by appropriate assumption on the utility function and on the action set, and the hypothesis of a single consumption good.

Rustichini and Siconolfi (2003) have recently revised the model of Prescott and Townsend, generalizing the conditions for existence and optimality of equilibrium.

In the above models, the only hidden action consumers make is choosing a , since their consumption choices can be monitored, hence enforced. The relevance of this assumption for the efficiency of competitive equilibrium has been stressed in a series of papers by Arnott and Stiglitz (1986), Greenwald and Stiglitz (1986) and Arnott et al. (1992). These authors claim that inefficient equilibria emerge if that assumption is removed. The claim is proved using a model of optimal taxation, where a benevolent planner is supposed to redistribute income through insurance contracts assigned at $t = 0$, and tax or subsidize trades on spot markets. In this set-up, inefficiency of equilibrium corresponds to non-zero taxes. Indeed, Arnott and Stiglitz (1983) find that this case arises if there are multiple consumption goods and preferences satisfy assumption H.

Yet, differences in the model set-up in part obscured the connection between the two strands of literature. As a matter of fact, the explicit comparison of the CM with the FM equilibrium is an attempt to clarify that connection, and in particular the importance of differences in the trade structure.

Under simpler assumptions, the FM equilibrium has been studied by Helpman and Laffont (1975), who obtained an efficiency result analogous to proposition 2 for the case of a single consumption good, and by Lisboa (2001), who obtained an efficiency result analogous to proposition 2 for the case of separable preferences.

APPENDIX

In this Appendix we derive equations (15a)–(15c). Recall that:

$$\partial_{ax_s} v = \partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u.$$

Since $p \cdot \partial_{\tau_s} x_s = 1$, we get the following chain of equalities:

$$\begin{aligned}
(\partial_{ax_s} v \cdot \partial_{\tau_s} x_s) p &= ((\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u) \cdot \partial_{\tau_s} x_s) p \\
&= (\partial_a \pi_s (\partial_{x_s} u \cdot \partial_{\tau_s} x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)) p \\
&= (\partial_a \pi_s \lambda_s (p \cdot \partial_{\tau_s} x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s)) p \\
&= \partial_a \pi_s \lambda_s p + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s) p \\
&= \partial_a \pi_s \partial_{x_s} u + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s) p \\
&= \partial_{ax_s} v - \pi_s \partial_{ax_s} u + \pi_s (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s) p \\
&= \partial_{ax_s} v - \pi_s (\partial_{ax_s} u - (\partial_{ax_s} u \cdot \partial_{\tau_s} x_s) p) . \tag{22}
\end{aligned}$$

Since $p \cdot \partial_a x_s = 0$, we also get the following chain of equalities:

$$\begin{aligned}
(\partial_{ax_s} v \cdot \partial_a x_s) &= (\partial_a \pi_s \partial_{x_s} u + \pi_s \partial_{ax_s} u) \cdot \partial_a x_s \\
&= \partial_a \pi_s (\partial_{x_s} u \cdot \partial_a x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_a x_s) \\
&= \partial_a \pi_s \lambda_s (p \cdot \partial_a x_s) + \pi_s (\partial_{ax_s} u \cdot \partial_a x_s) \\
&= \pi_s (\partial_{ax_s} u \cdot \partial_a x_s) . \tag{23}
\end{aligned}$$

Substituting (22) and (23) in (14a)–(14c) we get the equations in the text.

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